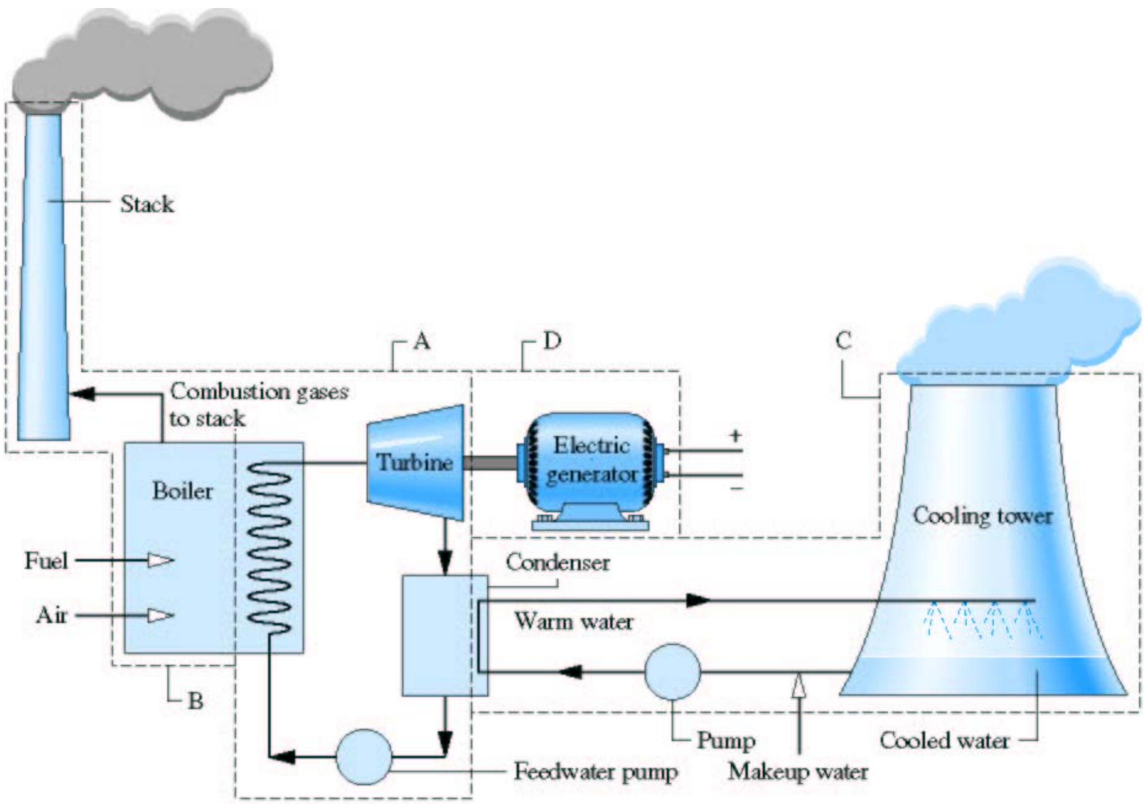
Application of Control Volume Energy Analysis

Most thermodynamic devices consist of a series of components operating in a cycle, e.g., steam power plant



Main components of the above cycle are:

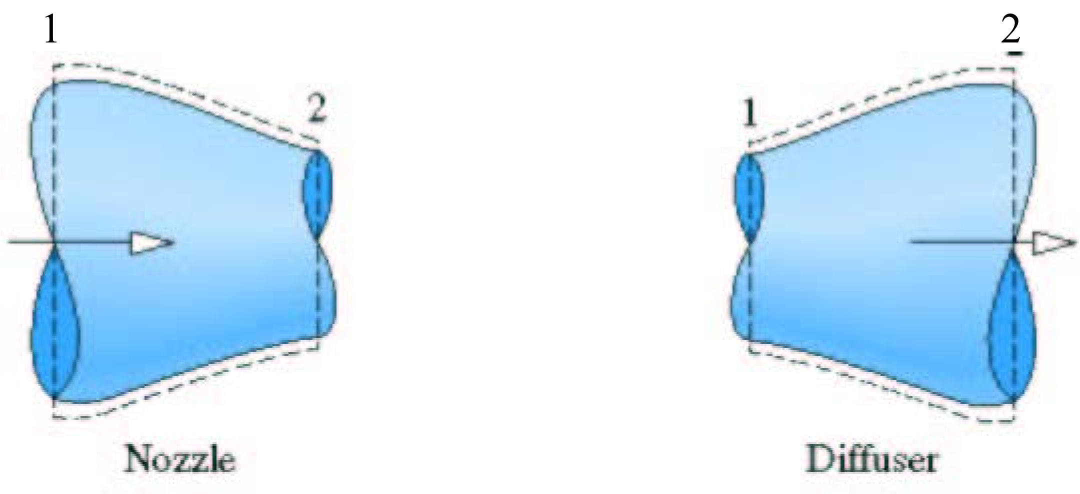
1) Boiler (steam generator) – heat exchanger
2) Turbine – generates work
3) Condenser – heat exchanger
4) Pump

Others components include:

nozzles, diffusers, throttling devices

Nozzles and Diffusers

Devices that increase or decrease the flow velocity by passing the flow through a variable area duct, $A_1 \neq A_2$



Applying conservation of mass assuming steady flow:

$$\cancel{\frac{dM_{CV}}{dt}} = \rho_1 A_1 V_1 - \rho_2 A_2 V_2$$

$$\rho_1 A_1 V_1 = \rho_2 A_2 V$$

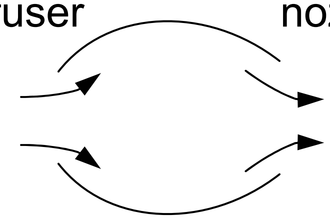
For low subsonic flow ($\rho_1 = \rho_2$)

$$\boxed{\frac{V_2}{V_1} = \frac{A_1}{A_2}}$$

Subsonic Nozzle: $A_2 < A_1 \rightarrow V_2 > V_1$

Subsonic Diffuser: $A_2 > A_1 \rightarrow V_2 < V_1$

Aircraft gas turbine



Applying the energy equation (assuming steady, no heat loss, ΔPE=0):

$$\cancel{\frac{dE}{dt}} = \cancel{\dot{q}} - \cancel{\dot{w}_s} + (h_i + V_i^2/2 + \cancel{gZ_i}) - (h_e + V_e^2/2 + \cancel{gZ_e}) = 0$$

$$h_1 + V_1^2/2 = h_2 + V_2^2/2$$

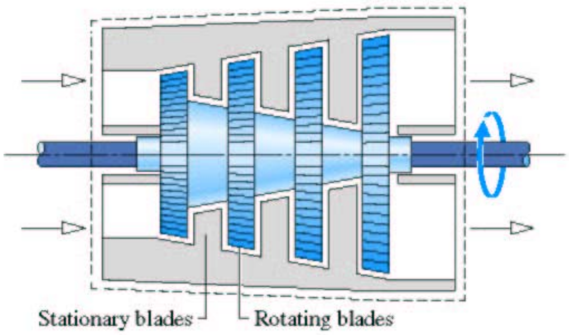
$$\boxed{V_2^2 = V_1^2 + 2(h_1 - h_2)}$$

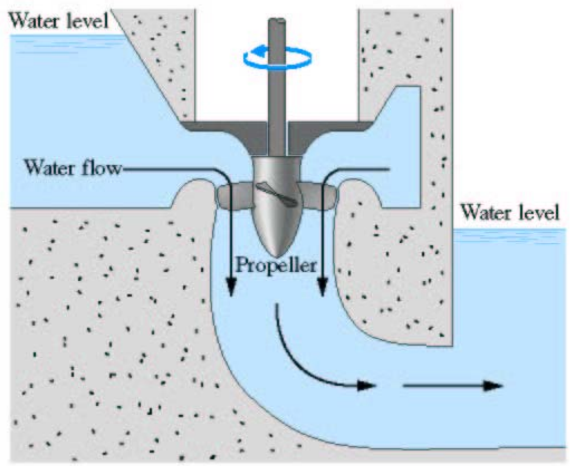
For a rocket nozzle $V_2 >> V_1$

$$\boxed{V_2^2 = 2(h_1 - h_2)}$$

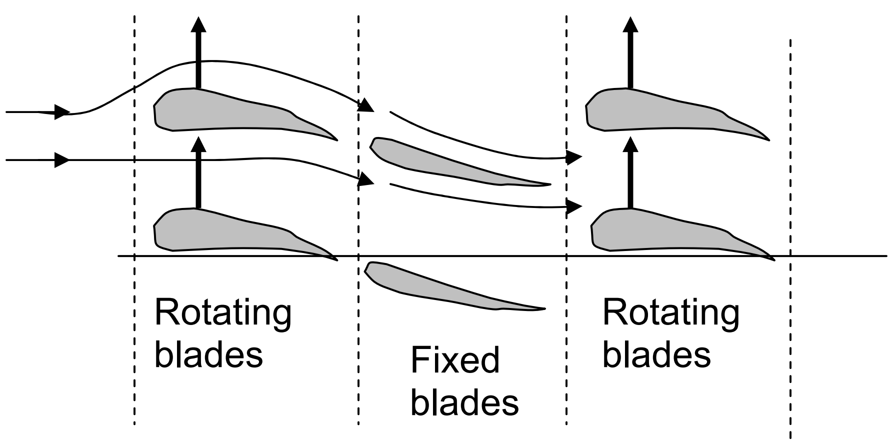
Turbine

A device in which shaft work is generated as a result of gas passing through a set of blades attached to a freely rotating shaft

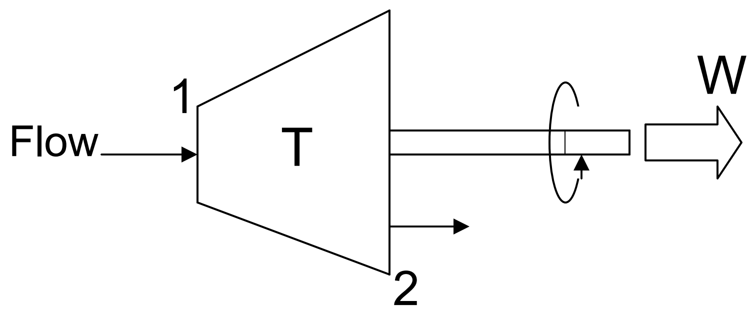




The rotating blades redirect the flow off axis, so you need a set of fixed blades that straighten out the flow before the next set of rotating blades



In this course we are not interested in the details of the flow through each blade, or row of blades. We are interested in the overall energy balance



Applying First Law (steady-state, neglect heat transfer)

$$0 = \cancel{\dot{q}} - \dot{w} + (h_1 + V_1^2/2) - (h_2 + V_2^2/2)$$

$$\boxed{\dot{w} = (h_1 - h_2) + (V_1^2/2 - V_2^2/2)}$$ work per unit mass

Often the change in KE is small compared to change in h i.e., $h_1 - h_2 >>> V_1^2/2 - V_2^2/2$

$$\dot{w} = h_1 - h_2$$

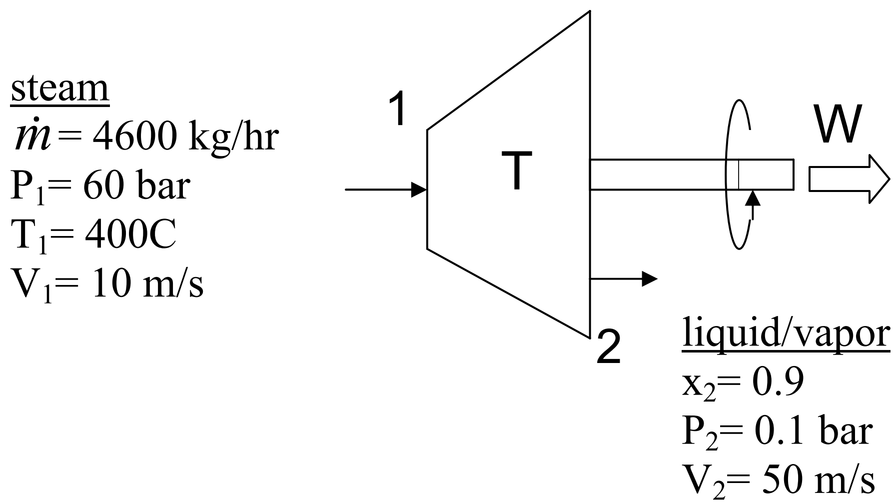
Note: work output $(\dot{w} > 0) \rightarrow h_1 > h_2$

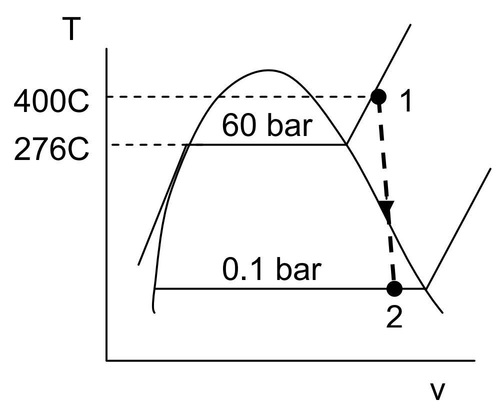
Power is work output per unit time

$$\boxed{\dot{W} = \dot{m}\dot{w} = \dot{m}(h_1 - h_2)}$$

EXAMPLE

Steam enters a turbine operating at steady-state with a mass flow rate of 4600 kg/h. The turbine develops a power output of 1000 kW. At the inlet the pressure is 60 bars, the temperature is 400C and the velocity is 10 m/s. At the exit the pressure is 0.1 bar, the quality is 0.9 and the velocity is 50 m/s. Calculate the rate of heat transfer between the turbine and the surroundings, in kW.





Assume steady-state and ΔPE is negligible

$$0 = \dot{q} - \dot{w} + (h_1 + V_1^2/2) - (h_2 + V_2^2/2)$$

$$\dot{q} = \dot{w} + (h_2 - h_1) + (V_2^2/2 - V_1^2/2)$$

Need enthalpy at states 1 and 2

State 1:

From saturated water Table A-3 T_{sat} (60 bar)= 275.6C

since $T_1 > T_{sat}$ at same pressure have superheated vapor

From superheated water vapor Table A-4

h(60bar, 400C)= 3177.2 kJ/kg

State 2:

From saturated water Table A-3

h_f(0.1 bar)= 191.8 kJ/kg h_g(0.1 bar)= 2584.7 kJ/kg

$h_2 = h_f + x_2(h_g - h_f)$ = 191.8 + 0.9(2392.8)= 2345.4 kJ/kg

so $h_2 - h_1$= 2345.4 - 3177.2 = -833.8 kJ/kg

$$0.5\left(V_2^2 - V_1^2\right) = 0.5(50^2 - 10^2) = 1200\ \mathrm{m^2/s^2}$$

$$= 1200\ \frac{\mathrm{m^2}}{\mathrm{s^2}}\left(\frac{1\ \mathrm{N}}{\mathrm{kg\cdot m/s^2}}\right)\left(\frac{1\ \mathrm{J}}{\mathrm{N\cdot m}}\right) = 1200\ \frac{\mathrm{J}}{\mathrm{kg}}$$

Collecting terms:

$$\dot{q} = \dot{w} + (h_2 - h_1) + (V_2^2/2 - V_1^2/2)$$

$$\dot{Q} = \dot{W} + \dot{m}(h_2 - h_1) + \dot{m}(V_2^2/2 - V_1^2/2)$$

$$= 1000\ \mathrm{kW} + 4600\ \frac{\mathrm{kg}}{\mathrm{hr}}\left(\frac{1\ \mathrm{hr}}{3600\ \mathrm{s}}\right)\left[-833.8\ \frac{\mathrm{kJ}}{\mathrm{kg}} + 1.2\ \frac{\mathrm{kJ}}{\mathrm{kg}}\right]$$

$$\boxed{\dot{Q} = -61.3\ \mathrm{kW}}$$

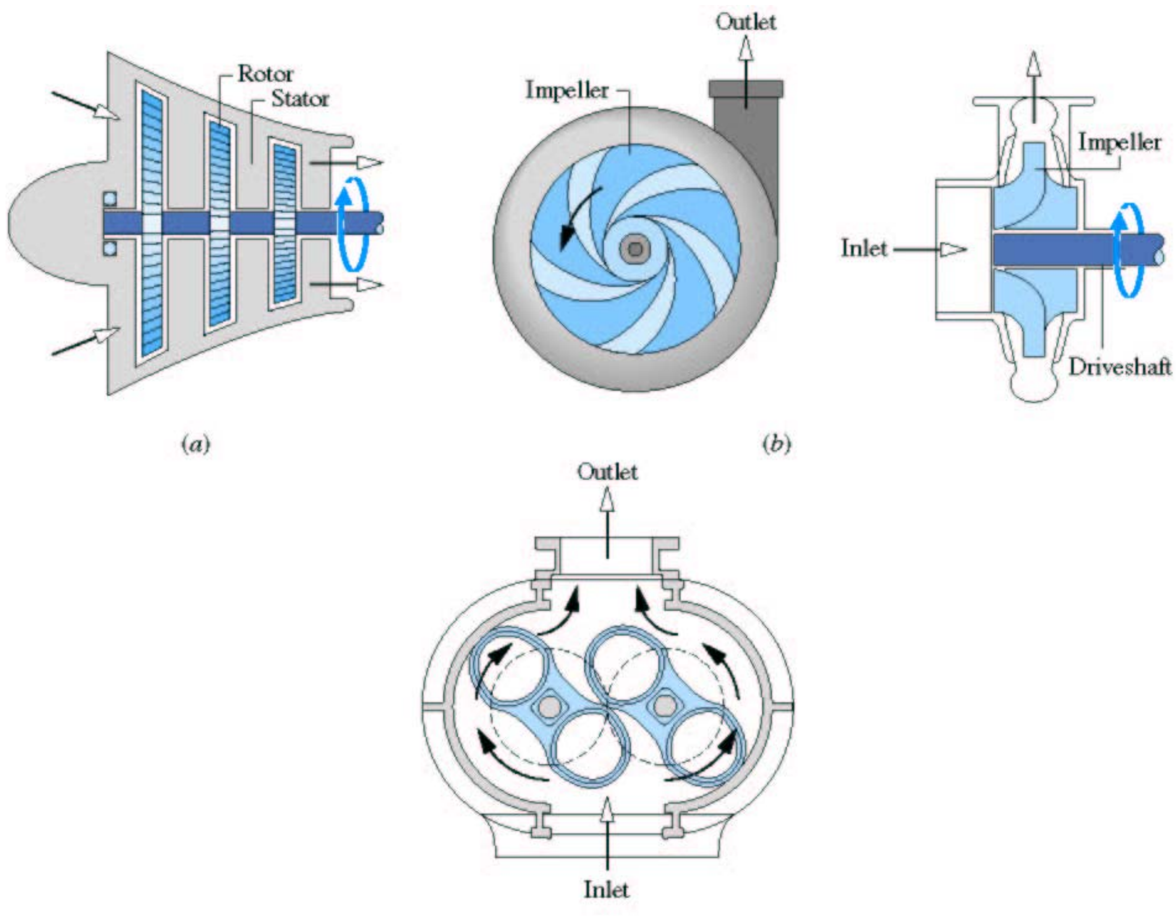
Negative sign implies heat loss from turbine

Note:

1) Difference in magnitude between Δh and Δke
2) Magnitude of heat loss $\dot{Q}$ (61 kW) compared to magnitude of power output $\dot{W}$ (1000 kW)

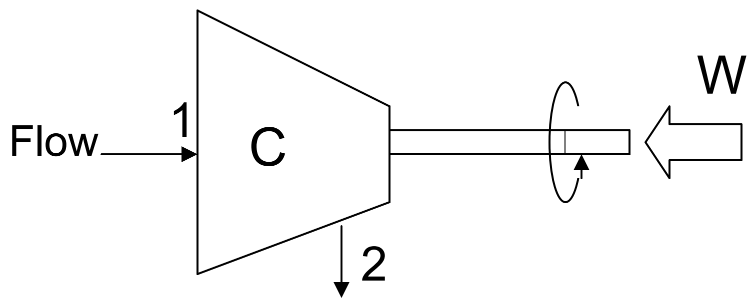
Compressor/pump

A device in which shaft work input is used to raise the pressure of a fluid (liquid or vapor)



(c)

Again we are not interested in the details of the flow through each blade or row of blades. We are interested in the overall energy balance



Applying First Law (steady-state, neglect heat transfer and ΔPE)

$$0 = \not{\dot{q}} - \dot{w} + (h_1 + V_1^2/2) - (h_2 + V_2^2/2)$$

$$\boxed{\dot{w} = (h_1 - h_2) + (V_1^2/2 - V_2^2/2)}$$ work per unit mass

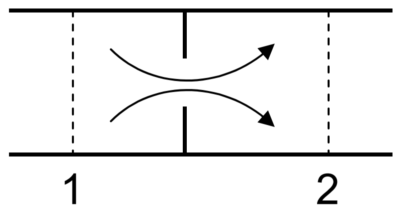
Often the change in KE is small compared to change in h i.e., $h_1 - h_2 >>> V_1^2/2 - V_2^2/2$

$$\boxed{\dot{w} = h_1 - h_2}$$

Note: work input $(\dot{w} < 0) \rightarrow h_2 > h_1$

Throttling Device

A device that generates a significant pressure drop via a flow restriction, e.g., partially closed valve.



Applying First Law (steady-state, neglect heat transfer and ΔPE)

$$0 = \dot{q} - \dot{w} + (h_1 + V_1^2/2) - (h_2 + V_2^2/2)$$

$$\boxed{h_1 + V_1^2/2 = h_2 + V_2^2/2}$$

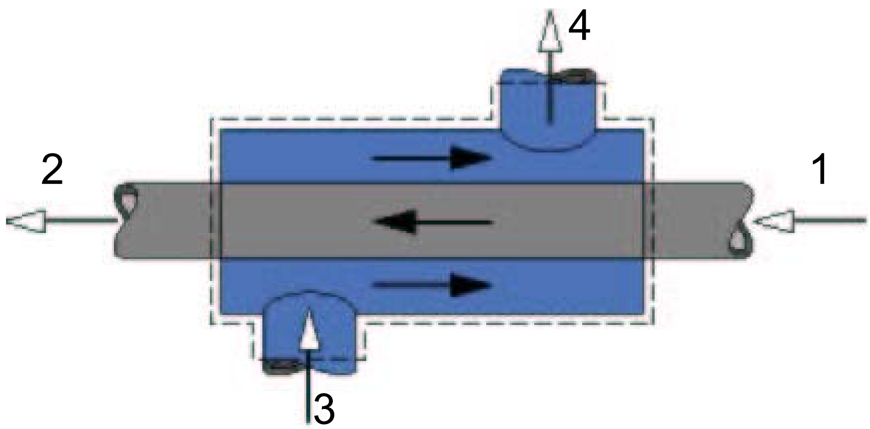
If the state 2 is taken far downstream from the blockage the change in velocity is negligible, i.e., $V_1 \approx V_2$

$$\boxed{h_1 = h_2}$$

Throttling process is characterized by constant enthalpy

Heat Exchangers

These are devices that transfer energy between fluid streams at different temperatures cool or heat one of the fluids. The following is a tube-in-tube heat exchanger



Can have cross-flow or parallel-low type

Applying First law to above cross flow heat exchanger assuming steady flow, no heat loss to the environment and ΔKE and ΔPE is negligible

$$0 = \not{\dot{Q}} - \not{\dot{W}} + \dot{m}_1(h_1 + \not{V}_1^2/2 + \not{g}Z_1) + \dot{m}_3(h_3 + \not{V}_3^2/2 + \not{g}Z_3)$$
$$- \dot{m}_2(h_2 + \not{V}_2^2/2 + g\not{Z}_2) - \dot{m}_4(h_4 + \not{V}_4^2/2 + g\not{Z}_4)$$

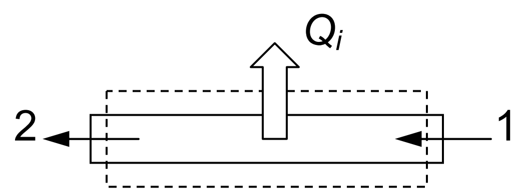
Steady flow so $\dot{m}_1 = \dot{m}_2$ and $\dot{m}_3 = \dot{m}_4$

$$0 = \dot{m}_1(h_1 - h_2) + \dot{m}_3(h_3 - h_4)$$

Solving we get

$$\frac{\dot{m}_1}{\dot{m}_3} = \frac{h_4 - h_3}{h_1 - h_2}$$

To get the rate of heat transfer from one stream to the other perform CV analysis on only the inner-tube (*assume inner-stream is hotter than outer-stream*)



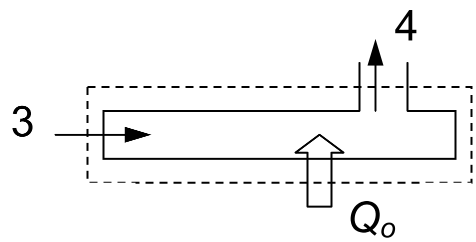
Again Applying First Law with same assumptions

$$0 = (-\dot{Q}_i) + \dot{m}_1(h_1 - h_2)$$

$$\frac{\dot{Q}_i}{\dot{m}_1} = (h_1 - h_2)$$

Since $\dot{Q}_i > 0 \rightarrow h_1 > h_2$, so $T_1 > T_2$ (fluid cools down)

A CV analysis of the outer-stream would give



$$0 = (+\dot{Q}_o) + \dot{m}_3 (h_3 - h_4)$$

$$\boxed{\frac{\dot{Q}_o}{\dot{m}_3} = (h_4 - h_3)}$$

Since $\dot{Q}_o > 0 \rightarrow h_4 > h_3$, so $T_4 > T_3$ (fluid heats up)

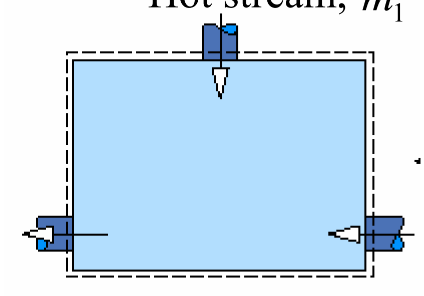
Note, the magnitude of the energy transfer rate from the inner stream $\dot{Q}_i$ equals the magnitude of the energy transfer rate into the outer-stream $\dot{Q}_o$

$$\dot{Q}_i = \dot{Q}_o$$
$$\dot{m}_1 (h_1 - h_2) = \dot{m}_3 (h_4 - h_3)$$

$$\boxed{\frac{\dot{m}_1}{\dot{m}_3} = \frac{h_4 - h_3}{h_1 - h_2}}$$

Note we recover the same relationship obtained using the global energy balance

Another common type of heat exchanger is a direct contact heat exchanger, e.g., open feed water heater.



This type of heat exchanger consists of a vessel where a hot stream and cold stream of the same fluid are mixed and exit at an intermediate temperature through a single outlet.

Apply conservation of mass and First Law to the CV and assuming steady state, negligible KE and PE change to get:

$$\cancel{\frac{dM_{CV}}{dt}} = \dot{m}_1 + \dot{m}_2 - \dot{m}_3$$

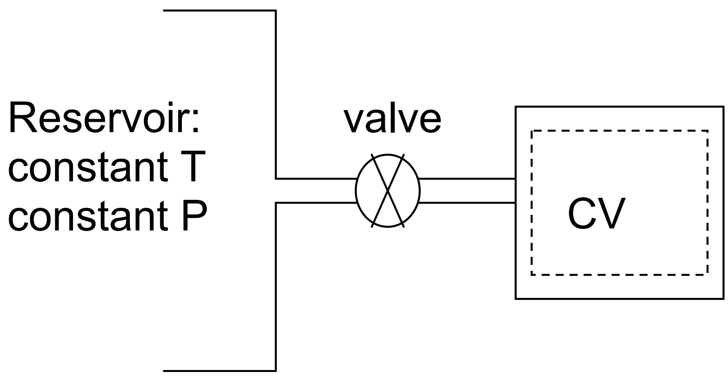
$$\boxed{0 = \dot{m}_1 + \dot{m}_2 - \dot{m}_3}$$

$$\cancel{\frac{dE_{CV}}{dt}} = \cancel{\dot{Q}} - \cancel{\dot{W}} + \dot{m}_1(h_1) + \dot{m}_2(h_2) - \dot{m}_3(h_3)$$

$$\boxed{0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3}$$

Transient Control Volume Analysis

Applies when the CV has only one inlet or one exit



Considering the filling of a rigid tank of volume V_{cv} with a gas supplied at a constant pressure and temperature

Applying conservation of mass

$$\frac{dM_{CV}}{dt} = \dot{m}_i - \cancel{\dot{m}_e}$$

Applying First Law, neglecting heat transfer to environment, KE and PE

$$\frac{dE_{CV}}{dt} = \frac{dU_{CV}}{dt} = \cancel{\dot{Q}} - \cancel{\dot{W}} + \dot{m}_i(h_i + \cancel{V_i^2}/2 + \cancel{g}Z_i) - \cancel{\dot{m}_e}(.....)$$

Note: constant enthalpy across the valve (throttling device) → gas specific enthalpy, h_i , into the CV equals the gas specific enthalpy in the reservoir, h_R

$$\frac{dU_{CV}}{dt} = \dot{m}_i h_i = \dot{m}_i h_R$$

Substituting

$$\frac{dU_{CV}}{dt} = h_R \frac{dM_{CV}}{dt}$$

Integrating from the initial state "i" to the final state "f"

$$\int_{U_i}^{U_f} dU = h_R \int_{M_i}^{M_f} dM$$

$$U_f - U_i = (M_f - M_i)h_R$$

$$M_f u_f - M_i u_i = (M_f - M_i)h_R$$

If the tank is initially empty (vacuum) $m_i = 0$

$$\cancel{M}_f u_f = \cancel{M}_f h_R$$

$$c_V T_f = c_P T_R$$

$$\boxed{T_f = \frac{c_P}{c_V} T_R = k T_R}$$

Work done getting gas into the CV results in a final tank gas temperature higher than the reservoir temperature