

Time: 3 hours

Max. Marks: 75

Answer any five questions
All questions carry equal marks

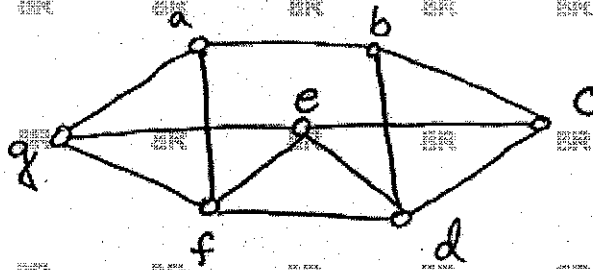
- 1.a) Prove the following are tautologies (Using truth tables):
- $\neg(P \vee Q) \vee (\neg P \wedge Q) \vee P$
 - $((P \vee Q) \rightarrow R) \wedge (\neg P) \rightarrow (Q \rightarrow R)$.
- b) Obtain a disjunctive normal form of $(Q \vee (P \wedge R)) \wedge \neg((P \vee R) \wedge Q)$ [8+7]
- 2.a) Show that $R \rightarrow S$ can be derived from the premises $P \rightarrow (Q \rightarrow S)$, $\neg R \vee P$ and Q .
- b) Write the following sentences in the symbolic form
- Arjun is a student
 - All students like easy courses
 - Sociology is an easy course.
- [8+7]
- 3.a) Define: Reflexive, symmetric and transitive relations and give one example for each.
- b) Let $f: R \rightarrow R$ and $g: R \rightarrow R$, where R is the set of real numbers be given by $f(x) = x^2 - 2$ and $g(x) = x + 4$. Find $f \circ g$ and $g \circ f$. State whether these functions are injective. [8+7]
- 4.a) Let H be a sub group of a group G and $a \in G$. Let $aHa^{-1} = \{aha^{-1} / h \in H\}$. Show that aHa^{-1} is a sub group of G and $O(H) = O(aHa^{-1})$.
- b) State and prove the fundamental theorem of homomorphism. [7+8]
- 5.a) Explain the inclusion-exclusion principle.
- b) Prove that if any 30 people are selected, then we may choose a sub set of 5 so that all 5 were born on the same day of the week. [7+8]
- 6.a) Solve $a_n = a_{n-1} + 2n + 1$ where $a_0 = 1$.
- b) Solve the following recurrence relation using the characteristic roots
- $$a_n - 4a_{n-1} + 4a_{n-2} = 0 \text{ for } n \geq 2 \text{ and } a_0 = \frac{5}{2}, a_1 = 8.$$
- [7+8]
- 7.a) Give three examples for complete graph for different n .
- b) What is the spanning tree? How to find a spanning tree in a graph? Explain with examples. [7+8]

8.a) Define Hamiltonian graph. Discuss basic rules for constructing Hamiltonian paths and cycles.

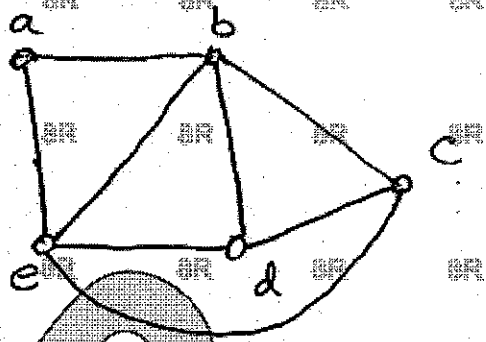
b) Find the chromatic number of the following graphs.

[7+8]

(i)



(ii)



8R