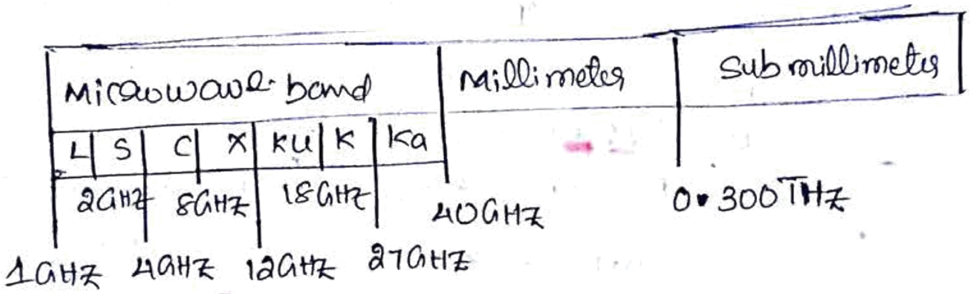
UNIT - I

* Microwave Transmission Lines - I *

⇒ Introduction :-

* Microwaves are electromagnetic waves whose frequencies range from 1 GHz to 1000 GHz (1 GHz = 10^9 Hz)

* Microwaves (MW's) are so called since they are defined in terms of their wavelength in the sense that ~~micro refers to~~ range from 30 cm (at 1 GHz) to 1 mm (300 GHz).



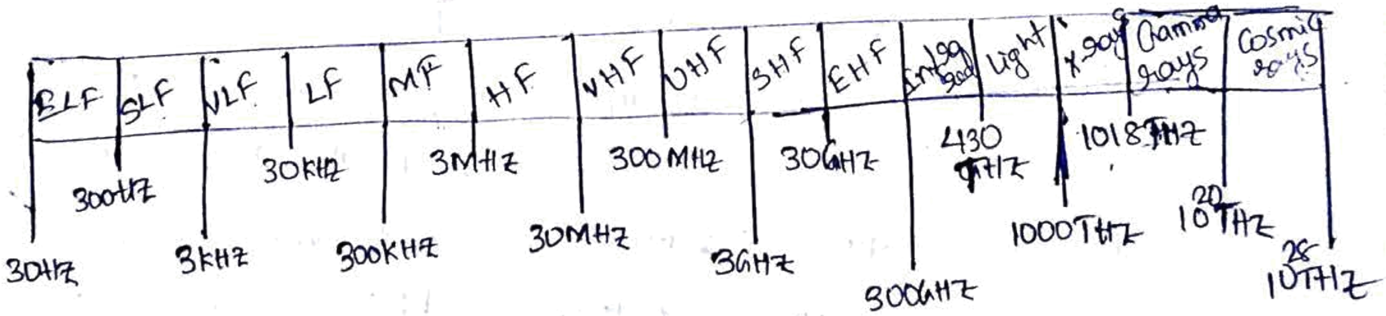


fig : Electromagnetic frequency spectrum

* figure shows the various available electromagnetic frequency spectrum.

Prefix	Power of Ten	Symbol
Exa	10^{18}	E
Peta	10^{15}	P
Tera	10^{12}	T
Giga	10^9	G
Mega	10^6	M

kilo	10^{3}	K
milli	10^{-3}	m
micro	10^{-6}	μ
nano	10^{-9}	n
pico	10^{-12}	p
femto	10^{-15}	f
atto	10^{-18}	a

Table 1:- Metric prefixes.

⇒ Microwave Region And Band Designations:-

Table 2. shows the microwave region in the electromagnetic spectrum along with other frequency ranges as per CCIR recommendations.

Table 2: CCIR Band designation

Band	Frequency Range	Band Designation
1	3HZ - 30HZ	Ultra Low frequency (ULF)
2.	30HZ - 300HZ	ELF
3.	300HZ - 3000HZ	Voice frequency (VF) Telephone / baseband fq's
4	3KHZ - 30KHZ	VLF (Very Low fq)
5	30KHZ - 300KHZ	LF (Low fq)
6	300KHZ - 3000KHZ	MF (Medium fq)
7	3MHZ - 30MHZ	HF (High fq)
8	30MHZ - 300MHZ	VHF (Very High fq)
9	300MHZ - 3000MHZ	UHF (Ultra high fq)
10	3GHZ - 30GHZ	SHF (Super high fq)
11	30GHZ - 300 GHZ	EHF (Extreme high fq)
12	300GHZ - 3000GHZ (3THZ)	Infrared light
13	3THZ - 30THZ.	Infrared light
14	30THZ - 300THZ	Infrared light

15	300THz - 3PHz	Visible light
16.	3PHz - 30PHz	Ultra voiled light
17.	30PHz - 300PHz	X-rays
18	300PHz - 3EHz	Gamma rays
19.	3EHz - 30EHz	Cosmic rays

* standard band designations for microwave frequen-cies listed as per Institute of Electrical and Electronic Engineering (IEEE) is the industry standard

Table 3: IEEE/ Industry standards

Band Designation		Frequency Range (GHz)
UHF		0.3 - 3.0
L		1.1 - 1.7
LS		1.7 - 2.6
S		2.6 - 3.9
C		3.9 - 8.0
X		8.0 - 12.5
Ku		12.5 - 18.0
K		18.0 - 26
Ka		26 - 40
Q		33 - 50
U		40 - 60
M		50 - 75
E	Milli	60 - 90
F	meter	90 - 140
G		140 - 220
R.		220 - 300
Sub-millimeter		>330

→ Advantages of Microwaves :—

* There are some unique advantages of microwaves over low frequencies.

1). Increased Bandwidth availability :—

* Microwaves have large bandwidths (1GHz - 103GHz) compared to the common bands namely MW, SW and VHF waves. The advantage of large bandwidths is that the frequency range of information channels will be a small percentage of the carrier frequency and more information can be transmitted in microwave frequency range. Microwave region is very useful since the lower band of frequency is already crowded.

* Infact, microwave region (1000 GHz) contains thousand sections of the frequency band 0-10^9 Hz and hence any one of these thousand sections may be used to transmit all the TV, radio and other communications that is presently transmitted by the 0-10^9 Hz bands. (Bandwidths of speech = 4KHz ; Music = 10 - 15 KHz, T.V. = 5-7 MHz ; Telegraph channel = 120 - 240 Hz).
i.e., greater bandwidth provides more room for stuff to be packed into the transmission.

* ~~The~~ Microwave are used in various long distance communication applications such as Telephone networks, TV network, Space communication, Telemetry, Defense, Railways etc. FM and digital modulation schemes also required higher bandwidths.

2) Improved directive properties :-

* As frequency increases, directivity increases and beam width decreases. Hence the beamwidth of radiation θ is proportional to λ/D.

* At low frequency bands, the size (diameter) of the antenna becomes very large if it is required to get sharp beams of radiation.

* However, at microwave frequencies, antenna size of several wavelength lead to smaller beamwidths and an extremely directed beam, just the same way as an optical lens focusses light rays. Therefore microwave frequencies are said to possess quasi-optical properties.

* For example we know that for a parabolic antenna

$$B = \frac{140^\circ}{(D/\lambda)}$$

* where D = diameter of antenna in cm

λ = wavelength in cm.

B = beamwidth in degrees.

* At 30GHz ($\lambda = 1cm$) for 1° beam width.

$$D = \frac{140}{B} \times \lambda = \frac{140}{1} \times 1 = 140cm.$$

* At 300MHz ($\lambda = 100cm$) for 1° beam width

$$D = \frac{140}{1} \times 100 = 14000 \ cm.$$

* From above example, it is clear that antenna size is small for microwave frequencies.

Power radiated is given by $\eta_0 \pi I_0^2 \left(\frac{l}{\lambda}\right)^2$

where l = length, I_0 = ac current carried.

* As frequency increases, λ decreases hence power radiated and gain increases.

* As gain (Power) is inversely proportional to λ^2, high gain is achievable at microwave frequencies i.e., high gain and directive antennas can be designed and fabricated more easily at microwave frequencies, which is highly impracticable at lower frequency bands.

3). Fading effect and reliability :— Fading effect due to variation in the transmission medium is more effective at low frequency. Due to "Line of sight (LOS)" propagation and high frequencies there is less fading effect and hence microwave communication is more reliable.

4) Power requirements :— Transmitter / receiver power requirements are ~~pretty~~ low at microwave frequencies compared to that at short wave band.

5) Transparency Property of microwave :— Microwave frequency band ranging from 300 MHz - 10 GHz are capable of freely propagating through the ionized layers surrounding the earth as well as through the atmosphere. The presence of such a transparent window in a microwave band ~~facilities~~ facilitates the study of microwave radiation from the sun and stars in radio astronomical research of space.

* It also makes possible for duplex communication

and exchange of information between ground stations and space vehicles. (4)

⇒ Applications of Microwaves:—

* Microwaves are used in long-distance communication systems, radars, radio astronomy, navigation etc., Broadly, the applications can be in the areas listed below.

1. Telecommunication:— International Telephone and T.V., space communication (earth-to-space and space-to-earth), telemetry, communication link for railways etc.,

2. Radars:— Detect aircraft, tracking/guided super-sonic missiles, observe and track weather patterns, air traffic control (ATC), burglar alarms, garage door openers, police speed detectors etc.,

3) Commercial and Industrial applications use heat property of microwaves.

(i) Microwave oven (2.45GHz, 600W)

(ii) Drying machine - textile, food and paper industry for drying clothes, potato chips, printed matter etc.,

(iii) Food processing industry:— Precooling/cooking, pasteurizing/sterility, heat frozen/refrigerated precooked meats, roasting of food grains/beans.

(iv) Rubber industry/plastics/chemical/forest product industries.

(v) Mining/public works, breaking rock, tunnel boring, drying/breaking up concrete, breaking up coal seams, curing of cement.

(vi) Drying inks, drying/sterilising grains, drying/sterilising pharmaceuticals, drying textiles, leather, tobacco, power transmission.

(vii) Biomedical applications (diagnostic/therapeutic) - diathermy for localized superficial heating, deep electromagnetic heating for treatment of cancer, hyper-thermia (local, regional (or) whole body for cancer therapy), electromagnetic transmission through human body has been used for monitoring of heart beat lung water detection etc.

4) Electronic warfare :- ECM/ECCM (Electronic Counter measure / Electronic counter counter measure) systems spread spectrum systems.

5) Identifying objects (or) ~~personals~~ personnel by non-contact method.

6) Light generated charge carriers in a microwave semi-conductor makes it possible to create a whole new world of microwave devices, fast ~~#~~ jitter - free switches, phase shifters, HF generation, tuning elements etc.,

⇒ Wave guides :- (Single Line) :-

* At high frequencies higher than 8 GHz, transmission of electromagnetic waves along transmission lines and cables becomes difficult mainly due to the losses that occur both in the solid dielectric needed to support the conductor and in the conductors themselves. A metallic

tube can be used to transmit electromagnetic wave at these frequencies.

* A hollow metallic tube of uniform cross-section for transmitting electromagnetic waves by successive reflections from the inner walls of the tube is called a "waveguide"

* They are used in UHF and microwave regions as an alternative to transmission lines. It may be noted that, no TEM. wave can exist in a waveguide but TE and TM. waves exist. Induced currents in the walls of the waveguide give rise to power losses and to minimize these losses, the waveguide wall resistance is made as low as possible. Hence the inner surface of the waveguide is usually coated with either gold (or) silver to improve the conductivity and to minimise losses inside the waveguide because of roughness.

* The waveguides are usually filled with air. They are superior to the coaxial cables at UHF and higher frequencies. can handle greater power and possess less resistance.

→ Comparison of waveguide with 2-wire Transmission Lines:-

1) waves travelling in a waveguide has a phase velocity and will be attenuated as in a transmission lines.

2) when the wave reaches the end of the waveguide. It is reflected unless the load impedance is adjusted to absorb the wave.

3) Any irregularity in a waveguide produces reflection just like an irregularity in a transmission line.

4) Reflected wave can be eliminated by proper impedance

matching as in a transmission line.

5) When both incident and reflected waves are present in a waveguide, a standing wave pattern results as in a transmission line.

→ **Dissimilarities:-**

1) There is a cutoff value for the frequency of transmission (f) ~~the~~ depending upon the dimensions and the shape of the waveguide. Only the waves having frequencies greater than cutoff frequency 'f_c' will be propagated. Hence the waveguide acts as a highpass filter with 'f_c' as the cutoff frequency.

In a 2-wire lossless transmission line all the frequencies can pass through.

2) Waveguide is a one conductor transmission system. The whole body of waveguide acts as ground and the wave propagates through multiple reflections from the walls of waveguide.

3) The velocity of propagation of the waves inside the waveguide is quite different from that through free space due to multiple reflections from the walls of the waveguide.

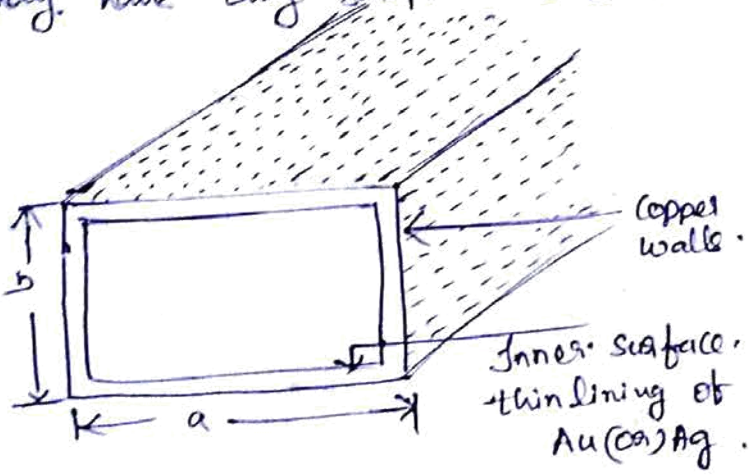
4) In a waveguide, we define what is called as the wave impedance (a function of frequency) which is analogous to the characteristic impedance Z_0 of 2-wire transmission system.

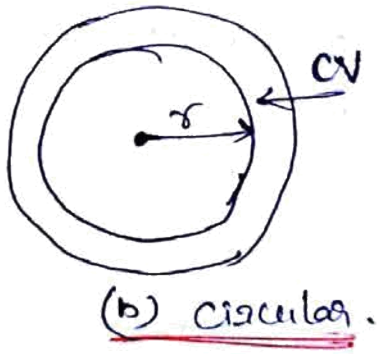
5) The system of propagation in waveguide is in accordance with 'Field theory' while that in transmission line is in accordance with 'circuit theory' and hence return conductor is not required in waveguide.

6) If one end of the waveguide is closed using a shorting plate, there will be reflections and hence standing waves will be formed. If the other end is also closed, then the hollow box so formed can support a signal which can bounce back and forth between the two shorting plates resulting in resonance. This is the principle of cavity resonators.

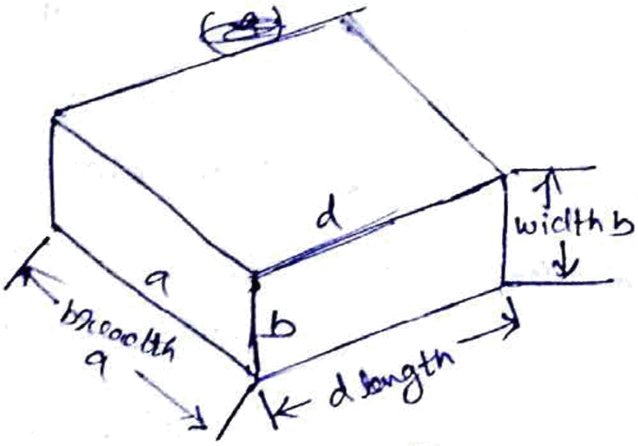
→ Types of waveguides :-

* Any shape of cross-section of a waveguide can support electromagnetic waves. But since irregular shapes are difficult to analyze and are rarely used, rectangular and circular waveguide have become more common. waveguide cross-section may have any shape as shown in fig. (1).



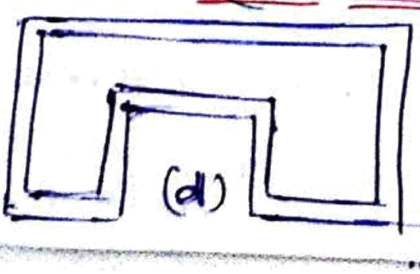


(b) Circular.

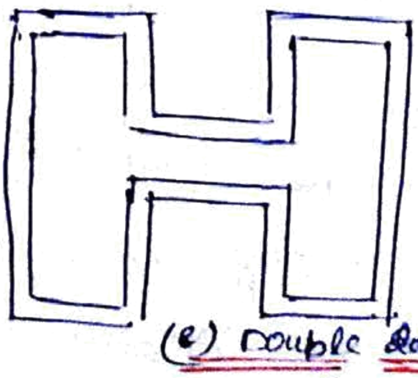


(a) Rectangular

(c). Eliptical



(d) Single Ridged



(e) Double Ridged.

* Rectangular waveguide (fig 2(a)) is most common. Circular waveguide tends to twist the waves as these travel through them. However circular waveguides (fig (2b)) are used with rotating antenna as in radars.

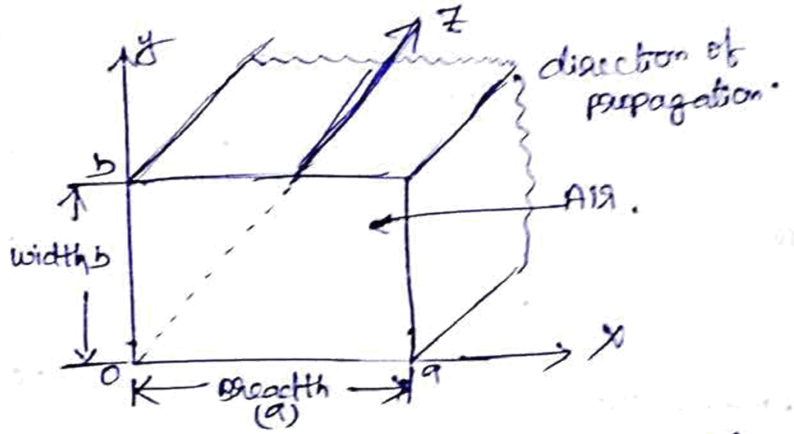
* Eliptical shape is preferred in flexible waveguides. (fig (2c)). Flexible waveguides will be required whenever the waveguide section should be cable of movement, i.e. bending, stretching (or) twisting. This must not cause the deterioration ~~of~~ in the ~~waveguide~~ performance of the waveguide. A copper tube having an elliptical cross-section is a good example of a flexible waveguide. These have smaller transverse corrugations and transitions to rectangular waveguide at the ends, which helps transform a TE_{11} mode in the flexible waveguide into the TE_{10} modes at either end. Joints and bends are not ~~difficult~~ required here. It has a polythene outer cover that bends quite easily but ~~too~~ twisting is difficult. Flexible waveguides have comparable power handling capability, attenuation and SWR as those of rectangular waveguides.

* Ridging is a convenient method of reducing the the waveguide dimensions and thereby increasing the critical wavelength. However, the presence of ridge has the disadvantage of increased attenuation, reduced power handling capacity and introducing distortions. Single and double ridge waveguides have been shown in fig. (2d), (2e). Also the useful frequency range of the waveguide is increased by ridging. It also helps in reducing the phase velocity. As the disadvantages outweigh the advantages, they are not used for standard applications.

Propagation of waves in Rectangular waveguides

* Consider a rectangular waveguide situated in the rectangular coordinate system with its breadth along x-axis, width along y-axis and the wave is assumed to propagate along the z-direction.

Waveguide is filled with air as dielectric as shown in fig (3).



fig(3):- Propagation through a rectangular waveguide.

* The wave equation for TE and TM waves are given by.

$$\nabla^2 H_z = -\omega^2 \mu \varepsilon H_z \quad \text{for TE wave } (E_z = 0)$$

$$\nabla^2 E_z = -\omega^2 \mu \varepsilon E_z \quad \text{for TM wave } (H_z = 0).$$

* Expanding $\nabla^2 E_z$ in rectangular coordinate system.

$$\boxed{\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} = -\omega^2 \mu \varepsilon E_z} \quad -(1)$$

* Since the wave is propagating in the 'z' direction we have the operator

$$\frac{\partial^2}{\partial z^2} = \gamma^2$$

* Substituting this operator in eq'n (1) we get

$$\boxed{\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \gamma^2 E_z = -\omega^2 \mu \varepsilon E_z} \quad -(2)$$

(or)

$$\boxed{\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + (\gamma^2 + \omega^2 \mu \varepsilon) E_z = 0} \quad -(3)$$

* Let $\gamma^2 + \omega^2\mu\varepsilon = h^2$, be a constant, then eq'n (3) can be rewritten as

$$\boxed{\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + (h^2 E_z) = 0} \text{ for TM wave} \quad -(4)$$

* Similarly, $\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \frac{\partial^2 H_z}{\partial z^2} = -\omega^2\mu\varepsilon H_z$.

$$\Rightarrow \boxed{\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \cancel{\frac{\partial^2 H_z}{\partial z^2}} + h^2 H_z = 0} \text{ for TM wave} \quad -(5)$$

* By solving the above partial differential equations, we get solutions for E_z and H_z using maxwell's equation, it is possible to find the various components along x and y directions $[E_x, E_y, H_x, H_y]$

* From Maxwell's 1'st equation, we have

$$\nabla \times H = j\omega\varepsilon E.$$

* Expanding $\nabla \times H$

i.e.,
$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = j\omega\varepsilon[\hat{i}E_x + \hat{j}E_y + \hat{k}E_z]$$

* replacing $\frac{\partial}{\partial z} = -\gamma$ (an operator), we get

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & -\gamma \\ H_x & H_y & H_z \end{vmatrix} = j\omega\varepsilon[\hat{i}E_x + \hat{j}E_y + \hat{k}E_z]$$

* Equating coefficients of $\hat{i}, \hat{j}, \hat{k}$ (after expanding), we get.

$$\frac{\partial H_z}{\partial y} + \gamma H_y = j\omega\varepsilon E_x \quad -(6)$$

$$\frac{\partial H_z}{\partial x} + \gamma H_x = -j\omega\varepsilon E_y \quad -(7)$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z \quad -(8)$$

* Similarly, Maxwell's 2^{nd} equation ; we have.

$$\nabla \times E = -j\omega\mu H$$

* Expanding $\nabla \times E$, we get

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & -\gamma \\ E_x & E_y & E_z \end{vmatrix} = -j\omega\mu\left(\hat{i} H_x + \hat{j} H_y + \hat{k} H_z\right)$$

* Expanding and equating coefficients of $\hat{i}$, $\hat{j}$ and $\hat{k}$

we get

$$\boxed{\frac{\partial E_z}{\partial y} + \gamma E_y = -j\omega\mu H_x} \quad (9)$$

$$\boxed{\frac{\partial E_z}{\partial x} + \gamma E_x = +j\omega\mu H_y} \quad - (10)$$

$$\boxed{\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z} \quad - (11)$$

* Combining eq'n. (6) and eq'n. (10) to eliminate H_y we get.

$$H_y = \frac{+1}{j\omega\mu}\frac{\partial E_z}{\partial x} + \frac{\gamma}{j\omega\mu}E_x .$$

* Substituting for H_y in eq'n (6) we get

$$\frac{\partial H_z}{\partial y} + \frac{\gamma}{j\omega\mu}\frac{\partial E_z}{\partial x} + \frac{\gamma^2}{j\omega\mu}E_x = j\omega\varepsilon E_x .$$

(or) $$E_x\left[j\omega\varepsilon - \frac{\gamma^2}{j\omega\mu}\right] = \frac{\gamma}{j\omega\mu}\frac{\partial E_z}{\partial x} + \frac{\partial H_z}{\partial y}$$

* Multiplying by $j\omega\mu$, we get.

$$E_x\left[-\omega^2\mu\varepsilon - \gamma^2\right] = \gamma\frac{\partial E_z}{\partial x} + j\omega\mu\frac{\partial H_z}{\partial y}$$

$$E_x\left[-(\gamma^2 + \omega^2\mu\varepsilon)\right] = \gamma\frac{\partial E_z}{\partial x} + j\omega\mu\frac{\partial H_z}{\partial y}$$

* where $\gamma^2 + \omega^2\mu\varepsilon = h^2$

* Dividing by $-h^2$ throughout, we get

$$\boxed{E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}} \quad - (12)$$

Similarly.

$$\boxed{\begin{aligned} E_y &= \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x} \quad (13)\\ \text{and } H_x &= \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x} + \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y} \quad (14)\\ \text{and } H_y &= \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x} \quad (15)\end{aligned}}$$

* These, equations give a general relationship for field components within a waveguide.

→ **Propagation of TEM waves:—**

* we know for a TEM wave

$$E_z = 0 \text{ and } H_z = 0.$$

* Substituting these values in eq'n (12) to (15) all the fields components along x and y directions E_x, E_y, H_x, H_y, vanish and hence a TEM wave cannot exist inside a waveguide.

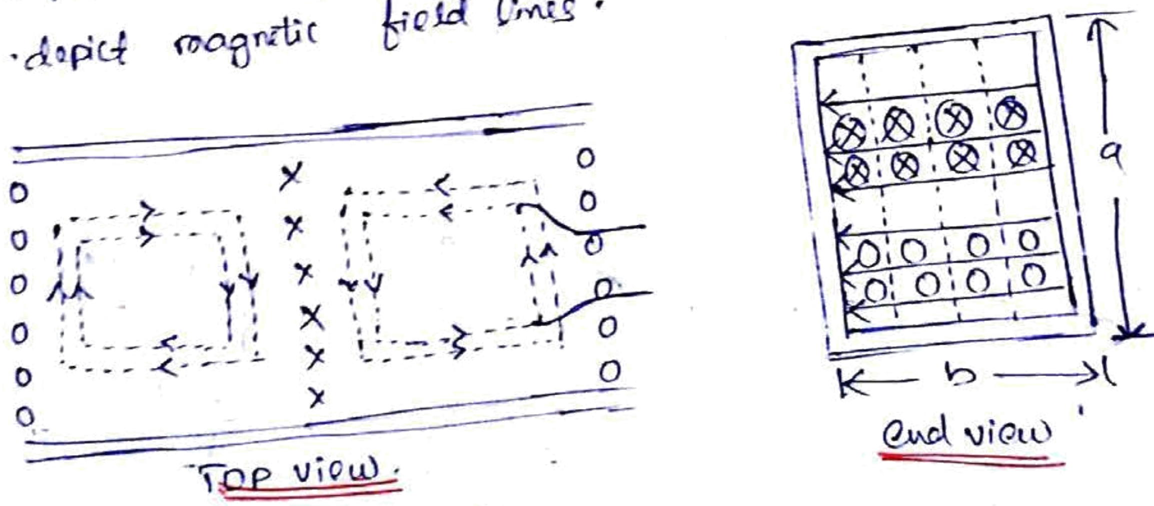
⇒ **TE and TM modes:—**

* The electromagnetic wave inside a waveguide can have an infinite number of patterns which are called modes. we know that an electromagnetic wave consists of magnetic and electric fields which are always perpendicular to each other. The fields in the waveguide which make up these mode patterns must obey certain physical laws. At the surface of a conductor the electric field cannot have a component parallel to the surface. This indicates that the electric field must always be perpendicular to the surface at a conductor. The magnetic field on the other hand is always parallel to the surface of the conductor and cannot have a component perpendicular to it at the surface.

* In general there are two kinds of modes in a waveguide. In the first type, the electric field is always transverse to the direction of propagation and is called the Transverse electric (or) TE wave. In the second type the magnetic field is always transverse to the direction of propagation and is called the Transverse magnetic (or) TM wave. Thus in a TE mode, no electric line is in direction of propagation i.e., $E_z = 0$, if 'z' is the direction of propagation. But $H_z \neq 0$. In a TM mode, no magnetic line is in direction of propagation i.e., $H_z = 0$ but $E_z \neq 0$

→ Field Patterns:—

* Figure(4) shows the field patterns for a TE wave. Solid lines depict electric field lines (or) voltage lines and dotted lines depict magnetic field lines.



Fig(4): Field pattern of TE wave.

* We use subscript for designating a particular mode TE_{mn} (or) TM_{mn} where 'm' is the number of half wave ~~loops~~ variations of the electric (or magnetic) field across the wider dimension 'a' of the wave guide and 'n' indicates the number across the narrow dimensions 'b'. Referring to TE pattern shown in fig(4) it can be seen that the ~~variations~~ voltage varies from '0' to maximum and maximum to '0' across

wide dimension 'a'. This is half variation. Hence m=1. Across the narrow dimension there is no variation in voltage 'v'. Hence n=0. Therefore this mode is TE_{10} mode.

The mode having the highest cutoff wavelength is known as dominant mode of the waveguide and all other modes are called higher modes. For example TE_{10} is the dominant mode for TE waves. It is the mode which is used for practically all electromagnetic transmission in a rectangular waveguide.

*Dominant mode is ~~a~~ almost always a low-loss, distortion less transmission and higher modes results in a significant loss of power and also undesirable harmonic ~~distro~~ distortion.

*The radiation pattern for TE mode is shown in fig(5). Sketches of some higher order TE mode are shown in fig 6)

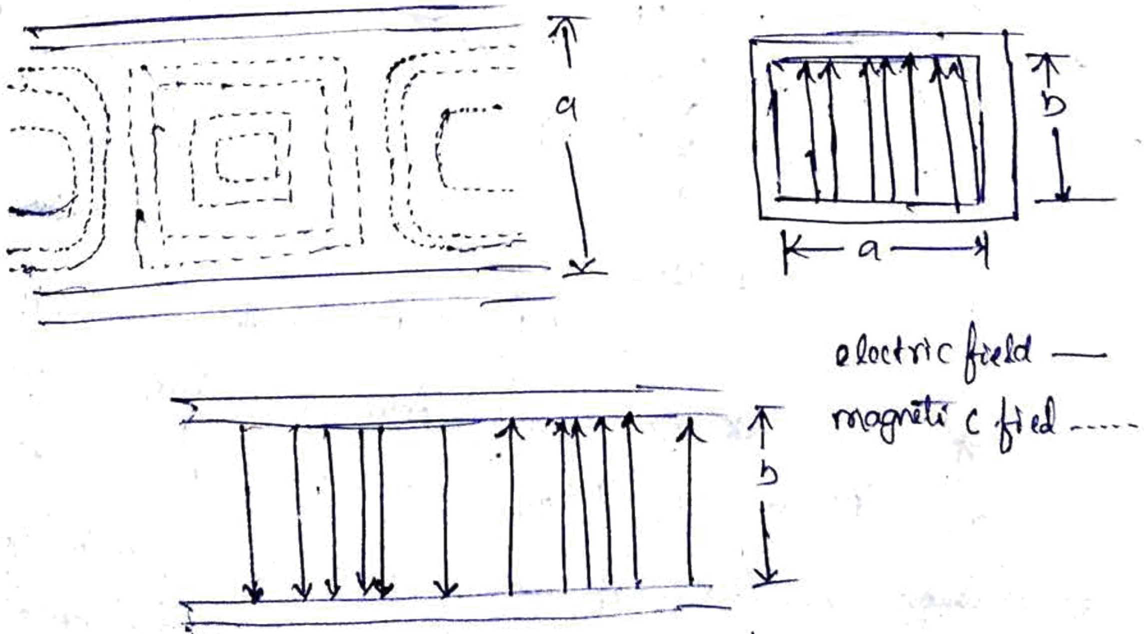
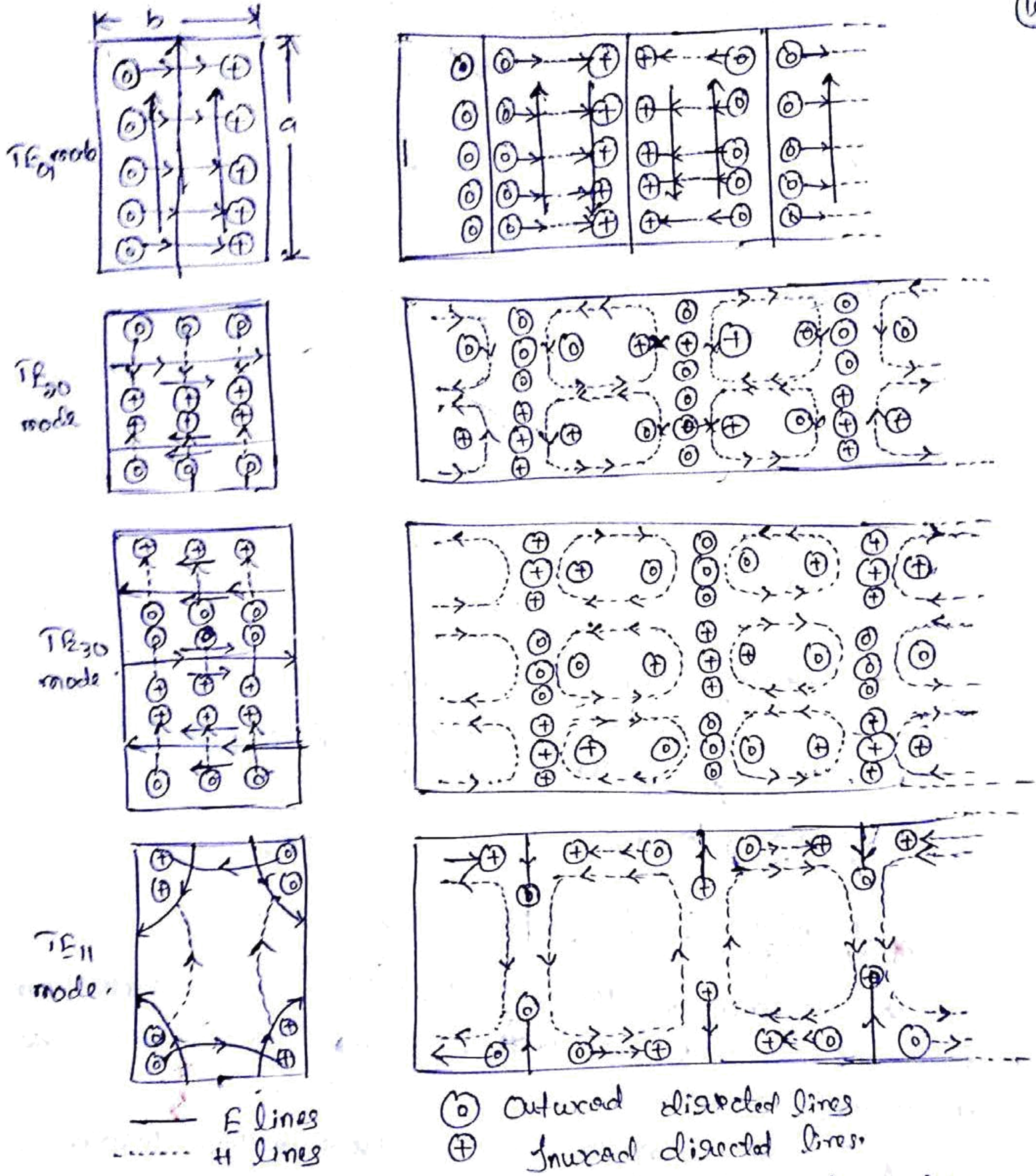


fig.(5) Radiation Pattern for TE_{10} mode.



fig(6):- Field pattern of higher order modes.

→ Propagation of TM waves in Rectangular wave guide:-

* For TM wave, $H_z = 0$; $E_z \neq 0$
* The wave equation of a TM wave is

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + h^2 E_z = 0 \quad \text{—} (1)$$

* This is a partial differential equation which can be solved to get the different field components E_x, E_y, H_x and H_y

by 'separation of variables method':

Let us assume a solution

$$E_z = XY \quad -(2)$$

* where, X is a pure function of 'x' only
Y is a pure function of 'y' only.

* Since x and y are independent variables.

$$\frac{\partial^2 E_z}{\partial x^2} = \frac{\partial^2 (XY)}{\partial x^2} = Y \cdot \frac{\partial^2 X}{\partial x^2}$$

$$\frac{\partial^2 E_z}{\partial y^2} = \frac{\partial^2 (XY)}{\partial y^2} = X \cdot \frac{\partial^2 Y}{\partial y^2}$$

* Using these two in eq'n (1) we get

$$\boxed{Y\frac{d^2X}{dx^2} + X\frac{d^2Y}{dy^2} + h^2XY = 0} \quad -(3)$$

* Dividing throughout by XY, we get

$$\boxed{\frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} + h^2 = 0} \quad -(4)$$

* $\frac{1}{X}\frac{d^2X}{dx^2}$ is a pure function of x only.

* $\frac{1}{Y}\frac{d^2Y}{dy^2}$ is a pure function of y only.

* The sum of these is a constant. Hence each term must be equal to a constant separately since 'x' and 'y' are independent variables.

* We use separation of variables method to solve the differential eq'n (4)

* Let $$\frac{1}{X}\frac{d^2X}{dx^2} = -B^2 \quad -(5)$$

* and $$\frac{1}{Y}\frac{d^2Y}{dy^2} = -A^2 \quad -(6)$$

* where $-A^2$ and $-B^2$ are constants

* Substituting eq'ns (5) and (6) in eq'n (4) we get

$$-B^2 - A^2 + h^2 = 0$$

(or) $$h^2 = A^2 + B^2 \quad -(7)$$

* equations (5) and (6) are ordinary. 2^{nd} order. differential (11) equations, the solutions of which are. given by

$$\boxed{X = C_1 \cos Bx + C_2 \sin By} \quad -(8)$$

$$\boxed{Y = C_3 \cos Ay + C_4 \sin Ay} \quad -(9)$$

* where C_1, C_2, C_3 and C_4 are constants which can be. evaluated by applying the boundary conditions.

* The complete solution is given by eq'n (2)

i.e.,) $E_z = X \cdot Y$.

* Substituting the values of X and Y from eq'n (8) and (9).

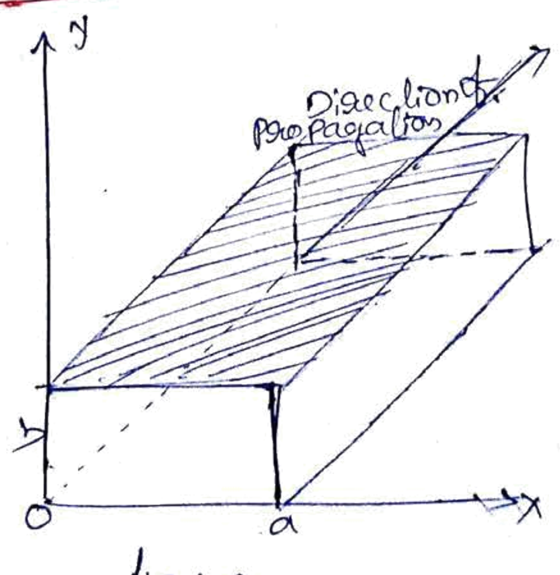
we get

$$\boxed{E_z = \left[C_1 \cos Bx + C_2 \sin Bx\right]\left[C_3 \cos Ay + C_4 \sin Ay\right]} \quad -(10)$$

→ Boundary conditions :—

Since the entire surface. of the rectangular waveguide. acts as a short circuit (or) ground for electric. field, $E_z = 0$ all along the ~~surface~~ boundary walls of the waveguide. Since there are four walls, as shown in fig (7) there are four boundary conditions.

1'st Boundary Condition :— [Bottom Plane (or) bottom wall] :—



fig(7) :—

* we know that $E_z = 0$, all along the Bottom wall.

i.e., $E_z = 0$ at $y = 0$ $\forall x \to 0$ to a

$\forall$ stands "for all" and $x \to a$ means x ~~vary~~ varying between 0 to 'a'.

→ **2nd Boundary condition:—** (left side plane (or) left side wall)

$E_z = 0$ at $x = 0$ $\forall$ $y \to 0$ to b.

→ **3rd boundary condition:—** (Top plane (or) top wall)

$E_z = 0$, at $y = b$ $\forall$ $x \to 0$ to a.

→ **4th boundary condition:—** (Right side wall (or) plane).

$E_z = 0$ at $x = a$ $\forall$ $y \to 0$ to b.

* (i) substituting 1st boundary condition in eq'n (10) given by,

$$E_z = [C_1 \cos Bx + C_2 \sin Bx][C_3 \cos Ay + C_4 \sin Ay]$$

* we have, $E_z = 0$ at $y = 0$ $\forall$ $x \to 0$ to a.

(or) $0 = [C_1 \cos Bx + C_2 \sin Bx] C_3$ $(\because \cos 0 = 1, \sin 0 = 0)$

* This is true for all ($\forall$) $x \to 0$ to a

$C_1 \cos Bx + C_2 \sin Bx \neq 0$; $\therefore C_3 = 0$

* Using this in eq (10), the solution reduces to

$$\boxed{E_z = [C_1 \cos Bx + C_2 \sin Bx][C_4 \sin Ay]} \quad \text{--- (11)}$$

* (ii) Substituting 2nd boundary condition in eq'n (11) we get

$E_z = 0 = C_1 C_4 \sin Ay$ $\forall$ $y \to 0$ to b $[\because \cos 0 = 1$ and $\sin 0 = 0]$

since $\sin Ay \neq 0$ and $C_4 \neq 0$

$C_1 = 0$

Now using this in eq'n (11) the solution further reduce to

$$\boxed{E_z = C_2 C_4 \sin Bx \sin Ay} \quad \text{--- (12)}$$

(iii) substituting 3rd boundary condition in eq'n (12) we get.

$E_z = 0 = C_2 C_4 \sin Bx \sin Ab$ [at $y = b$, $\forall$ $x \to 0$ to a]

since $\sin Bx \neq 0$, $C_4 \neq 0$, $C_2 \neq 0$ Otherwise there would be no solution.

$\sin Ab = 0$

(or) $Ab =$ a multiple of $\pi = n\pi$.

where 'n' is a constant, $n = 0, 1, 2, \ldots$

$A =$

$$\boxed{A = \frac{n\pi}{b}} \quad -(13)$$

(iv) substituting 4th boundary condition in eq'n (12)

$$E_z = 0 = C_2 C_4 \sin Ba \sin Ay \quad [\text{at } x = a,\ y \to 0 \text{ to } b]$$

since $\sin Ay \neq 0,\ C_4 \neq 0,\ C_2 \neq 0$

$$\sin Ba = 0$$

(or) $Ba = m\pi$

where $m = 0, 1, 2, \ldots$

$$\boxed{B = \frac{m\pi}{a}} \quad -(14)$$

* Now the complete solution is given by eq'n (12)

$$E_z = C_2 C_4 \sin Bx \sin Ay$$

* where A and B are as in eq'n (13) and (14)

i.e., $E_z = C_2 C_4 \sin\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y \cdot e^{-\gamma z} e^{j\omega t}$

* where $e^{-\gamma z}$ = propagation along 'z' direction

$e^{j\omega t}$ = sinusoidal variation w.r.to 't'

* Let $C = C_2 C_4$, some other constant

$$\therefore \boxed{E_z = C \sin\left(m\frac{\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y\, e^{j\omega t - \gamma z}} \quad -(15)$$

* Since E_z is known E_x, E_y, H_x and H_y are given by the following equations ~~(from eq'n~~

$$E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$

$$E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} \quad (\text{as for TM wave } H_z = 0)$$

$$\therefore \boxed{E_x = \frac{-\gamma}{h^2} C\left(\frac{m\pi}{a}\right)\cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y\, e^{j\omega t - \gamma z}} \quad -(16)$$

and

$$E_y = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$

$$\boxed{E_y = \frac{-\gamma}{h^2} C\left(\frac{n\pi}{b}\right)\sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y\, e^{j\omega t - \gamma z}} \quad -(17)$$

$$H_x = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x} + \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y}$$

$$Hx = \frac{\cancel{\gamma}}{h^2}$$

$$\therefore \boxed{Hx = \frac{j\omega\varepsilon}{h^2} C\left(\frac{n\pi}{b}\right) \sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y \, e^{j\omega t - \gamma z}} \quad (18)$$

$$Hy = \frac{-\gamma}{h^2} \frac{\partial Hz}{\partial y} \mp \frac{j\omega\varepsilon}{h^2} \frac{\partial Ez}{\partial x}$$

$$\therefore \boxed{Hy = \frac{+j\omega\varepsilon}{h^2} C\left(\frac{m\pi}{a}\right) \cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y \, e^{j\omega t - \gamma z}} \quad -(19)$$

→ TM modes in Rectangular waveguides:-

* depending on the values of 'm' and 'n', we have various mode in TM waves. In general, we represent the modes as TM_{mn} ~~where m and n are defined as~~

→ Various TM_{mn} modes:-

→ TM_{00} mode: m=0 and n=0

* If m=0 and n=0 are sustained in Ex, Ey, Hx and Hy all of them vanish and hence TM_{00} mode cannot exist.

→ TM_{01} mode: m=0 and n=1.

* again, all field components vanish hence TM_{01} mode cannot exist.

→ TM_{10} mode: m=1 and n=0

→ all field components vanish and hence TM_{10}

* Even now, all field components

mode cannot exist.

→ TM_{11} mode: m=1 and n=1

* Now we have all the four components Ex, Ey, Hx and Hy i.e., TM_{11} mode exist and for all higher value of m and n, the components exist i.e., all higher modes do exist.

→ Cutoff frequency of a waveguide (waveguide as a Highpass filter):-

From equations (13) and (14) we know that.

$$h^2 = \gamma^2 + \omega^2\mu\varepsilon = A^2 + B^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

i.e., $\gamma^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon$

(or) $\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon} = \alpha + j\beta$.

* At lower frequencies.

$$\omega^2\mu\varepsilon < \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

* γ then becomes real and positive and equal to the attenuation constant 'α' i.e., the wave is completely attenuated and there is no phase change. Hence the wave cannot propagate.

* However, at higher frequencies,

$$\omega^2\mu\varepsilon > \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

* γ becomes imaginary. there will be phase change 'β' and hence the wave propagates. At the transition, γ becomes zero and the propagation just starts. The frequency at which 'γ' just becomes zero is defined as the cutoff frequency (or threshold frequency) 'f_c'.

* i.e., At $f = f_c$, $\gamma = 0$ (or) $\omega = 2\pi f = 2\pi f_c = \omega_c$

$$0 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega_c^2\mu\varepsilon$$

(or) $$\omega_c = \frac{1}{\sqrt{\mu\varepsilon}}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^{1/2}$$

(or) $$f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^{1/2}$$

(or) $$f_c = \frac{c}{2\pi}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^{1/2} \quad \left(\because c = \frac{1}{\sqrt{\mu\varepsilon}}\right)$$

$$\therefore \boxed{f_c = \frac{c}{2}\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]^{1/2}}$$

* The cutoff wavelength (λ_c) is

$$\lambda_c = \frac{c}{f_c} = \frac{c}{\frac{c}{2}\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]^{1/2}}$$

(or) $\boxed{\lambda_{c_{m,n}} = \frac{2ab}{\sqrt{m^2b^2+n^2a^2}}}$

* All the wavelengths greater than λ_c are attenuated and those less than 'λ_c' are allowed to propagate inside the waveguide.

→ <u>Guide wavelength</u>, <u>Group and phase Velocity</u> :-

* Here we define the guide wavelength, group velocity and phase velocity relevant for transmission of a wave in a wave guide.

<u>Guide wavelength (λ_g)</u> :-

* It is defined as the distance travelled by the wave in order to undergo a phase shift of 2π radians.

* This is shown by fig(8).

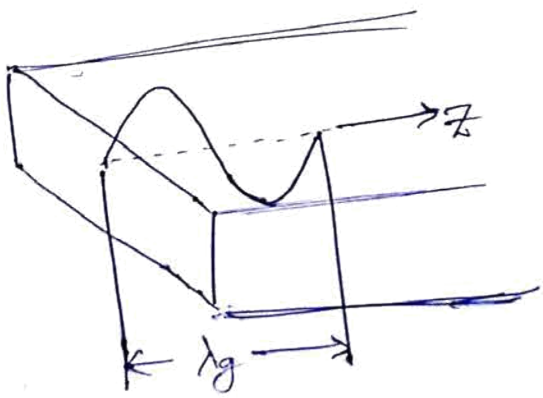


fig (8) :-

* It is related to the phase constant by the relation

$$\lambda_g = \frac{2\pi}{\beta}.$$

* The wavelength in the waveguide is different from the wavelength in free space. In fact, it is related to the free space wavelength 'λ_0' and cutoff wavelength λ_c by

$$\boxed{\frac{1}{\lambda_g^2} = \frac{1}{\lambda_0^2} - \frac{1}{\lambda_c^2}}$$ (to be proved) in

(or) $$\boxed{\lambda_g = \frac{\lambda_0}{\sqrt{1-(\lambda_0/\lambda_c)^2}}}$$

* This equation is true for any mode. In a waveguide of any cross-section, provided λ_c corresponds to the mode and the cross-section of the waveguide.

* From the above relation, it is clear that if $\lambda_0 << \lambda_c$, the denominator is approximately equal to '1'. and $\lambda_g = \lambda_0$. As λ_0 symbol λ_c, λ_g increases and reaches infinity when $\lambda_0 = \lambda_c$. when $\lambda_0 > \lambda_c$. it is evident that λ_g is imaginary which is nothing but no propagation in the waveguide.

Phase Velocity (V_p):—

* The wave propagates in the waveguide when guide wavelength 'λ_g' is greater than the free space wavelength 'λ_0'. Since the velocity of propagation is the product of λ and f, it follows that in a waveguide, $V_p = \lambda_g f$ where V_p is the phase velocity. But the speed of light is equal to product of 'λ_0' and f. This V_p is greater than the speed of light since $\lambda_g > \lambda_0$. This is contradicting since no signal can travel faster than the speed of light. However the wavelength in the guide is the length of the cycle and V_p represents the velocity of the phase. In fact it is defined as the rate at which the wave changes its phase in terms of the guide ~~waveguide~~ wavelength.

$$\text{i.e.,} \quad V_p = \frac{\lambda_0}{\text{unit time}} = \lambda_g f = \frac{2\pi f \lambda_g}{2\pi} = \frac{2\pi f}{2\pi/\lambda_g}$$

$$\text{i.e.,} \quad \boxed{V_p = \frac{\omega}{\beta}}$$

* where, $\omega = 2\pi f$, $\beta = \frac{2\pi}{\lambda_g}$

V_p is termed as "phase velocity".

→ Group Velocity (V_g):—

* If there is modulation in the carrier, the modulation envelope actually travels at velocity slower than that of carrier alone and of course slower than speed of light. The velocity of modulation envelope is called the "group velocity" 'V_g'. This happens when a modulated signal travels in a wave-guide. The modulation goes on slipping backward with respect to the carrier.

* It is defined as the rate at which the wave propagates through the waveguide and is given by.

$$\boxed{V_g = \frac{d\omega}{d\beta}}$$

→ Expression for phase velocity and Group Velocity:—

1. Expression for V_p :—

* from equation $\left(V_p = \frac{\omega}{\beta}\right)$ — (1)

and $h^2 = \gamma^2 + \omega^2 \mu \varepsilon = A^2 + B^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$

and $\gamma = \alpha + j\beta$.

for wave propagation $\gamma = j\beta$. ($\because$ attenuation, $\alpha = 0$)

$$\therefore \gamma^2 = (j\beta)^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu \varepsilon \quad — (2)$$

At $f = f_c$, $\omega = \omega_c$, $\gamma = 0$

$$\therefore \omega^2 \mu \varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \quad — (3)$$

using eq'n. (2) in (3) we get

$$\gamma^2 = (j\beta)^2 = \omega_c^2 \mu \varepsilon - \omega^2 \mu \varepsilon$$

$$\therefore \gamma^2 = \beta^2 = \omega^2 \mu \varepsilon - \omega_c^2 \mu \varepsilon$$

(or) $$\beta = \sqrt{\omega^2 \mu \varepsilon - \omega_c^2 \mu \varepsilon}$$

$$\beta = \sqrt{\mu \varepsilon (\omega^2 - \omega_c^2)} = \sqrt{\mu \varepsilon} \sqrt{(\omega^2 - \omega_c^2)} \quad — (4)$$

$$\therefore V_p = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\mu\varepsilon}\sqrt{\omega^2-\omega_c^2}} = \frac{1}{\sqrt{\mu\varepsilon}}\frac{1}{\sqrt{1-(\omega_c/\omega)^2}}$$

i.e.) $V_p = \dfrac{c}{\sqrt{1+(f_c/f)^2}}$

* we also know that, f (any frequency) = c/λ_0, where λ_0 is free space wavelength and f_c is cut off frequency = c/λ_c
* where λ_c is cut-off wavelength.

$$\frac{f_c}{f} = \frac{c}{\lambda_c}\cdot\frac{\lambda_0}{c} = \frac{\lambda_0}{\lambda_c}$$

$$\therefore \boxed{V_p = \frac{c}{\sqrt{1-(\lambda_0/\lambda_c)^2}}} \quad -(5)$$

2) Expression for V_g:-

* from equation $V_g = \dfrac{d\omega}{d\beta}$

and from the equation $\beta = \sqrt{\mu\varepsilon(\omega^2-\omega_c^2)}$

* Now differentiating β w.r.to 'ω', we get.

$$\frac{d\beta}{d\omega} = \frac{1}{2\sqrt{(\omega^2-\omega_c^2)\mu\varepsilon}} \times 2\omega\mu\varepsilon$$

$$\frac{d\beta}{d\omega} = \frac{\sqrt{\mu\varepsilon}}{\sqrt{1-(\omega_c/\omega)^2}} = \frac{\sqrt{\mu\varepsilon}}{\sqrt{1-(f_c/f)^2}}$$

(or) $V_g = \dfrac{d\omega}{d\beta} = \dfrac{\sqrt{1-(f_c/f)^2}}{\sqrt{\mu\varepsilon}}$

(or) $V_g = c\sqrt{1-\left(\dfrac{\lambda_0}{\lambda_c}\right)^2}$

* Consider the product of V_p and V_g.

i.e.) $V_p V_g = \dfrac{c}{\sqrt{1-(\lambda_0/\lambda_c)^2}}\cdot c\sqrt{1-(\lambda_0/\lambda_c)^2}$

$$\therefore \boxed{V_p V_g = c^2}$$

* Now we give another method for ~~giving~~ proving $v_p \cdot v_g = c^2$.

* As per earlier ~~product~~ discussion we know that v_p is greater than the speed of ~~high~~ light by the ratio (λ_g/λ_0)

~~As per earlier.~~ $\therefore v_p = \frac{\lambda_g}{\lambda_0} \cdot c$

* The group velocity v_g is shortened by the same ratio

i.e., $v_g = \frac{\lambda_0}{\lambda_g} \cdot c$

$$\therefore v_p \cdot v_g = \frac{\lambda_g}{\lambda_0} c \cdot \frac{\lambda_0}{\lambda_g} \cdot c = c^2$$

⇒ Relation between λ_g, λ_0 and λ_c :-

* we know that $v_p = \lambda_g \cdot f = \frac{\lambda_g}{\lambda_0} \cdot c$

$$v_p = \frac{c}{\sqrt{1-(\lambda_0/\lambda_c)^2}}$$

$$\therefore \frac{c}{\sqrt{1-(\lambda_0/\lambda_c)^2}} = \frac{\lambda_g}{\lambda_0} \cdot c.$$

(or)

$$\boxed{\lambda_g = \frac{\lambda_0}{\sqrt{1-(\lambda_0/\lambda_c)^2}}}$$

⇒ Physical Interpretation of phase and Group velocity :-

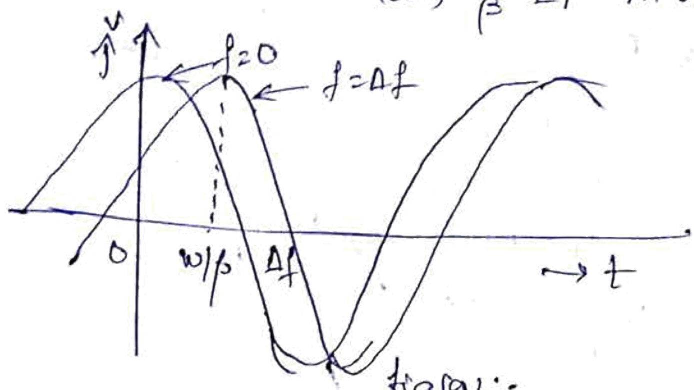
* The wave propagating between conducting parallel planes will encounter two different velocities, phase velocity and group velocity.

* Any electromagnetic wave has two velocities, one with which it propagates and the other with which it changes its phase. In free space these are naturally the same and equal to velocity of light.

* Consider a simple, single frequency wave which has the mathematical form.

$$V = V_m \cos(\omega t - \beta z)$$

* The wave is shown in fig(9) as a function of 't', for $t=0$, and $t=\Delta t$. The wave propagates through a distance of $\lambda_g \Delta t$ in a time Δt (or) $\frac{\omega}{\beta}\Delta t$ in a time Δt. (16)



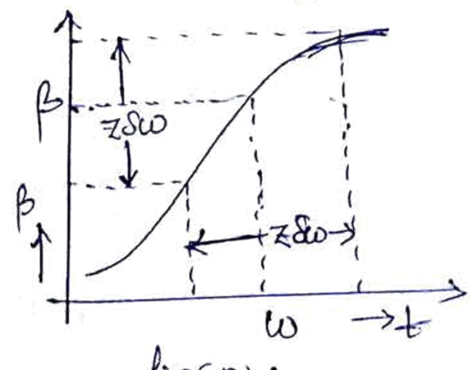
fig(9):-

$\therefore$ Velocity $= \dfrac{\omega/\beta \cdot \Delta t}{\Delta t} = \omega/\beta$.

(or) velocity $= \dfrac{\lambda g}{t} = \lambda g \cdot f = \dfrac{\lambda g 2\pi f}{2\pi} = \dfrac{\omega}{2\pi/\lambda g} = \dfrac{\omega}{\beta}$

* The velocity defined as the phase velocity, V_p is the velocity at which the whole wavefront of equiphase moves in the z direction.

* Now consider a wave made up of two frequency components of equal amplitude one with a frequency less than ω_0 and other at a frequency slightly greater than ω_0 with phase constant β as shown in fig(10).



fig(10):

$$e = A\cos[(\omega_0 - d\omega)t - (\beta_0 - d\beta)z] + [(\omega_0 + d\omega)t - (\beta_0 + d\beta)z]$$

The above equation can be written as

$$e = A\cos[(\omega_0 t - \beta_0 z) - (d\omega t - d\beta z)] + A\cos[(\omega_0 t - \beta_0 z) + (d\omega t - d\beta z)]$$

$$= 2A\cos(d\omega t - d\beta z)\cos(\omega_0 t - \beta_0 z)$$

* The first cosine term may be thought of as a modulation coefficient, modulating a carrier at a frequency ω_0 as shown in fig(11).

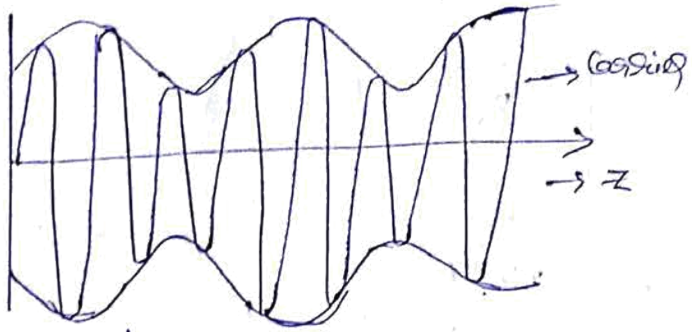


fig (11).

* The modulation envelope of the wave moves at a velocity $d\omega/d\beta$ and the carrier at a velocity of ω/β. Hence the envelope velocity is defined and the group velocity V_g given by

$$V_g = d\omega/d\beta.$$

* If there exists a group of many waves with small dispersion in frequency the resultant envelope will move at the group velocity.

* phase and group velocities are same when β is a linear function of ω (e.g., velocity in a lossless line). But in a waveguide β is given by

$$\beta = \omega\sqrt{\mu\varepsilon}\sqrt{1-(\omega_c/\omega)^2}$$

$$V_p = \frac{\omega}{\beta} = \frac{V_c}{\sqrt{1-(\omega_c/\omega)^2}} = \frac{V_c}{\sqrt{1-(\lambda_0/\lambda_c)^2}} > V_c$$

$$V_g = \frac{d\omega}{d\beta} = \frac{1}{\left(\frac{d\beta}{d\omega}\right)} = V_c\sqrt{1-(\lambda_0/\lambda_c)^2} < V_c$$

* The geometric mean of V_p and V_g is always the free space velocity (velocity of light) i.e., $V_g \cdot V_p = V_c^2$.

* The group velocity has an interesting interpretation as the velocity of energy flow in the waveguide system. This concept can be verified for TE_{10} mode in rectangular waveguide.

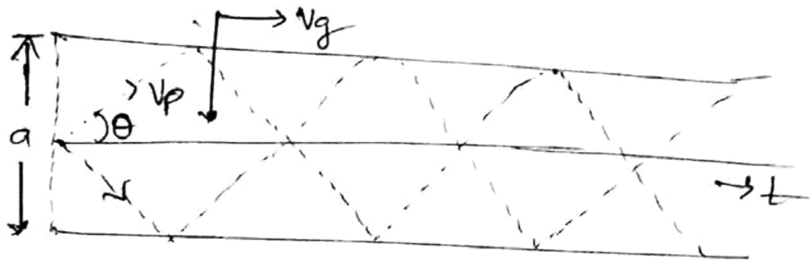
Here the propagating waves are seen to bounce off at side walls obliquely with an angle of incidence θ, as shown in fig.(12) with multiple reflections from the walls of the wave guide. (17)

* From fig.(13) it is seen that:

$$\left.\begin{array}{l}\lambda_g = \lambda_p = \lambda_0/\sin\theta \\ \lambda_n = \lambda_0/\cos\theta\end{array}\right\} \text{ Here } \lambda_0 > \lambda_g.$$

$$V_g = V_0 \sin\theta$$

$$V_n = V_c \cos\theta$$



fig(12):

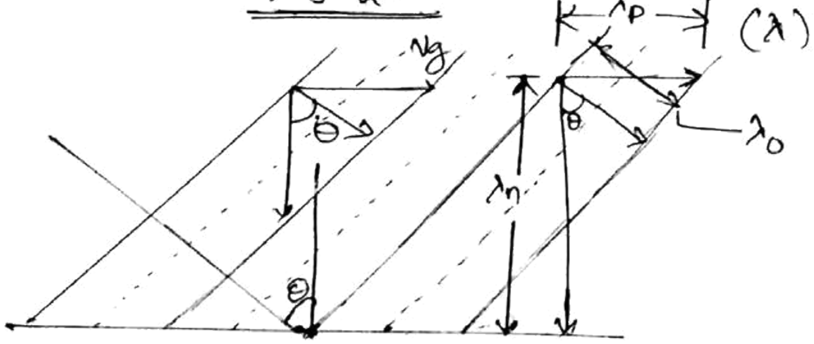


fig (13):

* As seen the wavelength measured in some direction is greater than the value measured in the direction of propagation.

* The velocity of propagation in a direction parallel to the conducting surface is $V_g = V_c \sin\theta$ and the wavelength in this direction is $\lambda_g = \lambda_0/\sin\theta$. If the frequency is '$f$' it follows that the velocity (phase velocity) with which the wave changes phase in a direction parallel to the conducting surface is given by the product of the frequency and wavelength λ_p.

i.e., $V_p = f\lambda_p$ (or) $f\lambda_g = \frac{f.\lambda_0}{\sin\theta} = \frac{V_c}{\sin\theta}$.

combining eq'ns above.

$V_g = V_c \sin\theta$ and $V_p = \frac{V_c}{\sin\theta}$, $\therefore V_p \cdot V_g = c^2$.

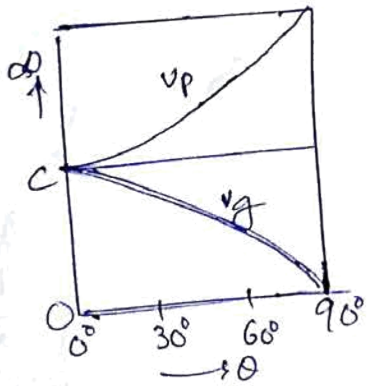
* Phase velocity is the velocity with which the wave changes phase at a plane boundary and not the velocity with which it travels along the boundary. Another example of the phase velocity of a wave train corresponding to sea waves approaching a beach at an angle rather than straight in. The phenomenon which accompanies this is that the edge of the wave appears to sweep along the beach much faster than the wave is really travelling. It is the phase velocity that provides this effect.

* A plot of V_p and V_g is shown in fig.(14) as a function of wave angle.

* It is seen from the plot that as θ decreases.

1) V_p increase and becomes equal to ∞ at $\theta = 90°$

2) V_g decreases and becomes equal to zero at $\theta = 90°$



fig(14) :-

* Thus the group velocity with which the energy is actually transmitted can almost be equal to 'C' the velocity of light.

* At cutoff frequencies, V_p becomes infinite and V_g becomes zero. This means that there is no propagation of energy along the waveguide. The wave simply bounces back and forth between parallel plates and normal to them.

⇒ TM mode in rectangular waveguides:-

* Consider the various TM_{mn} modes for various values of m and n.

1) TM_{11} mode:- minimum possible mode $m=1$ and $n=1$

from equation $\lambda_{mn} = \frac{2ab}{\sqrt{m^2b^2 + n^2a^2}}$

Now putting m = n = 1 we get

$$\boxed{\lambda_{c11} = \frac{2ab}{\sqrt{a^2+b^2}}}$$

2) **TM$_{12}$ mode :-** Now putting m=1, n=2 in the above equation we get:

$$\boxed{\lambda_{c12} = \frac{2ab}{\sqrt{b^2+4a^2}}}$$

3) **TM$_{21}$ mode :-** Now m=2, n=1.

$$\therefore \boxed{\lambda_{c21} = \frac{2ab}{\sqrt{4b^2+a^2}}}$$

* By inspection is clear that $\lambda_{c11} > \lambda_{c12}$ and $\lambda_{c11} > \lambda_{c21}$ and so on.

* The field patterns of TM$_{11}$ mode is shown in fig (15).

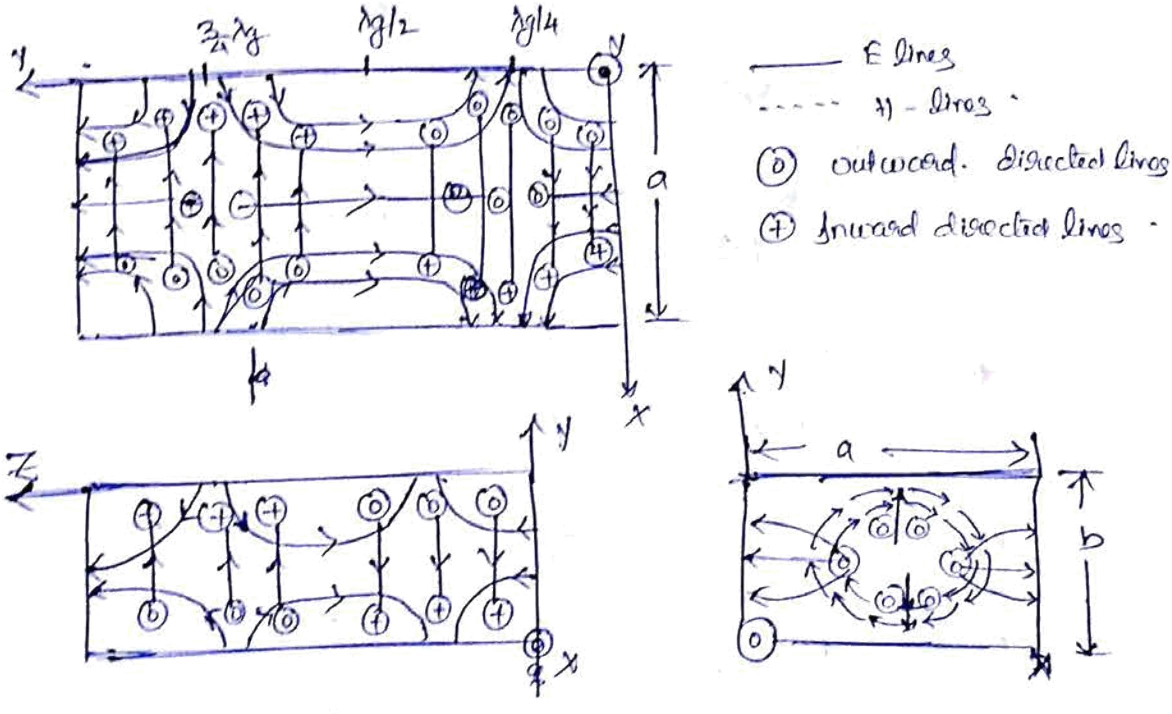


fig (15) :-

⇒ **Propagation of TE waves in a Rectangular waveguide:-**

* The TE$_{mn}$ modes in a rectangular waveguide are characterized by $E_z = 0$. In other words the 'z' component of the magnetic field H_z must exist in order to have energy transmission in the guide.

* The wave equation (Helmholtz equation) for TE wave is given by.

Now putting m = n = 1 we get

$$\boxed{\lambda_{11} = \frac{2ab}{\sqrt{a^2+b^2}}}$$

2) TM$_{12}$ mode :- Now putting m=1, n=2 in the above equation we get:

$$\boxed{\lambda_{c12} = \frac{2ab}{\sqrt{b^2+4a^2}}}$$

3) TM$_{21}$ mode :- Now m=2, n=1.

$$\therefore \boxed{\lambda_{c21} = \frac{2ab}{\sqrt{4b^2+a^2}}}$$

* By inspection is clear that $\lambda_{c11} > \lambda_{c12}$ and $\lambda_{c11} > \lambda_{c21}$ and so on.

* The field patterns of TM$_{11}$ mode is shown in fig (15).

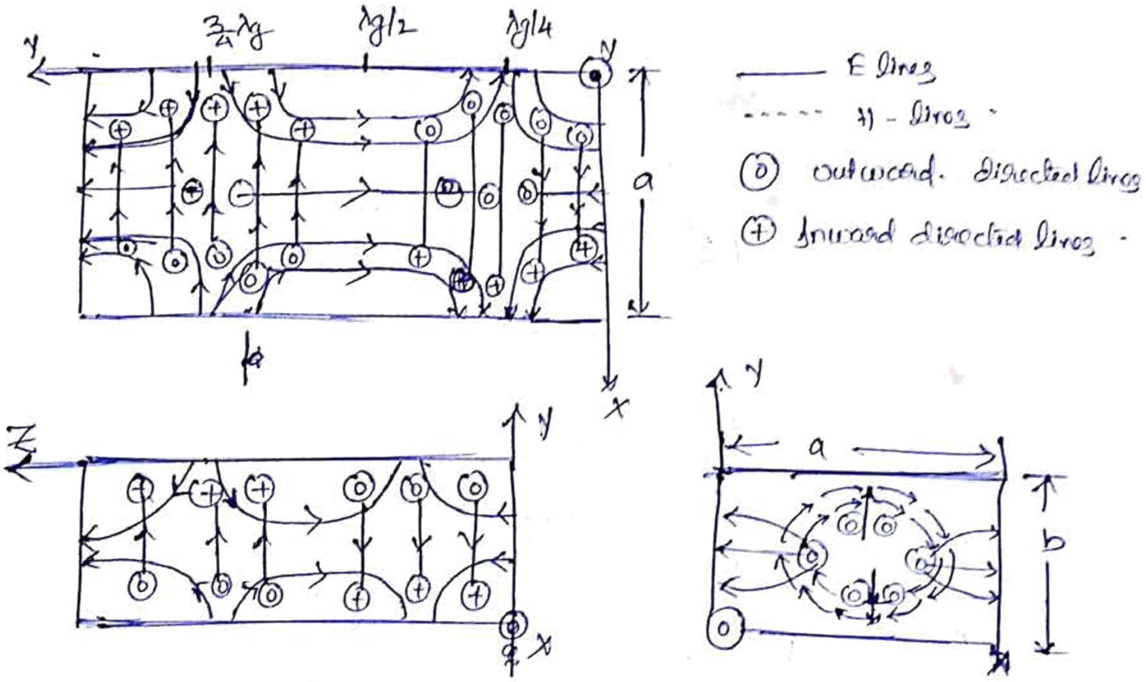


fig (15) :-

⇒ Propagation of TE waves in a Rectangular waveguide :-

* The TE$_{mn}$ modes in a rectangular waveguide are characterised by $E_z = 0$. In other words the 'z' component of the magnetic field H_z must exist in order to have energy transmission in the guide.

* The wave equation (Helmholtz equation) for TE wave is given by.

$$\nabla^2 H_z = -\omega^2 \mu \varepsilon H_z$$

i.e.,) $$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \frac{\partial^2 H_z}{\partial z^2} = -\omega^2 \mu \varepsilon H_z$$

(or) $$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \gamma^2 H_z + \omega^2 \mu \varepsilon H_z = 0. \quad \left[\because \frac{\partial^2}{\partial z^2} = \gamma^2\right]$$

(or) $$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + (\gamma^2 + \omega^2 \mu \varepsilon) H_z = 0$$

(or) $$\boxed{\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + h^2 H_z = 0} \quad - (1)$$

* This is a partial differential equation whose solution can be assumed as $H_z = XY$. where
 X is a pure function of 'x' only
 Y is a pure function of 'y' only.

* Substituting for H_z in eq'n (1) we get

$$Y\frac{d^2 X}{dx^2} + X\frac{d^2 Y}{dy^2} + h^2 XY = 0$$

* Dividing throughout by XY, we get

$$\frac{1}{X}\frac{d^2 X}{dx^2} + \frac{1}{Y}\frac{d^2 Y}{dy^2} + h^2 = 0 \quad - (2)$$

* Here $\frac{1}{X}\frac{d^2 X}{dx^2}$ is purely a function of x

* and $\frac{1}{Y}\frac{d^2 Y}{dy^2}$ is purely a function of y.

* Equating each of these items to a constant, we get

$$\frac{1}{X}\frac{d^2 X}{dx^2} = -B^2$$

and $$\frac{1}{Y}\frac{d^2 Y}{dy^2} = -A^2$$

* where $-B^2$ and $-A^2$ are constants

* Substituting these in eq'n (2), we get

$$-B^2 - A^2 + h^2 = 0$$

$$\therefore \quad h^2 = A^2 + B^2 \quad - (3)$$

* Solving for X and Y by separation of variable method

$X = C_1 \cos Bx + C_2 \sin Bx$

$Y = C_3 \cos Ay + C_4 \sin Ay$.

* Therefore the complete solution of $H_z = XY$ is

i.e., $H_z = (C_1 \cos Bx + C_2 \sin Bx)(C_3 \cos Ay + C_4 \sin Ay)$ .....(4)

* where C_1, C_2, C_3 and C_4 are constants which can be evaluated by applying boundary conditions.

⇒ Boundary Conditions :—

* As in case of TM waves, we have four boundaries for TE waves also, as shown in fig(16).

* Here since we are considering a TE wave,

$E_z = 0$ but we have components along x and y direction.

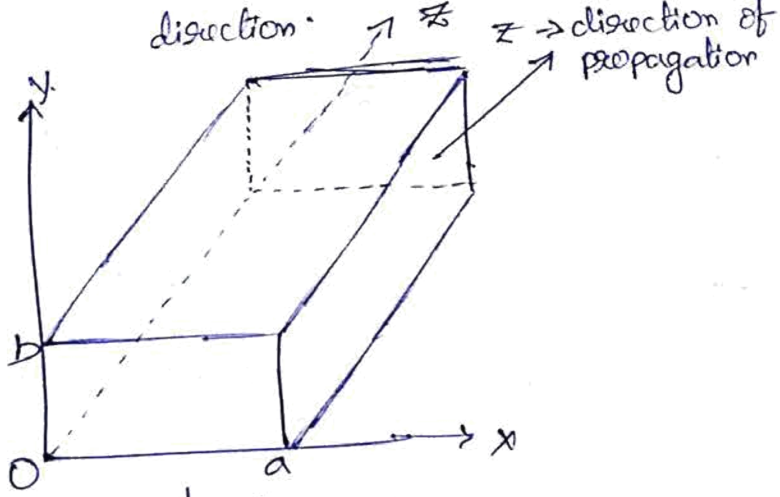


fig. (16):—

* $E_x = 0$ all along bottom and top walls of the waveguide
* $E_y = 0$ all along left and right wall of the waveguide

1st boundary Condition :—

$E_x = 0$ at $y = 0$ ∀ $x \to 0$ to a. (bottom wall)

2nd boundary condition :—

$E_x = 0$ at $y = b$ ∀ $x \to 0$ to a (top wall)

3rd boundary condition :—

$E_x = 0$ at $x = 0$ ∀ $y \to 0$ to b (left side wall)

4th boundary condition:-

$E_y = 0$. at $x = a$ $\forall$ $y \to 0$ to b (Right side wall)

(i) substituting 1st boundary condition in eq'n - (4)

* Since 1st boundary condition is

$E_x = 0$ at $y = 0$ $\forall$ $x \to 0$ to a, let us write E_x in terms of H_z.

as $$E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$

* Since $E_z = 0$, the 1st term $= 0$

$$\therefore E_x = \frac{-j\omega\mu}{h^2}\frac{\partial}{\partial y}\left[(C_1 \cos Bx + C_2 \sin Bx)(C_3 \cos Ay + C_4 \sin Ay)\right]$$

i.e., $$E_x = \frac{-j\omega\mu}{h^2}(C_1 \cos Bx + C_2 \sin Bx)(-A\, C_3 \sin Ay + A\, C_4 \cos Ay)$$

* Substituting 1st boundary condition in the ~~waveguide~~ above equation we get

$$0 = \frac{-j\omega\mu}{h^2}(C_1 \cos Bx + C_2 \sin Bx)(0 + AC_4)$$

* Since $(C_1 \cos Bx + C_2 \sin Bx) \neq 0$, $A \neq 0$

$$C_4 = 0$$

* Substituting the value of C_4 in eq'n (4) the solution reduces to

$$\boxed{H_z = (C_1 \cos Bx + C_2 \sin Bx)(C_3 \cos Ay)} \quad - (5)$$

(ii) 3rd boundary condition :-

$E_y = 0$ at $x = 0$ $\forall$ $y \to 0$ to b.

* from the below equation

$$\boxed{E_y = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{-j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}}$$

* Since $E_z = 0$ and substituting the value of H_z from eq'n (5) we get

$$E_y = \frac{j\omega\mu}{h^2}\frac{\partial}{\partial x}\left[(C_1 \cos Bx + C_2 \sin Bx)(C_3 \cos Ay)\right]$$

i.e. $$E_y = \frac{j\omega\mu}{h^2}(0 + BC_2)$$

i.e., $E_y = \frac{j\omega\mu}{h^2}\left[(-BC_1 \sin Bx + BC_2 \cos Bx) C_3 \cos Ay\right]$

* Substituting the 3rd boundary condition.

$x = 0$, $\forall\ y \to 0$ to b in the above equation

$$0 = \frac{j\omega\mu}{h^2}(0 + BC_2)\, C_3 \cos Ay$$

* Since, $\cos Ay \neq 0$, $B \neq 0$, $C_3 \neq 0$.

$$C_2 = 0.$$

* Substituting the value of C_2 in eq'n. (5), the solution now reduces to

$$\boxed{H_z = C_1 C_3 \cos Bx \cos Ay} \quad -(6)$$

(iii) 2nd Boundary Condition :-

- $E_x = 0$ at $y = b$ $\forall\ x \to 0$ to a.

* from the below equation we have

$$E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$

$$= -\frac{j\omega\mu}{h^2}\frac{\partial}{\partial y}\left[C_1 C_3 \cos Bx \cos Ay\right] \quad (\because E_z = 0)$$

$$= \frac{+j\omega\mu}{h^2} C_1 C_3 A \cos Bx \sin Ay.$$

* Substituting 2nd boundary condition, in the above equation, we get

$$0 = \frac{j\omega\mu}{h^2} C_1 C_3 A \cos Bx \sin Ab.$$

$$\cos Bx \neq 0,\ C_1, C_3 \neq 0.$$

$\therefore$ $\sin Ab = 0$ (or) $Ab = n\pi$.

* where $n = 0, 1, 2, \ldots \infty$

(or) $\boxed{A = \frac{n\pi}{b}}$ $\quad -(7)$

(iv) 4th boundary condition :-

$E_y = 0$ at $x = a \; \forall \; y \to 0$ to b

$$E_y = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}.$$

$$E_y = \frac{j\omega\mu}{h^2}\frac{\partial}{\partial x}\left[C_1 C_3 \cos Bx \cos Ay\right]$$

$(\because E_z = 0$ and $H_z = C_1 C_3 \cos Bx \cos Ay)$

* i.e., $E_y = \frac{-j\omega\mu}{h^2} C_1 C_3 B \sin Bx \cos Ay.$

* Substituting the boundary condition.

$$0 = \frac{-j\omega\mu}{h^2} C_1 C_3 B \sin Bx \cos Ay \Rightarrow \forall \; y \to 0 \text{ to } b$$

$\cos Ay \neq 0$, $C_1, C_3 \neq 0$

$\therefore \sin Ba = 0$

$\therefore Ba = m\pi$ where $m = 0, 1, 2, \ldots, \infty$

$$\therefore \boxed{B = \frac{m\pi}{a}} \quad \text{— (8)}$$

* The complete solution is

$$H_z = C_1 C_3 \, A \cos Bx . \cos Ay.$$

* Substituting for A and B from eq'ns (7) and (8)

we get

$$H_z = C_1 C_3 \cos\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y$$

Let $C_1 C_3 = C$ (another constant)

$$\therefore \boxed{H_z = C \cos\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y \, e^{(j\omega t - \gamma z)}} \quad \text{— (9)}$$

* Thus it can be seen that for a TM wave E_z has sine-sine components and for a TE wave H_z has cosine-cosine components.

* Field components

$$E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}.$$

* Here 1st term $= 0$ since $E_z = 0$ for TM wave.

i.e.,
$$\boxed{E_x = \frac{j\omega\mu}{h^2} \cdot C \cdot \left(\frac{n\pi}{b}\right) \cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y \cdot e^{(j\omega t - \bar{\gamma} z)}} \quad (10)$$

$$E_y = -\frac{\bar{\gamma}}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}.$$

again 1st term is 0 since $E_z = 0$ for TM wave

$$\therefore \boxed{E_y = \frac{-j\omega\mu}{h^2} C\left(\frac{m\pi}{a}\right) \sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y \, e^{(j\omega t - \bar{\gamma} z)}} \quad (11)$$

similarly. $H_x = \frac{-\bar{\gamma}}{h^2}\frac{\partial H_z}{\partial x} + \frac{j\omega\epsilon}{h^2}\frac{\partial E_z}{\partial y}$

$$\therefore \boxed{H_x = \frac{+\bar{\gamma}}{h^2} C\left(\frac{m\pi}{a}\right) \sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y \, e^{(j\omega t - \bar{\gamma} z)}} \quad (12)$$

and $H_y = \frac{-\bar{\gamma}}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\epsilon}{h^2}\frac{\partial E_z}{\partial x}.$

$$\therefore \boxed{H_y = \frac{-\bar{\gamma}}{h^2} C\left(\frac{n\pi}{b}\right) \cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y \, e^{(j\omega t - \bar{\gamma} z)}} \quad (13)$$

→ **TE Modes in rectangular wave guide:-**

* TE_{mn} is the general mode and specific modes are given by various values of m and n

(a) **TE_{00} mode :-** m=0 and n=0

All field components vanish therefore it can not exist.

(b) **TE_{10} mode :-** m=1, n=0

$E_x = 0$, $H_y = 0$, E_y and H_x exist

Therefore TE_{10} mode exists

(c) **TE_{01} mode:-** m=0, n=1.

$E_y = 0$, $H_x = 0$ E_x and H_y exist.

(d) **TE_{11} mode :-** m=1, n=1

This also exists and even higher modes.

→ **Dominant mode :-**

* Dominant mode is that mode for which the cutoff wavelength (λ_c) assumes a maximum value.

* we know that $\lambda_{cmn} = \dfrac{2ab}{\sqrt{m^2b^2+n^2a^2}}$

- for TE_{01} mode $\lambda_{c01} = \dfrac{2ab}{\sqrt{a^2}} = 2b$
- for TE_{10} mode $\lambda_{c10} = \dfrac{2ab}{\sqrt{b^2}} = 2a$
- for TE_{11} mode $\lambda_{c11} = \dfrac{2ab}{\sqrt{a^2+b^2}}$

* λ_{c10} has the maximum value since 'a' is the larger dimension. Hence TE_{10} mode is the dominant mode in rectangular waveguides.

* The other expression of β, v_p, v_g and λ_g remain the same as for TM waves.

i.e., $\beta = \dfrac{2\pi}{\lambda_g} = \sqrt{\omega^2\mu\epsilon - \omega_c^2\mu\epsilon}$

$$v_p = \frac{c}{\sqrt{1-(\lambda_0/\lambda_c)^2}}$$

$$v_g = c\sqrt{1-(\lambda_0/\lambda_c)^2}$$

and. $$\lambda_g = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

⇒ Wave Impedance (Z_z) in TM and TE waves:-

* wave impedance is defined as the ratio of the strength of electric field in one transverse direction to the strength of the magnetic field along the other transverse direction as shown in figure.

i.e., $Z_z = \dfrac{E_x}{H_y} = \dfrac{-E_y}{H_x}$.

(or) $$\boxed{Z_z = \frac{\sqrt{E_x^2+E_y^2}}{\sqrt{H_x^2+H_y^2}}} \quad -(14)$$

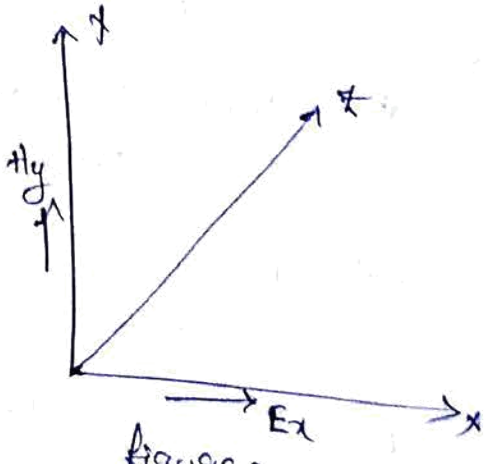


figure:

1) wave impedance of a TM wave in rectangular waveguide :-

$$Z_z = Z_{TM} = \frac{E_x}{H_y} = \frac{\frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}}{\frac{-\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}}.$$

* for a TM wave $H_z = 0$ and $\gamma = j\beta$.

$$\therefore Z_{TM} = \frac{\frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x}}{\frac{-j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}} = \frac{\gamma}{j\omega\varepsilon} = \frac{j\beta}{j\omega\varepsilon}$$

(or) $Z_{TM} = \dfrac{\beta}{\omega\varepsilon}$

* we know that $\beta = \sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}$

$$\therefore Z_{TM} = \frac{\sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}}{\omega\varepsilon} = \sqrt{\frac{\mu}{\varepsilon}}\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

(or) $Z_{TM} = \sqrt{\dfrac{\mu}{\varepsilon}}\sqrt{1 - (\lambda_0/\lambda_c)^2}$

For air $\sqrt{\dfrac{\mu}{\varepsilon}} = \sqrt{\dfrac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}}$

$$= \sqrt{\frac{4\pi \times 10^{-7}}{\frac{1}{36\pi}\times 10^{-9}}} = \sqrt{4\pi \times 36\pi \times 10^{2}}$$

$$= 2 \times 6\pi \times 10 = 120\pi = 377\,\Omega = \eta$$

* where η is the intrinsic impedance of free space.

$$\therefore \boxed{Z_{TM} = \eta\sqrt{1 - (\lambda_0/\lambda_c)^2}} \quad -(15)$$

$\lambda = c/f$

* since λ_0 is always less than λ_c for wave propagation
$f_0 > f_c$

$Z_{TM} < \eta$

* This shows that wave impedance for a TM wave is always less than free space impedance.

$$Z_z = Z_{TE} = \frac{E_x}{H_y} = \frac{\left(\frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}\right)}{\left(\frac{-\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}\right)}$$

* For TE waves $E_z = 0$ and $\gamma = j\beta$

$$\therefore Z_{TE} = \frac{\frac{-j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}}{\frac{-\gamma}{h^2}\frac{\partial H_z}{\partial y}} = \frac{j\omega\mu}{\gamma}$$

$$= \frac{j\omega\mu}{j\beta} = \frac{\omega\mu}{\beta}.$$

$$\therefore \quad Z_{TE} = \frac{\omega\mu}{\sqrt{\mu\varepsilon}\sqrt{\omega^2-\omega_c^2}}$$

$$= \frac{\eta}{\sqrt{1-(\omega_c/\omega)^2}} = \frac{\eta}{\sqrt{1-(f_c/f)^2}}$$

(or) $$\boxed{Z_{TE} = \frac{\eta}{\sqrt{1-(\lambda_0/\lambda_c)^2}}} \quad -(16)$$

Therefore $Z_{TE} > \eta$ as $\lambda_0 < \lambda_c$ for wave propagation. This shows that wave impedance for a TE wave is always greater than free space impedance.

For TEM waves between parallel planes (or) an ordinary parallel wire (or) coaxial transmission lines the cutoff frequency is zero and wave impedance for TEM wave is the free space impedance itself.

i.e., $Z_z = (TEM) = \eta$

Also when the waveguide has a dielectric other than air say with a dielectric constant ε_r then the behavior of the waveguide gets changed.

For air dielectric, ~~we know~~ the guide wavelength is given by

$$\lambda_g = \frac{\lambda_0}{\sqrt{1-(\lambda_0/\lambda_c)^2}}$$

For the case of waveguide with dielectric constant ε_r,

$$\lambda_g = \frac{\lambda_0}{\sqrt{\varepsilon_r - (\lambda_0/\lambda_c)^2}} \quad \text{and} \quad \lambda_{g\,dielectric} = \frac{\lambda_{g\,air}}{\sqrt{\varepsilon_r}}$$

Since $\varepsilon_r > 1$ and $\lambda_{dielectric} < \lambda_{air}$, hence frequencies less than cutoff values can pass through the same guide.

Power Transmission In Rectangular waveguide:-

* The power transmitted through a waveguide and the power loss in the guide walls can be calculated by means of complex. poynting theorem. we assume that the waveguide is ~~infinitely~~ terminated in such a way that there is no reflection from the receiving end (or) that the waveguide is ~~de~~ infinitely long as compared to its wavelength.

* The power transmitted through a waveguide is given by

$$\boxed{P_{tr} = \int P \cdot ds = \int \frac{1}{2}(E \times H^*) \cdot ds} \quad -(1)$$

* For a loss less dielectric, the time avege power flow through a rectangular waveguide is

$$P_{tr} = \frac{1}{2Z_z}\int_a |E|^2 da = \frac{Z_z}{2}\int_a |H|^2 da.$$

* where $Z_z = \frac{E_x}{H_y} = \frac{-E_y}{H_x}$.

$$|E|^2 = |E_x|^2 + |E_y|^2$$

$$|H|^2 = |H_x|^2 + |H_y|^2$$

* For TMmn mode, the average power transmitted through a rectangular waveguide of dimensions a and b is

$$\boxed{P_{tr} = \frac{1}{2\eta\sqrt{1-(\lambda_0/\lambda_c)^2}}\int_0^b\int_0^a |E_x|^2 + |E_y|^2 \, dx\, dy} \quad -(2)$$

* For ~~TM~~ TEmn modes $Z_z = \frac{\eta}{\sqrt{1-(\lambda_0/\lambda_c)^2}}$

$$\therefore \boxed{P_{tr} = \frac{\sqrt{1-(\lambda_0/\lambda_c)^2}}{2\eta}\int_0^b\int_0^a |E_x|^2 + |E_y|^2 \, dx\, dy} \quad -(3)$$

⇒ Power losses in a waveguide:—

Conducting

* Losses in a waveguide can be due to attenuation below cutoff and losses associated with attenuation due to the waveguide walls and ~~di~~ the dielectric dissipation within the waveguide.

* At frequencies below the cutoff frequency ($f<f_c$) the propagation constant 'γ' will have only the attenuation term 'α' ($\gamma=\alpha+j\beta$) that is to say that the phase constant 'β' itself becomes imaginary implying wave attenuation.

$$\text{Hence } \beta = \frac{j2\pi}{\lambda_g}$$

$$\text{we know that } \lambda_g = \frac{\lambda}{\cos\theta} = \frac{\lambda}{\sqrt{1-(f_c/f)^2}}$$

$$\text{Therefore } \beta = j\frac{2\pi}{\lambda}\sqrt{(f_c/f)^2-1} = j\frac{2\pi f_c}{c}\sqrt{1-(f/f_c)^2}$$

$$\beta = j\alpha$$

* Hence the cutoff attenuation constant 'α' is given by

$$\boxed{\alpha = \frac{54.6}{\lambda_c}\sqrt{1-(f_0/f_c)^2}} \text{ dB/length.}$$

* In fact this is the stop band attenuation of the waveguide high pass filter. For $f>f_c$, the waveguide exhibits very low loss and for $f<f_c$ the attenuation is high and results in full reflection of the wave i.e., cutoff attenuation is basically the reflection loss.

* Attenuation constant due to an imperfect, non magnetic dielectric in the waveguide is given by.

$$\alpha_d = \frac{27.3\sqrt{\varepsilon_R}\tan\delta}{\lambda_0\sqrt{1-(f_c/f)^2}} \text{ dB/length.}$$

* where $\tan\delta$ = dielectric loss ~~lang~~ tangent of the insulating material (dielectric).

* The attenuation constant due to the imperfect conducting walls for TE_{10} mode is given by:

$$\boxed{\alpha_c = \frac{R_s}{b\eta_0} \cdot \frac{1 + \frac{2b}{a}(f_c/f)^2}{\sqrt{1-(f_c/f)^2}}}$$ Np/length.

* where R_s = sheet resistivity in Ohm/m^2

η_0 = intrinsic impedance of free space (377Ω)

Thus $\boxed{R_s = \frac{1}{\sigma \delta_s}}$

* where σ is the conductivity of the metallic walls in S/m and skin depth is

$$\boxed{\delta_s = \frac{1}{\sqrt{\pi f \mu_r \mu_0 \sigma}}}$$

* where f = frequency,

μ_r = relative permeability (typically $\mu_r=1$)

μ_0 = permeability of free. ($4\pi \times 10^{-7}$ H/m)

⇒ Dominant mode and Degenerate mode in rectangular waveguides :-

The walls of the waveguides can be considered as nearly perfect conductors. Therefore, the boundary conditions require that electric field be normal i.e., perpendicular to the wave guide walls. The magnetic field must be tangential i.e., ~~normal~~ parallel to the waveguide walls.

Because of these boundary conditions a zero subscript can exist in the TE mode but not in the TM mode. For e.g. TE_{10}, TE_{01}, T_{20} etc modes. can exist in ~~the TE mode~~ a rectangular waveguide but ~~but not in the TM mode.~~ but only the TM_{11}, TM_{12}, TM_{21} etc. modes can exist. The physical size of ~~the~~ the waveguide determines the propagation

of modes depending on the values of m and n.

* The minimum cutoff frequency for a rectangular waveguide is obtained for a dimension a>b for m=1 and n=0, i.e., TE_{10} mode is the dominant mode for a rectangular waveguide. (since for TM_{mn} modes m≠0 (or) n≠0, the lowest order mode TE_{10} is the dominant mode for a>b).

* Some of the higher order modes having the same cutoff frequency ~~ca~~ are called "degenerate modes". For a rectangular waveguide TE_{mn}/TM_{mn} modes for which both m≠0, n≠0 will alway be degenerate modes.

* For a square guide in which a=b, all the TE_{pq}, TE_{qp} ~~an~~, TM_{pq} and TM_{qp} modes are together degenerate modes. It is ~~access~~ necessary that higher order degenerate modes are not supported by the guide in the operating band of frequencies to avoid undesirable components appearing at the output alongwith losses.

* Also it may be necessary to prevent the conversion of a particular waveguide mode to another. Such mode conversion usually results from waveguide irregularities (or) from impedance structure used in transmission line. such mode conversion can be supported by

(i) choosing suitable waveguide dimension (the undesired mode/modes having cutoff frequency above the desired modes can be suppressed)

(ii) Using mode filter (undesired modes can be suppressed by providing a metallic plate (or) vane in the waveguide where undesired modes have tangential electric field lines).

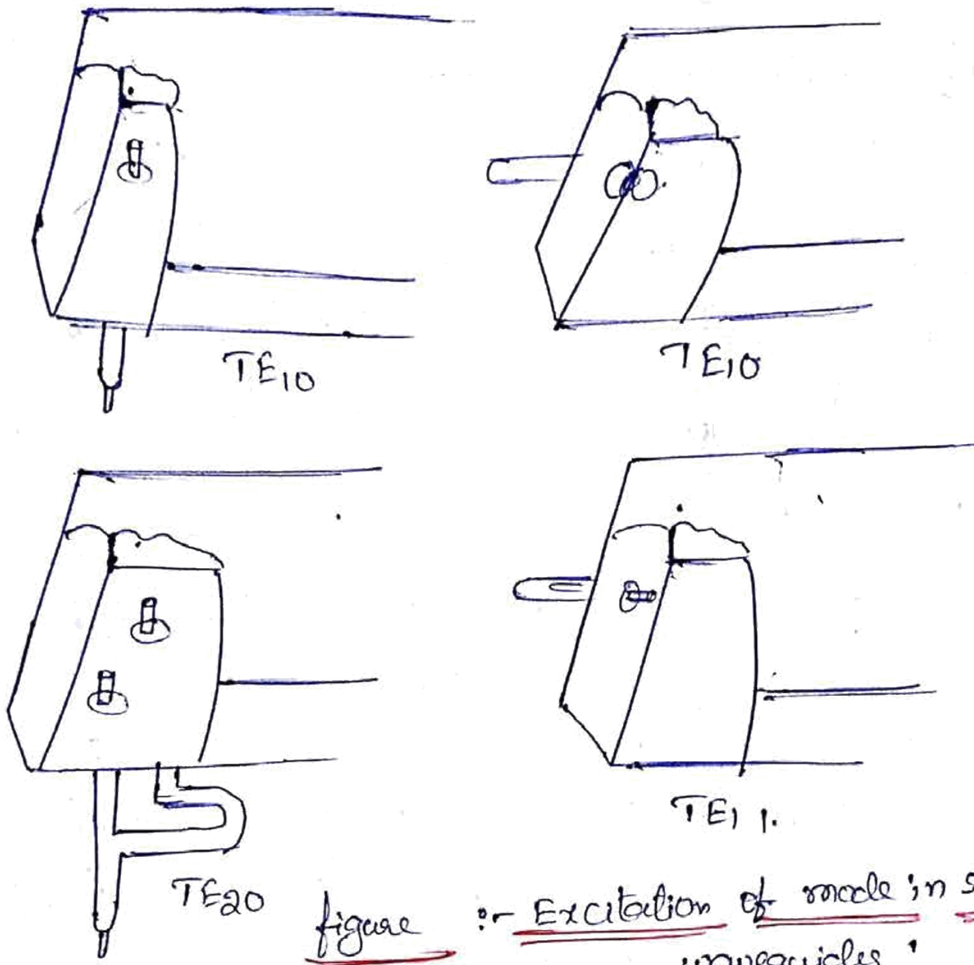


figure :- Excitation of mode in rectangular waveguides '

★ The various modes in a waveguides can be excited by various launching devices. Figure shows how the TE10, TE20 and TE11 modes are launched in rectangular waveguides. These launching devices are in fact, antennas. At the receiving end of the waveguide a similar launching device (receiving antenna) is used to convert the e.m fields within the waveguide back to voltage and currents on a transmission line. Hence, one must know which waveguide mode was used to launch the e.m fields at the transmitting ends of the waveguide. If more than one mode exists at a particular frequency in the waveguide, then discontinuities such as bends, joints etc., would cause e.m energy to be transferred from one mode to another. This results.

For a rectangular guide 2.5 cm x 1.0 cm . cross section having $H_z = 20\cos(\pi x/a)\, Exp[j\, 10^{11} t + \beta z)$ A/m., Identify the Propagating mode & the direction of propagation. Also determine its phase constant & Z_0

Sol:- given that

For a rectangular wave guide,

$a = 2.5, b = 1.0\,cm, \beta = ?\ , Z_0 = ?$

given $H_z = 20\cos\left(\frac{\pi x}{b}\right) \cdot Exp[j(10^{11} t + \beta z)]$

$$H_z = 20\cos\left(\frac{\pi x}{b}\right) \cdot \cos\left(\frac{0 \cdot y}{a}\right) \cdot Exp[j(10^{11} t + \beta z)] \quad \text{— (1)}$$

The general expression for magnetic field moving in positive z-direction in a rectangular wave guide is given by

$$H_z = H_{oz} \cos\left(\frac{m\pi x}{a}\right) \cdot \cos\left(\frac{n\pi y}{a}\right) e^{j(\omega t - \beta t)}$$

by comparing. $m = 1, n = 0$

$$\beta = \omega\sqrt{\mu\varepsilon}\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

$\omega = 10^{11}$

$2\pi f = 10^{11}$

$f = \frac{10^{11}}{2 \times \pi}$

$f = 15916\,GHz$.

$$f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2}$$

$$f_c = \frac{1}{2\pi\sqrt{\mu_0\varepsilon_0}}\sqrt{\left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2}$$

$$f_c = \frac{1}{2\pi\sqrt{4\pi\times10^{-9}\times\frac{1}{36\pi}\times10^{-4}}}\sqrt{\left[\left[\frac{1\cdot\pi}{2.5}\right]^2 + \left[\frac{0\cdot\pi}{1.0}\right]^2\right]}$$

$$f_c = \cancel{6Hz}\ 6GHz.$$

$$\beta = 10^{11}\sqrt{4\pi\times10^{-7}\times\frac{1}{36\pi}\times10^{-9}}\sqrt{1-\left[\frac{6\times10^9}{15.916\times10^9}\right]}$$

$$= 10^{11}\times\frac{1}{3}\times10^{-8}\times0.9286$$

$$\beta = 308.667\ rad/m.$$

& $$Z_0 = \frac{377}{\sqrt{1-(f_c/f)^2}}$$

$$= \frac{377}{\sqrt{1-\left(\frac{6\times10^9}{15.916\times10^9}\right)^2}}$$

$$= \frac{377}{0.926}$$

$$Z_0 = 407.127\,\Omega.$$

UNIT - II

Micro wave Transmission lines - II

1) Power Transmission in Rectangular waveguide :-

* The power transmitted through a waveguide and the power loss in the guide walls can be calculated by means of the complex poynting theorem.

* Assumptions :-

* It is assumed that the guide is terminated in such a way that there is no reflections from the receiving end (or) that the guide is infinitely long compared with the wavelength.

* From the poynting theorem the power transmitted through a guide is given by

$$P_{tr} = \oint P \cdot ds = \oint \frac{1}{2} (E \times H^*) \cdot ds \quad (1)$$

* For a lossless dielectric, the time average power flow through a rectangular guide is given by

$$P_{tr} = \frac{1}{2Z_g} \int |E|^2 da = \frac{Z_g}{2} \int |H|^2 da$$

$$\text{where } Z_g = \frac{E_x}{H_y} = \frac{-H_y}{H_x}$$

* Only mutually perpendicular components of E & H contribute to power flow

$$|E|^2 = |E_x|^2 + |E_y|^2$$

$$|H|^2 = |H_x|^2 + |H_y|^2$$

* For $TE_{m,n}$ mode, the avg power transmitted through a rectangular waveguide is given by

$$\boxed{P_{tr} = \frac{1}{2\eta\sqrt{1-(f_c/f)^2}} \int_0^b\int_0^a \left(|E_x|^2 + |E_y|^2\right) dx\,dy}$$

* For TM_{mn} modes, the avg power transmitted through a rectangular waveguide is given by.

$$\boxed{P_{tr} = \frac{1}{2\eta\sqrt{1-(f_c/f)^2}} \int_0^b\int_0^a \left(|E_x|^2 + |E_y|^2\right) dx\,dy}$$

2) Power losses in a rectangular waveguide:-

* As the EM wave propagates through a waveguide the wave intencively gets attenuated because of losses in the waveguides. There are three types of losses in the waveguide They are

(1) power loss in dielectric filling:-

* In a low-loss dielectric ($\sigma << \omega\epsilon$), the attenuation constant for a plane wave travelling in an unbounded lossy ~~dielect~~ dielectric is given as.

$$\boxed{\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}} = \frac{\eta\sigma}{2}} \quad \text{--- (1)}$$

* The attenuation caused by the low-loss dielectric in the rectangular waveguide for the TE_{mn} (or) TM_{mn} modes is given by

$$\boxed{\alpha_g = \frac{\sigma\eta}{2\sqrt{1-(f_c/f)^2}}} \leftarrow \text{for TE mode}$$

$$\boxed{\alpha_g = \frac{\sigma\eta}{2}\sqrt{1-(f_c/f)^2}} \leftarrow \text{for TM mode.}$$

* If the operating frequency (f) is greater than f_c i.e., ($f >> f_c$), then the attenuation constant in the guide approaches that for the unbounded dielectric. i.e., eq'n (1) (2)

* If the operating frequency (f) is less than f_c i.e., $\boxed{f << f_c}$, then ~~its~~ the attenuation constant becomes very large and non-propagation occurs.

(2) Power loss in waveguide walls :-

* In a waveguide the wave is propagated by reflections from walls. The Tangential component of electric field and ~~of~~ normal component of magnetic field develops losses in the walls. Due to this the average power in the waveguide is dissipated.

* The attenuation in waveguide "α_g" is given as

$$\boxed{\alpha_g = \frac{P_L}{2P_{tr}}}$$

P_L : power loss / unit length

P_{tr} : power transmitted through its waveguide.

(3) Misaligned waveguide sections :-

When the waveguide sections are joined and if the joint is not proper, (or) misaligned, there will be some loss due to the reflections.

⇒ Microwave transmission Lines :—

* Microwave propagates through components and devices that acts as a section of microwave transmission lines. which are broadly called as waveguides.

* The conventional openwire transmission lines are not suitable for microwave transmission.

* At microwave frequencies the following transmission lines will be used.

(i) Multi conductor lines:—

* Transmission lines, coaxial cables, strip line, microstrip lines and coplanar lines etc.,

(ii) Single Conductor lines (waveguides):-

* Rectangular waveguides and Ridge waveguides etc.,

(iii) Open Boundary Structures:—

* Dielectric rods and open waveguides etc.,

* Multi conductor lines support TEM (or) Quasi TEM modes propagation.

* The single conductor lines support TE (or) TM waves (Hybride HE modes as they are called).

3) Strip Lines and Micro Strip Lines :-

* Both strip lines and micro strip lines are miniature transmission lines which were developed mainly to takes full advantage of miniaturization

(i) Strip lines :-

* Strip lines has evolved from a coaxial cable.

* These are basically planar transmission lines that are widely used at frequencies from "100MHz to 100GHz".

*Construction :- It consists of a pair of flat metallic ground planes separated by a ~~diele~~ dielectric in the middle of which is embedded a thin metallic strip.

* Strip of width "w" which is greater than its thickness "T".

* The dielectric used is often Teflon, Aluminium (or) ~~silli~~ Silicon.

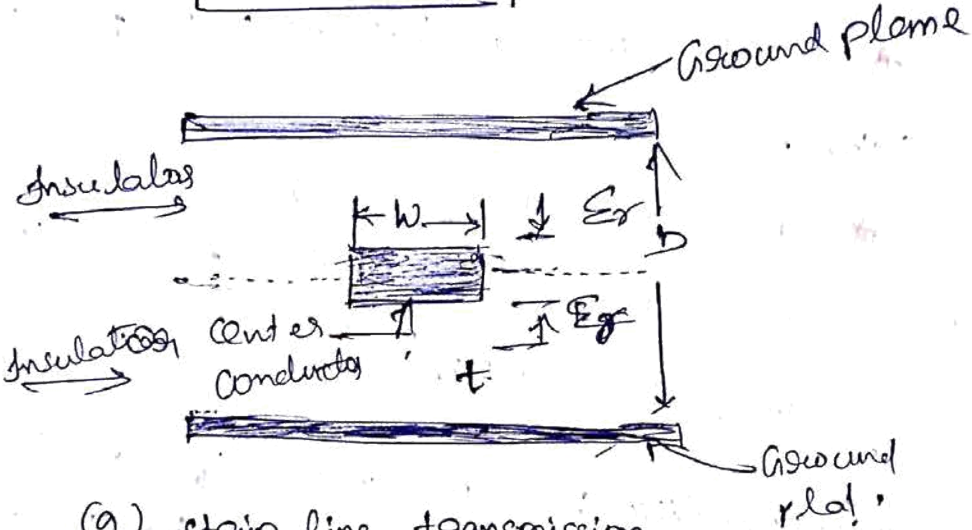
* The dominant mode for the stripline is TEM mode and the fields are ~~con~~ confined with in the transmission line with no radiation losses.

* Like coaxial cable and waveguides strip lines can also be used to build other microwave

microwave components such as - Isolators, circulators, Duplexers etc.,

* The guide wavelength (λ_g) in case of striplines is given by

$$\boxed{\lambda_g = \frac{\lambda_0}{\sqrt{\varepsilon_r}}}$$

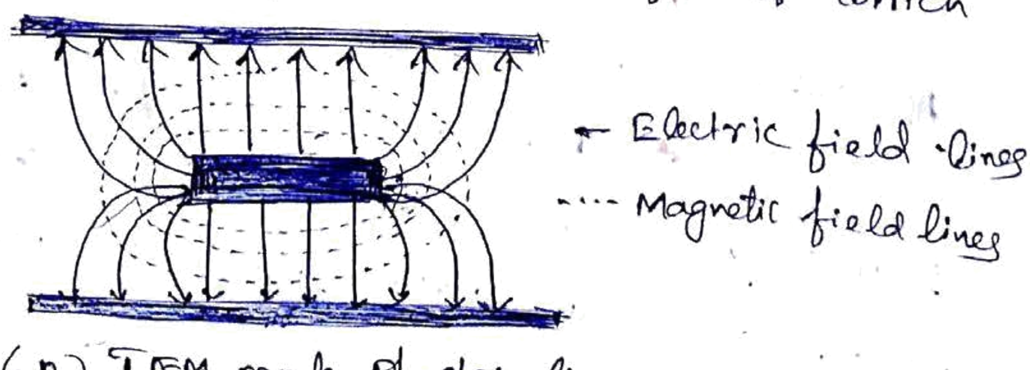


(a) strip line transmission Line.

→ Micro strips:-

* Microstrip has evolved from a parallel wire line.

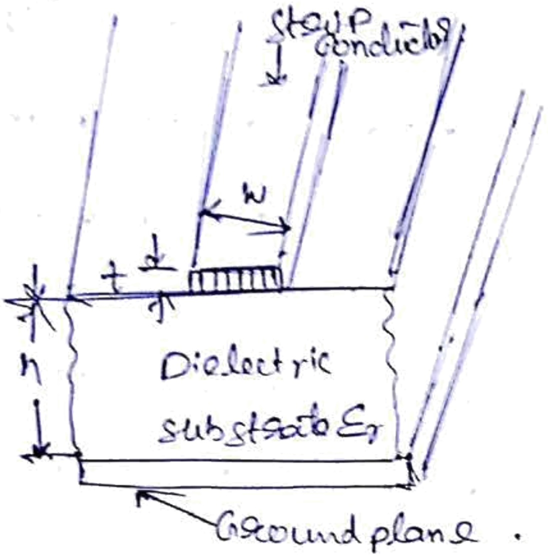
* It is unsymmetrical ~~stripedline~~ stripline i.e., nothing but a parallel plate transmission line having dielectric substrate, the one face of which



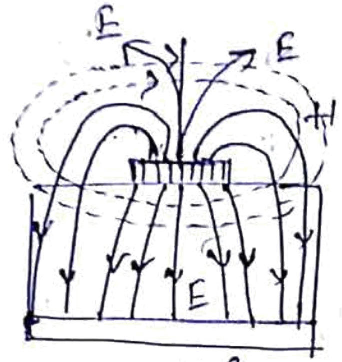
(b) TEM mode of strip line.

is metalized ground and other face has a thin conducting strip of certain width 'w' and thickness 't'.

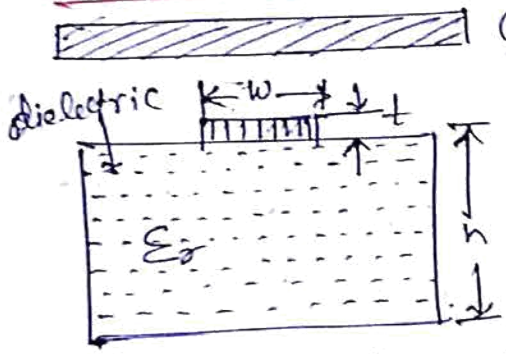
* The top ground plane is not present in a microstrip as compared to the strip line.



(a) Microstrip lines



(b) Field lines



fig(c) Microstrip line with a cover plate.

* Sometimes a coverplate is used for shielding purposes but it is kept much farther away from the ground plane. so as not to effect the microstrip field lines as shown in fig (c):-

* Disadvantages of striplines:-

→ The circuit is not accessible during development for adjustment and tuning

→ It is difficult to mount discrete and active components like transistors, diodes, circulators, chip resistors, chip capacitors etc.,

→ Advantages of Microstrip lines Over strip lines:-

1) Complete conductor pattern may be deposited and passed on a single dielectric substrate which is supported by single metal ground plane, thus fabrication is much easy and cost would be lower.

2) Due to the plane nature of microstrip structure any type of conductor chips can be ~~attenuated~~ attached to the microstrip element.

3) Also there is an easy access to the top surface making it easy to mount passive (or) active discrete devices and also for minor adjustments after the circuit has been fabricated.

⇒ Disadvantages:-

1) Due to the openness of microstrip line structure the higher radiation losses (or) interference due to the near by conductors will occur. this can be reduced by choosing thicker substrates with high dielectric constant.

→ 4) Z_0 Relations:-

* effective dielectric constant (ϵ_{re}):-

For a homogeneous dielectric medium, the propagation delay time per unit length is

$$T_{dh} = \sqrt{\mu \epsilon}$$

* where $\mu \rightarrow$ permittivity of the medium.

$\varepsilon \rightarrow$ permitivity of the medium.

* In free space, the propagation delay time is

$$\boxed{T_{df} = \sqrt{\mu_0 \varepsilon_0} = 3.333\ ns/m\ (or)\ 1.016\ ns/ft}$$

* where

$\mu_0 \rightarrow 4\pi \times 10^{-7}\ H/m\ (or)\ 3.83 \times 10^{-7}\ H/ft$

$\varepsilon_0 \rightarrow 8.854 \times 10^{-12}\ F/m\ (or)\ 2.69 \times 10^{-12}\ F/ft$

* In transmission lines in a non magnetic medium the propagation delay time is

$$\boxed{T_d = 1.016\sqrt{\varepsilon_r}\ ns/ft}$$

$$\left(\because T_d = \sqrt{\mu\varepsilon} = \sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r} = 1.016\sqrt{\varepsilon_r} \right)$$

* Digiacomo and his coworkers ~~disconnected~~ discovered an emperical eq'n for the effective relative dielectric constant (ε_{re}) of a microstip lines by measuring the propagation delay time and the relative dielectric constant (ε_r) of several board materials.

* The empirical equation is

$$\boxed{\varepsilon_{re} = 0.475\,\varepsilon_r + 0.67} \quad \text{--- (1)}$$

* where $\varepsilon_r \rightarrow$ Relative ~~permittivity~~ of dielectric constant of the board material.

$\varepsilon_{re} \rightarrow$ effective relative dielectric constant for a microstrip line.

line and the ground plane (h), and the homogeneous dielectric constant of the board material (ε_r) (6)

* The method used to derive characteristic impedance (Z_o) equation of a microstrip line from a well known equation & make some changes is called a "Comparative (or) an indirect method"

* The well known equation of the characteristic impedance (Z_o) of a wire-over-ground transmission line is given as

$$\boxed{Z_o = \frac{60}{\sqrt{\varepsilon_r}} \ln\left(\frac{4h}{d}\right)} \quad \text{for } h >> d \quad (2)$$

* where

$\varepsilon_r \rightarrow$ dielectric constant of the ambient medium

$h \rightarrow$ the height from the centre of the wire to the ground plane.

$d \rightarrow$ Diameter of the wire.

* If the effective (or) equivalent values of the relative dielectric constant (ε_r) of the ambient medium and the diameter 'd' of the wire can be determined for the microstrip line, the characteristic impedance (Z_o) of the microstrip line can be calculated.

* Transformation of a rectangular conductor into an equivalent circular conductor:-

* The cross-section of a microstrip line is rectangular so the rectangular conductor must be transformed into an equivalent circular conductor spring field discovered an