UNIT-I

Communication

It is the process of establishing connection or link between two points. It is the process of conveying message at a distance or the basic process of exchanging information. The electronic equipment used for communication purpose is called communication equipment. The equipment when assembled together forms a communication system.

Ex:- Line Telephony & Telegraphy, Radio broadcasting, point to point communication, mobile communication Computer communication, Radar communication, TV broadcasting etc.

Elements of a Communication system

Communication involves the transmission of information from one point source to another through a succession of process listed below.

i, the generation of a thought pattern or image in the mind of an originator.

ii, Description of that image with a certain measure of precision, by a set of aural or visual symbols.

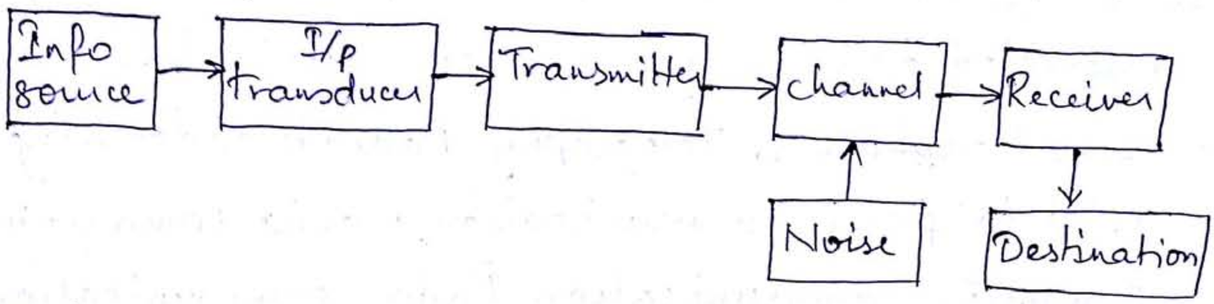
iii, encoding of these symbols in a form that is suitable for transmission over a physical medium.

iv, the decoding and reproduction of the original symbols.

v, the recreation of the original thought pattern or image with a definable degradation in quality, in the mind of a recipient.

Hence, the purpose of a communication system is to transmit an information bearing signal, from a source, located at one point, to a user or destination, located at another point some distance away.

Figure shows the block diagram of a general communication system in which the different elements are represented by blocks.



Info. source :- Communication system serves to communicate a message or information. This message or information originates in the information source. There can be various messages in the form of words, groups of words, code symbols, sound signal etc. Out of these messages, only the desired message is selected and conveyed or communicated.

I/p Transducer :- Transducer is a device which converts one form of energy into another form. The message from the information source may or may not be in electrical form. If the message is not electrical in form, an input transducer is used to convert it into a time varying electrical signal.

Ex:- In radio broadcasting, a microphone converts the information or message which is in the form of sound waves into electrical signals.

Transmitter :- The transmitter processes the electrical signal. For example, in radio broadcasting, the electrical signal obtained from sound signal is processed to restrict its range of audio frequencies (upto 5KHz in AM radio broadcast) and is amplified. In wire telephony, no processing is needed. In long distance radio communication or broadcast, signal amplification is necessary before modulation.

Modulation is the main function of transmitter. In modulation, the message signal is superimposed upon the high frequency carrier. Hence inside the transmitter, signal processing such as restriction of range of audio frequencies, amplification & modulation are achieved to ease the transmission of the signal through the channel.

Channel and noise :- Channel means the medium through which the message travels from the transmitter to the receiver. The channel provides a physical connection between the transmitter and the receiver.

During the process of transmission and reception the signal gets distorted due to noise which is an unwanted signal which tends to interfere with the required signal. It is random in nature.

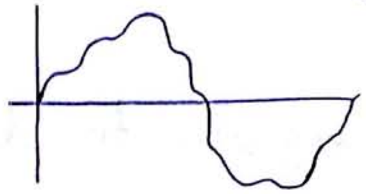
Receiver :- It reproduces the message signal in electrical form from the distorted received signal. This reproduction is accomplished by a process known as demodulation or detection. This is the reverse process of modulation carried out in transmitter.

Destination :- This is the final stage which is used to convert an electrical message signal into its original form. Ex:- In radio broadcasting, the destination is a loudspeaker which works as a transducer i.e it converts the electrical signal into original sound signal.

Analog and digital signals

Analog signal

It varies smoothly and continuously with time, i.e the analog signals are defined for every value of time and they take continuous values in a given time interval. Hence analog messages are characterized by data whose values vary over a continuous range.

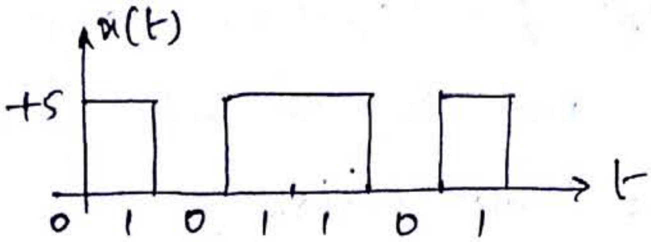


Ex:- Temperature or atmospheric pressure of a location may vary over a continuous range and may assume infinite no. of possible values.

Digital signal

Signal representation of a sequence of numbers, each number representing the signal magnitude at an instant of time is known as a digital signal.

Ex:- Printed language consists of 26 letters, 10 numbers, a space and several punctuation marks. Hence any text is a digital message constructed from about 50 symbols.



Since a digital signal is represented only by digits, we can use number system to represent a digital signal. But practically binary system is used to represent a digital signal, where each digit takes on only one of two possible values 0 and 1.

The above waveform is a pulse train with 0V representing 0 or logic 0 and +5V representing logic 1.

Conversion of analog signals to digital signals

In communication systems, if analog signal is available and a digital signal is to be transmitted, in such cases, we have to convert an analog signal into digital signal. i.e a continuous time signal is to be converted in the form of digits.

Consider the analog signal shown below in fig (a)

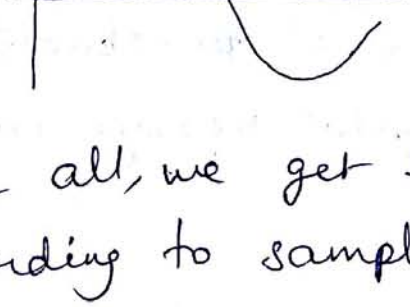
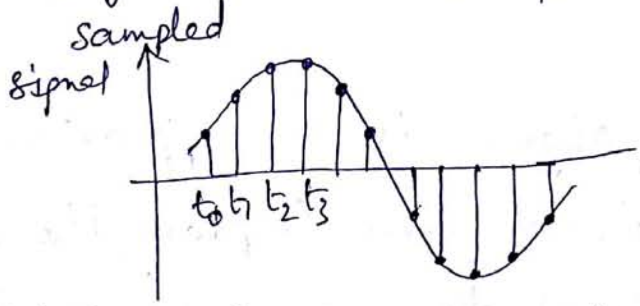


fig (a) Analog signal

First of all, we get the sample of this signal according to sampling theorem. Mark the time instants t_0, t_1, t_2 and so on, at equal time intervals along the time axis.

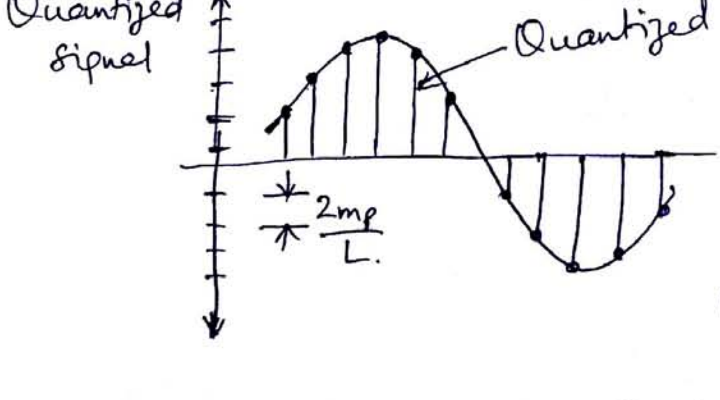
At each of these time instants, the magnitude of the signal is measured and thus samples of the signal are taken. fig(b) below shows a representation of the signal in terms of its samples.



(b). samples of analog signal

The figure(b) shows the discrete time signal. However, since the magnitude of each sample can take any value in a continuous range, the signal is still an analog signal. This analog signal can be converted into digital form by a process known as quantization. In quantization, the total amplitude range which the signal may occupy is divided into a number of standard levels.

As shown in fig(c), amplitudes of the signal $x(t)$ lie in the range $(-m_p, m_p)$ which is partitioned into L intervals, each of magnitude $\Delta v = \frac{2m_p}{L}$



(c) Quantized signal

Now each sample is approximated or rounded off to the nearest quantized level as shown in fig (c). Since each sample is now approximated to one of the L numbers, therefore the information is digitized.

The quantized signal is an approximation of the original one. we can improve the accuracy of the quantized signal to any desired degree by increasing the number of levels L.

Analog and Digital Communication

Depending upon the message signal, communication may be classified as

i) Analog communication

ii) Digital communication.

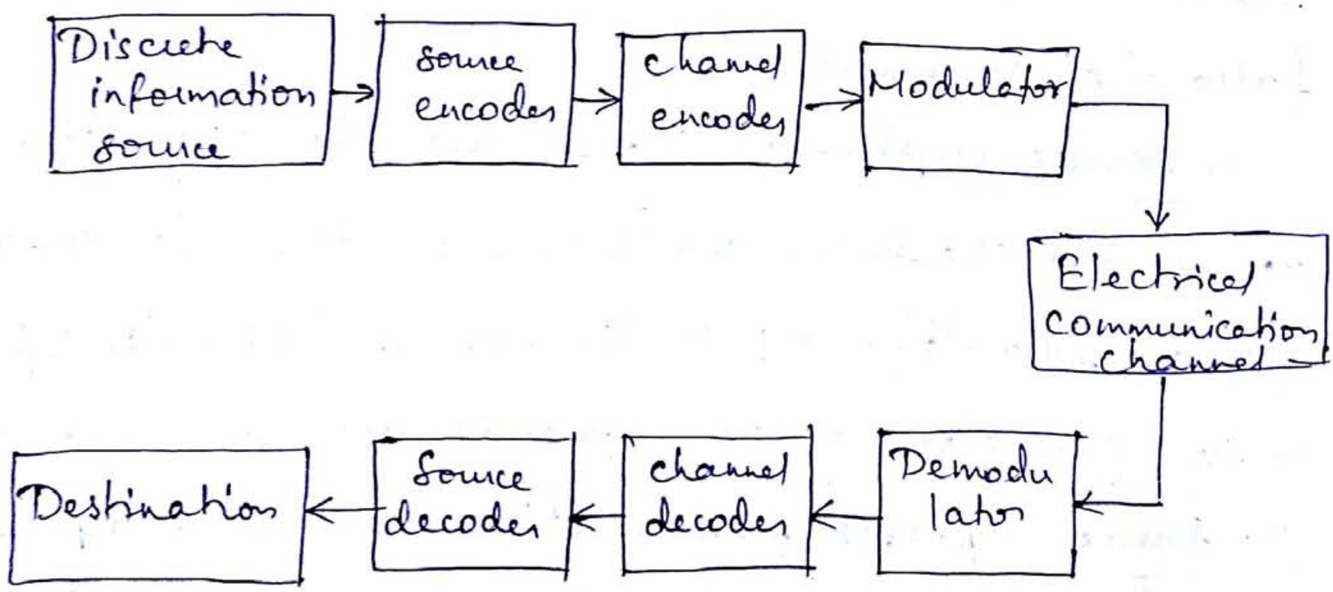
In Analog communication, the message or information to be transmitted is analog in nature. The analog message signal modulates a high freq. carrier to produce modulated signal.

Digital Communication

In digital communication, the message signal to be transmitted is digital in nature.

Model of a Digital Communication system

Fig. shows the model of a digital communication system. The overall purpose of the system is to transmit the message or sequences of symbols coming out of a source to a destination point at a high rate and accuracy. The source & destination point are physically separated in space and a communication channel connects the source to the destination point.



Model of a digital communication system

Discrete info. source :- Info. source may be classified as analog and discrete information source. In case of analog communication, the information source is analog. Analog information sources such as microphone actuated by speech emit one or more continuous amplitude signals.

In case of digital communication, the info source produces a message signal which is continuously varying with time. An analog info source may be transformed into a discrete info source through the process of sampling & quantization. Discrete info. sources are characterized by the following parameters.

i) source alphabet :- These are the letters, digits or special characters available from the info. source

ii) symbol rates :- It is the rate at which the info source generates source alphabets, units are symbols/sec

iii) Source alphabet probabilities :- Each source alphabet from the source has independent occurence rate in the sequence. Ex:- the letters A, E, I etc. occur frequently capacity of each source alphabet is different in a particular sequence. This parameter defines average info content of the symbols. Entropy describes the average info content per symbol, it is defined in terms of bits per symbol. The source information rate is the product of symbol rate and source entropy.

$$\underset{\text{bits/sec}}{\text{Info. rate}} = \underset{\text{symbols/sec}}{\text{symbol rate}} \times \underset{\text{bits/symbol}}{\text{source entropy}}$$

Source encoder and decoder

The symbols produced by the info. source are given to the source encoder. These symbols cannot be transmitted directly. They are first converted into digital form (binary sequence of 1's and 0's) by the source encoder. Each binary '1' and '0' is known as a bit. The group of bits is called codeword.

The source encoder assigns codewords to the symbols. For each distinct symbol, there is a unique codeword. The codeword can be of 4, 8, 16 or 32 bit length. As the number of bits are increased in each codeword, the symbols that may be represented are also increased.

Ex:- 8 bits would have 2^8 i.e 256 distinct codewords.

This means that 8 bits may be used to represent 256 symbols and 16 bits represent $2^{16} = 65536$ symbols. Typical source encoders are pulse code modulators, delta modulator etc. Source encoders must have the following important parameters.

i) Block size :- Block size describes the maximum number of distinct codewords which can be represented by a source encoder. This depends on the number of bits in the codeword. As an example, the block size of 8-bits source encoder will be 2^8 i.e 256 code words.

ii) codeword length :- Codeword is the number of bits used to represent each codeword. As an example if 8 bits are assigned to each code word, then the codeword length will be 8 bits.

iii) Average data rate :- Average data rate is the output bits per second from the source encoder. The source encoder assigns multiple number of bits to each input symbol.

Ex:- If we consider that the symbols are given to the source encoder and the length of the code word is 8 bits, then the o/p data rate from source encoder is

$$\text{Data rate} = \text{symbol rate} \times \text{code word length}$$
$$= 10 \times 8 = 80 \text{ bits/second}$$

iv) <u>Efficiency of the encoder</u> :- The efficiency of the encoder is the ratio of minimum source information rate to the actual output data rate of the source encoder.

<u>Source decoder</u> :- At the receiver end, some sort of decoder is used to perform the reverse operation to that of source encoder. It converts the binary output of the channel decoder into a symbol sequence. Some decoders also use memory to store 'codewords. The decoders and encoders can be synchronous or asynchronous.

<u>Channel encoder and Decoder</u> :-

After converting the message or info. signal in the form of binary sequence by the source encoder, the signal is transmitted through the channel. The channel adds noise and interference to the signal being transmitted. Hence errors are introduced.

Hence channel coding is done to avoid errors. The channel encoder adds some redundant binary bits to the input sequence. These redundant bits are added with some predefined logic.

For example, let us consider that the codeword from the source encoder to make it 4-bits long. This fourth bit is added (i.e 1 or 0) in such a manner that the number of 1's in the encoded word

remain even (also known as even parity)
The table below gives the output of a source encoder, the fourth bit depending on the parity and the output of channel encoder.

o/p of source encoder			Bit to be added by the channel encoder for even parity	o/p of channel encoder			
b_3	b_2	b_1	b_0	b_3	b_2	b_1	b_0
1	0	0	0	1	1	0	0
0	1	0	1	0	1	0	1
0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1

It may be observed from the table that each codeword at the o/p of channel encoder contains even number of 1's. Now at the receiver end, if odd number of 1's are detected, then the receiver comes to know that there is an error in the received signal.

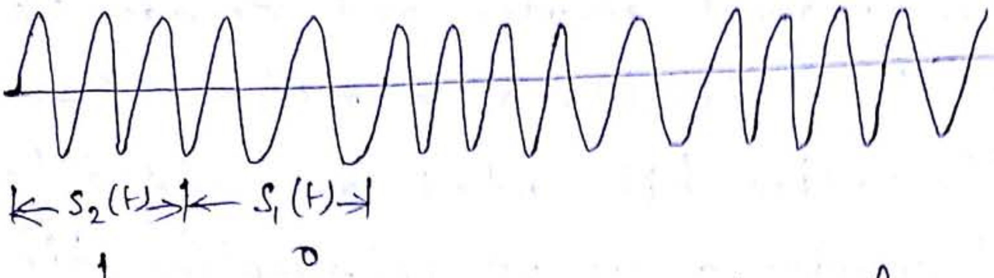
The channel decoder at the receiver is thus able to reconstruct error free accurate bit sequences and reduce the effects of channel noise and distortion.

Hence the channel encoder and decoder serve to increase the reliability of a received signal. However the extra bits which are added by the channel encoder carry no information, they are only used by the channel decoder to detect and correct errors, if any.

Digital modulators and demodulators :-

If the modulating signal is digital (binary code) then digital communication techniques are used. The carrier signal used by digital modulators is always continuous sinusoidal wave of high frequency. The digital modulators map the input binary sequence of 1's and 0's to the analog signal waveforms.

For example, if one bit at a time is transmitted, then digital modulator signal is $s_1(t)$ to transmit binary '0' and $s_2(t)$ for binary '1' as shown in figure below.



Here the signal $S_1(t)$ has low frequency compared to $S_2(t)$. Hence, even though the modulated signal seems to be continuous, the modulation is discrete (i.e in steps). This means that a signal carrier is converted into two wave forms $S_1(t)$ and $S_2(t)$ because of digital modulation.

Now if the codeword consists of 2 bits and they are to be transmitted at a time, then there would be 2^2 i.e 4 distinct symbols i.e code words. Thus these code words will require four distinct waveforms for transmission. Such types of modulators are known as M-ary modulators. Amplitude shift keying (ASK), phase shift keying (PSK), frequency shift keying (FSK), differential phase shift keying (DPSK) and minimum shift keying (MSK) are examples of various digital modulators.

At the receiver end, the digital demodulator converts the input modulated signal into the sequence of binary bits.

Communication channel

The connection between transmitter & receiver is established through a communication channel. The communication can take place through wire lines, wireless or fibre optic channels. The other media such as optical disks, magnetic tapes and disks etc. may also be called as a communication channel since they can also carry data through them.

The problems in a comm. channel are

i) Signal attenuation :- Attenuation in channel occurs due to the internal resistance of channel and fading of the signal.

ii) Amplitude and phase distortion :- The transmitted signal is distorted in amplitude and phase due to non linear characteristics of the channel.

iii) Additive noise interference :- Interference is produced due to internal solid state devices.

iv) Multipath distortion :- Multipath distortion occurs mostly in wireless communication channels.

Advantages and Disadvantages of digital Communication

Advantages

1) the digital communication systems are simpler and cheaper compared to analog communication systems because of the advances made in IC technology.
2) In digital communication, the speech, video and other data may be merged and transmitted over a common channel using multiplexing.
3) Using data encryption, only permitted receivers may be allowed to detect the transmitted data. This property is of most importance in military applications.
4) Since the transmitted signal is digital in nature, large amount of noise interferences may be tolerated.
5) Since the transmission is digital and channel encoding is used, the noise does not accumulate from repeater to repeater in long distance communication.
6) Since channel coding is used, the errors may be detected and corrected in receivers.

7) Digital Communication is adaptive to other advanced branches of data processing such as Digital Signal Processing, image processing and data compression etc.

Disadvantages

1) Due to analog to digital conversion, the data rate becomes high. therefore more transmission bandwidth is required for digital communication.

2) Digital Communication needs synchronization in case of synchronous modulation.

Certain issues in Digital Transmission

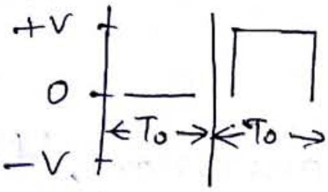
I) Line coding

Analog signal is represented in binary in form of two symbols 0 and 1. For electrical voltages to be transmitted over an electrical line, line coding is done

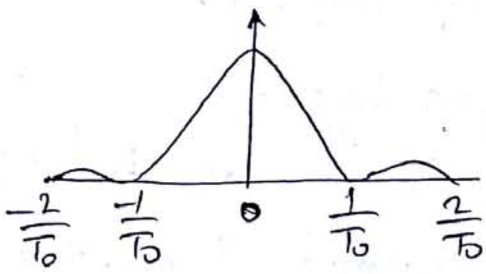
Digital presentations of data

a) Unipolar NRZ :- This code is unipolar as logic '0' is represented by 0V and logic '1' by a constant signal level, say +V during its entire bit interval and hence called non return to zero.

This code has high dc as well as low freq components. A long string of 1's and 0's make clock recovery difficult and cause synchronization problem.



a) Timing diagram



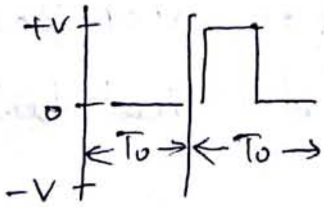
b) power spectral density

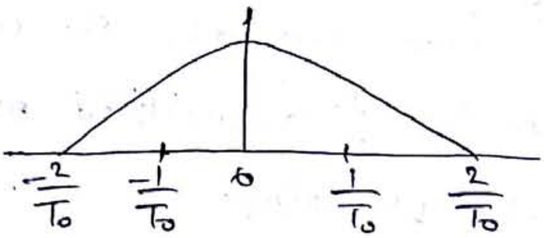
b) Unipolar RZ

This code is unipolar as the excursion is between 0V and +V. But logic 1 is represented by a pulse which returns to zero after half bit period.

a) Timing diagram

b) PSD

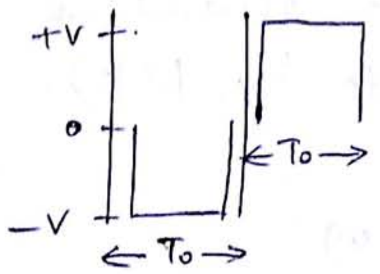




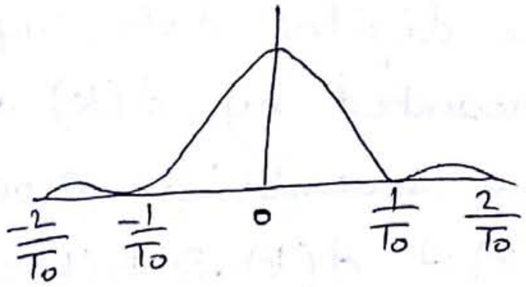
c) Bipolar NRZ

This code is bipolar as the excursion is between +V and -V. But the pulses do not return to zero and stay at that level for the entire bit duration. This code performs better in presence of noise.

a) Timing diagram



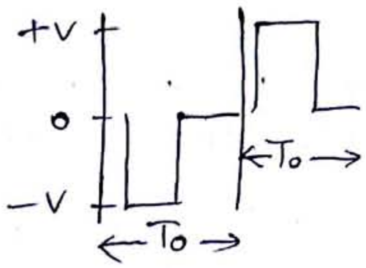
b) PSD



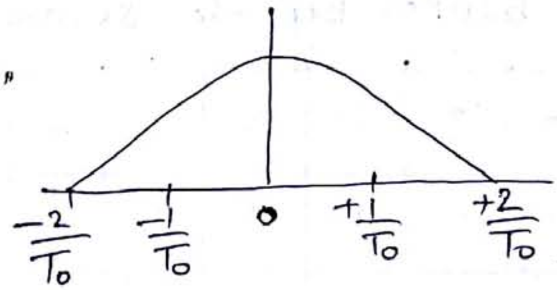
d) Bipolar RZ

This code is bipolar as the excursion is between +v and −v. Both logic 1 and logic 0 are represented by pulses that return to zero within the bit interval. This code performs better in presence of noise

a) Timing diagram



b) PSD



II) Scrambling

Scrambling is a process by which the data is randomized. It is difficult to decipher a scrambled data unless we do its reverse operation through unscrambling. Scrambler removes a pattern while randomizing the data. Thus it removes long string of 1's or 0's which can help clock recovery and synchronization.

Example

Let the digital data input to the scrambler be represented by d(k) and its o/p by b(k). Define a scrambling process as

$$b(k) = d(k) \oplus b(k-2) \oplus b(k-4)$$

where b(k-j) represents the j^{th} last value of b(k). Consider the input data as 1's of 14-bits and past values of b(k) are all zeros.

Sol:- The table shows the o/p of the scrambler. The EX-OR operation of three variables of above equation gives b(k)=1 whenever there are odd no. of 1's present. We can see that the long string of 1's are broken by the scrambler.

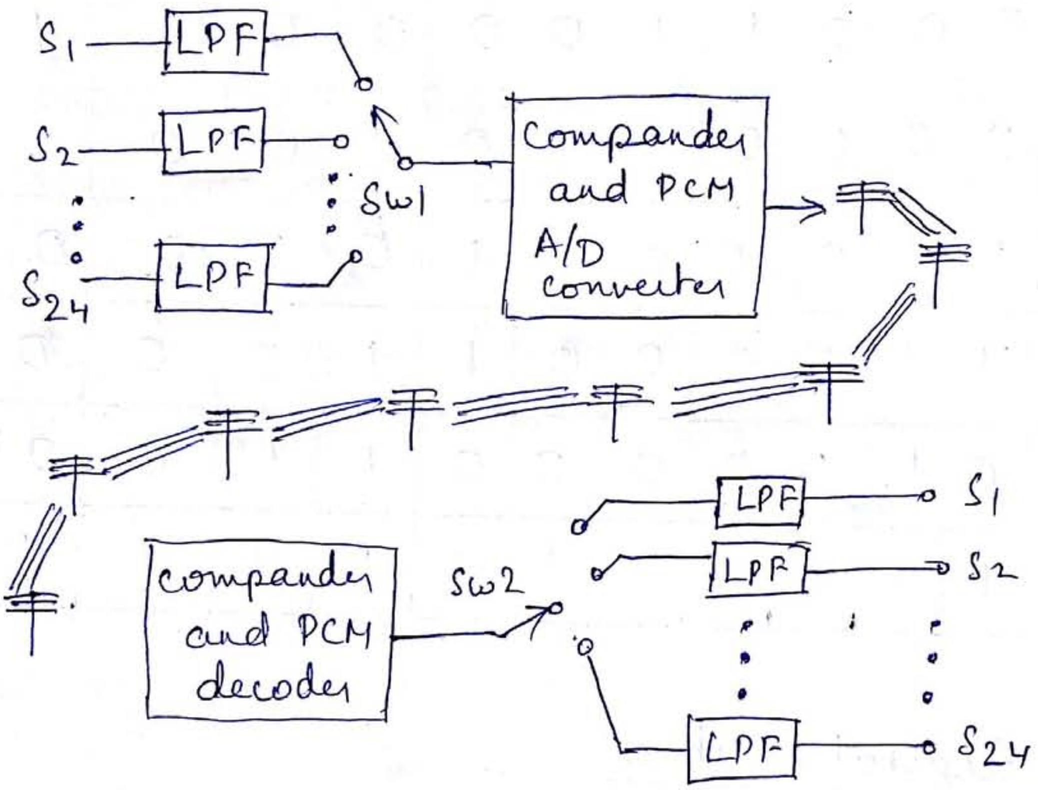
K	1	2	3	4	5	6	7	8	9	10	11	12	13	14
d(k)	1	1	1	1	1	1	1	1	1	1	1	1	1	1
b(k-4)	0	0	0	0	1	1	0	0	0	0	1	1	0	0
b(k-3)	0	0	0	1	1	0	0	0	0	1	1	0	0	0
b(k-2)	0	0	1	1	0	0	0	0	1	1	0	0	0	0
b(k-1)	0	1	1	0	0	0	0	1	1	0	0	0	0	1
b(k)	1	1	0	0	0	0	1	1	0	0	0	0	1	1

The unscrambling process is the reverse of the scrambling process obtained as

$$\hat{d}(k) = b(k) \oplus b(k-2) \oplus b(k-4)$$

K	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$b(k)$	1	1	0	0	0	0	1	1	0	0	0	0	1	1
$b(k-1)$	0	1	1	0	0	0	0	1	1	0	0	0	0	1
$b(k-2)$	0	0	1	1	0	0	0	0	1	1	0	0	0	0
$b(k-3)$	0	0	0	1	1	0	0	0	0	1	1	0	0	0
$b(k-4)$	0	0	0	0	1	1	0	0	0	0	1	1	0	0
$\hat{d}(k)$	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Each of the TDM signals are A/D converted and companded (compress + expand). The resulting digital waveform is transmitted over a coaxial cable. The cable minimizes signal distortion and suppresses noise from external sources.



At approximately 6000 feet intervals, the signal is regenerated by amplifiers called repeaters and then sent on to the destination. The repeater eliminates the effect of distortion introduced by the channel and it also eliminates noise.

IV) Multiplexing the T1 lines – The T1, T2, T3 and T4 lines

To take further advantage of the merits of TDM and digital transmission, the common carriers are further multiplexed. Four T1 lines are multiplexed in an M_{12} multiplexer to generate a T2 transmission system, seven T2 lines convert to a T3 line in an M_{23} multiplexer and six T3 lines convert to a T4 line in an M34 multiplexer.

Advantages of digital Communication systems

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2) In digital comm., the speech, video and other data may be merged and transmitted over a common channel using multiplexing.
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7) Digital comm. is adaptive to advanced branches of data processing such as digital signal processing, image processing and data encryption.

Disadvantages

1) Due to analog to digital conversion, the data rate becomes high, hence more transmission bandwidth is required for digital communication.

2) Digital Comm. needs synchronization in case of synchronous modulation.

Shannon's theorem on channel capacity

Hartley-Shannon law

When Shannon's theorem of channel capacity is applied specifically to a channel in which the noise is Gaussian, it is known as Hartley-Shannon law.

$$C = B \log_2 \left(1 + \frac{S}{N}\right)$$

Signal power

$$S = \int_{-B}^{B} \text{Power spectral density of signal}$$

$$\text{PSD of white gaussian noise} = \frac{N_0}{2}$$

$$N = \int_{-B}^{B} \frac{N_0}{2}\, df = \frac{N_0}{2} \int_{-B}^{B} df = \frac{N_0}{2} \left[f\right]_{-B}^{B} = N_0 B$$

Bandwidth – S/N ratio Trade off

The channel capacity of the gaussian channel is given as $C = B \log_2 \left(1 + \frac{S}{N}\right)$

The channel capacity depends on 2 factors

(i) Bandwidth (B)

(ii) S/N ratio

Noiseless channel

If there is no noise in the channel, then $N = 0$

Hence $\frac{S}{N} = \infty$, Such a channel is called a noiseless channel. The capacity of such channel is

$$C = B \log_2 (1 + \infty) = \infty$$

Thus a noiseless channel has infinite capacity.

Noisy channel

Here $N \neq 0$

and $\frac{S}{N} \neq \infty$

$\therefore \quad C = B \log_2 \left(1 + \frac{S}{N}\right)$

Hence the noisy channel has a finite capacity.

problem

Data is to be transmitted at the rate of 10,000 bits/sec. over a channel having bandwidth B of 3 KHz. Determine the S/N ratio required. If the bandwidth is increased to 10 KHz, then determine the S/N ratio.

sol:- The data is to be transmitted at the rate of 10,000 bits/sec, hence the channel capacity must be atleast 10,000 bits/sec for error free transmission.

$$C = B \log_2 \left(1 + \frac{S}{N}\right)$$

$$C = 10000 \quad , B = 3000$$

$$10000 = 3000 \log_2 \left(1 + \frac{S}{N}\right) \; ; S/N = 9$$

If $B = 10000$

$$10000 = 10000 \log_2 \left(1 + \frac{S}{N}\right) \quad , S/N = 1$$

Hence if Bandwidth is increased, S/N is reduced by 9 times. i.e if signal power is reduced, band width is increased.

Sampling theorem

Statement of sampling theorem

1) A bandlimited signal of finite energy which has no frequency components higher than W Hertz is completely described by specifying the values of the signal at instants of time called sampling instants.
2) A bandlimited signal of finite energy which has no frequency components higher than W Hz may be completely recovered from the knowledge of its samples taken at the rate of 2W samples per second.

The first part of the statement tells about the sampling of the signal and the second part tells about the reconstruction of the signal.

Combined statement

A continuous time signal can be completely represented in its samples and recovered back if the sampling frequency is twice of the highest frequency content of the signal i.e

$$f_s \geq 2W$$

where f_s = sampling frequency

W = higher frequency content

proof of sampling theorem

Part I

Representation of $x(t)$ in terms of its samples

Step 1 :- Define $x_\delta(t)$

The sampled signal $x_\delta(t)$ is given as

$$x_\delta(t) = \sum_{n=-\infty}^{\infty} x(t)\,\delta(t-nT_s)$$

$x_\delta(t)$ is the product of $x(t)$ and impulse train $\delta(t-nT_s)$, which indicates samples placed at $\pm T_s, \pm 2T_s, \pm 3T_s$ and so on.

Step 2 :- Fourier transforms.

$$x(t) \xleftrightarrow{FT} X(f)$$

$$x_\delta(t) \xleftrightarrow{FT} \sum_{n=-\infty}^{\infty} x(t)\,\delta(t-nT_s)$$

$$\delta(t-nT_s) \longleftrightarrow f_s \sum_{n=-\infty}^{\infty} \delta(f-nf_s)$$

$$\therefore X_\delta(f) = FT\left\{x(t)\,\delta(t-nT_s)\right\}$$

$$= FT\{\text{product of } x(t) \text{ and impulse train}\}$$

Fourier transform of product in time domain becomes convolution in frequency domain.

$$X_\delta(f) = FT\left\{x(t)\right\} * FT\left\{\delta(t-nT_s)\right\}$$

$$= X(f) * f_s \sum_{n=-\infty}^{\infty} \delta(f-nf_s)$$

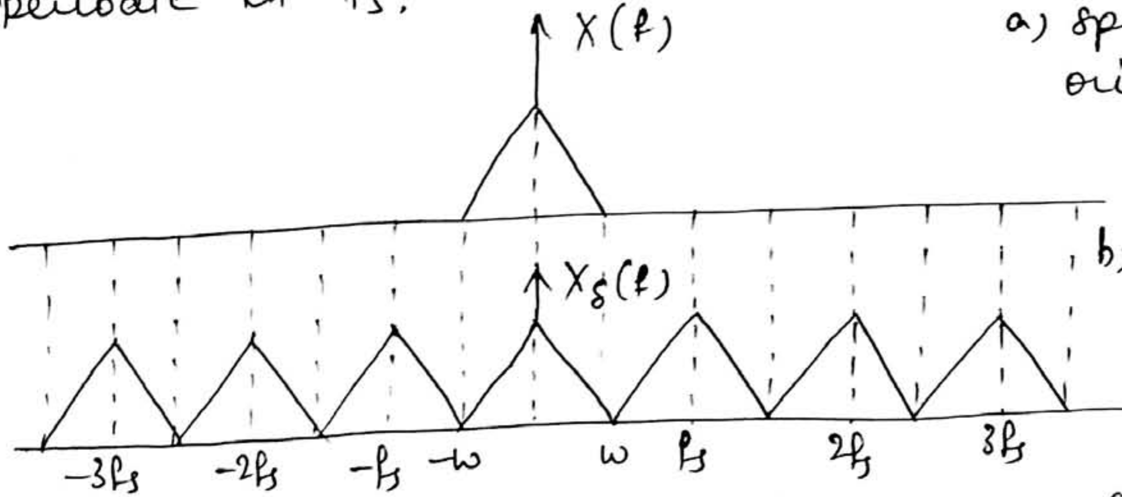
Applying the shifting property of impulse function and linearity property

$$X_\delta(f) = f_s \sum_{n=-\infty}^{\infty} X(f) * \delta(f - nf_s)$$

$$= f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)$$

$$= \text{----}\ f_s X(f+2f_s) + f_s X(f+f_s) + f_s X(f) + f_s X(f-f_s) + f_s X(f-2f_s) + \text{-----}$$

The above equation shows that $X(f)$ is placed at $\pm f_s, \pm 2f_s, \pm 3f_s$ --- which means that $X(f)$ is periodic in f_s.



a) spectrum of original signal

b) spectrum of sampled signal

<u>step 3</u> :- Relation between $X(f)$ and $X_\delta(f)$

Let us assume $f_s = 2w$

$$X_\delta(f) = f_s X(f)$$

or $$X(f) = \frac{1}{f_s} X_\delta(f)$$

<u>step 4</u> :- Relation between $x(t)$ and $x(nT_s)$

$$X(f) = \sum_{n=-\infty}^{\infty} x(t)\, e^{-j2\pi f t}$$

In this equation f = freq of CT signal (continuous time)

$\frac{f}{f_s}$ = freq of discrete time signal

Substitute $x(t) = x(nT_s)$ (samples of $x(t)$

$$X_\delta(f) = \sum_{n=-\infty}^{\infty} x(nT_s)\, e^{-j2\pi \frac{f}{f_s} \cancel{t}} \qquad T_s = \frac{1}{f_s}$$

$$t = n$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s)\, e^{-j2\pi f n T_s}$$

The above equation holds for $f_s = 2w$, i.e if samples are taken at the rate of $2w$ or higher. $x(t)$ can be completely represented by its samples.

Part II

Reconstruction of $x(t)$ from its samples.

Step 1 :- Take inverse fourier transform of $X(f)$ which is in terms of $X_\delta(f)$.

$$X_\delta(f) = \sum_{n=-\infty}^{\infty} x(nT_s)\, e^{-j2\pi f n T_s}$$

$$X(f) = \frac{1}{f_s} \sum_{n=-\infty}^{\infty} x(nT_s)\, e^{-j2\pi f n T_s}$$

$$x(t) = IFT\{X(f)\} = IFT\left\{\frac{1}{f_s} \sum_{n=-\infty}^{\infty} x(nT_s)\, e^{-j2\pi f n T_s}\right\}$$

$$= \int_{-\infty}^{\infty} \left\{\frac{1}{f_s} \sum_{n=-\infty}^{\infty} x(nT_s)\, e^{-j2\pi f n T_s}\right\} e^{j2\pi f t}\, df$$

Interchanging the order of summation and integration

$$\cancel{x(t) = \int_{-\infty}^{\infty} \frac{1}{f_s} \sum_{n=-\infty}^{\infty}}$$

Integration can be taken from $-w \le f \le w$.

$$x(t) = \sum_{n=-\infty}^{\infty} x(nT_s) \frac{1}{f_s} \int_{-w}^{w} e^{j2\pi f.(t-nT_s)} \, df.$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s) \cdot \frac{1}{f_s} \left[\frac{e^{j2\pi f(t-nT_s)}}{j2\pi(t-nT_s)} \right]_{-w}^{w}$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s) \cdot \frac{1}{f_s} \left[\frac{e^{j2\pi w(t-nT_s)} - e^{-j2\pi w(t-nT_s)}}{j2\pi(t-nT_s)} \right]$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s) \cdot \frac{1}{f_s} \frac{\sin 2\pi w(t-nT_s)}{\pi(t-nT_s)}$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s) \frac{\sin \pi(2wt - 2wnT_s)}{\pi(f_s t - f_s nT_s)}.$$

Here $f_s = 2w$, $T_s = \frac{1}{f_s} = \frac{1}{2w}$

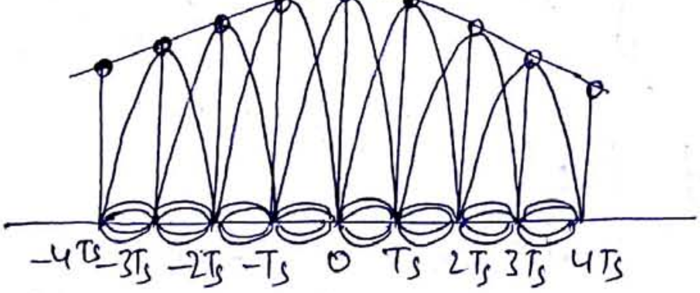
$$x(t) = \sum_{n=-\infty}^{\infty} x(nT_s) \frac{\sin \pi(2wt - n)}{\pi(2wt - n)}$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s) \operatorname{sinc}(2wt - n) \qquad \left[\because \frac{\sin \pi\theta}{\pi\theta} = \operatorname{sinc}\theta \right]$$

<u>Step 2</u> Expanding the above equation

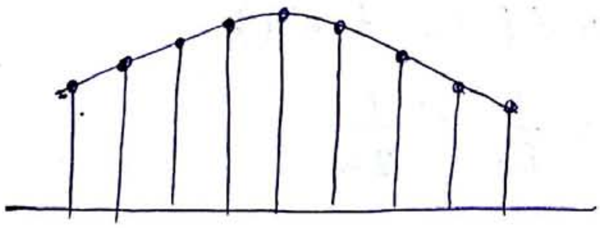
$$x(t) = \cdots x(-2T_s)\operatorname{sinc}(2wt+2) + x(-T_s)\operatorname{sinc}(2wt+1) + x(0)\operatorname{sinc}(2wt) + x(T_s)\operatorname{sinc}(2wt-1) + x(2T_s)\operatorname{sinc}(2wt-2) + \cdots$$

Here the samples are weighted by sinc functions. The sinc function is the interpolating function. The fig shows how $x(t)$ is interpolated.



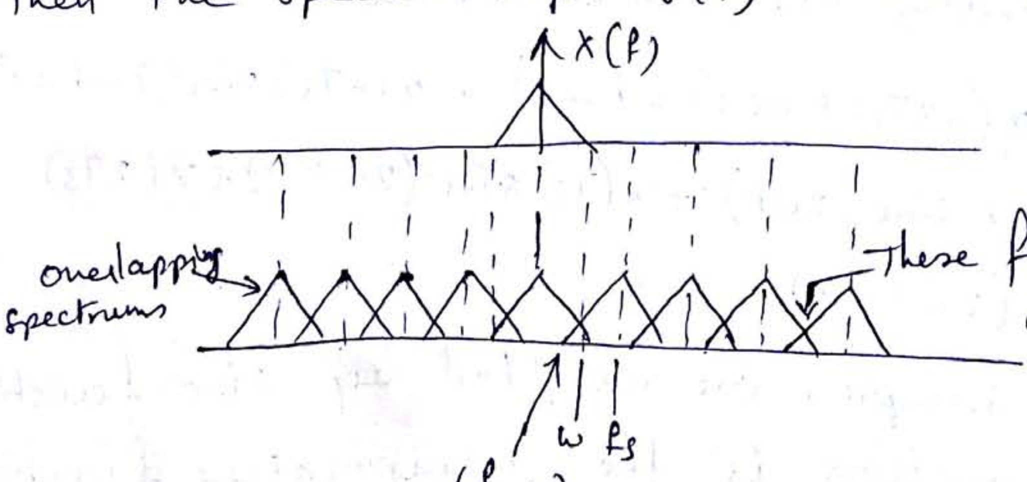
Step 3 :- Reconstruction of $x(t)$ by low pass filter.

when the interpolated signal is passed through the low pass filter of bandwidth $-w \le f \le w$ then the reconstructed wave form is obtained



Effects of Undersampling (Aliasing)

while proving the sampling theorem, we considered that $f_s = 2w$. Consider the case of $f_s < 2w$ Then the spectrum of $X_\delta(f)$ will be as follows.



The spectrums located at $X(f)$, $X(f-f_s)$, $X(f-2f_s)$ overlap each other.

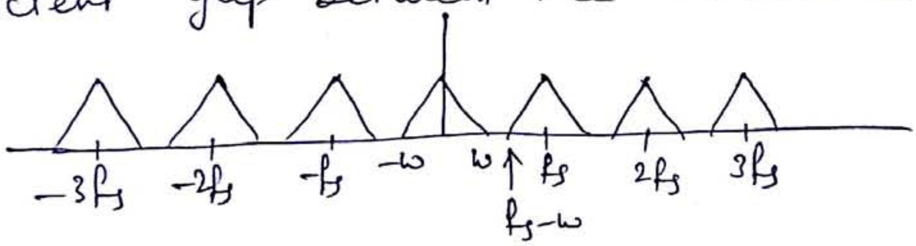
<u>Aliasing</u> :- When the high frequency interferes with low frequency and appears as a low frequency, the phenomenon is called aliasing.

<u>Effects of Aliasing</u> :-

Aliasing can be avoided by two methods

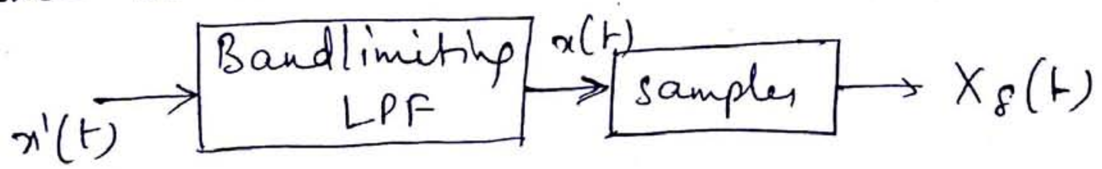
i) <u>Sampling rate $f_s \geq 2w$</u>

When the sampling rate is made higher than $2w$ then the spectrum will not overlap, there will be sufficient gap between the individual spectrums.



ii) <u>Bandlimiting the signal</u>

The sampling rate is $f_s = 2w$. At this rate there should be no aliasing but there can be a few components higher than $2w$ which create aliasing. Hence a LPF is used before a sampler.



The o/p of LPF is strictly bandlimited and there are no frequency components higher than w, hence there will be no aliasing.