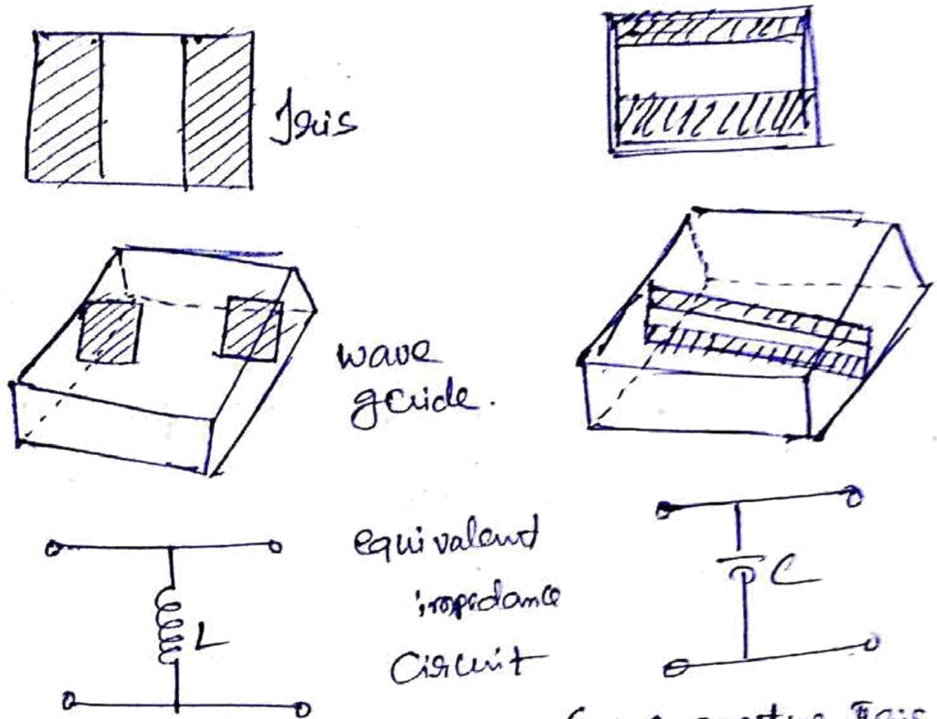
* UNIT – III *

* Wave Guide Components AND Applications – I *

⇒ Wave guide Irises :-

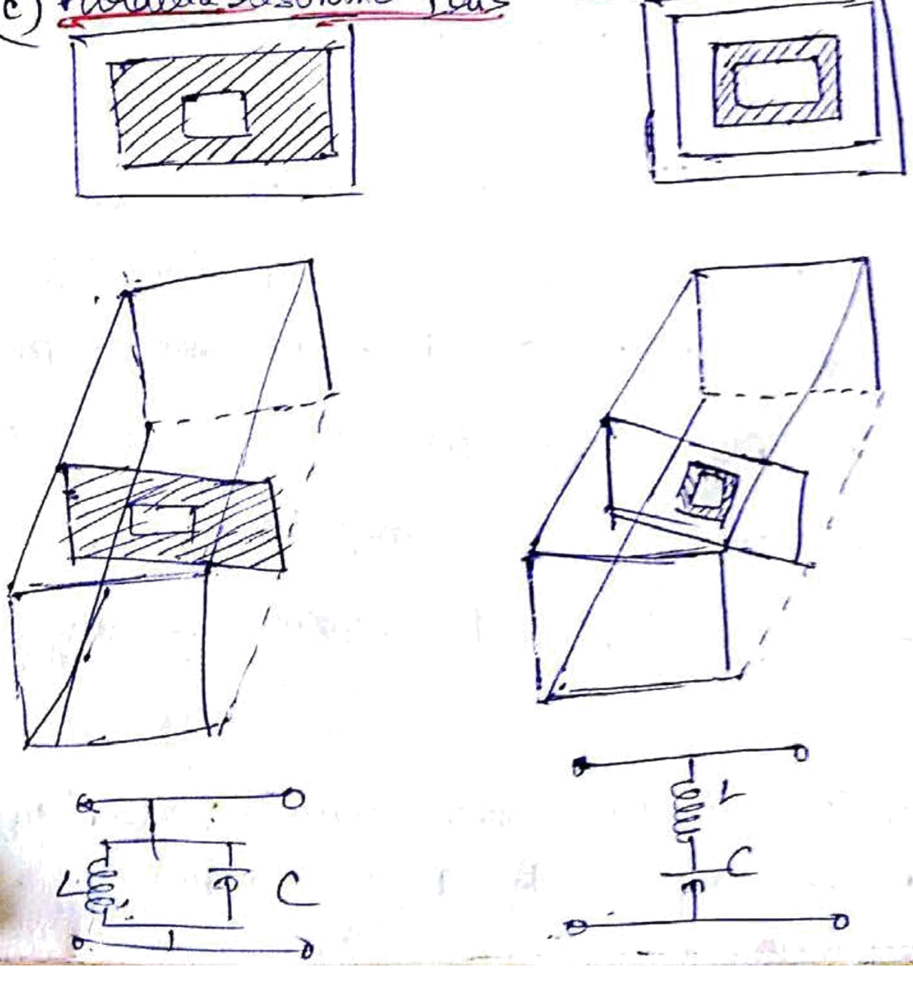


(a) Inductive Iris.

(b) Capacitive Iris

(c) Parallel resonance Iris

(d) Series resonance Iris



* Any susceptance appearing across the guide causing mismatch needs to be cancelled by introducing another susceptance of the same magnitude but of opposite nature.

* An inductive Iris allows a current to flow where none flowed before. The plane of polarization of electric field is parallel to the plane of iris, the current flow due to iris causes a magnetic field to be set up. Energy storage of magnetic field ~~due~~ takes place and there is an increase in inductance at that point of the waveguide.

* In capactive iris, it is seen that the potential which is existed between the top and bottom walls of the waveguide now exists between surfaces which are closer and therefore capacitance has increased at that point.

* The inductive and capacitive irises. ~~of~~ if combined suitably the inductive and capactive reactance introduced, will be equal and iris becomes a parallel resonant circuit. Figure shows a series resonant ~~circuit~~ iris which is supported by non-metallic material and its transparent to the flow of microwave energy.

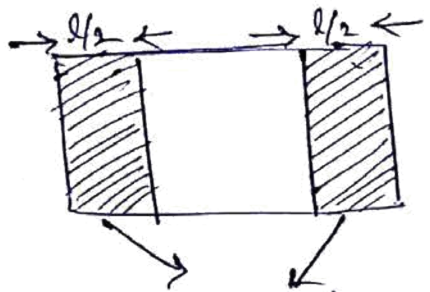
→ Windows :-

* waveguide windows also known as diaphragms, apertures (or) Irises are used to provide impedance

impedance matching in the waveguides. Three types of windows are available.

1) Inductive window
2) Capacitive window
3) Resonant windows.

→ 1) Inductive windows:—



projection from side walls of guide.

* An inductive window allows the current flow where non flown before.

* Conducting diaphragm extending in a waveguide from side walls have the effect of adding an an inductive susceptance across the waveguide at the point at which diaphragms are placed.

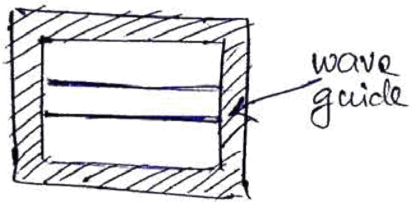
* These diaphragms ~~are~~ has to be placed in a position where magnetic field is strong and electric field is relatively weak.

* The electric field that advanced before now has a conducting surface in its plane, which permits current flow.

* Thus some energy is stored in the magnetic field which leads to an increased inductance at that point of the waveguide.

→ 2) Capacitive windows:—

Conducting diaphragms extending in a ~~g~~ wave guide from top and bottom walls have the effect of adding a capacitive susceptance across the waveguide at the point at which diaphragms are placed.



capacitive windows.

* In capacitive Iris a potential which had existed earlier between the top and bottom walls of waveguide now exists between surfaces that are closer. This results in an increased capacitance at that points.

B) Resonant windows:-

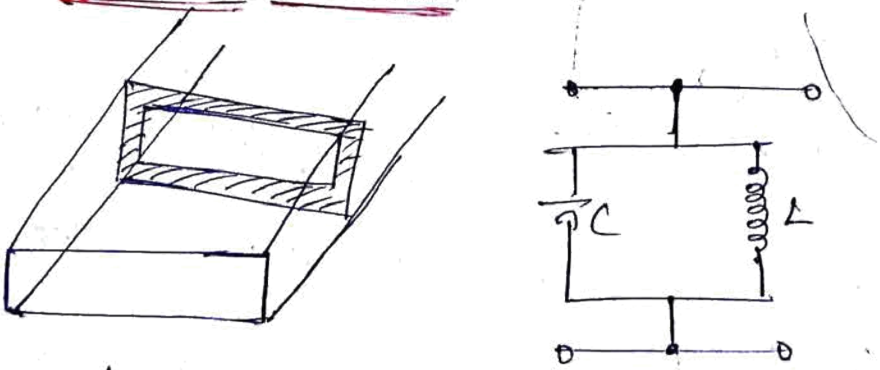


fig:- Resonant window.

* A conducting diaphragm of the form shown in figure gives the effect of a parallel tuned (LC) circuit connected across the guide at point where diaphragm is placed.

A resonant window may be considered as a combination of an inductive and capacitive window, at the same point in the waveguide.

If the inner dimensions of aperture are properly chosen, the frequency range covered is large.

However a limit of minimum aperture size prevents any further changes.

* Windows are usually employed only to ~~match~~ correct a permanent mismatch rather than to provide adjustable matching.

Coupling probes and Loops:-

* Probe coupling and loop coupling are the two commonly used techniques for coupling microwave signal to the waveguides. Both probe and loop coupling can be used to launch a particular mode in a waveguide.

* Output can also be seen from the probes / loops.

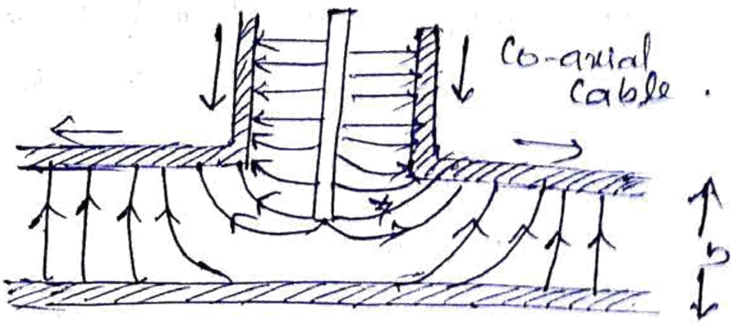
* probes couple primarily to the electric field and loops to magnetic field but in each case both are set up because electric and magnetic field are inseparable.

Coupling Probes:-

* A coaxial line may be coupled to a wave-guide by means of either a probe parallel to the electric field at (or) near the point where electric field has maximum value

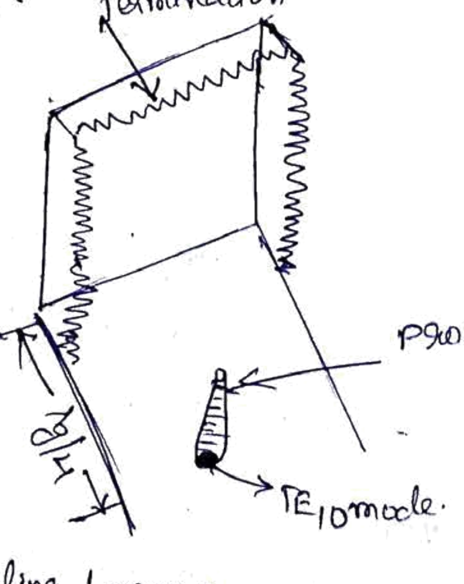
The figure below shows the progress of set of two lines of electric field at a point in the wave as the wave passes through the junction of co-axial line and the waveguide.

Generally a short circuit terminates the waveguide and the probe is placed approximately



a quarter wavelength from the Terminations.

* The figure below shows the correct positioning of the coupling probes for launching dominant mode (TE_{10}). The probe is placed at a distance of '$\lambda g/4$' from the shorted end of the waveguide and the centre of broader dimensions of the wave guide because at that point electric field is maximum.



⇒ Coupling Loops :-

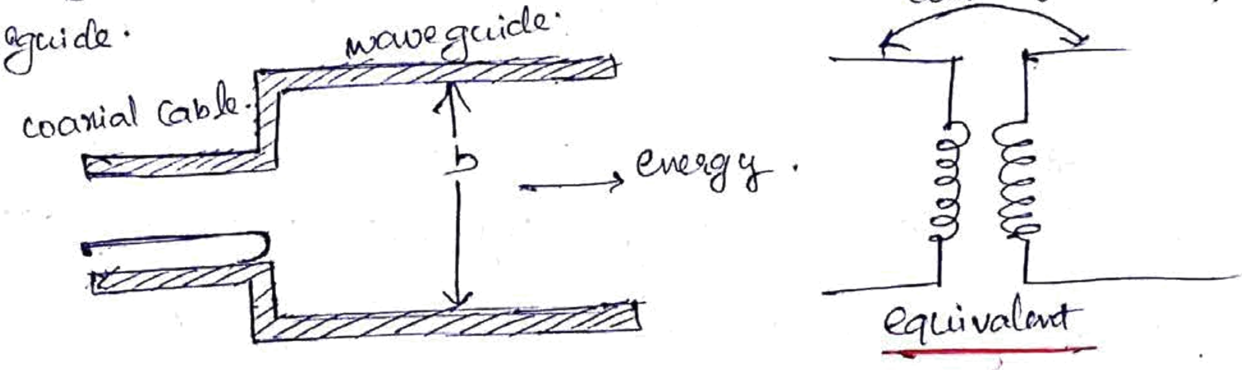
* The loop coupling is basically magnetic, so the loop must be placed at (or) near the point of maximum magnetic field strength and tuned in such a direction that its plane is normal to flux lines.

* The loop can be mounted at the end of the shorted waveguide (or) in the middle of top (or) bottom wall at a distance $(n\lambda/2)$ from the short circuited end, where

$n \rightarrow$ integer

$\lambda \rightarrow$ wavelength.

* The figure below shows the use of a coupling loop in a junction b/n co-axial line and waveguide.



* The plane of the loop should be perpen-dicular to H field for maximum coupling. The degree of coupling obtained with a loop depends upon its size, shape and orientation, and in general increases with the area of loop.

⇒ Tuning Posts and Screws :-

* Tuning Posts and screws are also used for impedance matching.

* When a metallic cylindrical post is introduced into the broader (wider) side of waveguide, it produces the same effect as an loi's in providing lumped reactance at that point.

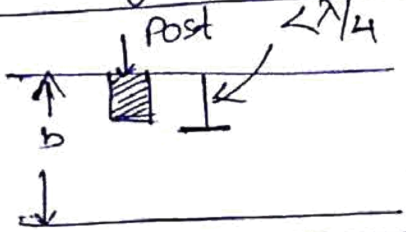
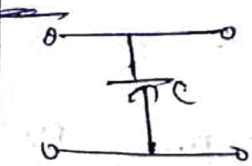
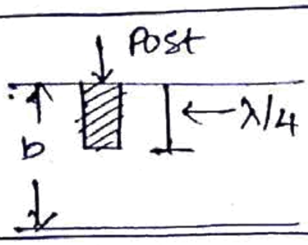
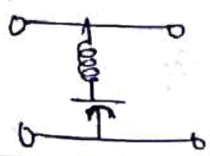
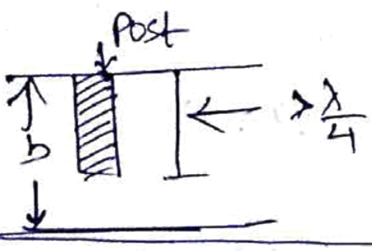
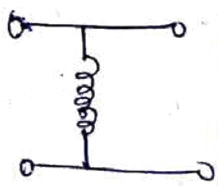
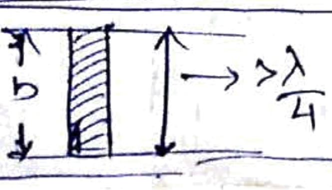
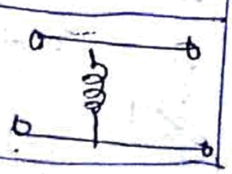
* If the post extends only a short distance ($< \lambda g/4$) into the waveguide, it behaves capacitively

and this capacitive susceptance increases with the depth of penetration

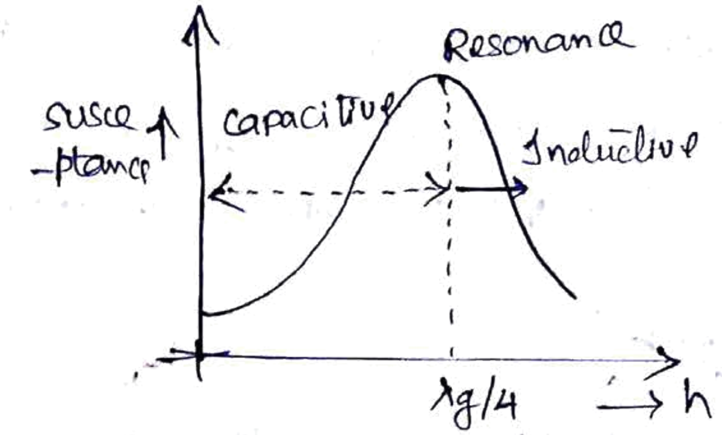
→ when the depth is equal to $\lambda g/4$, the post acts as a series resonant circuit

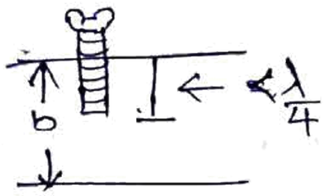
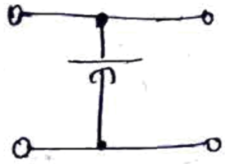
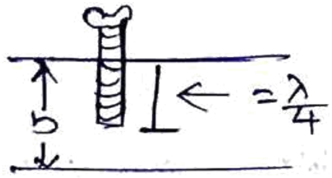
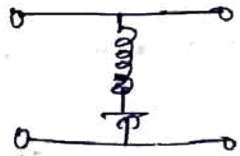
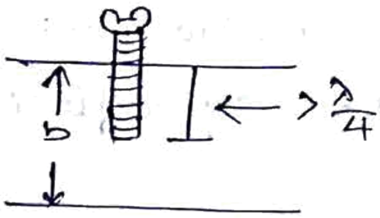
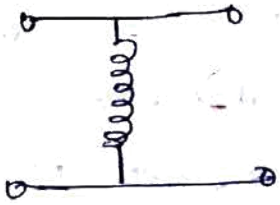
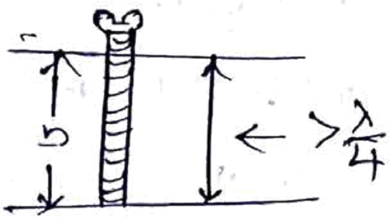
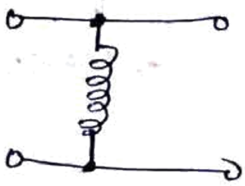
⇒ If it is $>\lambda g/4$, the post behaves inductively and this inductive susceptance decreases when the post is moved further away from the centre of the waveguide. when the post is extended completely across the waveguide, the becomes inductive.

* The big advantage of the post over an iris is that it is readily adjustable. An adjustable post is known as a screw (or) slug. As in case of posts depending upon the depth of penetration, the tuning screw may introduce inductive (or) capacitive susceptance.

SNO	Tuning Post (side view)	Condition	equivalent circuit
1		$<\lambda/4$	
2		$=\frac{\lambda}{4}$	
3)		$>\lambda/4$	
4)		$>\frac{\lambda}{4}$	

* Susceptance vs penetration :-



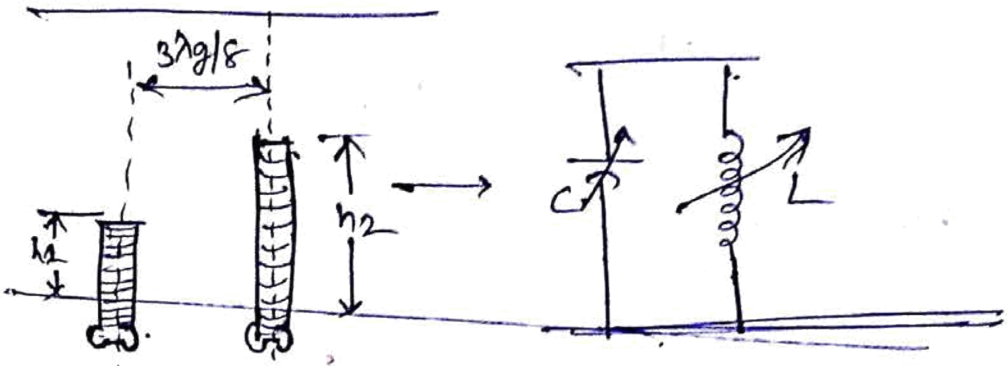
S.NO	Tuning screw side view	Condition	equivalent circuit
1		$< \lambda/4$	
2)		$= \frac{\lambda}{4}$	
3		$> \frac{\lambda}{4}$	
4		$> \frac{\lambda}{4}$	

⇒ Matched Load :-

*. A Matched load (or) termination is a one port component that absorbs all the power incident upon it. This requires that its impedance equal the characteristic impedance of ~~the line~~ line. to

to which it is connected.

* A combination of two screws $3\lambda_g/8$ a part can be used to match a waveguide to its load similar to use of two fixed stubs in a Tx line



⇒ **Wave guide Attenuators:-**

* Microwave attenuators are used in every type of equipement involving transmission, control and measurement of microwave energy.

* For perfect matching sometimes we require that the microwave power in a waveguide be absorbed completely without any reflection. For this we make use of attenuators.

* **Attenuators are commonly used for:-**

→ Measuring power gain (or) loss in dB

→ Providing isolation between instruments,

→ Reducing the power i/p to a particular stage to prevent overloading.

* There are two broad categories of attenuators

(i) Fixed attenuators

(ii) Variable attenuators

Attenuators

- Fixed Attenuators
- Variable Attenuators [Continuous (or) stepwise variable attenuation]
  - Flap-type attenuator
  - movable vane attenuator
  - Rotary vane attenuator.

→ Fixed Attenuators:—

* Fixed attenuators are used where fixed amount of attenuation is to be provided. If such a fixed attenuator absorbs all the energy entering into it, we call it as a "waveguide terminator"

* Fixed attenuators are commonly used in two types of applications.

→ One is in a calibration channel to establish a known signal level (like C.R.O)

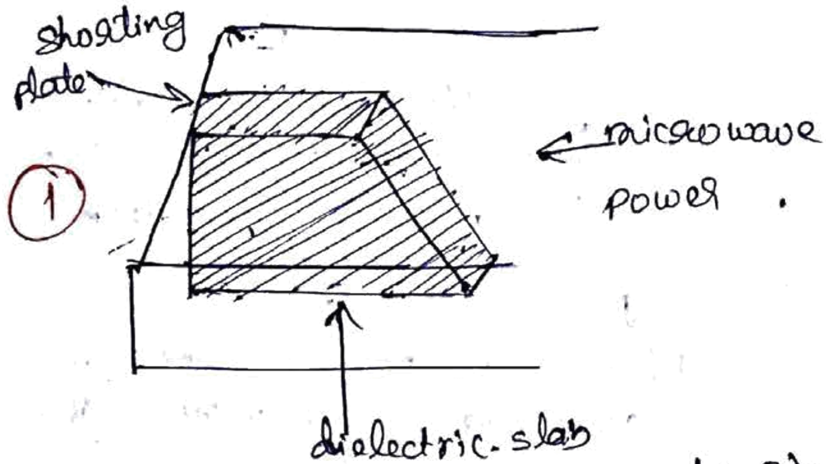
→ Flatness over required frequency range is important here

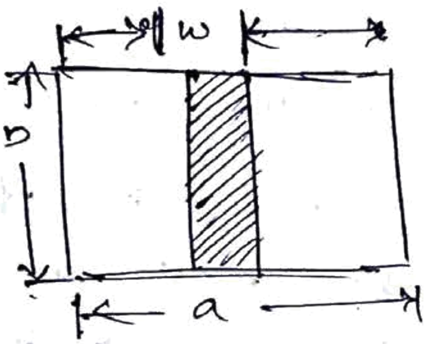
→ In the second type, the device is used for impedance matching (or) as a buffer to prevent interaction b/n two devices.

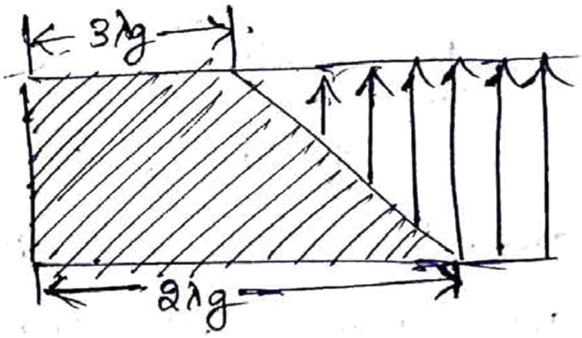
→ Here low VSWR is an important requirement

* The figure below shows a fixed attenuator where a dielectric slab consisting of glass ~~slab~~ slab coated with aquadag (or) carbon film has been used

as a plug.







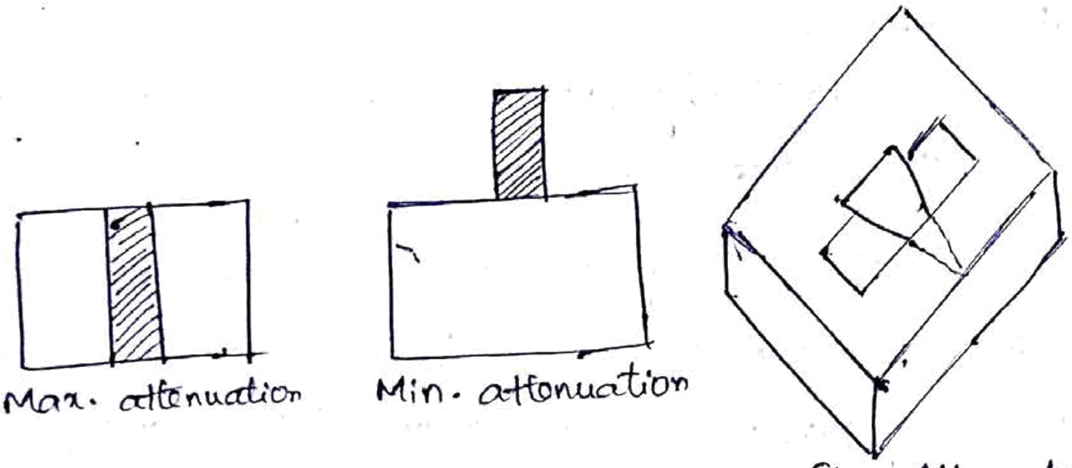
→ Variable Attenuator:—

* Variable attenuators provide continuous (or) stepwise variable attenuation. For rectangular wave guides, these attenuators can be "flap-type" (or) "vane type".

→ Flap-type attenuator:—

The following figure shows the flap-type attenuator. It consists of a resistive element (or) disc inserted into a longitudinal slot cut at the centre of the wider dimension of the guide.

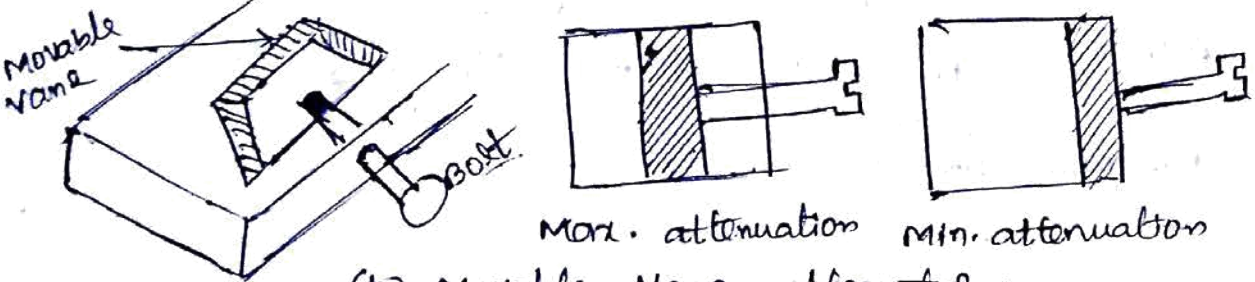
The flap is mounted on the hinged arm allowing it to descend into the centre of the wave guide, the degree of attenuation is determined by the depth of insertion of the flap.



→ Vane type Attenuator:-

* It basically consists of a glass vane with a coating of aquadag (or) carbon similar to a fixed vane attenuator. If the vane used at the centre is made movable, it can be used as a variable attenuator.

* The vane is tapered at both ends for matching the attenuator to the waveguide. An adequate match is obtained if the taper length is made equal to $\lambda g/2$.



(b) Movable Vane attenuator.

for circular waveguides, these attenuators can be "Rotary Vane type attenuators"

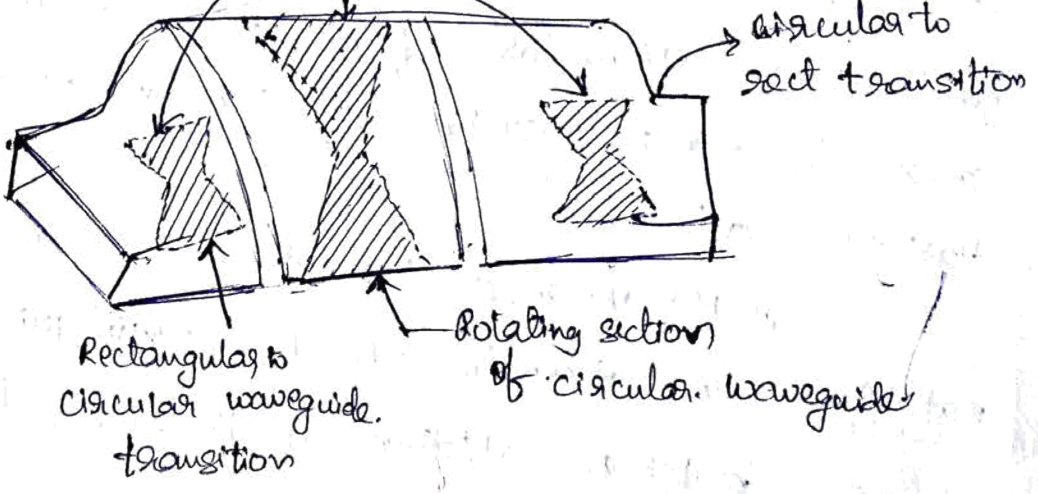
⇒ Rotary Vane type Attenuators:—

It provides a

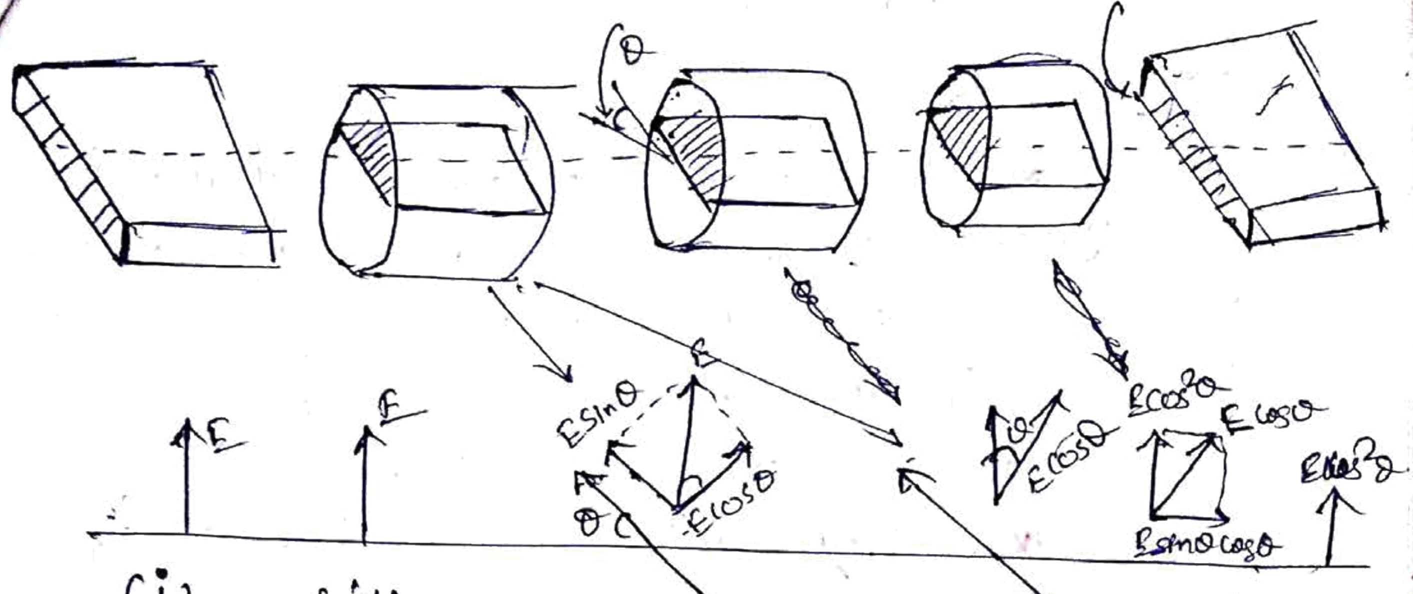
* ~~It is a directing reading~~ precision attenuator with an accuracy of ±2% of the indicated attenuation over the operating fq range.

Basically, it consists of three sections of waveguide (one behind the other).

* A rectangular to circular waveguide transition containing a horizontal attenuator strip is connected to a rotatable circular waveguide containing an attenuator strip and this in turn is connected to a circular to rectangular waveguide transition containing a horizontal attenuator strip.



* A functional diagram indicating the operating principle of this type of attenuator is shown in the figure below.



Stage (i):– when all strips are allined ~~to plane of~~ the electric field of the applied wave is normal to the strips and hence no current flows in the attenuation strips and therefore no attenuation occurs

stage (ii):– In a position where the central attenuation strip is rotated by angle θ, the electric field of the applied wave can be resolved into two factors one perpendicular ($E\cos\theta$) and other parallel ($E\sin\theta$) to the resistive card.

* The portion which is parallel ($E\sin\theta$) will be absorbed, where as the portion which is perpendicular ($E\cos\theta$) will be transmitted.

stage (iii):– Again "$E\cos\theta$" is resolved into two ~~years~~ factors, namely, "$E\cos^2\theta$" and "$E\cos\theta\sin\theta$"

stage (iv):– The final o/p of the attenuator becomes "$E\cos^2\theta$" which has the same polarization as the input wave.

⇒ Phase Shifters :-

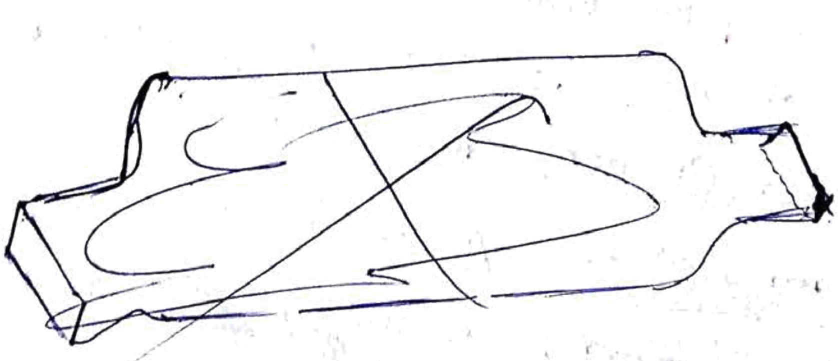
* This is an instrument that produces an adjustable change in the phase angle of the wave transmitted through it. Ideally, it should be perfectly matched to i/p & o/p lines, and should produce zero attenuation.

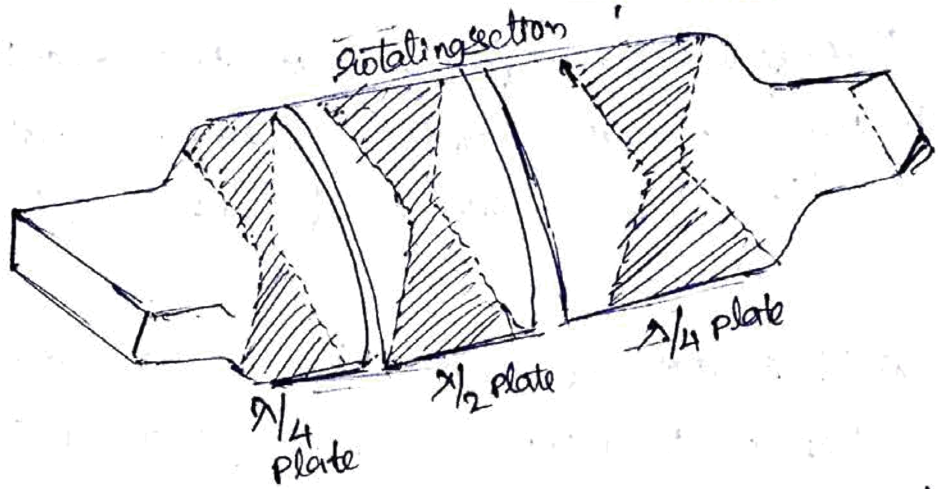
* Many applications require phase shifts to be introduced b/n two given positions in a waveguide system. The ~~plane~~ phase shift required to may be fixed (or) variable.

* Two types of phase shifters namely, rotary phase shifter and a dielectric phase shifter.

→(i) Rotary phase shifter :-

* It is similar in construction to a rotary attenuator except that the central resistive card is replaced by a half wave plate and the two ~~other~~ outer resistive cards are replaced by quarter wave plates.





* A slab of dielectric material is used for the construction of a quarter wave plate. The ends of the dielectric slab are tapered to reduce reflections to a negligible value.

* The length of the quarter wave plate (l) is adjusted that we get a differential phase change of 90°.

* The half wave plate is similar in construction except that its length is increased to produce a differential phase change of 180°.

* The above figure shows a rotary phase shifter. The quarter wave plates are oriented at an angle 45° relative to the broad wall of the rectangular waveguide. The rotation of half wave plate through an angle changes the phase of the transmitted wave by an amount equal to 2θ.

→ Dielectric phase shifter:-

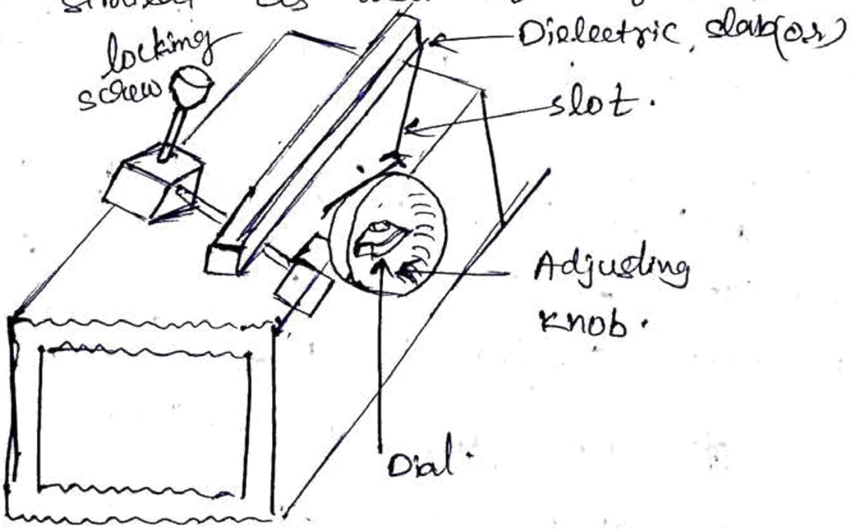
* The principle of dielectric phase shifter is that the difference in phase shift between two points is determined by the velocity of propagation

and therefore is a function of the medium.

* Thus insertion of a dielectric into the waveguide shifts the phase of the wave propagating through it.

* It consists of a dielectric slab (or) Vane specially shaped to minimize reflection effects, inserted through longitudinal slot cut along the wider dimension of the wave guide as shown in the figure below.

* A physical construction of a phase ~~attenu~~ Shifter ~~also~~ is similar as that of a flap attenuator



Dielectric Vane (Variable) phase shifter.

* If the vane is inserted deeper, there is more change in the medium & there is a greater phase shift.

* The amount of phase shift is max when the slab is at the centre and minimum when it is adjacent to the wall of the wave guide.

phase shift form

the page

* In the above dielectric vane variable. (10)
phase shifters, the phase shift is changed continuously from one value to another. and hence they are also termed as analog phase shifters.

* If the fixed phase shift is produced, then they are known as digital / discrete phase shifter.

→ Digital phase shifter are mostly used in phase array antennas

→ Analog phase shifters are used in bridges and instruments.

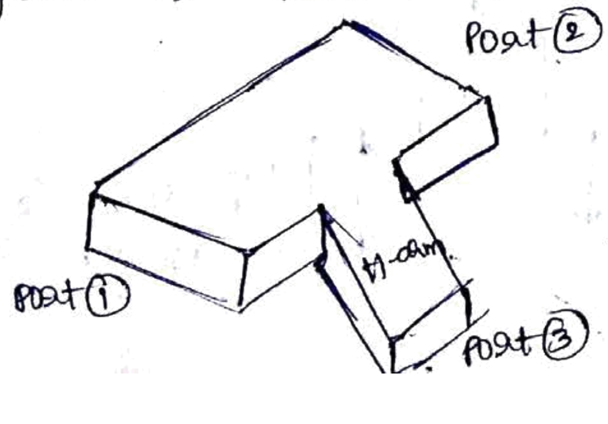
⇒ WAVE GUIDE JUNCTIONS :-

* Microwaves Tees are the junctions (or) networks having 3 (or) more ports, tees are used for the purpose of connecting a branch section of waveguide in series (or) parallel with the main waveguide.

→ E-Plane Tee
→ H-plane Tee
→ Magic (or) Hybrid Tee (EH plane Tee).

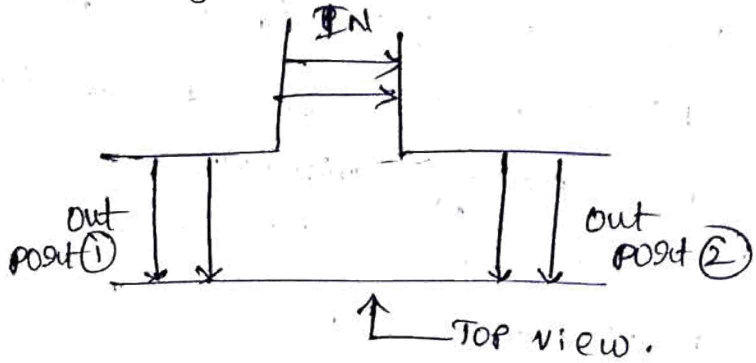
(1) H-Plane Tee (or) Shunt Tee :-

A wave guide in which the axis of its side arm is parallel to the ~~field~~ H-field of the main guide is known as H-plane Tee.



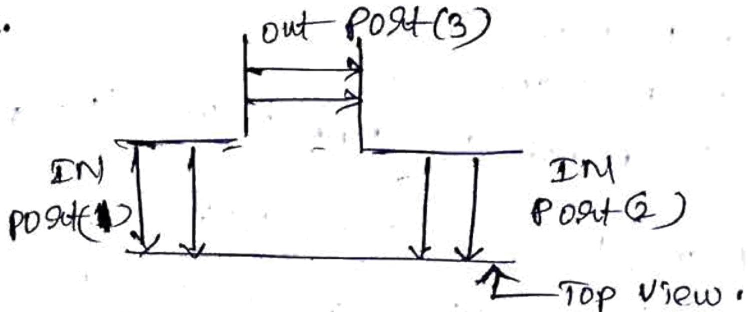
Properties :—

(i) when the i/p is applied at the side arm i.e., at the port (3), the o/p's are obtained from the collinear arms i.e., port (1) and port (2) and are equal magnitude and phase.

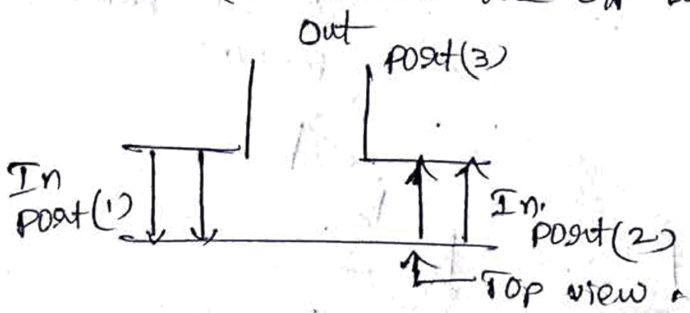


(ii) when the i/p is applied at collinear arms i.e, at port (1) & (2), the o/p obtained from the side arm depends on the phase of the inputs applied at collinear arms i.e.,

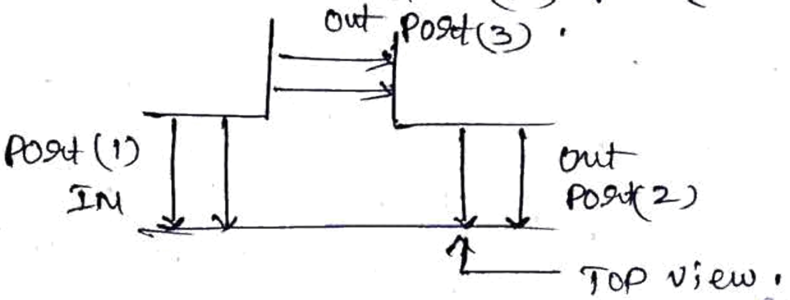
→ If phased i/p's are applied at port (1) & port (2), then maximum power is obtained at port (3).



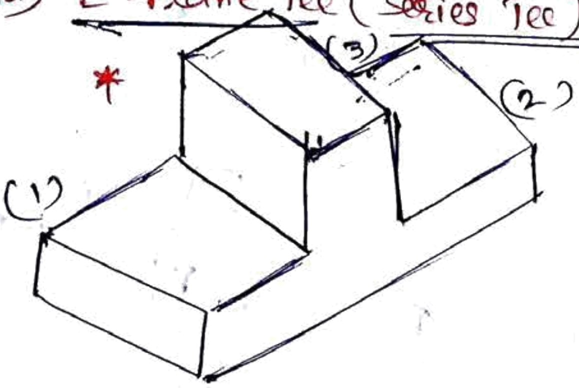
→ If a 180° phase shift is applied between port (1) and port (2), then the o/p at port (3) is zero.



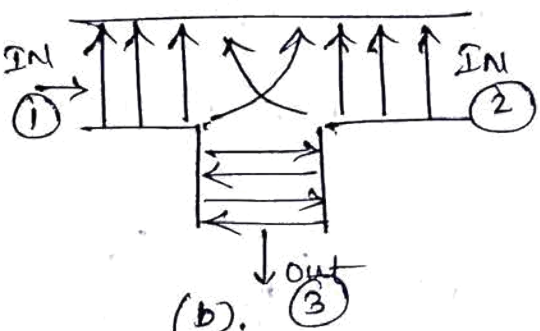
(iii) when the i/p is applied at one of collinear arms i.e., port(1) (or) port(2), the respective o/p's are obtained at port(2) (or) port(1) and side arm.



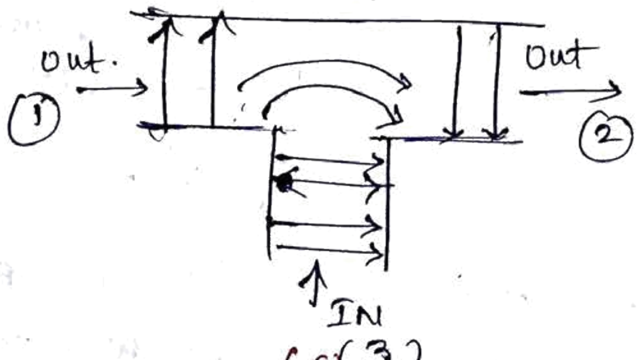
⇒ 2) E-plane Tee (Series Tee):-



(a)



(b).



(c)

(d)

* If the input arm of tee comes from broadwall then the junction is called an "E-plane tee."

* E plane tee is a voltage (or) series junction symmetric about the centre arm. so the signal to be split up is fed to it.

* The waves entering the junction from side arm split up and leaves the main arm with equal magnitude ~~and~~ but of opposite phase as shown in fig (b).

* Similarly the wave entering the junction from main arm leaves the side arm, the resulting field being proportional to the difference between instantaneous field entered the junctions from opposite directions as shown in fig (b) and (c).

⇒ Hybrid Junction (or) Magic Tee :-

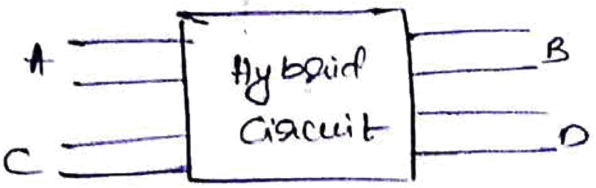


fig (a)

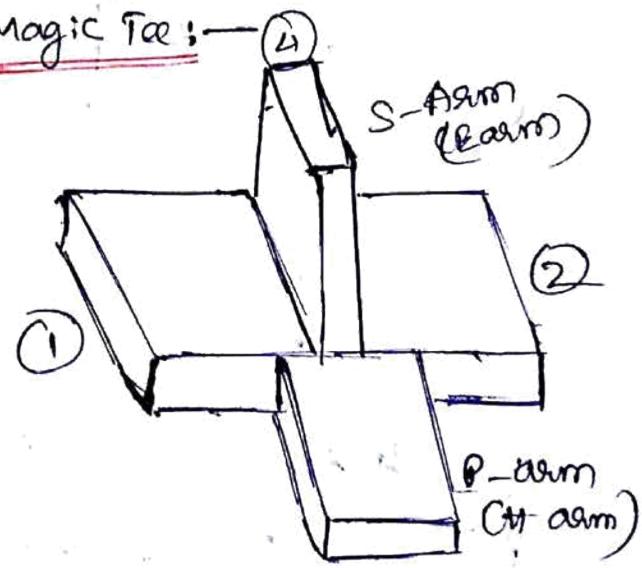


fig (b) 3.

* A magic tee is a combination of E-plane tee and H-plane tee. It acts as a 4-port circuit as shown in fig a. If power enters through arms A and C, then the power is delivered entirely to arms B and D with no power transmission from port A and C and vice versa.

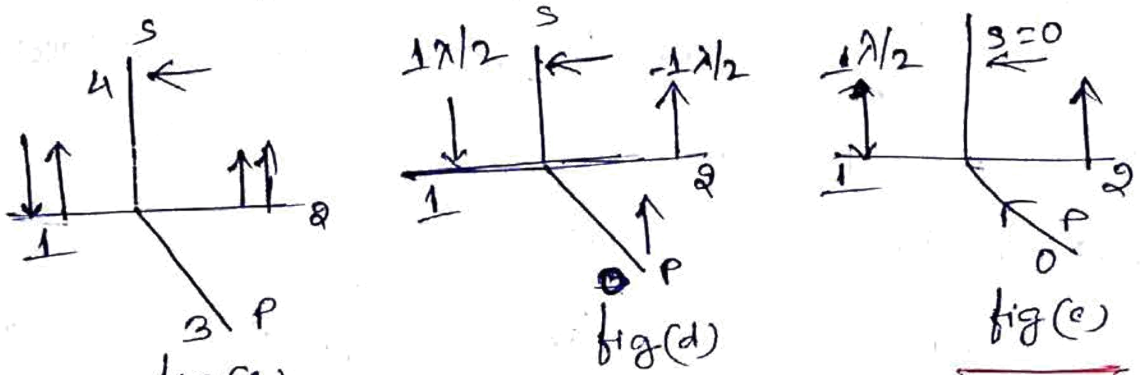
* Similarly, power entering B (or) D is delivered entirely to arms A (or) C, with no power transmission from B to D (or) D to B.

→ Operation of Magic Tee:-

* H field in parallel arm 'P' is normal to 'E'-field in series arm
* There is no different direct transmission between series and parallel arms.
* when waves of equal amplitude and phase enters 'P' and 'S' arms, the E-fields cancel in one of the side arms and add in the other as shown in fig (b).
* The energy applied to 'P' (or) 'S' arm is divided equally between 1 and 2 arms $1/\sqrt{2}$ as shown in figure (c) and (d)

⇒ Effects of reflections:-

* Power will not divide equally between side arms 1 and 2 when power enters through s(or)p.
* Imbalance exists between arms 1 and 2
* Reflections must be avoided (or) compensated.



fig(c) fig(d) fig(e)

→ Applications of Magic Tee:-

→ As an isolator

→ As a matching device

→ In balanced Mixer

→ As a switch.

→ **Magic Tee as a duplexer:-**

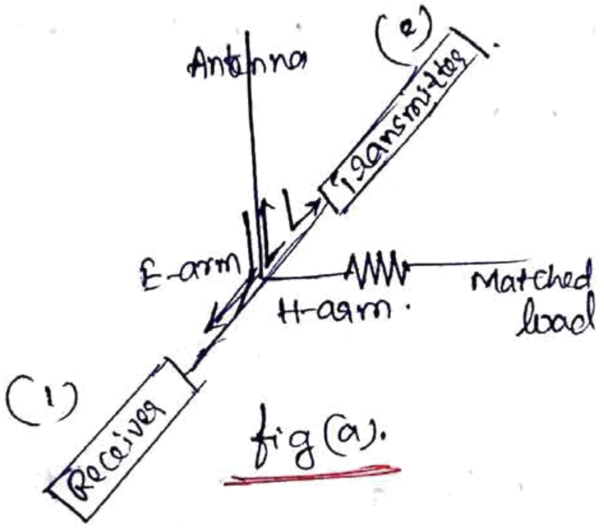


fig (a).

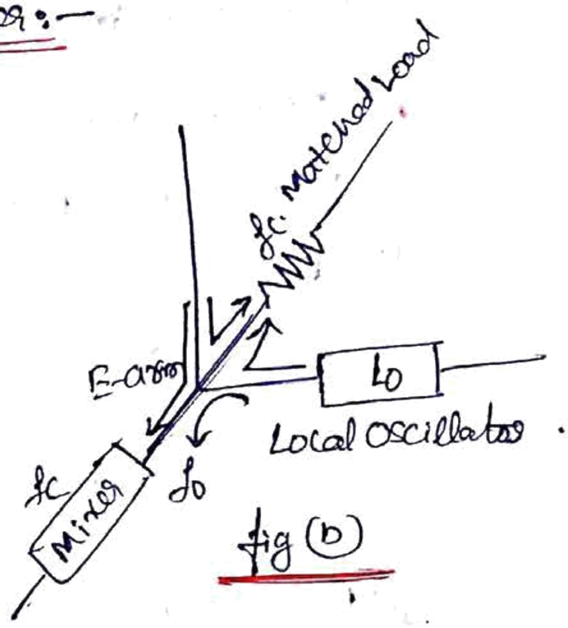


fig (b)

* Port s (1) and (2) are isolated ports. Also E and H are isolated ports. Thus no transmitted power ~~reaches~~ reaches the receiver i.e., during transmission, the transmitted power divides equally and thus. Only half the transmitted power is radiated into space via the antenna. The remaining half power is wasted in the matched load. During reception, the received power by antenna divides equally and half power reaches the receiver where it is processed.

→ **Magic Tee as a Mixer:-**

* E and H are isolated ports. Thus oscillator power does not reach the antenna. Also received signal fc by the antenna will not reach the local oscillator. As shown, half the local oscillator signals fo and half the received signal fc by antenna reach the mixer

that half the local oscillator signal and the receiver signal are wasted in matched load as shown in fig(b).

→ Directional Coupler :-

* A directional coupler is a useful hybrid wave guide joint which couples power in an auxiliary wave guide arm in one direction.

* It is a 4 port device but one of the port is terminated into a matched load.

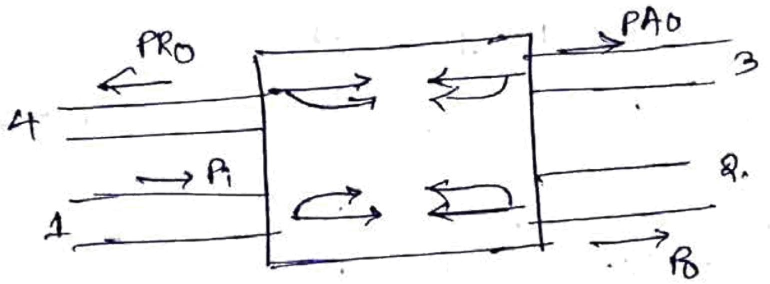


fig :- Directional coupler as a 4 port device.

→ characteristics of a directional coupler :-

* An ideal bidirectional coupler has the following characteristics.

(i) If power is fed into port (1), the power is coupled in ports (2) and (3) i.e., power flows in the forward direction but no power couples in port (4) i.e. in backward direction. Similarly power fed in (2) couples in ports (1) and port (4) not in (3)

(ii) If power couples in reverse direction i.e., power fed in (1) appears in ports (2) and (4) and nothing in (3), then such type of coupler is known as backward directional coupler.

The important parameters of a directional coupler are:

(a) coupling factor (c) coefficient.
(b) directivity (d) Bandwidth
(c) Insertion loss (e) Frequency sensitivity.

Coupling factor:- It is the ratio of power supplied to the main line input (P_i) to the forward power (P_f) at the auxiliary line output.

$$\boxed{C = 10\log(P_i/P_f)\ dB}$$

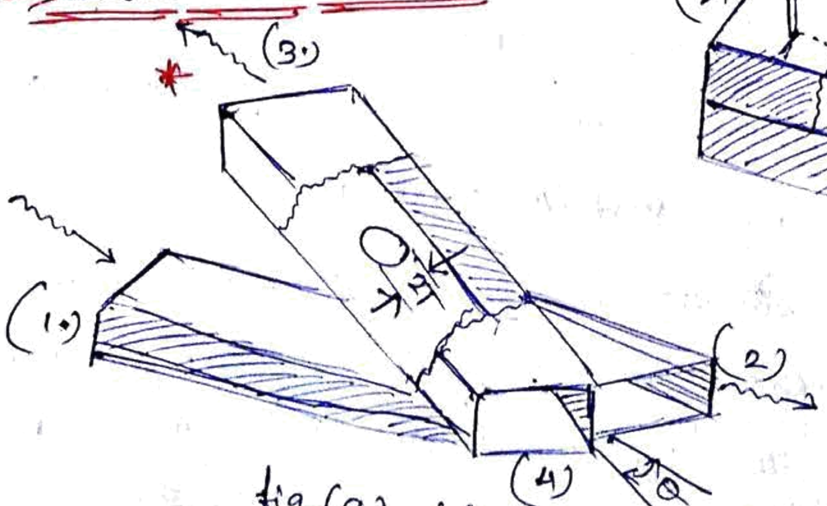
Directivity:- It is the ratio of forward power to the backward power.

$$\boxed{D = 10\log(P_f/P_b)}$$

Types of couplers:-

1) Bethe hole Directional coupler (single hole)
2) Long slot directional couples.
3) ~~slow~~ schwinger reverse phase couples
4) capacitance - loop directional coupler.
5) Double - hole (or) multi - hole couples.

Bethe hole couples:-



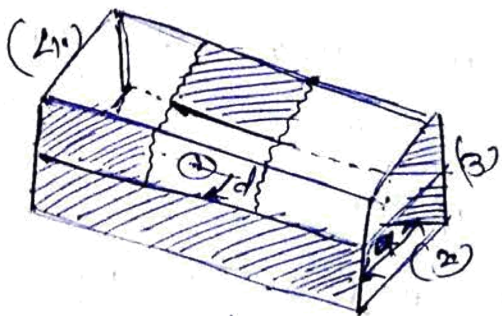


fig (b)

fig (a) & (b) Bethe hole directional coupler.

* It consists of two rectangular waveguides coupled by means of a circular aperture located at the center of the common board wall. To achieve directional coupling the axes of the 2 wave guides are inclined at an angle θ. Fig. (a) ~~shows~~ determined by frequency and the dimensions of the hole. If θ is zero then the hole is to be taken offset and not at the center. This construction can be understood by considering TE_{10} mode incident on port 1, produces a normal electric dipole in the aperture plus a tangential magnetic dipole, whose direction is same as that of the ~~...~~ incident wave and magnitude proportional to the incident wave strength.

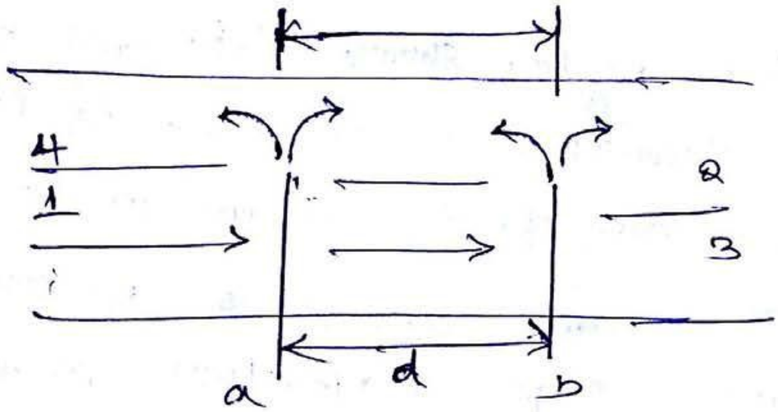
* Now in the auxiliary arm (upper waveguide) the normal electric dipole and axial component of magnetic dipole radiate symmetrically in both the directions. But the transverse component of magnetic dipole radiates anti symmetrically.

* So, proper adjustments of the axes of two wave guide at an angle θ cause the complete cancellation of backward power in the auxiliary arm. This cancellation may also be achieved by offsetting the hole as shown in fig (b).

→ Two Hole (or) multi hole couplers:-

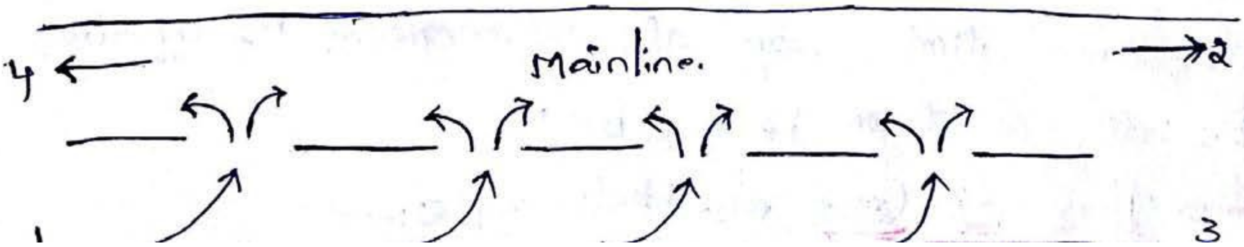
* Two hole couplers are constructed by coupling two identical rectangular wave guides by means of 2.

identical apertures spaced by a distance of $\lambda_g/4$, i.e., quarter guide wavelength apart. In this type of coupler as shown in fig(a). The signal which travels back to the first ~~of~~ hole from the second hole is 180° out



fig(a): Two hole type coupler.

of phase and so the 2 signals ~~fed~~ tend to cancel in the ~~background~~ backward direction. The signals in the forward directions are in phase and hence they add together. If the distance between the 2 holes is other $\lambda_g/4$, the said characteristics of coupling and directivity can not be achieved and so this type of coupler is a narrow band coupler. It is to be noted that directivity and coupling parameters are the sum of inherent property of the single bethe hole and the one associated with arrays.



* Multi hole couplers. fig (b) operate on the same basic principles. as 2 hole coupler. The coupling array. holes are each. separated by a distance of $\lambda g/4$.

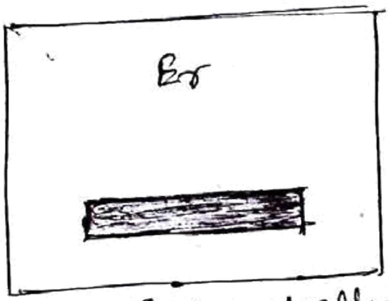
* The hole dimensions are so varied so that a wide frequency band is covered. these coupling versus frequency variations are + or -0.5db over the entire wave guide range.

* The holes are drilled in the narrow side of the wave guide for high power operation.

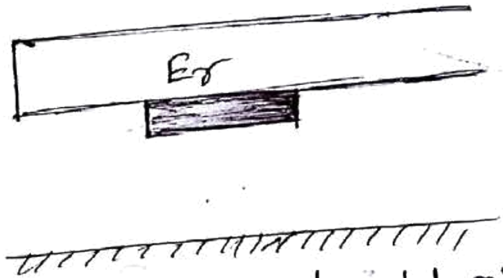
Types of Microstrip Lines :-

* There are many varieties of microstrip lines that have been used in practice such as embedded microstrip, standard inverted microstrip, suspended microstrip and slotted transmission line.

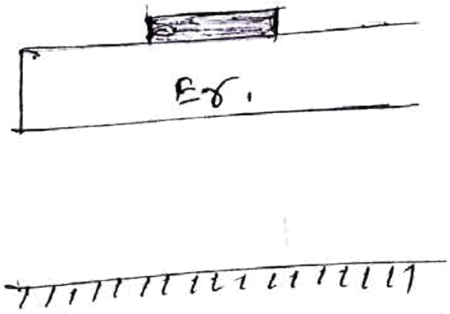
* The cross-sectional view of these are shown in fig



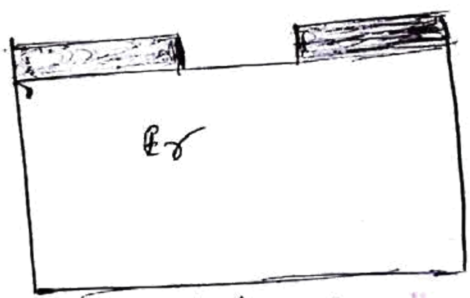
(a) embedded microstrip



(b) Inverted microstrip



(c) suspended microstrip

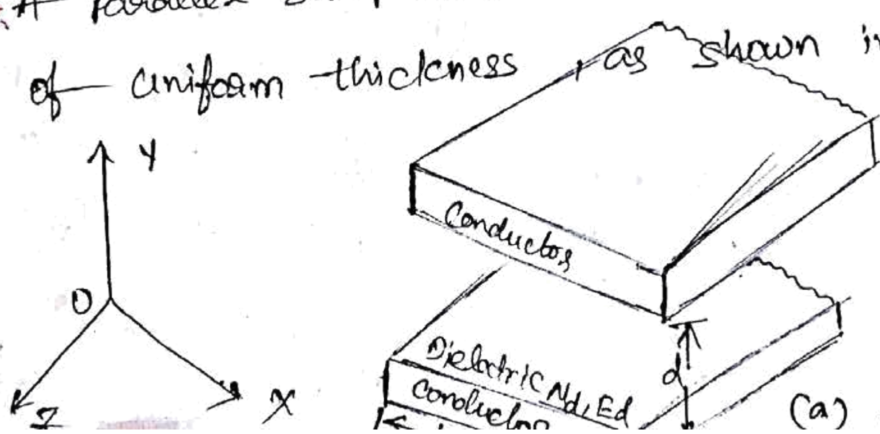


(d) slotted microstrip

* Some other TEM lines such as parallel strip lines, coplanar strip lines, also have been used for MIC's (microwave Integrate Circuits).

Parallel strip Lines :-

* A parallel strip line consists of two perfect dielectric slabs of uniform thickness as shown in fig.

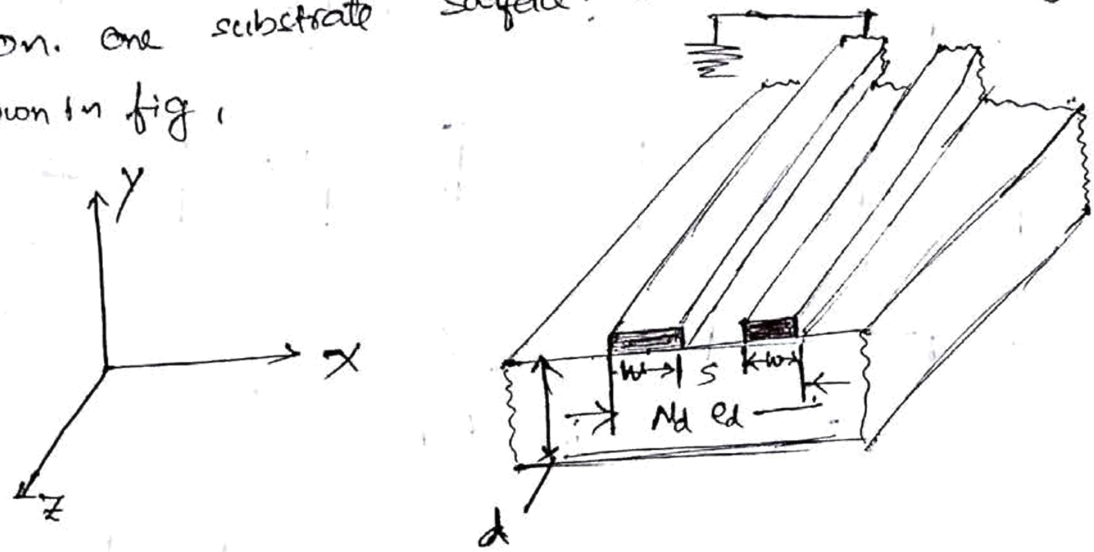


(a) Parallel strip lines

* The parallel strip line is similar to a two conductor transmission line, with the result it can support a quasi - TEM mode.

→ Coplanar strip lines:-

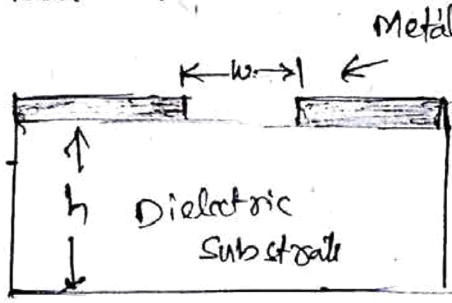
* A coplanar strip line consists of two conducting strips on one substrate surface with one strip grounded as shown in fig.



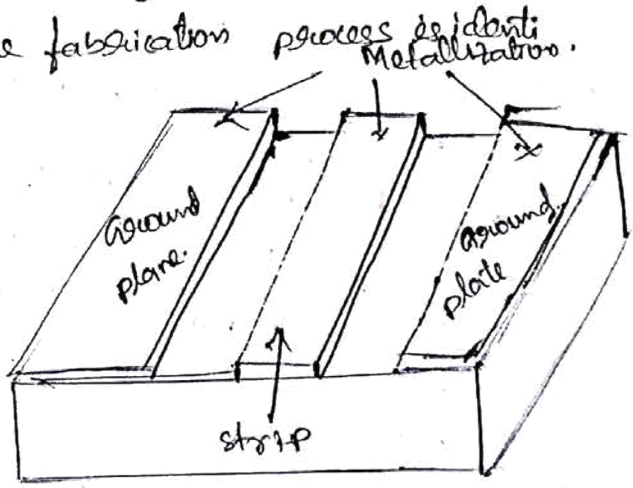
coplanar strip lines.

⇒ slot line and coplanar waveguide:-

* A slot line consists of a slot or gap in a conducting coating on a dielectric substrate. The fabrication process is identical to that of microstrip lines.



(a) slot line



(b) Coplanar waveguide

* a coplanar wave guide consist of a strip of thin metallic film deposited on the surface of a dielectric slab with two ground electrodes running adjacent and parallel to the strip on the same surface.

* UNIT - IV *

*Wave Guide Components And Applications-II *

→ Ferrite Devices:- Composition & Characteristics:-

* Ferrites are non-metallic with resistivities (ρ) nearly 10^4 times greater than metals and with dielectric constants (ϵ_r) around 10-15 and relative permeabilities of the order of 1000. They have magnetic properties similar to those of ferrous metals. They are oxide based compounds having general composition of the form $MeO \cdot Fe_2O_3$ i.e., a mixture of a metallic and ferric oxide where MeO represents any divalent metallic oxide. Such as MnO, ZnO, CdO, NiO (or) a mixture of these. They are obtained by firing powdered oxides of materials at 1100°C (or) more and pressing them into different shapes. This processing gives them the added characteristics of ceramic insulators so that they can be used at microwave frequencies.

* Ferrites have atoms with large number of spinning electrons resulting in strong magnetic properties. These magnetic properties are due to the magnetic dipole moment associated with the electron spin. Because of the above properties, ferrites find application in a number of microwave devices to reduce reflected power, for modulation purposes and in switching circuits. Because of high resistivity they can be used upto 1000GHz.

* One more property of ferrites which is useful at microwave frequencies i.e., the "non-reciprocal" property. When two circularly polarized wave one rotating clockwise and other anticlockwise are made to propagate through ferrite, the material reacts differently to the two rotating fields, thereby presenting different effective permeabilities to both the waves i.e., ϵ_{r1}, μ_{r1}, β_1 for left circularly polarized wave and ϵ_{r2}, μ_{r2}, β_2 for the right circularly polarized wave.

→ Faraday Rotation in Ferrites :-

* Consider an infinite lossless medium. A static field B_0 is applied along the z-direction. A plane TEM wave that is linearly polarized along the x-axis at t=0 is made to propagate through the ferrite in the z-direction.

The plane of polarization of this wave will rotate with distance, a phenomenon known as "Faraday rotation".

Any linearly polarised wave can be regarded as the vector sum of two counter rotating circularly polarised wave. ($E_0/2$ vectors shown in figure (a)) The ferrite material offers different characteristics to these waves, with the result that the phase change for one wave is larger than the other wave resulting in rotation 'θ' of the linearly polarised wave at $Z = l$.

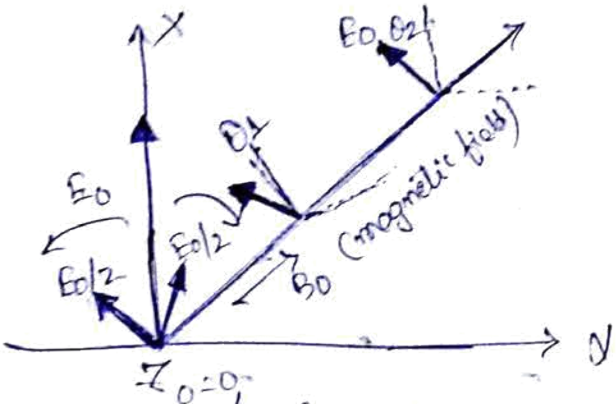


fig (a) Faraday rotation.

* It is observed that a rotation of 100 degrees (or) more per cm of ferrite length is typical for ferrites at a frequency of 10 GHz. If the direction of propagation is reversed, the plane of polarization continues to rotate in the same direction i.e., $Z = l$ to $Z = 0$, the wave will arrive back at $Z = 0$ polarized at an angle 2θ relative to x-axis.

* In fact the angle of rotation 'θ' given by

$$\theta = \frac{l}{2}(\beta_+ \sim \beta_-)$$

* where l = length of the ferrite rod

β_+ = phase shift for the right circularly polarised (component in clockwise direction) wave w.r.to some reference.

β_- = phase shift for the left circularly polarised (component in anticlockwise direction) wave w.r.to the same reference.

* In a ferrite medium, There will be finite losses. The propagation constant for circularly polarised

wave will have unequal attenuation constants and unequal phase constant. Due to this, the direction of faraday rotation will be different in the two regions above and below the resonant frequency (ω_0).

* A two port ferrite device is shown in fig(2). when a wave is transmitted from port ①

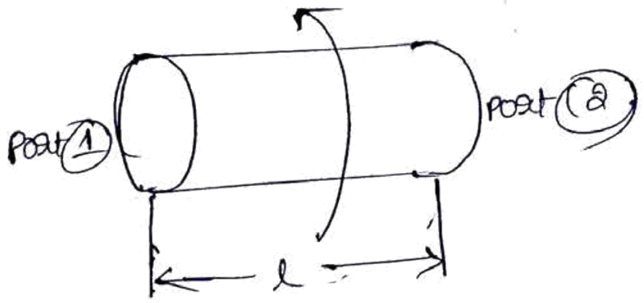


fig (2):

port ② it undergoes rotation in the anticlockwise direction as shown. Even if the same wave is allowed to propagate from ~~prot~~ port ② port ①, it will undergo rotation in the same direction (anticlockwise). Hence the direction of rotation of linearly polarised wave is independent of the direction of propagation of the wave.

⇒ Microwave Devices which make use of Faraday rotation:-

a) Gyrator
b) Isolator
c) circulator.

a) Gyrator :-

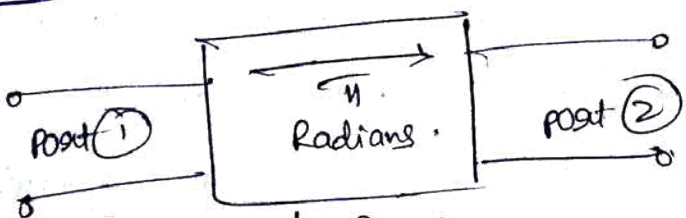


fig (a)

* It is a two port device that has relative (3) phase difference of 180° for transmission from port (1) port (2) and 'no' phase shift (0° phase shift) for transmission from port (2) to port (1) as shown in fig(a).

* The construction of a gyrator is as shown in fig(b). It consists of a piece of circular waveguide carrying the dominant TE_{11} mode with transistors to a standard rectangular waveguide with dominant mode (TE_{10}) at both ends. A thin circular ferrite rod tapered at both ends is located inside the circular waveguide supported by polyfoam and the waveguide is surrounded by a permanent magnet which generates dc magnetic field for proper operation of ferrite. To the input end a 90° twisted rectangular wave guide is connected as shown.

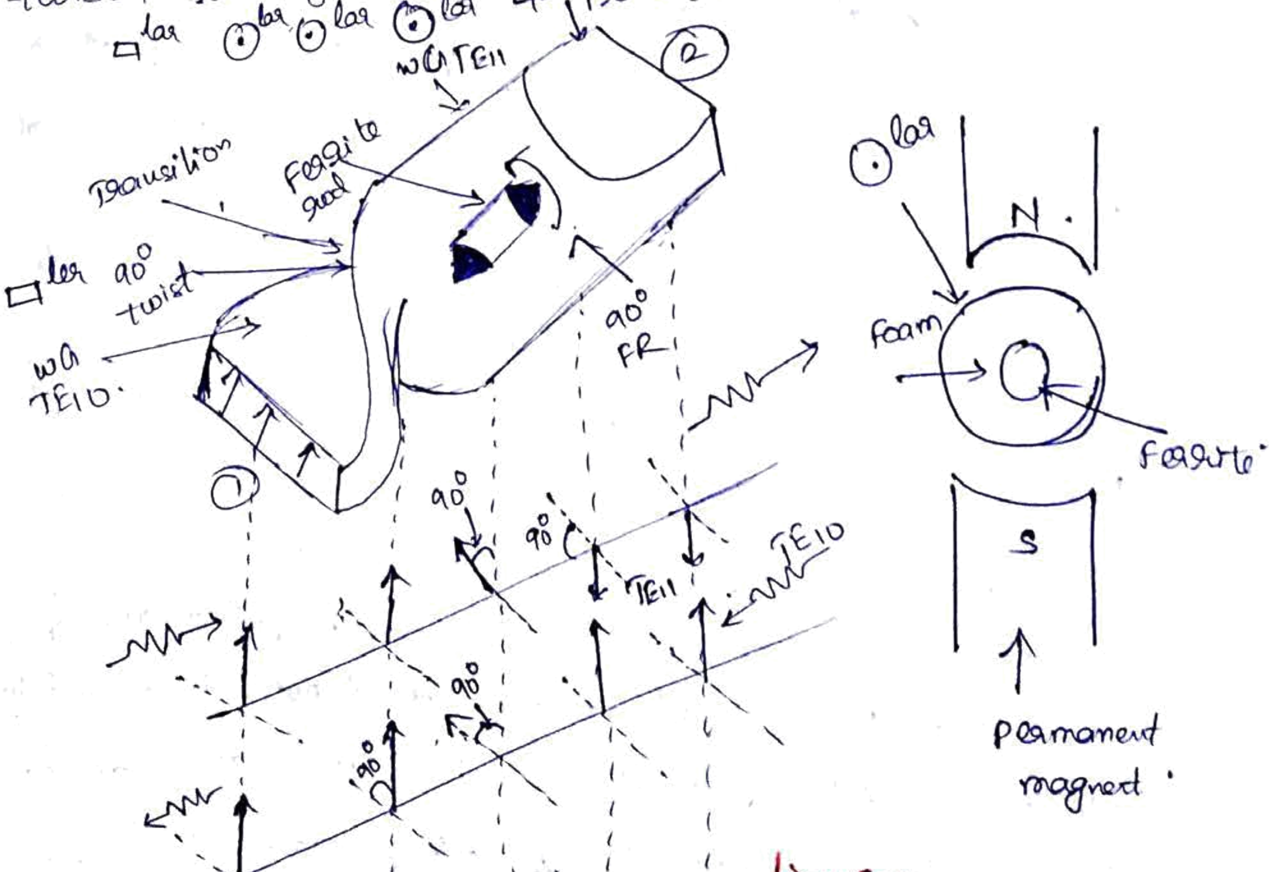


fig (b)

→ Operation :—

* When a wave enters port ① its plane of polarization rotates by 90° because of the twist in the wave guide. It again undergoes faraday rotation through 90° because of ferrite rod and the wave which comes out of port ② will have a phase shift of 180° compared to wave entering the port ①

* But when the same wave enters port ② (TE_{10} mode signal), it undergoes faraday rotation through 90° in the same anticlockwise direction. Because of the twist, this wave gets rotated back by 90° comes out of port ① with 0° phase shift. as shown in fig (a) Hence a wave at port ① undergoes a phase shift of π radians (or 180°)

→ ~~Isolator~~ :— but a wave fed from port ② does not change its phase in a gyrator.

→ Isolator :—

* An isolator is a 2 port device which provides very small amount of attenuation for transmission from port ① to port ② but provides maximum attenuation for transmission from port ② to port ① This requirement is very much desirable when we want to match a source with a variable load.

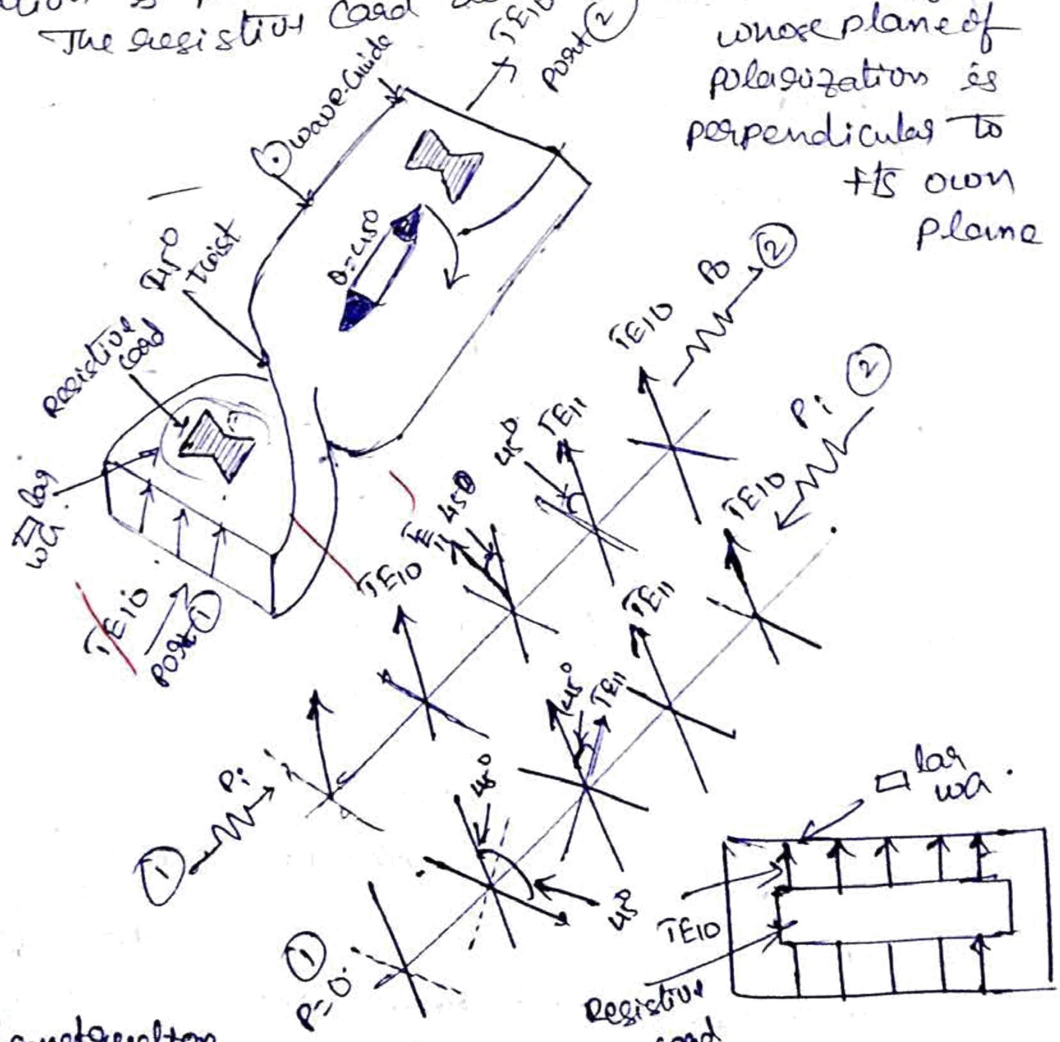
* In most new generators, the o/p amplitude and frequency tend to fluctuate very significantly with changes in load impedance. This is due to mismatch of generator o/p to the load resulting in reflected wave from load. But these reflected waves should not be allowed to reach the MW generator, which will cause amplitude and frequency instability of MW generator.

* when an isolator is placed b/n generator & load the (1) generator is coupled to the load with '0' attenuation and any reflections from load are absorbed by the isolator without effecting generator's O/P and hence acts as matched for all loads with no change in fq and O/P power.

fig (a).

Construction:—

* The construction of Isolator (fig (b)) is similar to gyrator except that an isolator makes use of 45° twisted R.W.G (Rectangular W.G) and 45° faraday rotation ferrite rod. a resistive card is placed along the larger dimension of the □lar W.G. so as to absorb a wave whose plane of polarisation is parallel to the plane of resistive card. The resistive card does not absorb any wave whose plane of polarization is perpendicular to its own plane.



fig(b): construction of isolator.

Operation :-

* A TE_{10} wave passing from port (1) through the resistive card and is not attenuated. After coming out of the card, the wave gets shifted by 45^0 because of the twist, in anticlockwise direction and then by another 45^0 in clockwise direction because of the ferrite rod and hence comes out of port (2) with the same polarization as port (1) without any attenuation.

* But a TE_{10} wave fed from P(2) gets a pass from the resistive card placed near P(2) since the plane of polarization of wave is $\perp^{lar}$ to the plane of resistive card. Then the wave gets rotated by 45^0 due to faraday rotation in clockwise direction and further get rotated by 45^0 in clockwise direction due to the twist in the waveguide.

* The plane of polarization of the wave will be parallel ~~to~~ with that of the resistive card and hence the wave will be completely absorbed by the Resistive card and the o/p at port (1) will be zero. This power is dissipated in the card as heat

⇒ Circulator :-

* A circulator is a four port MW device which has a peculiar property that each terminal is connected only to the next clockwise terminal i.e., port (1) is connected to port (2) only and not to port (3) and (4) and port (2) is connected only to port (3) etc. This is shown in fig (a) ~~is com~~ Although

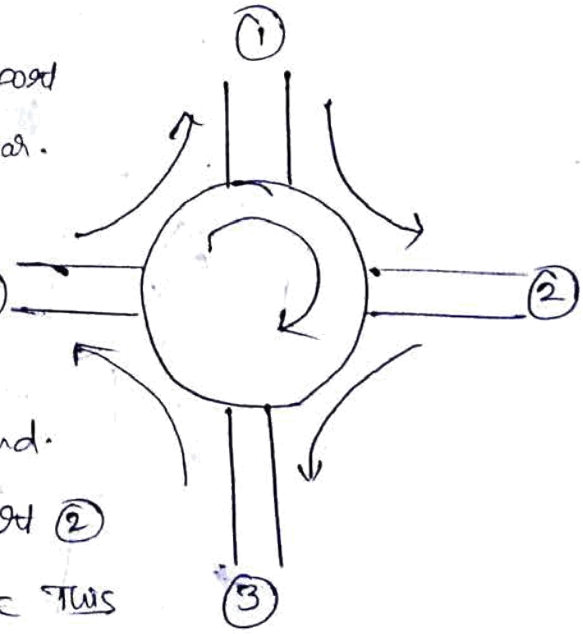


fig (a)

there is no restriction on the number of ports, four ports are ~~mainly. com~~ most ~~commonly~~ commonly used. They are useful parametric amplifiers, tunned diodes.

amplifiers, and multiplexers in radars.

→ Construction :-

A four port faraday rotation circulator is shown in fig(b). The power entering port(1) is TE_{10} mode and is converted to TE_{11} mode because of gradual rectangular to circular transition. This power passes port(3) unaffected since the electric field is not significantly cut and is rotated through 45^0 due to the ferrite, passes port(4) uneffected (for the same reasons as it passes port(3)) and finally emerges out of port(2). Power ~~form~~ from port(2) ~~port~~ will have plane of polarization already ~~by~~ tilted by 45^0 with respect to port(1). This

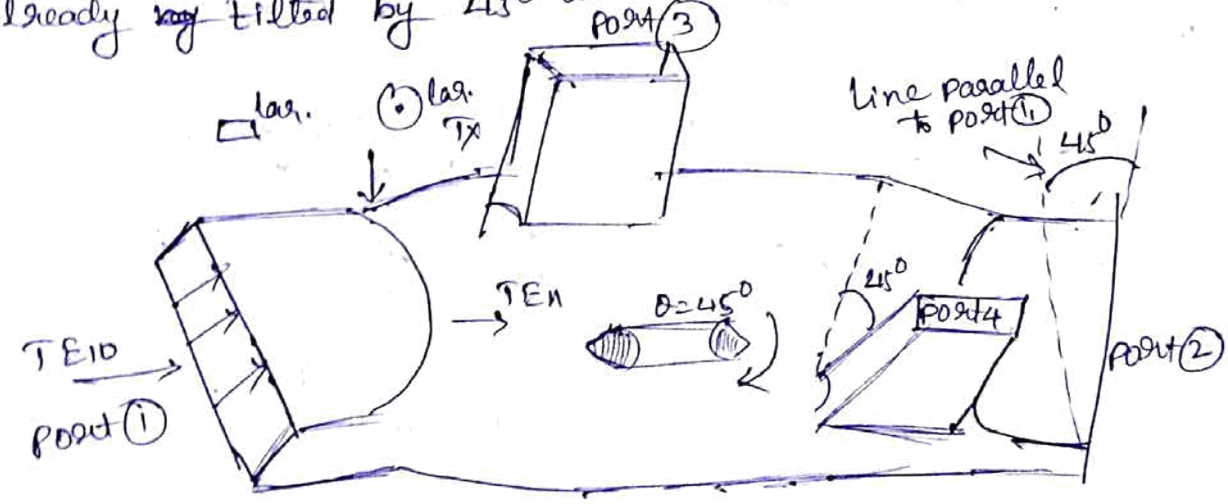
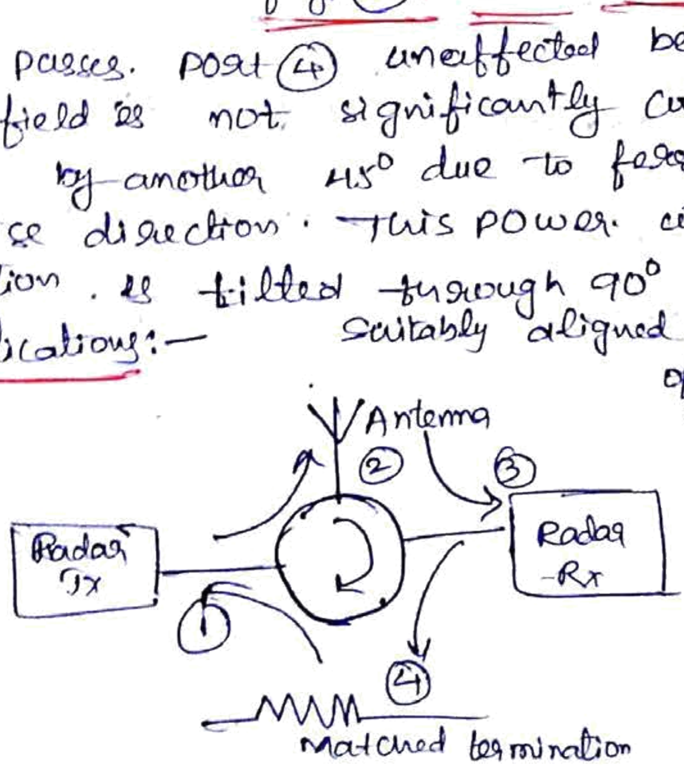


fig. (b): Four port Circulator

power passes port(4) unaffected because ~~of~~ again the electric field is not significantly cut. This wave gets rotated by another 45^0 due to ferrite rod in the clockwise direction. This power whose plane of polarization is tilted through 90^0 finds port(3) suitably aligned and emerges out of it. Similarly P(3) is coupled only to port(4) and P(4) to P(1).

→ Applications :-

1)



fig(a).

A circulator can be used as a duplexer for a radar antenna system as shown in fig(a).

* Transmitter feeds the antenna while the received energy is directed to the receiver. The powerful radar transmitter is isolated from the sensitive receiver and also the same antenna can be used for both transmission and reception. This is the duplexer action being performed by a circulator.

2) we can have three port circulators, strip lines circulators that can have several applications.

Two three port circulators can be used in tunnel

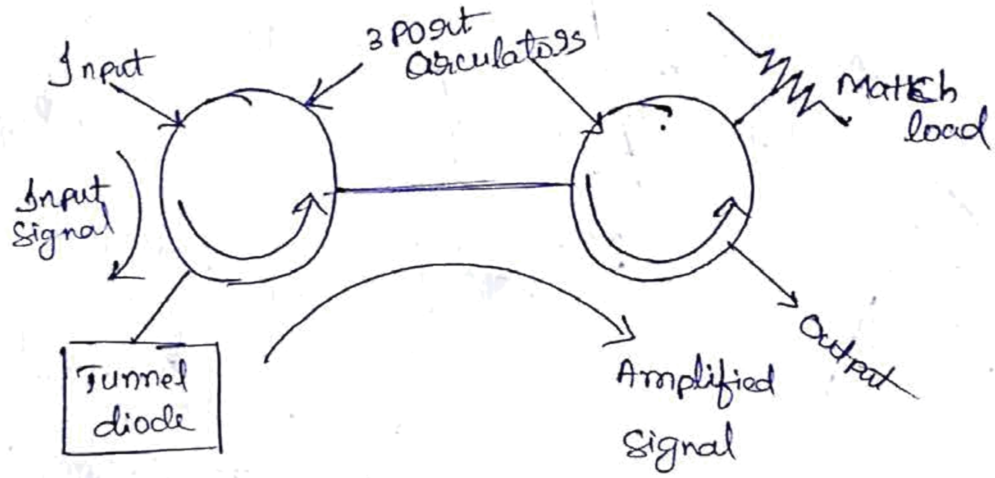


fig (b).

diode (or) parametric amplifier as shown in fig(b).

3) Circulators can be used as low power devices as they can handle low powers only.

Wave guide Microwave Junctions :-

* Sometimes in a waveguide system it is neccessary to split all (or) part of the microwave energy into particular directions. This is achieved by wave guides, or in general by microwave junctions. These are combined to form coupler units that direct the energy as required. Alternately the same junction may be used to combine two (or) more signals.

* In general, a microwave junction is an interconnection of two (or) more microwave components as shown in fig (1).

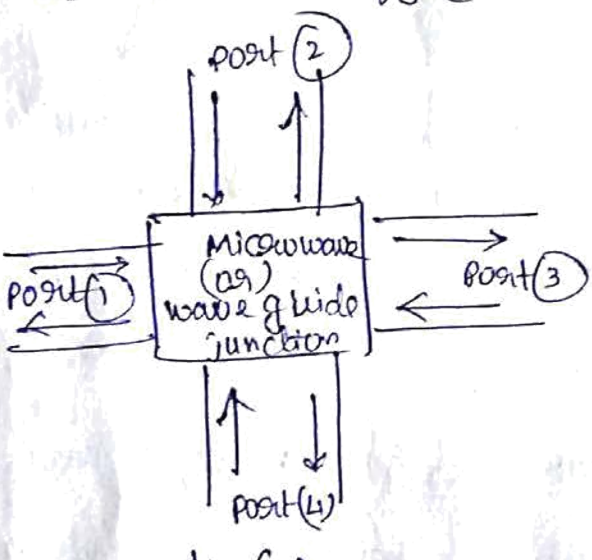


fig (1) :

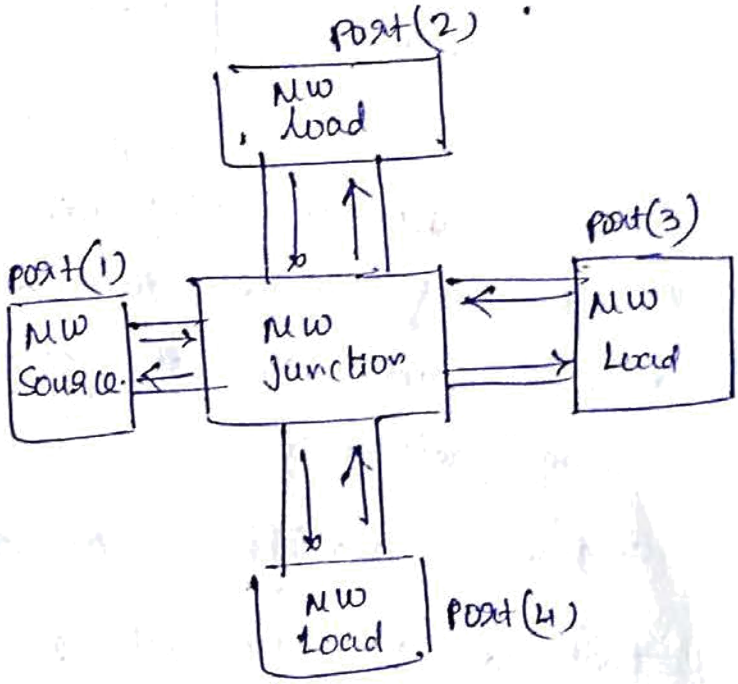


fig (2) :

~~The microwave~~

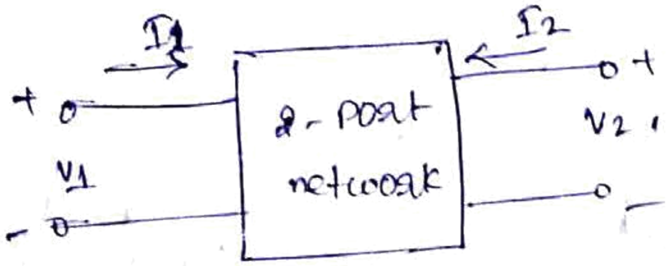
* This junction has four ports similar to low frequency two-port network. Fig (2) shows a microwave source at port (1) and microwave loads at port (2), (3) & (4).

* When input from microwave source is applied at port (1). a part of it comes out of port (2) another part out of port (3) some part out of port (4).

and the remaining port may some out of port (1) it due to mismatch between port (1) and microwave junctio coeffic

→ **Scattering (or) (S) Parameters :-**

* Low frequency circuits can be described by two port networks and their parameters such as Z, Y, H, ABCD etc., as per network theory. Here the network parameters relate the total voltages and total currents as shown in fig (3)



fig(3)

* In a similar way, at new frequencies, we talk of travelling waves with associated power instead of voltages and currents and the microwave junction can be defined by what are called as S-parameters (or) "scattering parameters" as shown in fig (2) It can be seen that for an input at one port, we have four outputs. Similarly if we apply inputs to all the ports, we will have 16 outputs, which are represented in a matrix form and that matrix is called as a "scattering matrix".

* It is a square matrix which gives all the combinations of power relationships between the various input and output ports of a microwave junction.

* The elements of matrix are called scattering coefficients (or) scattering (s) parameters.

* To obtain the relationship between the scattering matrix and the input / output powers at different ports consider a junction of 'n' number of transmission lines wherein the ith line is terminated in a source as shown in fig (4):

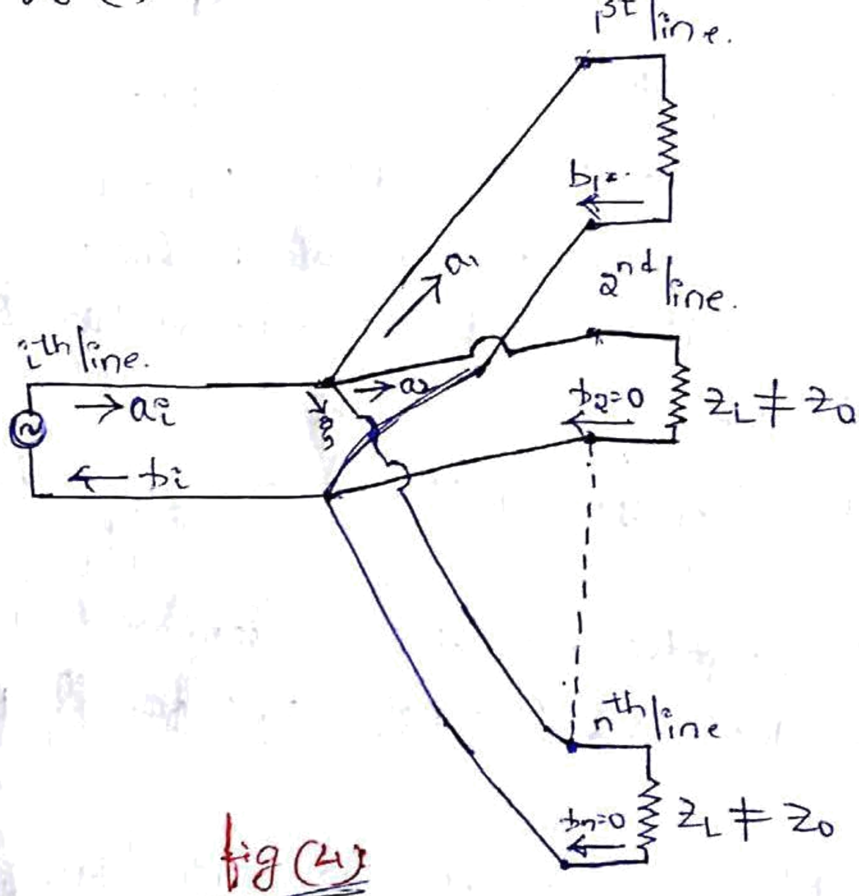


fig (4)

Case (1) :- let the first line be terminated in an impedance other than the characteristic impedance (i.e., Z_L) ($Z_L \neq Z_0$) and all the remaining lines (from 2^{nd} to n^{th} line) in an impedance equal to Z_0 (i.e., $Z_L = Z_0$).

* If a_i be the incident wave at the junction due to a source at i^{th} line, then it divides itself among (n-1) number of lines as $a_1, a_2, \ldots a_n$ as shown

in fig(4). There will be no reflections from 2nd to n line and the incident waves are absorbed. since their impedances are equal to characteristic impedance (Z_0). But there is a mismatch at the 1st line and hence there will be a reflected wave b_1 going back into the junction.

b_1 is related to a_1 by

b_1 = (reflection coefficient) $a_1 = S_{11} a_1$

* where S_{i1} = reflection coefficient of 1st line.

1 → reflection from 1st line and

i → source connected at ith line.

* Hence, the contribution to the outward travelling wave in the ith line is given by

$$b_i = S_{i1} a_1 \quad (\because b_2 = b_3 = \cdots = b_n = 0).$$

→ Case (2):— let all the (n-1) lines be terminated in an impedance other than Z_0 (i.e., $Z_L \neq Z_0$ for all the lines)

* Then there will be reflections into the junction from every lines and hence the total contribution to the outward travelling wave in the ith line is given by

$$b_i = S_{i1} a_1 + S_{i2} a_2 + S_{i3} \cdot a_3 + \cdots + S_{in} a_n$$

* i = 1 to n. since 'i' can be any line from 1 to n

therefore, we have

$$b_1 = S_{11} a_1 + S_{12} a_2 + S_{13} a_3 + \cdots + S_{1n} a_n$$

$$b_2 = S_{21} a_1 + S_{22} a_2 + S_{23} a_3 + \cdots + S_{2n} a_n$$

$$b_n = S_{n1} a_1 + S_{n2} a_2 + S_{n3} a_3 + \cdots + S_{nn} a_n$$

In matrix form:

$$\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1n} \\ S_{21} & S_{22} & \cdots & S_{2n} \\ \vdots & \vdots & & \vdots \\ S_{n1} & S_{n2} & \cdots & S_{nn} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \quad \text{— (2)}$$

column matrix [b] corresponding to reflected waves (or) o/p

Scattering column Matrix [S] of order $n \times n$

Matrix [a] corresponding to Incident waves (or) Input

$$\therefore [b] = [S][a] \quad \text{— (3)}$$

When a junction of n number of waveguides is considered.

→ a's represent inputs to particular ports

→ b's represent outputs out of various ports.

→ S_{ij} represents scattering coefficients resulting due to i/p at i^{th} port and output taken out of j^{th} port.

→ S_{ii} denotes how much of power is reflected back from the junction into the i^{th} port when i/p power is applied at the i^{th} port itself.

⇒ **Properties of S-Matrix:-**

1) [S] is always a square matrix of order $(n \times n)$

2) [S] is a symmetric matrix

$$S_{ij} = S_{ji}$$

3) [S] is a unitary matrix.

i.e., $[S][S]^* = [I]$

where $[S]^*$ = complex conjugate of [S]

[I] = unit matrix or Identity matrix of the same order as that of [S]

4) The sum of the products of each term of any row (or) column multiplied by the complex conjugate of the corresponding terms of any other row (or) column is zero.

i.e., $\sum_{i=1}^{n} S_{ik} S_{ij}^* = 0 \quad k \neq j \quad \begin{pmatrix} k = 1,2,3 \ldots n \\ j = 1,2,3 \ldots n \end{pmatrix}$

5) If any of the Terminal (or) reference planes (say kth port) are moved away from the junction by an electric distance $\beta_k l_k$, each of the coefficients S_{ij} involving k will be multiplied by the factor $e^{-j\beta_k l_k}$

H-plane Tee Junction:-

A T-junction is an intersection of three wave guides in the form of English alphabet "T". There are several types of Tee junction.

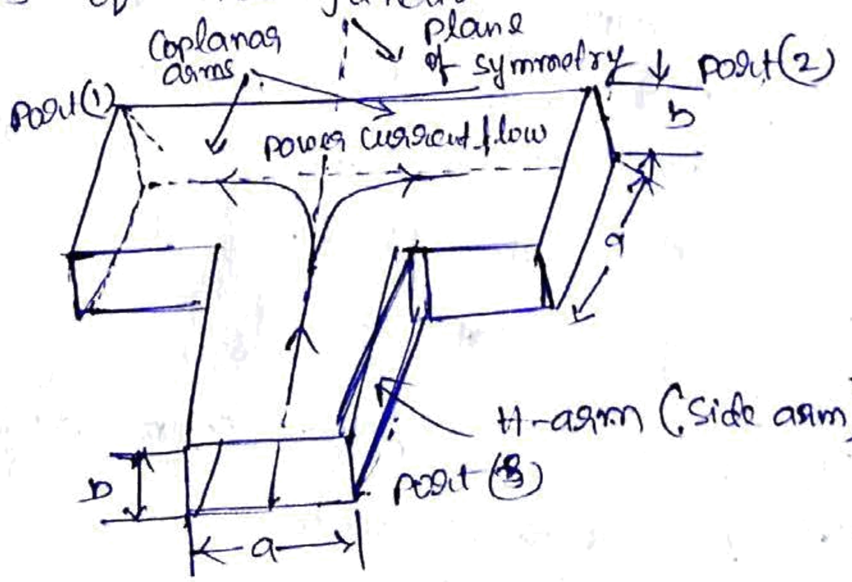


fig. (5)

* A H-plane junction is formed by cutting a rectangular slot along the width of a main waveguide and attaching another waveguide - the side arm - called the H-arm as shown in fig (5). The port (1) and port (2) of the main waveguide are called collinear ports and port (3) is the H-arm (or) side arm.

* H-plane Tee is so called because the axis of the side arm is parallel to the planes of the main transmission line. As all three arms of H-plane Tee lie in the plane of magnetic field, the magnetic field divides itself into the arms. Therefore this is also called a current junction.

* The properties of a H-plane Tee can be completely defined by its [S] matrix. The order of scattering matrix is 3X3 since there are three possible inputs and 3 possible outputs.

$$\therefore [S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \quad -(4)$$

* Now we determine the S-parameters S_{ij}, $i \to 1,2,3$ and $j \to 1,2,3$ by applying the properties of [S].

1) Because of plane of symmetry the junction scattering coefficient S_{13} and S_{23} must be equal.

$$\therefore \boxed{S_{13} = S_{23}},$$

2) From the symmetry property, $S_{ij} = S_{ji}$

$$\therefore \quad S_{12} = S_{21}, \quad S_{23} = S_{32} = S_{13}$$

$$S_{13} = S_{31}$$

3) Since port is perfectly matched to the junction

$S_{33} = 0$.

With these properties [S] matrix of eq'n (4) becomes

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix} \quad -(5)$$

i.e., we have four unknowns

4) From the unitary property

$$[S][S]^* = [I]$$

i.e.,
$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & S_{13}^* \\ S_{13}^* & S_{13}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Multiplying we get

$R_1C_1 : S_{11}S_{11}^* + S_{12}S_{12}^* + S_{13}S_{13}^* = 1$ (R_1C_1 = row 1, column 1)

(or) $|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1$ — (6)

R_2C_2 similarly $|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1$ — (7)

R_3C_3 : $|S_{13}|^2 + |S_{13}|^2 = 1$ — (8)

R_3C_1 : $S_{13}S_{11}^* + S_{13}S_{12}^* = 0$ — (9)

from eq'n (8)

$2|S_{13}|^2 = 1$ (or) $S_{13} = \frac{1}{\sqrt{2}}$ — (10)

Comparing eq'ns 6 and 7 we get

$|S_{11}|^2 = |S_{22}|^2$

$\therefore$ $S_{11} = S_{22}$ — (11)

from eq'n (9) $S_{13} \neq 0$, ($S_{11}^* + S_{12}^* = 0$ (or) $S_{11}^* = -S_{12}^*$)

(or) $S_{11} = -S_{12}$ (or) $S_{12} = -S_{11}$

Using these eq'ns in (6)

$|S_{11}|^2 + |S_{11}|^2 + \frac{1}{2} = 1$

(or) $2 \cdot |S_{11}|^2 = \frac{1}{2}$ (or) $S_{11} = \frac{1}{2}$. — (13)

∴ from eq'n (11) and (12)

$$S_{12} = -\frac{1}{2}. \quad — (14)$$

and $$S_{22} = \frac{1}{2}. \quad — (15)$$

* Substituting for S_{13}, S_{11}, S_{12} and S_{22} from eq'n (10) and eq'n (13) to (15) in eq'n (5) we get.

$$[S] = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix} \quad — (16)$$

* we know that $[b] = [S][a]$ from eq'n (3)

$$\therefore \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

$$b_1 = \frac{1}{2}a_1 - \frac{1}{2}a_2 + \frac{1}{\sqrt{2}}a_3 \quad — (17)$$

$$b_2 = -\frac{1}{2}a_1 + \frac{1}{2}a_2 + \frac{1}{\sqrt{2}}a_3 \quad — (18)$$

$$b_3 = \frac{1}{\sqrt{2}}a_1 + \frac{1}{\sqrt{2}}a_2 \quad — (19)$$

$a_3 \neq 0$, $a_1 = a_2 = 0$.

→ Case (1) :— (i.e.,) Input is given at port (3) and no inputs at port (1) and port (2).

* Substituting these in eq'ns (17), (18) and (19) we get.

$$b_1 = \frac{a_3}{\sqrt{2}}, \quad b_2 = \frac{a_3}{\sqrt{2}}, \quad b_3 = 0.$$

* Let P_3 be the power input at port (3). Then this power divides equally between port (1) and (2) in phase i.e.) $P_1 = P_2$

$$P_3 = P_1 + P_2 = 2P_1 = 2P_2$$

* The amount of power coming out of port (1) (or) port (2) due to input at port (3)

$$= 10 \log_{10} \frac{P_1}{P_3} = 10 \log_{10} \frac{P_1}{2P_1} = 10 \log_{10} (1/2)$$

$$= -10 \log_{10} 2 = -10(0.3010) = -3\text{dB}.$$

* Hence the power coming out of port (1) and port (2) is 3dB down with respect to input power at port (3) hence the H-plane Tee is called as 3-dB splitter.

* Further when TE_{10} mode is allowed to propagate into port (3), the electric field lines do not change their direction when they come out of port (1) and (2) hence called H-plane tee i.e., The waves that come out of port's (1) and (2) are equal to in magnitude and phase.

Case 2:— $a_1 = a_2 = a$, $a_3 = 0$.

$$b_1 = \frac{a}{2} - \frac{a}{2} + \frac{1}{\sqrt{2}} a_3 = \frac{a_3}{\sqrt{2}} = 0$$

$$b_2 = -\frac{a}{2} + \frac{a}{2} + \frac{1}{\sqrt{2}} a_3 = \frac{a_3}{\sqrt{2}} = 0.$$

$$b_3 = \frac{a_1}{\sqrt{2}} + \frac{a_2}{\sqrt{2}} = \frac{a}{\sqrt{2}} + \frac{a}{\sqrt{2}}$$

* i.e., the output at port (3) is addition of the two inputs at port (1) and port (2) and these are added in phase.

E-Plane Tee :-

* A rectangular slot is cut along the broader dimension of a long waveguide and a side arm is attached as shown in figure port(1) and port(2) are the collinear arms and port(3) is the E-arm.

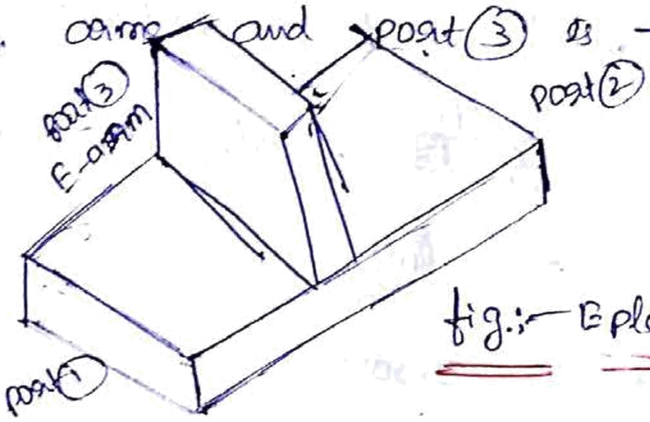


fig:- E plane Tee.

* When TE_{10} mode is made to propagate into port (3), the two outputs at port (1) and port (2) will have a phase shift of 180° as shown in figure. Since the electric field lines change their direction when they come out of port (1) and (2), it is called a E-plane Tee.
E-plane Tee is a voltage (or) series junction symmetrical about the central arm. Hence any ~~two~~ signal that is to be split (or) any two signal that are to be combined will be fed from the E-arm.

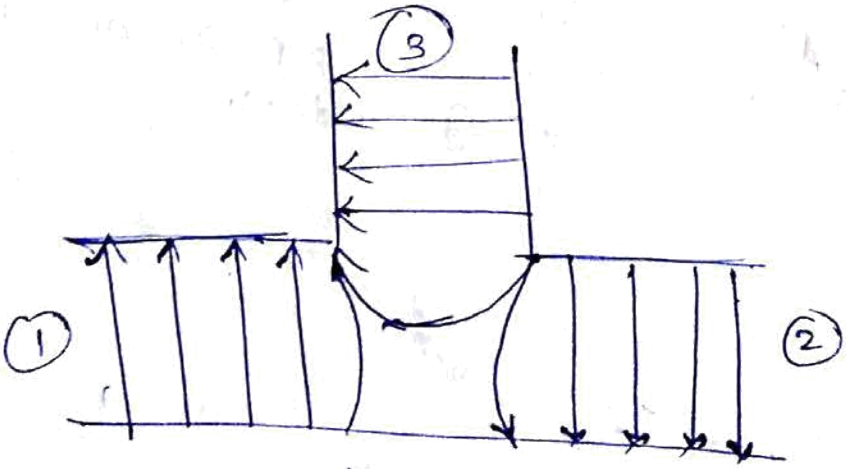


fig.

① and ② are in phase i.e., $P_1 = P_2$ (power outputs at the respective ports corresponding to b_1 and b_2), and port (comes

But $P_3 = P_1 + P_2 = 2P_1 = 2P_2$.

The amount of power coming out of port (1) (or) port (2) due to input at port ③

$$= \log_{10}\left(\frac{P_1}{P_3}\right) = 10 \log_{10} \frac{P_1}{2P_1} = 10 \log_{10} (1/2).$$

$$= -10 \log_{10} 2$$

$$= -10 \ (0.3010)$$

$$= -3dB.$$

Since the power coming out of port ② is 3dB down with respect to input power at port ③. Hence the H-

* The scattering matrix of an E-plane Tee. Can be used to describe its properties. The power out of port ③ (side (or) E-arm) is proportional to the difference between instantaneous powers entering from ports ① and port ②

* The effective value of the power leaving the E-arm is proportional to the phasor difference between the power entering ports ① and ②.

* when powers entering the main arms (port ① and port ②) are inphase - Opposition, maximum energy comes out of port ③ (or) E-arm.

* since it is a three port junction the scattering matrix can be derived as given below.

* [s] is a 3x3 matrix. since there are 3 ports.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \quad -①$$

* The scattering matrix coefficient

$$S_{23} = -S_{13} \quad -②$$

* ~~There~~ since the output at ports ① and port ② are out of phase by 180° with an input at port ③

* If port ③ is perfectly matched to the junction

$$S_{33} = 0 \quad -③$$

* From symmetry property $S_{ij} = S_{ji}$

$$S_{12} = S_{21},\ S_{13} = S_{31},\ S_{23} = S_{32}$$

* Substituting all the above parameters in eq'n ①

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix} \quad - ⓐ$$

* From unitary property $[S]\cdot[S]^* = [I]$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & -S_{13}^* \\ S_{13}^* & -S_{13}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

R_1C_1 : $S_{11}S_{11}^* + S_{12}S_{12}^* + S_{13}S_{13}^* = 1$

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1 \quad -④$$

R_2C_2: $S_{12} S_{12}^* + S_{22} \cdot S_{22}^* + S_{13} \cdot S_{13}^* = 1$

$$|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1. \quad (5)$$

R_3C_3: $S_{13} S_{13}^* + S_{13} S_{13}^* = 1$

$$|S_{13}|^2 + |S_{13}|^2 = 1. \quad (6)$$

R_3C_1: $S_{13} \cdot S_{11}^* - S_{13} S_{12}^* = 0 \quad (7)$

equating (4) and (5)

$$\cancel{|S_{11}|^2} + \cancel{|S_{12}|^2} + \cancel{|S_{13}|^2} = \cancel{|S_{12}|^2} + |S_{22}|^2 + |S_{13}|^2$$

$$S_{11} = S_{22} \quad (8)$$

* from equation (6)

$$2|S_{13}|^2 = 1$$

$$\boxed{S_{13} = 1/\sqrt{2}} \quad (9)$$

* from equation (7)

$$S_{13}(S_{11}^* - S_{12}^*) = 0$$

(or) $\boxed{S_{11} = S_{12} = S_{22}}$ — 10

* Substituting all these values in eq'n (4)

$$|S_{11}|^2 + |S_{11}|^2 + \frac{1}{2} = 1$$

$$2|S_{11}|^2 = \frac{1}{2} \quad \text{(or)} \quad \boxed{S_{11} = 1/2} \quad (11)$$

* Substituting all s-parameters in eq'n (a)

$$[S] = \begin{bmatrix} 1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & 1/2 & -1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \end{bmatrix}$$

* We know $[b] = [S][a]$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & 1/2 & -1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \quad (12)$$

$$b_1 = \frac{1}{2}a_1 + \frac{1}{2}a_2 + \frac{1}{\sqrt{2}}a_3$$

$$b_2 = \frac{1}{2}a_1 + \frac{1}{2}a_2 - \frac{1}{\sqrt{2}}a_3$$

$$b_3 = \frac{1}{\sqrt{2}}a_1 - \frac{1}{\sqrt{2}}a_2 .$$

Case (I) :-

$a_1 = a_2 = 0$, $a_3 \neq 0$ substituting in eq'n (12)

$b_1 = \frac{1}{\sqrt{2}}a_3$, $b_2 = -1/\sqrt{2}\, a_3$, $b_3 = 0$

* An input at port (3) usually divides between (1) and (2) but introduces a phase shift of 180^0 between the two outputs.
* Hence E-plane Tee also acts as a 3-dB splitter.

Case (II) :- $a_1 = a_2 = a$, $a_3 = 0$.

Substituting in eq'n (12)

$$b_1 = \frac{a}{2} + \frac{a}{2}$$

$$b_2 = \frac{a}{2} + \frac{a}{2}$$

$$b_3 = \frac{1}{\sqrt{2}}a - \frac{1}{\sqrt{2}}a = 0$$

~~Since~~ * ∴, equal inputs at port (1) and port (2) result in no o/p at port (3).

Case (III) :- $a_1 \neq 0$, $a_2 = 0$, $a_3 = 0$

$$b_1 = \frac{a_1}{2}, \quad b_2 = \frac{a_1}{2}, \quad b_3 = \frac{-a_1}{\sqrt{2}} .$$

* Similarly we can have all combinations of inputs and outputs.

→ **E-H plane (or) Hybrid (or) Magic Tee :-**

* Here rectangular slots are cut both along the width and breadth of a long waveguide and

side arms are attached as shown in figure. Port (1) and (2) are collinear arms, port (3) is the H-arm and port (4) is the E-arm. Such a device became necessary because of the difficulty of obtaining a completely matched three port Tee junction. combines the power dividing properties of both H-plane Tee and E-plane Tee as shown in figure and has the advantage of being completely matched at all its ports. This has several useful applications. using the properties of E-H plane Tee, its scattering matrix can be obtained as follows.

* [S] is a 4x4 matrix since there are 4 ports

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \quad \text{--- (1)}$$

* Because of H-plane Tee section

$$S_{23} = S_{13} \quad \text{--- (2)}$$

* Because of E-plane Tee section

$$S_{24} = -S_{14} \quad \text{--- (3)}$$

* Because of geometry of the junction an input at port (3) cannot come out of port (4) since they are isolated ports and vice versa.

$$S_{34} = S_{43} = 0 \quad \text{--- (4)}$$

* From symmetric property $S_{ij} = S_{ji}$

$$S_{12} = S_{21}, \; S_{13} = S_{31}, \; S_{23} = S_{32}$$

$S_{34} = S_{43}$, $S_{24} = S_{42}$, $S_{41} = S_{14}$

* If ports (3) and (4) are perfectly matched to the junction $S_{33} = S_{44} = 0$ — (5)

* substituting the above properties

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \quad — (6)$$

* from unitary property $[S][S]^* = [I]$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* & S_{14}^* \\ S_{12}^* & S_{22}^* & S_{13}^* & -S_{14}^* \\ S_{13}^* & S_{13}^* & 0 & 0 \\ S_{14}^* & -S_{14}^* & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

R_1C_1: $|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1$ — (7)

R_2C_2: $|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1$ — (8)

R_3C_3: $|S_{13}|^2 + |S_{13}|^2 = 1$ → (9)

R_4C_4: $|S_{14}|^2 + |S_{14}|^2 = 1$ — (10)

* from eq'n (9) and (10)

$S_{13} = 1/\sqrt{2}$ — (11)

$S_{14} = 1/\sqrt{2}$ — (12)

* Equating (7) and (8)

$$|S_{11}|^2 + \cancel{|S_{12}|^2} + \cancel{|S_{13}|^2} + \cancel{|S_{14}|^2} = \cancel{|S_{12}|^2} + |S_{22}|^2 + \cancel{|S_{13}|^2} + \cancel{|S_{14}|^2}$$

$S_{11} = S_{22}$ — (13)

* substituting all these value in equation (7)

$$|S_{11}|^2 + |S_{12}|^2 + 1/2 + 1/2 = 1$$

$$|S_{11}|^2 + |S_{12}|^2 + 1 = 1$$

$$|S_{11}|^2 + |S_{12}|^2 = 0$$

$$S_{11} = S_{12} = 0 \quad - (14)$$

* Substituting all those value in equation (8)

$$|S_{12}|^2 + |S_{22}|^2 + 1/2 + 1/2 = 1$$

$$|S_{12}|^2 + |S_{22}|^2 + 1 = 1$$

$$|S_{12}|^2 + |S_{22}|^2 = 0$$

$$S_{12} = S_{22} = 0 \quad - (15)$$

* This means ports (1) and (2) are also perfectly matched to the junction. Hence in any four port junction, if any two ports are perfectly matched to the junction, then the remaining two ports are automatically matched to the junction, such a junction where in all the four ports are perfectly matched to the junction is called a "Magic Tee."

Substituting the scattering parameters in eq'n (6)

$$[S] = \begin{bmatrix} 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 \end{bmatrix} \quad - (16)$$

we know that $[b] = [S][a]$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$$

$$\left.\begin{aligned} b_1 &= \frac{1}{\sqrt{2}}(a_3 + a_4) \\ b_2 &= \frac{1}{\sqrt{2}}(a_3 - a_4) \\ b_3 &= \frac{1}{\sqrt{2}}(a_1 + a_2) \\ b_4 &= \frac{1}{\sqrt{2}}(a_1 - a_2) \end{aligned}\right\} \quad -(17)$$

Case(i):- $a_3 \neq 0, a_1 = a_2 = a_4 = 0$

* Substituting these in eq'n (17) we get

$$b_1 = \frac{a_3}{\sqrt{2}}, \quad b_2 = \frac{a_3}{\sqrt{2}}, \quad b_3 = b_4 = 0$$

* This is the property of H-plane Tee.

Case(ii):- $a_4 \neq 0, a_1 = a_2 = a_3 = 0$

$$b_1 = \frac{a_4}{\sqrt{2}}, \quad b_2 = \frac{-a_4}{\sqrt{2}}, \quad b_3 = b_4 = 0$$

* This is the property of E-plane Tee.

Case(iii):- $a_1 \neq 0, a_2 = a_3 = a_4 = 0$

$$b_1 = 0, \quad b_2 = 0, \quad b_3 = \frac{a_1}{\sqrt{2}}, \quad b_4 = \frac{a_1}{\sqrt{2}}$$

* When power is fed into port (1), nothing comes out of port (2) even though they are collinear ports (Magic Tee). Hence ports (1) and (2) are called isolated ports. Similarly, an input at port (2) cannot come out at port (1). Similarly E and H ports are isolated ports.

Case(iv):- $a_3 = a_4$, $a_1 = a_2 = 0$

$$b_1 = \frac{1}{\sqrt{2}}(2a_3), \quad b_2 = 0, \quad b_3 = b_4 = 0$$

* This is nothing but the additive property. equal inputs at ports 3 and 4, result in an output at port 1 (in phase and equal in amplitude).

Case(v):- $a_1 = a_2$, $a_3 = a_4 = 0$

$$b_1 = 0, \quad b_2 = 0, \quad b_3 = \frac{1}{\sqrt{2}}(2a_1), \quad b_4 = 0$$

* that is equal inputs at ports 1 and 2 results in an output port 3 (additive property) and no outputs at ports 1, 2 and 4. This is similar to case(4).

→ ~~Directional Coupler~~:-

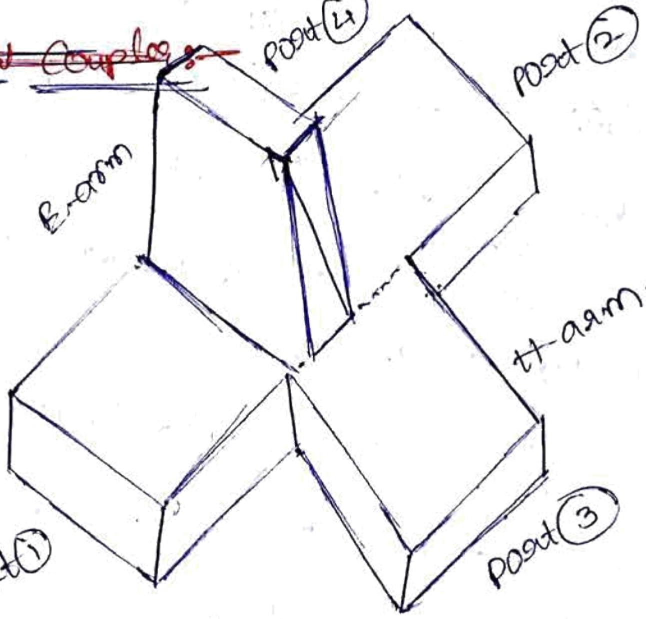


fig (a).

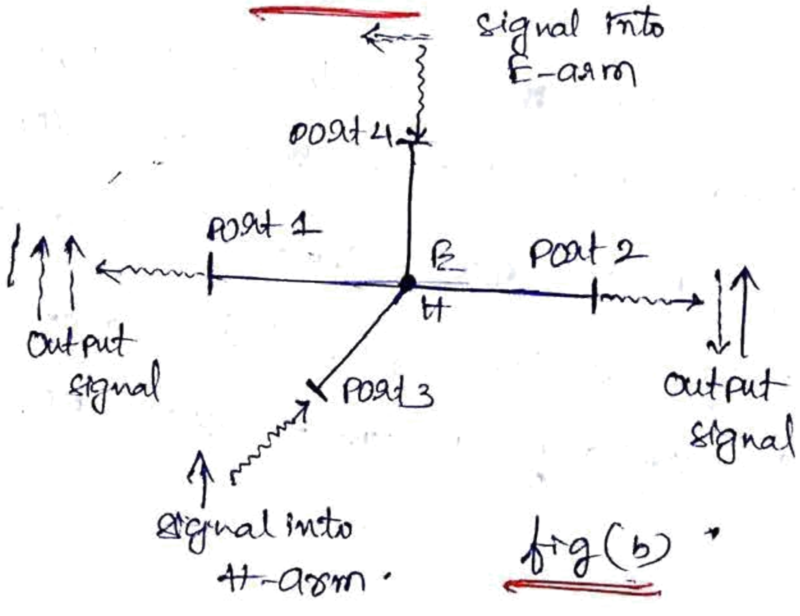


fig (b).

Directional Coupler:-

P_i = incident power at port 1

P_r = received power at port 2.

P_f = forward coupled power at port 4

P_b = back power at port 3.

* The performance of a directional coupler is usually defined in terms of two parameters which are defined as follows.

⇒ Coupling factor C:-

* The coupling factor of a directional coupler (D.C) is defined as the ratio of the incident power 'P_i' to the forward power 'P_f' measured in dB.

$$C = 10\log_{10}\left(\frac{P_i}{P_f}\right) dB \quad \text{— (1)}$$

⇒ Directivity (D) :-

* The directivity of a D.C is defined as the ratio of forward power 'P_f' to the back power 'P_b' expressed in dB.

$$D = 10\log_{10}\left(\frac{P_f}{P_b}\right) dB \quad \text{— (2)}$$

* For a typical D.C C = 20dB, D = 60dB.

* From eq'n (1)

$$20 = 10\log_{10}\left(\frac{P_i}{P_f}\right)$$

$$\frac{P_i}{P_f} = 10^2 = 100$$

$$P_f = \frac{P_i}{100}$$

* From equation (2)

$$60 = 10\log_{10}\left(\frac{P_f}{P_b}\right)$$

$$\frac{P_f}{P_b} = 10^6$$

$$P_b = \frac{P_f}{10^6} \quad \text{Substituting } P_f = \frac{P_i}{100}$$

$$= \frac{P_i}{10^8} = P_i 10^{-8}$$

* ∴ P_b is very small., power coming out of port (3) can be neglected.

* The coupling factor is a measure of how well the directional coupler distinguishes between the forward and reverse travelling powers.

→ **Isolation :-**

* Another parameter called isolation is defined as the directive properties of a directional coupler. It is defined as the ratio of the incident power P_f to the back power P_b expressed in dB.

$$I = 10 \log_{10} \frac{P_i}{P_b} \text{ dB.}$$

Isolation = coupling factor + directivity

→ **Scattering Matrix of a Directional Coupler :-**

* We use the properties of the directional coupler to arrive at the [S] matrix.

Directional coupler is a four port network. Hence [S] is a 4×4 matrix

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \quad (1)$$

* In a directional coupler all four ports are perfectly matched to the junction. Hence the diagonal elements are zero (17)

$$S_{11} = S_{22} = S_{33} = S_{44} = 0$$

* From symmetric property $S_{ij} = S_{ji}$

$$S_{12} = S_{21},\ S_{23} = S_{32},\ S_{13} = S_{31},\ S_{24} = S_{42},\ S_{34} = S_{43},\ S_{41} = S_{14}.$$

* Ideally back power is zero ($P_b = 0$), i.e., there is no coupling between port (1) and port (3)

$$S_{13} = S_{31} = 0.$$

* Also there is no coupling between port (2) and port (4)

$$S_{24} = S_{42} = 0$$

* Substituting the values of scattering parameters in equation (1) we get

$$[S] = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix} \quad \text{--- (2)}$$

* Since $[S][S]^* = I$ we get

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{12}^* & 0 & S_{14}^* \\ S_{12}^* & 0 & S_{23}^* & 0 \\ 0 & S_{23}^* & 0 & S_{34}^* \\ S_{14}^* & 0 & S_{34}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

R_1C_1:- $0 + S_{12}S_{12}^* + 0 + S_{14}S_{14}^* = 1$

$$|S_{12}|^2 + |S_{14}|^2 = 1 \quad \text{--- (3)}$$

R_2C_2:- $0 + S_{12}S_{12}^* + 0 + S_{23}S_{23}^* = 1$

$$|S_{12}|^2 + |S_{23}|^2 = 1 \quad \rightarrow (4)$$

R_3C_3:- $0 + S_{23}S_{23}^* + 0 + S_{34}S_{34}^* = 1$

$$|S_{23}|^2 + |S_{34}|^2 = 1 \quad - (5)$$

R_1C_3: $0 + S_{12}S_{23}^* + 0 + S_{14}S_{34}^* = 0$

$$S_{12}S_{23}^* + S_{14}S_{34}^* = 0 \quad - (6)$$

* equating (3) and (4)

$$\cancel{|S_{12}|^2} + |S_{14}|^2 = \cancel{|S_{12}|^2} + |S_{23}|^2$$

$$S_{14} = S_{23} \quad - (7)$$

equating (4) and (5)

$$|S_{12}|^2 + \cancel{|S_{23}|^2} = \cancel{|S_{23}|^2} + |S_{34}|^2$$

$$S_{12} = S_{34} \quad - (8)$$

* Let us assume that S_{12} is real and positive ='p'

$$S_{12} = S_{34} = S_{34}^* = P \quad - (9)$$

* From equations (6) and (9)

$$P S_{23}^* + S_{23} P = 0$$

$$P[S_{23} + S_{23}^*] = 0$$

$$P \neq 0, \quad S_{23} + S_{23}^* = 0$$

$$S_{23} = jy, \quad S_{23}^* = -jy.$$

i.e., S_{23} must be imaginary.

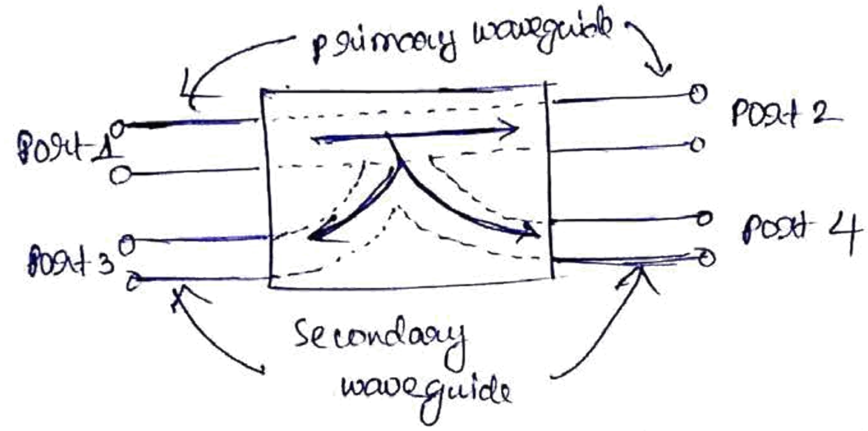
$$S_{14} = S_{23} = jq'$$

$$\therefore S_{12} = S_{34} = P$$

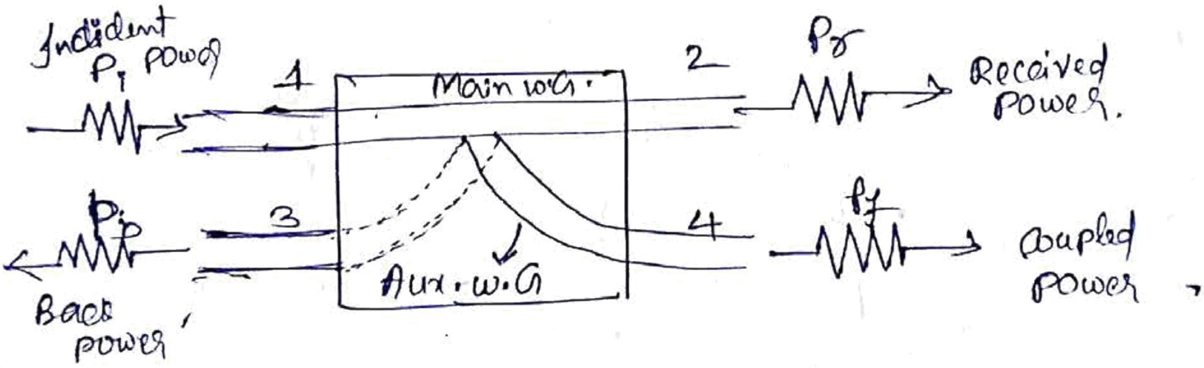
$$S_{23} = S_{14} = jq, \text{ Also } P^2 + q^2 = 1$$

* Substituting these values in eq'n (2), [S] matrix of a directional coupler is reduced to

$$[S] = \begin{bmatrix} 0 & P & 0 & jq \\ P & 0 & jq & 0 \\ 0 & jq & 0 & P \\ jq & 0 & P & 0 \end{bmatrix}.$$



fig(a) :- A schematic of a directional coupler.



fig(b) : Directional coupler, indicating powers.