9.1 INTRODUCTION

"Kinetics" is the study of bodies which are in motion by considering the forces causing the motion. In this chapter we consider the relation between forces and masses of the bodies. Similar to "Kinematics" in "Kinetics" also we consider the three types of motion i.e, Rectilinear, rotatory and plane motions.

9.2 RECTILINEAR MOTION – D'ALEMBERTS PRINCIPLE:

From Newton's second law we know that,

$$F = ma$$

If a system of forces act on the body, then the above equation can be modified as,

$$\Sigma F = ma \quad \text{... (1)}$$

where ΣF = Resultant of all farces

Now, equation (1) can be rewritten as,

$$\Sigma F - ma = 0 \quad \text{... (2)}$$

Here the term "$-ma$" can be treated as the opposite force acting to the direction of ΣF. This concept was developed by D'Alembert.

D'Alemberts Principle: The system of forces acting on a body in motion is in "dynamic equilibrium" with the inertia force of the body.

$\Rightarrow \quad \Sigma F - ma = 0$

ΣF = resultant force

ma = inertia force

Using this principle, we can solve the problems assuming that the moving bodies are in "dynamic equilibrium".

Example 1: For a lift of weight 'W'

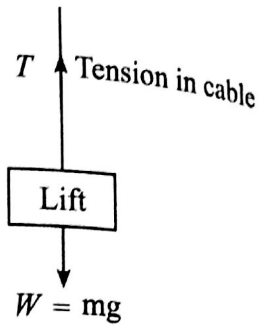


Fig. 9.1

Case (i): when lift is moving upwards.

Resultant force = $(T - W)$ and this force produces an acceleration 'a' to the lift.

$$\Rightarrow \quad (T - W) = ma$$

$$T = W + ma = W + \frac{W}{g}(a)$$

$$T = W\left(1 + \frac{a}{g}\right)$$

Case (ii) when the lift is moving downwards

$$T = W\left(1 - \frac{a}{g}\right)$$

Example 2: A lift carries a weight of 150 N and is moving with a uniform acceleration of 2.45 m/s^2. Determine the tension in the cable

(i) When lift is moving upwards

(ii) When lift is moving downwards

Ans: Given $W = 150$ N, $a = 2.45$ m/s^2

(i) Upward

$$T = W\left(1 + \frac{a}{g}\right)$$

$$T = 150\left(1 + \frac{2.45}{9.8}\right)$$

$$T = 187.5\ N$$

(ii) Downward

$$T = W\left(1 - \frac{a}{g}\right)$$

$$= 150\left(1 - \frac{2.45}{9.8}\right) = 112.5\ N$$

Example 3: A lift has an upward acceleration of 1.5 m/s^2. What pressure will a man weighing 500 N exert on the floor of the lift? What pressure would he exert if the lift had an acceleration 1.5 m/s^2 downwards? What upward acceleration would cause his weight to exert a pressure of 600 N on the floor of the lift?

Ans: Given $W = 500$ N, $a = 1.5$ m/s^2

Case (i): upward: $T = W\left(1 + \frac{a}{g}\right) = 500\left(1 + \frac{1.5}{9.8}\right) = 576.5\ N.$

Case (ii): Downward: $T = W\left(1 - \frac{a}{g}\right) = 500\left(1 - \frac{1.5}{9.8}\right) = 423.46\ N.$

Case (iii): Given $T = 600\ N$, $a = ?$

Upward: $T = W\left(1 + \frac{a}{g}\right) \quad \Rightarrow \quad 600 = 500\left(1 + \frac{a}{9.8}\right) = 1.96$ m/s^2

Example 4: A block weighing 1.5 kN rests on a horizontal plane as shown in Fig. 9.2(a) Find the magnitude of the force 'P' required to give the block an acceleration of 3 m/s² to the right. (Given $\mu = 0.25$).

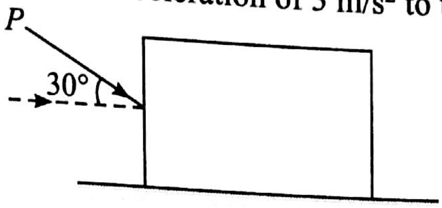


Fig. 9.2(a)

Ans: Given $W = 1.5$ kN = 1500 N

$a = 3$ m/s²

$\mu = 0.25$

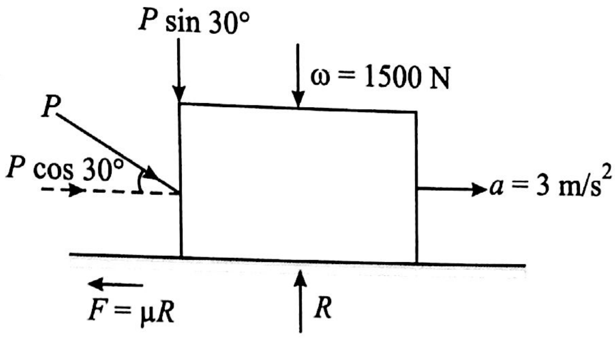


Fig. 9.2(b)

From *FBD*,

$$\Sigma F_y = ma_y$$

$$1500 + P \sin 30 - R = m\,(0) \quad [\because a_y = 0]$$

$$\Rightarrow \quad R = 1500 + \frac{P}{2} \qquad \ldots (1)$$

$$\Sigma F_x = ma_x$$

$$P \cos 30 - \mu R = ma$$

$$P\cos 30 - \left[0.25\left(1500 + \frac{P}{2}\right)\right] = \frac{1500}{9.8} \times 3$$

$$0.866\,P - 375 - 0.125\,P = 459.18$$

$$0.741\,P = 834.18$$

$$\boxed{P = 1125.74 \text{ N}}$$

Example 5: A block weighing 2000 N rests ona horizontal plane for which coefficient of friction is 0.2. This block is pushed by a force of 1000 N, which is acting at an angle of 30° to the horizontal. Find the velocity of the block after it moves 30 m, stating from rest.

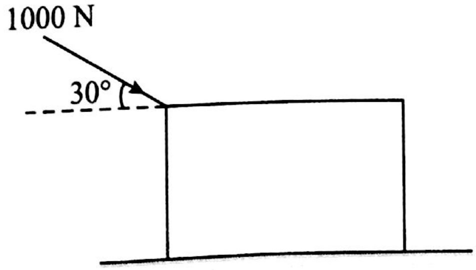


Fig. 9.3(a)

Ans: Given $W = 2000\ N$

$\mu = 0.2$

$v = ?$

$s = 30$ m

$u = 0$

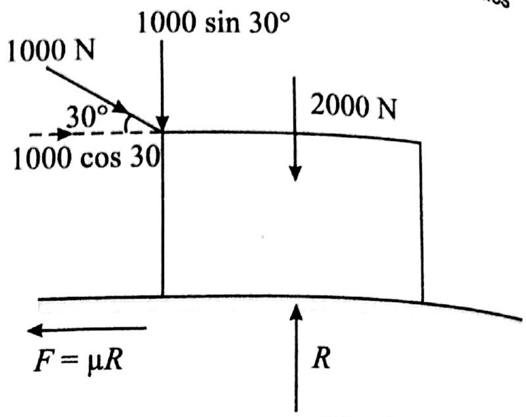


Fig. 9.3(b)

FBD of the given system is shown in Fig. 9.3(b)

From D'Alemberts Principal, $\Sigma F = ma$.

(i) $\Sigma F_y = ma_y$

$$1000 \sin 30 + 2000 - R = \frac{2000}{9.8} \times 0 \quad [\because \text{ movement is along } x\text{-axis, } a_y = 0]$$

$\Rightarrow$ $\boxed{R = 2500 \text{ N}}$

$F = \mu R = 0.2 \times 2500 = 500\ N.$

(ii) $\Sigma F_x = ma_x$

$$1000 \cos 30 - F = \frac{2000}{9.8} \times a$$

$$866.02 - 500 = 204.08\ a$$

$\Rightarrow$ $\boxed{a = 1.79 \text{ m/s}^2}$

We know, $v^2 - u^2 = 2as$.

$v^2 - 0 = 2 \times 1.79 \times 30$

$\boxed{v = 10.36 \text{ m/s}}$

Example 6: Problems on connected bodies (with and without friction)

(a) Horizontal surface without friction ($\mu = 0$) with weights W_1 and W_2:

From *FBD* of the given system, W_1 is moving vertically downwards and W_2 is moving horizontally on a smooth surface.

Here the velocity and acceleration of W_1 and W_2 are same i.e., 'a'.

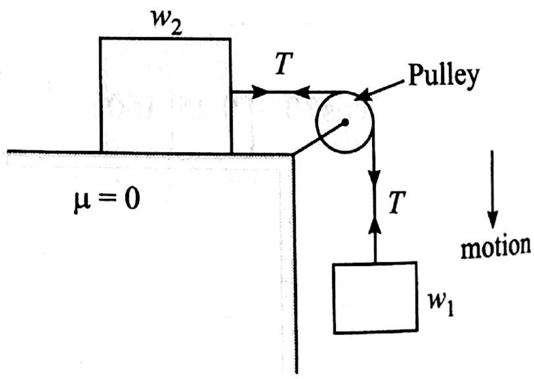


Fig. 9.4(a)

(*i*) For W_1	(*ii*) For W_2
$W_1 - t = m_1 a$ $W_1 - t = \frac{W_1}{g} a$ ...(1)	$T = m_2 a$ $T = \frac{W_2}{g} a$ ... (2)

Substitute 'T' value in eq (1)

$$W_1 - \frac{W_2}{g} a = \frac{W_1}{g} a$$

$$W_1 = \frac{W_1}{g} a + \frac{W_2}{g} a$$

$$W_1 = \frac{a}{g}(W_1 + W_2)$$

$$\Rightarrow \quad \boxed{a = \frac{W_1 \, g}{(W_1 + W_2)}}$$

Substitute 'a' value in eq (2)

$$\Rightarrow \quad T = \frac{W_1 W_2}{W_1 + W_2}$$

(b) Horizontal surface with friction 'μ' ($\mu \neq 0$) for bodies with weights W_1 and W_2.

From *FBD* of the given system is shown in Fig. 9.4(b),

(i) For W_1

$$W_1 - T = m_1 a$$

$$W_1 - T = \frac{W_1}{g} a \qquad \ldots(1)$$

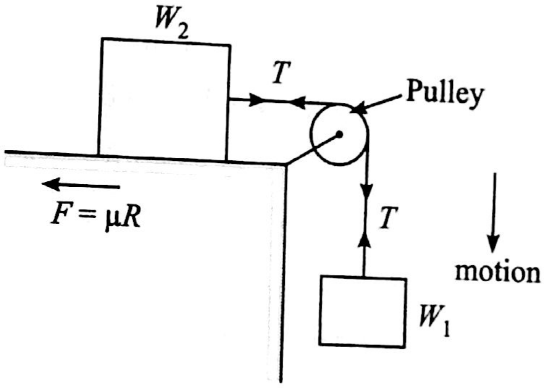


Fig. 9.4(b)

(ii) For W_2

$$T - F = m_2 a$$

$$T - \mu W_2 = \frac{W_2}{g} a \qquad \ldots(2)$$

Adding eq (1) and eq (2)

$$W_1 - T + T - \mu W_2 = \frac{W_1}{g} a + \frac{W_2}{g}$$

$$W_1 - \mu W_2 = \frac{a}{g}(W_1 + W_2)$$

$$\Rightarrow \quad \boxed{a = \frac{(W_1 - \mu W_2) g}{W_1 + W_2}}$$

Substitute the value of 'a' in eq (1)

$$W_1 - T = \frac{W_1}{g} \cdot \frac{(W_1 - \mu W_2) g}{W_1 + W_2}$$

$$T = W_1 - \left[\frac{W_1^2 - \mu W_1 W_2}{W_1 + W_2} \right]$$

$$= \frac{W_1^2 + W_1 W_2 - W_1^2 + \mu W_1 W_2}{W_1 + W_2}$$

$$\boxed{T = \frac{W_1 W_2 (1 + \mu)}{W_1 + W_2}}$$

(c) Inclined surface without friction ($\mu = 0$) with weights W_1 and W_2:

From *FBD* of the given system is shown in Fig. 9.4(c)

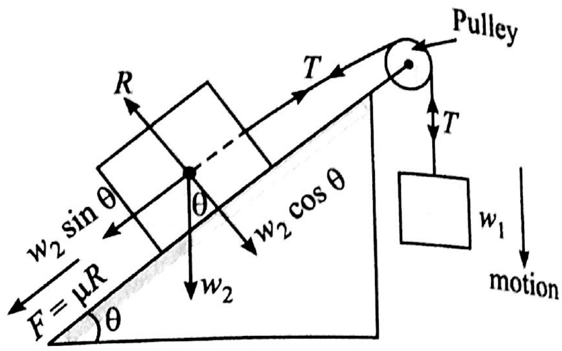


Fig. 9.4(c)

(i) $W_1 - T = m_1 a$

$$W_1 - T = \frac{W_1}{g} a \qquad \ldots (1)$$

(ii) For W_2

$$T - W_2 \sin\theta = m_2 a$$

$$T - W_2 \sin\theta = \frac{W_2}{g} a \qquad \ldots (2)$$

Adding equations (1) and (2)

$$W_1 - T + T - W_2 \sin\theta = \frac{W_1}{g} a + \frac{W_2}{g} a$$

$$W_1 - W_2 \sin\theta = \frac{a}{g}(W_1 + W_2)$$

$$\boxed{a = \frac{(W_1 - W_2 \sin\theta)g}{W_1 + W_2}}$$

Substitute '*a*' value in eq (1)

$$W_1 - T = \frac{W_1}{\not{g}} \cdot \frac{\not{g}(W_1 - W_2 \sin\theta)}{W_1 + W_2}$$

$$T = W_1 - \left[\frac{W_1^2 - W_1 W_2 \sin\theta}{W_1 + W_2}\right]$$

$$= \frac{W_1^2 + W_1 W_2 - W_1^2 + W_1 W_2 \sin\theta}{W_1 + W_2}$$

$$\boxed{T = \frac{W_1 W_2 (1 + \sin\theta)}{W_1 + W_2}}$$

(d) Inclined surface with friction ($\mu \neq 0$) with weights W_1 and W_2:

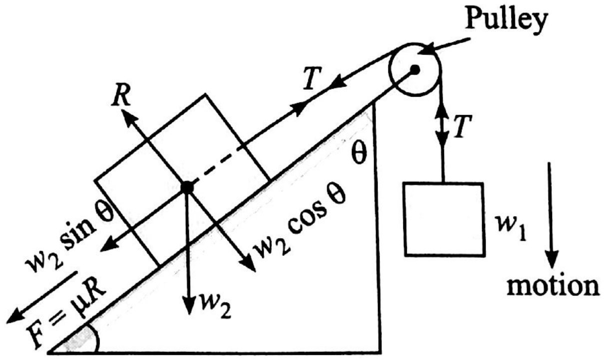


Fig. 9.4(d)

From *FBD* of the given system is shown in Fig. 9.4(d)

(i) **For W_1**

$$W_1 - T = m_1 a$$

$$W_1 - T = \frac{W_1}{g} a \ldots (1)$$

(ii) **For W_2**

$$T - W_2 \sin\theta - F = m_2 a$$

$$T - W_2 \sin\theta - \mu W_2 \cos\theta = \frac{W_2}{g} a \ \ldots(2) \ (\because \ F = \mu R = \mu W_2 \cos\theta)$$

Adding eq. (1) and (2)

$$W_1 - T + T - W_2 \sin\theta - \mu W_2 \cos\theta = \frac{W_1}{g} a + \frac{W_2}{g} a$$

$$\Rightarrow \boxed{a = \frac{g(W_1 - W_2 \sin\theta - \mu W_2 \cos\theta)}{W_1 + W_2}}$$

Substitute 'a' value in eq ...(1)

$$W_1 - T = \frac{W_1}{g} \cdot \frac{g(W_1 - W_2 \sin\theta - \mu W_2 \sin\theta)}{W_1 + W_2}$$

$$T = \frac{W_1^2 + W_1 W_2 - W_1^2 + W_1 W_2 \sin\theta + \mu W_1 W_2 \cos\theta}{W_1 + W_2}$$

$$\boxed{T = \frac{W_1 W_2 (1 + \sin\theta + \mu \cos\theta)}{W_1 + W_2}}$$

Example 7: Two bodies of weights 30 N and 20 N are connected to the ends of a light inextensible string passing over a smooth pulley. The weight of 30 N is placed on a smooth horizontal plane while the weight 20 N is hanging in air. Find (i) the acceleration of the system (ii) the tension in the sting.

Ans: Given $W_1 = 20$ N, $W_2 = 30$ N, $\mu = 0$

We know, (i) $a = \dfrac{g W_1}{W_1 + W_2}$

$$= \frac{9.8 \times 20}{(20 + 30)}$$

$$= 3.92 \text{ m/s}^2$$

(ii) $T = \dfrac{W_1 W_2}{W_1 + W_2} = \dfrac{20 \times 30}{20 + 30}$

$$\boxed{T = 12 \text{ N}}$$

Example 8: Two bodies of weights 20 N and 30 N are connected to the ends of a light inextensible string passing over a smooth pulley. If the coefficient of friction between 30 N body and surface is 0.3, determine 'a' and 'T'.

Ans: Given $W_1 = 20$ N, $W_2 = 30$ N, $\mu = 0.3$.

We know, $a = \frac{g(W_1 - \mu W_2)}{W_1 + W_2} = \frac{9.8(20-(0.3\times30)}{20+30}$

$$\boxed{a = 2.15 \text{ m/s}^2}$$

$$T = \frac{W_1 W_2 (1+\mu)}{W_1+W_2} = \frac{(20\times30)(1+0.3)}{20+30}$$

$$\boxed{T = 15.6 \text{ N}}$$

Example 9: Two blocks shown in Fig. 9.5 have weights $A = 30$ N and $B = 20$ N and coefficient of friction between block 'A' and horizontal surface is 0.5. It the system is released from rest and the block 'B' moves a vertical distance 2 m. What is the velocity of block 'B'? Neglect the friction in pulley and the extension of the string.

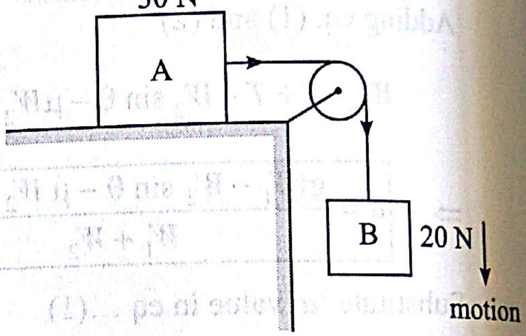


Fig. 9.5

Ans: Given $W_1 = 20$ N, $W_2 = 30$ N, $\mu = 0.5$, $u = 0$; s = 2m.

We know, $\boxed{a = \frac{(W_1 - \mu W_2)g}{W_1+W_2}} = \frac{9.8(20-(0.5\times30))}{20+30}$

$$\boxed{a = 0.98 \text{ m/s}^2}$$

$$T = \frac{W_1 W_2 (1+\mu)}{W_1+W_2} = \frac{(20\times30)(1+0.5)}{20+30}$$

$$\boxed{T = 18 \text{ N}}$$

We know, $v^2 - u^2 = 2as$

$v^2 - 0 = 2 \times 0.98 \times 2$

$$\boxed{v = 1.97 \text{ m/s}}$$

Example 10: Two bodies of weights 40 N and 25 N are connected to the two ends of an inextensible string as shown in Fig. 9.6 The weight 40 N is placed on a smooth inclined plane, and if the angle of inclined plane is 15°. Determine (i) acceleration (ii) tension

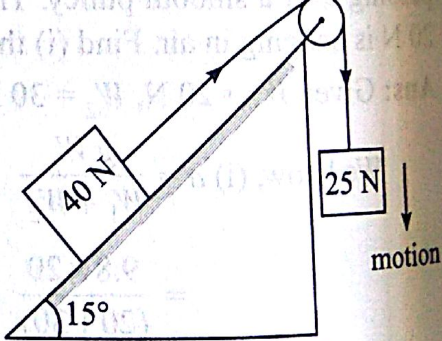


Fig. 9.6

Ans: Given $W_1 = 25$ N, $W_2 = 40$ N, $\theta = 15°$

We know, $a = \frac{g(W_1 - W_2 \sin\theta)}{W_1+W_2} = \frac{9.8(25-40\sin 15)}{25+40} = 2.2 \text{ m/s}^2$

$$\boxed{T = \frac{W_1W_2(1+\sin\theta)}{W_1+W_2}} = \frac{(25\times40)(1+\sin 15)}{25+40} = 19.36 \text{ N}$$

Example 11: Two bodies weighing 45 N and 30 N are connected to the ends of an inextensible string, which passes over a smooth pulley. The weight 45 N is placed on a 20° inclined plane while the weight 30 N is hanging over the pulley. Given $\mu = 0.3$. Determine (i) acceleration (ii) Tension.

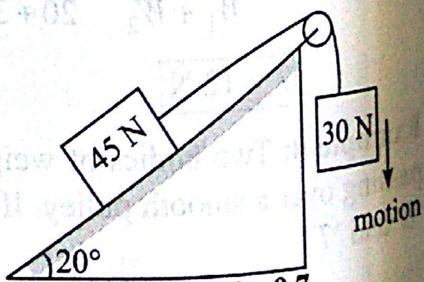


Fig. 9.7

Ans: Given $W_1 = 30$ N, $W_2 = 45$ N, $\theta = 20°$, $\mu = 0.3$

We know, $$\boxed{a = \frac{g(W_1 - W_2 \sin\theta - \mu W_2 \cos\theta)}{W_1 + W_2}}$$

$$= \frac{9.8(30 - 45\sin 20 - (0.3 \times 45\cos 20))}{30 + 45}$$

$$\boxed{a = 0.25 \text{ m/s}^2}$$

$$T = \frac{W_1 W_2}{W_1 + W_2}(1 + \sin\theta + \mu\cos\theta)$$

$$= \frac{30 \times 45}{30 + 45}(1 + \sin 20 + (0.3 \times \cos 20))$$

$$\boxed{T = 29.23 \text{ N}}$$

Example 12: From the following Fig. 9.8. Determine

(i) The acceleration of the system

(ii) The tension in the string

(iii) The distance moved by the weight 30 N in 4 seconds from rest, if $\mu = 0.2$ (between 50 N and surface)

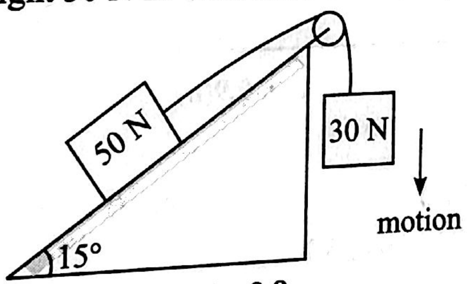


Fig. 9.8

Ans: Given $W_1 = 30$ N, $W_2 = 50$ N, $\mu = 0.2$, $\theta = 15°$

(i) $$\boxed{a = \frac{g(W_1 - W_2 \sin\theta - \mu W_2 \cos\theta)}{W_1 + W_2}} = \frac{9.8(30 - 50\sin 15 - (0.2 \times 50\cos 15))}{30 + 15}$$

$$\boxed{a = 0.90 \text{ m/s}^2}$$

(ii) $$T = \frac{W_1 W_2}{(W_1 + W_2)}(1 + \sin\theta + \mu\cos\theta)$$

$$= \frac{30 \times 50}{(30 + 50)}(1 + \sin 15 + 0.2\cos 15) \quad \boxed{T = 27.22 \text{ N}}$$

(iii) Given $u = 0$, $t = 4$ s, $a = 0.9$ m/s^2

We know, $$s = ut + \frac{1}{2}at^2$$

$$= (0 \times 4) + \frac{1}{2}(0.9 \times 4^2)$$

$$\boxed{s = 7.2 \text{ m}}$$

$T_2 = 9.58$ N

9.3 ROTATORY MOTION-FIXED AXIS ROTATION

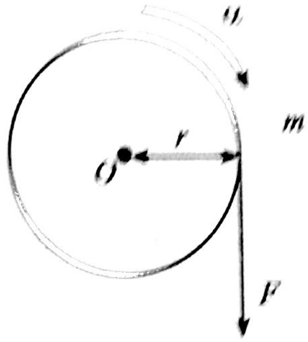


Fig. 9.13

Let us consider a wheel (or) pulley rotating about a fixed axis (passing through the point '0') in clockwise with an angular acceleration 'α' and mass 'm' as shown in Fig. 9.13. Let '*r*' be the radius of the wheel (or pulley.

Now similar to D'Alemberts principle in rectilinear motion, we can write

$$\Sigma M = I\alpha$$

$$\Rightarrow \quad \Sigma M - I\alpha = 0$$

Where ΣM = resultant moment = $F \times r$ (from Fig. 9.13)

I = Mass Moment of Inertia of wheel (or) pulley

$$\Rightarrow \quad I = \frac{mr^2}{2} \text{ (or) } mk^2$$

where m = mass of the wheel (or) pulley

k = radius of Gyration of the wheel (or) pulley

α = angular acceleration.

Example 19: A flywheel weighing 60 kN and having radius of gyration 1 *m* loses its speed from 400 rpm to 280 rpm in 2 minutes. Calculate

(i) The moment (or) retarding torque on it.

(ii) Change in its kinetic energy during the above period.

(iii) Change in its angular momentum during the same period.

Ans: Given $N_0 = 400$ rpm

$$\omega_0 = \frac{2\pi N_0}{60} = \frac{2\times3.14\times400}{60} = 41.86 \text{ rad/s}$$

$N = 280$ rpm

$$\omega = \frac{2\pi N}{60} = \frac{2\times3.14\times280}{60} = 29.32 \text{ rad/s.}$$

$t = 2$ min $= 120$ s.

We know, $\omega = \omega_0 + \alpha t$

$$\alpha = \frac{\omega-\omega_0}{t} = \frac{29.32-41.86}{120}$$

$\alpha = -0.104$ rad/s^2

$W = 60$ kN $= 60000$ N, $k = 1$ m

$$m = \frac{W}{g} = \frac{60000}{9.8} = 6122.4 \text{ kg}$$

$I = mk^2 = 6122.4 \times 1 = 6122.4$ kg – m^2

(i) Moment (or) Torque $= I\alpha$

$= 6122.4 \times (-0.104) = -636.72$ Nm.

(ii) Change in K.E. = Initial K.E. – Final K.E.

$$= \frac{1}{2}I(\omega_0^2 - \omega^2) = \frac{1}{2}\times6122.4(41.86^2 - 29.32^2) = 2732418.54 \text{ Nm.}$$

(iii) Change in Angular momentum $= I\omega_0 - I\omega$

$= 6122.4\ (41.86 - 29.32) = 76774.89$ kgm^2 rad/s

Example 20: A pulley of weight 400 N has a radius of 0.6 m. A block of 800 N is suspended by a rope wound round the pulley as shown in the Fig. 9.14. Determine the resultant acceleration in weight and tension in the rope.

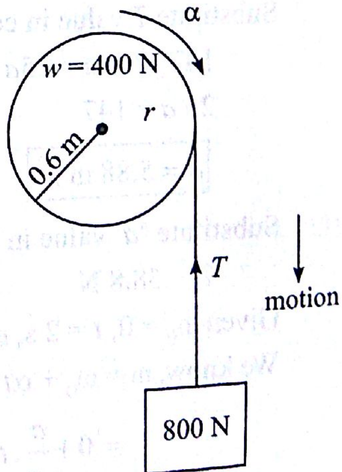


Fig. 9.14

Ans: Given $r = 0.6$ m

(i) Linear:

$\Sigma F = ma$

$$800 - T = \frac{800}{9.8}a \quad \text{... (1)}$$

$800 - T = 81.63\ a$

(ii) **Rotatory:**

$\Sigma M = I\alpha$

Force × ⊥r distance $= I\alpha$

$$T \times 0.6 = \frac{mr^2}{2}\cdot\frac{a}{0.6}$$

$$0.6\ T = \frac{400}{9.8}\times\frac{(0.6)^2}{2}\times\frac{a}{0.6} \quad \text{... (2)}$$

$\boxed{T = 20.4\ a}$

$\boxed{T = 159.93\text{ N}}$

Example 21: A cylinder of mass 20 kg rotates in a frictionless barring from rest, is under the action of mass 15 kg carried by a rope wrapped around the cylinder as shown in Fig. 9.15 If the diameter is 1 m, What will be the angular velocity of the cylinder after 2 s from the start of motion ($r = 0.5$ m)

Ans:

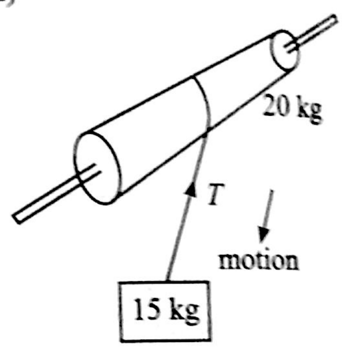


Fig. 9.15

For linear:

$\Sigma F = ma$

$(15 \times 9.8) - T = 15a$

$147 - T = 15\,a$ ...(1)

For rotatory:

$\Sigma M = I\alpha$

Force $\times \perp r$ distance $= I\alpha$

$$T \times 0.5 = \frac{mr^2}{2} \cdot \frac{a}{r} \quad [\because a = r\alpha]$$

$$0.5\,T = \frac{20 \times 0.5^2}{2} \cdot \frac{a}{0.5}$$

$\boxed{T = 10a}$ ... (2)

Substitute T value in eq. (1)

$147 - 10\,a = 15a$

$25\,a = 147$

$\boxed{a = 5.88\text{ m/s}^2}$

Substitute 'a' value in eq. (2)

$T\quad 58.8$ N

Given $\omega_0 = 0$, $t = 2$ s, $\omega = ?$

We know, $\omega = \omega_0 + \alpha t$

$$= 0 + \frac{a}{r} \cdot t = 0 + \frac{5.88}{0.5} \times 2$$

$\omega = 23.52$ rad/s

Example 22: The composite pulley as shown in Fig. 9.16. Its weightis 900 N and its radius of gyration is 0.7 m. The 3000 N, 5000 N blocks are connected to the pulley as shown in Fig. 9.16. Determine the tensions in the string and angular acceleration of the pulley.

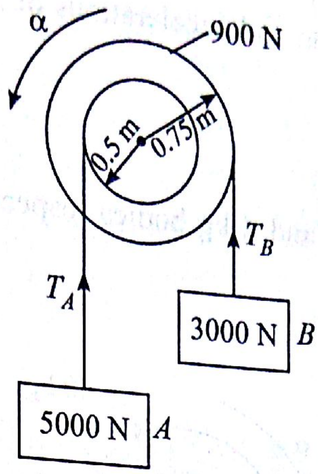


Fig. 9.16

Ans: Given $W_A = 5000$ N, $r_A = 0.5$ m

$W_B = 3000$ N, $r_B = 0.75$ m

Let T_A be the tension of 5000 N body and T_B be the tension of 3000 N body. Let a_A, a_B be the accelerations of 5000 N, 3000 N bodies respectively.

(A) For translator motion:

$\Sigma F = ma$

(i) **For 5000 N:**	(ii) **For 3000 N:**
$5000 - T_A = \frac{5000}{9.8} a_A$	$T_B - 3000 = \frac{3000}{9.8} a_B$
But $a_A = r_A \alpha = 0.5$ a	But $a_B = r_B \alpha = 0.75$ a
$\therefore\ 5000 - T_A = \frac{5000}{9.8} \times 0.5\,\alpha$	$\therefore\ T_B = 3000 + \left(\frac{3000}{9.8} \times 0.75\alpha\right)$
$\boxed{T_A = 5000 - 255.10\,\alpha}$ ...(1)	$\boxed{T_B = 3000 + 229.59\,\alpha}$ ...(2)

(B) For rotatory motion:

$\Sigma M = I$a

$$I = \text{mk}^2 = \frac{900}{9.8} \times 0.7^2 = 45 \text{ kg m}^2$$

$$\Sigma M = (T_A \times 0.5) - (T_B \times 0.75)$$

$$\therefore \quad \boxed{0.5\,T_A - 0.75\,T_B = 45\,\alpha} \quad \text{... (3)}$$

Substitute T_A and T_B values in eq. (3) from eq. (1) and eq. (2)

$0.5\,(5000 - 255.10\,\alpha) - 0.75\,(3000 + 229.59\,\alpha) = 45\,\alpha.$

$2500 - 127.55\,\alpha - 2250 - 172.19\,\alpha = 45\,\alpha.$

$344.74\,\alpha = 250$

$$\boxed{\alpha = 0.72 \text{ rad/s}^2}$$

Substitute 'α' value in eq. (1) and eq. (2)

$$\boxed{T_A = 4816.32 \text{ N}}$$

$$\boxed{T_B = 3165.30 \text{ N}}$$

Example 23: Find the angular acceleration and accelerations of A and B as shown in Fig. 9.17. Radius of gyration is 0.5 m.

Ans: Given $m_A = 60$ kg, $m_B = 20$ kg

$r_A = 0.25$ m, $r_B = 0.5$ m

Let T_A and T_B are tensions of 60 kg and 2 kg bodies respectively. Let a_A, a_B are the accelerations of 60 kg and 30 kg bodies.

(A) For translator motion:

$\Sigma F = ma.$

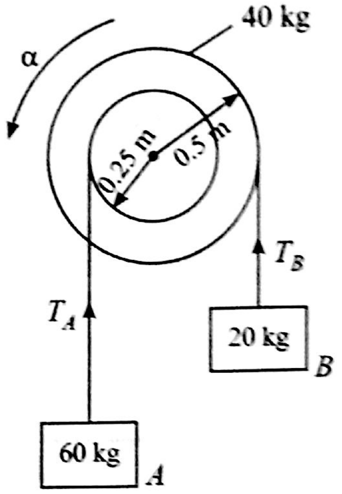


Fig. 9.17

(i) For 60 kg	(ii)For 30 kg
$(60 \times 9.8) - T_A = 60\, a_A$. But $a_A = r_A \alpha = 0.25\, \alpha$ $\therefore\ 588 - T_A = 60 \times 0.25\, \alpha$ $588 - T_A = 15\, \alpha$. $\boxed{T_A = 588 - 15\,\alpha} \to (1)$	$T_B - (20 \times 9.8) = 20\, a_B$ But $a_B = r_B \alpha = 0.5\, \alpha$ $\therefore\ T_B - 196 = 20 \times 0.5 \times \alpha$ $\boxed{T_B = 196 + 10\,\alpha} \to (2)$

(B) For rotatory motion:

$\Sigma M = I\alpha.$

$I = mk^2 = 40 \times 0.5^2 = 10$ kgm^2

$\Sigma M = (T_A \times 0.25) - (T_B \times 0.5)$

$\therefore\ 0.25\, T_A - 0.5 T_B = 10$ a

Substitute T_A and T_B values in eq. (1) and eq. (2)

$0.25\,(588 - 15\,\alpha) - 0.5\,(196 + 10\,\alpha) = 10\,\alpha.$

$147 - 3.75\,\alpha - 98 - 5\alpha = 10\,\alpha.$

$49 = 18.75$ a

$\boxed{\alpha = 2.61 \text{ rad/s}}$

Substitute 'α' value in eq. (1) and eq. (2)

$T_A = 588 - (15 \times 2.61)$

$\boxed{T_A = 548.85 \text{ N}}$

$T_B = 196 + (10 \times 2.61)$

$\boxed{T_B = 222.1\text{ N}}$

$a_A = 0.25\ \alpha = 0.25 \times 2.61$

$\boxed{a_A = 0.65\text{ m/s}^2}$

$a_B = 0.5\ \alpha = 0.5 \times 2.61$

$\boxed{a_B = 1.3\text{ m/s}^2}$

9.4 PLANE MOTION – EQUATION OF PLANE MOTION AND ROLLING BODIES

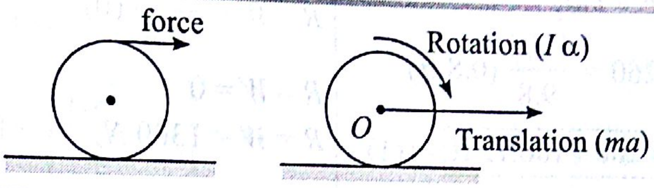


Fig. 9.18(a) **Fig. 9.18(b)**

When a force is applied on a wheel (or) pulley (or) cylinder as shown in Fig. 9.18(a), then the body starts to roll on the surface. This rolling body will have both translatory (rectilinear) and rotatory motions. (Fig. 9.18 (b))

Hence we can write the equations of a rolling body in plane motion as,

$\Sigma F - ma = 0$ and $\Sigma M - I\alpha = 0$

Note: The rolling of the body must be "without slipping"on the surface, then only we can use the above equations.

Example 24: A solid cylinder weighing 1300 N is acted upon by a force 'P' horizontally as shown in Fig. 9.19(a). Determine the maximum value of 'P' for which there will be rolling without slipping (Given $\mu = 0.2$)

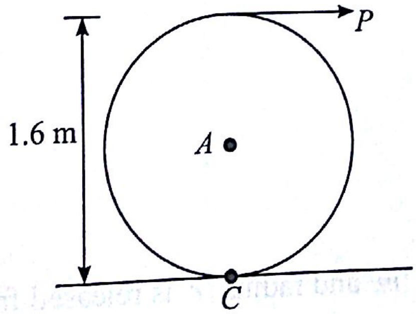


Fig. 9.19(a)

Ans:

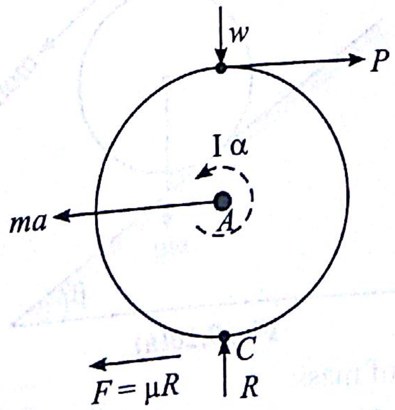


Fig. 9.19(b)

Given $D = 1.6$ m

$r = 0.8$ m

$a = r\alpha$

$a = 0.8\ \alpha.$

$$I = \frac{mr^2}{2} = \frac{1}{2}\left(\frac{1300}{9.8}\right)(0.8)^2$$

$I = 42.44$ kg m^2

$F = \mu R = 0.2 \times 1300 = 260$ N.

$\Sigma Fx = ma_x$ $P - F = ma_x$ $P - 260 = \frac{1300}{9.8}(0.8\ \alpha)$ $\boxed{P = 260 + 106.12\ \alpha} \rightarrow (1)$	$\Sigma Fy = ma_y$ $R - W = \frac{1300}{9.8}(0)$ $R - W = 0$ $R = W = 1300\ N$

By taking moment about 'C'

$(P \times 1.6) - (ma \times 0.8) - I\alpha = 0$

$1.6\ P - 84.89\ \alpha - 42.44\ \alpha = 0$

$1.6\ P - 127.33\ \alpha = 0$

$\boxed{1.6\ P = 127.33\ \alpha} \rightarrow (2)$

Substitute 'P' value in eq. (2)

$1.6\ (260 + 106.12\ \alpha) = 127.33\ \alpha$

$416 + 169.79\ \alpha = 127.33\ \alpha$

$42.46\ \alpha = 416$

$\boxed{\alpha = -9.79\ \text{rad/s}^2}$

Substitute 'α' value in eq. (1)

$\boxed{P = 778.9\ N}$

Example 25: A solid cylinder of mass 'm' and radius 'r' is released from rest and rolls down on an inclined plane as shown in Fig. 9.20(a).

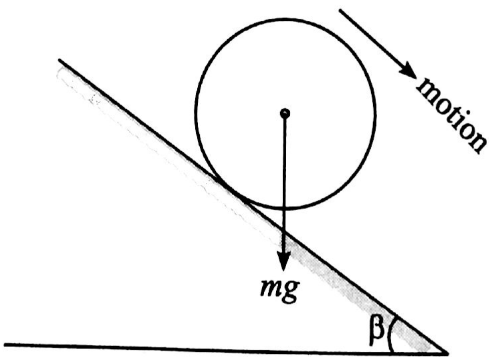


Fig. 9.20(a)

Find (i) Acceleration of the centre of mass

(ii) The maximum inclination for which rolling without slipping occurs. ($\mu = 0.2$)

Ans: From *FBD*,

$$\Sigma F_x = ma_x$$
$$(mg \sin \beta - F) = ma$$
$$F = mg \sin \beta - ma$$
$$F = m(g \sin \beta - a) \rightarrow (1)$$
$$\Sigma F_y - ma_y = 0$$
$$\Rightarrow \quad \Sigma F_y = ma_y$$
$$\boxed{mg \cos \beta - R = 0} \rightarrow (2)$$

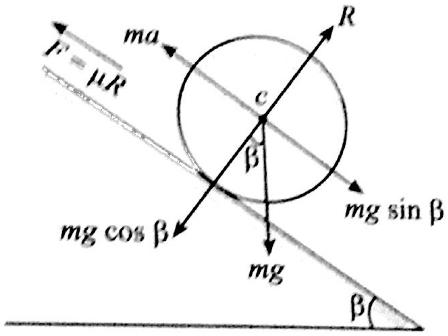


Fig. 9.20(b)

Now, $\Sigma M - I\alpha = 0$

Now taking moments about centre 'C'.

$$\boxed{(F \times r) - I\alpha = 0} \rightarrow (3)$$

$$(m(g \sin \beta - a) \times r) - \left[\frac{mr^2}{2} \times \alpha\right] = 0$$

$$m(g \sin \beta - a)r = \frac{mr^2}{2} \times \frac{a}{r}$$

$$(g \sin \beta - a) = \frac{a}{2}$$

$$g \sin \beta = \frac{3a}{2}$$

$$\Rightarrow \quad \boxed{a = \frac{2g}{3} \sin \beta} \rightarrow (4)$$

(ii) Given $\mu = 0.2$.

From eq. (3) $\Rightarrow F \times r = I\alpha$.

$$\mu R \times r = \frac{mr^2}{2} \times \frac{a}{r}$$

$$\mu \times mg \cos \beta \times r = \frac{\cancel{mr^2}}{\cancel{2}} \cdot \frac{\cancel{2}g \sin \beta}{\cancel{a} \times 3} \qquad \left[\because a = \frac{2g}{3} \sin \beta\right]$$

$$0.23. \cos \beta = \frac{1}{3} \sin \beta$$

$$\frac{\sin \beta}{\cos \beta} = 3 \times 0.23$$

$$\tan \beta = 0.69$$

$$\boxed{\beta = 34.6}$$

Substitute 'β' value in eq. (4)

$$a = \frac{2 \times 9.8}{3} \times \sin 34.6 \qquad \boxed{a = 3.7 \text{ m/s}^2}$$

Example 26: A solid cylinder of weight 'W' and radius 'r' rolls down on an inclined plane which makes an angle of θ with thehorizontal axis. Determine the minimum coefficient of friction and the acceleration of the mass centre, for rolling without slipping.

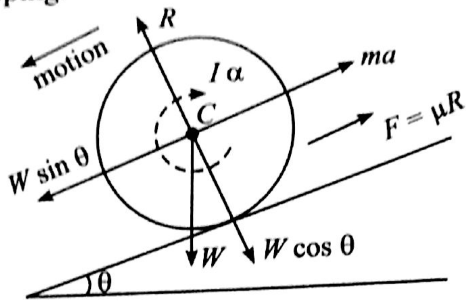


Fig. 9.21

Ans. From *FBD*:

$$\Sigma F_x - ma_x = 0$$

$$W \sin \theta - ma - F = 0$$

$$F = W \sin \theta - ma \rightarrow (1)$$

$$\Sigma Fy - ma_y = 0 \qquad [\because ay = 0]$$

$$R = W \cos \theta \rightarrow (2)$$

$$\Sigma M - I\alpha = 0 \rightarrow (3)$$

Taking moment about 'C'

$$(F \times r) = I\alpha.$$

$$(W \sin \theta - ma) \times r = \frac{mr^2}{2} \times \frac{a}{r} = \boxed{T = 157.97 \text{ N}}$$

$$W \sin \theta - ma = \frac{ma}{2}$$

$$W \sin \theta = \frac{3ma}{2}$$

$$a = \frac{2}{3} \cdot \frac{mg \sin \theta}{m} \qquad [\because W = mg]$$

$$\boxed{a = \frac{2}{3} g \sin \theta} \rightarrow (4)$$

Substitute 'a' value in eq. (1)

$$F = W \sin \theta - m \cdot \frac{2}{3} g \sin \theta$$

$$= W \sin - \frac{2}{3} W \sin \theta$$

$$F = \frac{1}{3} W \sin \theta \rightarrow (5)$$

But $F = \mu R = \mu\ W \cos\theta \rightarrow (6)$

From eqs. (5) and (6)

$$\mu W \cos\theta = \frac{1}{3} W\ \sin\theta$$

$$3\,\mu = \frac{\sin\theta}{\cos\theta}$$

$$\Rightarrow \boxed{\mu = \frac{1}{3}\tan\theta}$$

9.5 CENTRAL FORCE MOTION

"The forces which are always directed towards a fixed point are known as central forces and the resulting motion of the body due to these faces is known as central force motion".

Examples: Motion of planets around sum, motion of satellites around earth etc.

Let us consider a body of mass '*m*' making a central force motion around a planet of mass '*M*' as shown in Fig. 9.22(a).

Let the radius of the planet be '*R*' and the body of mass '*m*' is at a distance of '*r*' from the centre '*O*' of the planet.

The force acting on the body of mass '*m*' is given by

$$F = G\frac{Mm}{r^2}$$

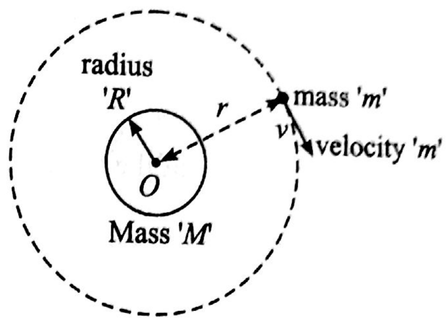


Fig. 9.22(a)

where G = Universal gravitational constant $= 6.67 \times 10^{-11} \dfrac{\text{Nm}^2}{\text{kg}^2}$

Now, if the body is placed on the planet surface, then the above force is given by,

$$F = G\frac{Mm}{R^2} \qquad (\because\ r = R)$$

here $F = mg$ ($\because$ the body is placed on the planet)

$$\Rightarrow\quad mg = G\frac{Mm}{R^2}$$

$$\Rightarrow\quad gR^2 = GM$$

$$\Rightarrow\quad M = \frac{gR^2}{G}$$

By knowing the value of R, we can know the Mass of the planet.

Example: If radius of earth $R = 6000$ km, then its mass $M = \dfrac{9.8 \times (6000 \times 10^3\ \text{m})^2}{6.67 \times 10^{-11}} = 5.2 \times 10^{24}$ kg.

In a central face motion:

10.1 INTRODUCTION

In this chapter, Work-Energy method is used to solve the "kinetic" problems. This method is easy over D'Alemberts method when the problems involve velocities rather than acceleration.

Work: "It is the product of displacement and the force in the direction of displacement".

$$\Rightarrow \quad W = \vec{F} \cdot \vec{s} = F\, s \cos\theta$$

Unit: Joule

Energy: - It is the ability to do work. The energy exists in different forms i.e., electrical energy, mechanical energy, Thermal energy, potential energy, kinetic energy etc.

Unit: Joule

Power: It is the rate at which the work is done.

$$\Rightarrow \text{power} = \frac{\text{work done}}{\text{time}}$$

Unit: Joule/s (or) watt.

10.2 WORK-ENERGY EQUATION FOR TRANSLATION (RECTILINEAR MOTION)

Statement: - The amount of work done is equal to the change in kinetic energy of the body.

$$\Rightarrow \quad \text{Work done} = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

Proof:

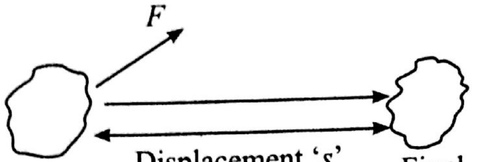


Fig. 10.1

Let us consider a net force F acting on a rigid body to displace it through a distance s as shows in Fig. 10.1; Let the initial and final velocities be u and v respectively.

From Newton's second law,

$$F = ma$$

$$\Rightarrow \quad F = m\frac{dv}{dt}\left(\because a = \frac{dv}{dt}\right)$$

$$\Rightarrow \quad F = m.\frac{dv}{dt}\times\frac{ds}{ds} = m.\frac{dv}{ds}\times\frac{ds}{dt}$$

$$\Rightarrow \quad F = m.\frac{dv}{ds}.v\left(\because \frac{ds}{dt} = v\right)$$

$$\Rightarrow \quad F.\,ds = mv\,dv$$

By integrating on both sides we get,

$$\int_O^s F.\,ds = \int_u^v mv\,dv$$

$$\Rightarrow \quad F.s = m\left(\frac{v^2}{2}\right)_u^v$$

$$\Rightarrow \quad F.s = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$\Rightarrow$ Work done = Change in kinetic energy.

The above equation is known as Work-Energy equation, used to solve the kinetic problems.

Note:

(1) Potential energy = mgh

(2) Kinetic energy = $\frac{1}{2}mv^2$

(3) Total energy = potential energy + kinetic energy = $mgh + \frac{1}{2}mv^2$

(4) Work done by a spring is given by,

$$W = -\frac{1}{2}k\,(x_2^2 - x_1^2)$$

where k = force constant of the spring

x_1 = Initial position

x_2 = Final position

10.3 WORK-ENERGY EQUATION FOR FIXED AXIS ROTATION (ROTATORY MOTION)

Statement: - The amount of work done is equal to the change in kinetic energy of the body.

$$\Rightarrow \quad \text{Work done} = \frac{1}{2} I\omega^2 - \frac{1}{2} I\omega_0^2 .$$

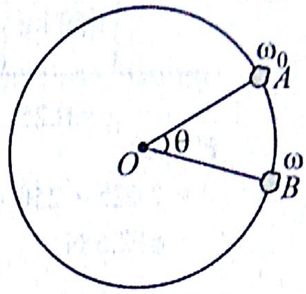


Fig. 10.2

Let us consider a body executing rotatory motion about a fixed point '*O*' as shown in Fig. 10.2. Let the angular displacement of the body be 'θ' (from *A* to *B*) and the initial and final angular velocities of the body be ω_o and ω respectively.

Now similar to $F = ma$, here we can write $M = I\alpha$

$$\Rightarrow \quad M = I\frac{d\omega}{dt} \left(\because \alpha = \frac{d\omega}{dt}\right)$$

$$\Rightarrow \quad M = I.\frac{d\omega}{dt} \times \frac{d\theta}{d\theta}$$

$$\Rightarrow \quad M = I.\frac{d\omega}{d\theta} \times \frac{d\theta}{dt}$$

$$\Rightarrow \quad M = I.\frac{d\omega}{d\theta}.\omega \left(\because \frac{d\theta}{dt} = \omega\right)$$

$$\Rightarrow \quad M.\,d\theta = I\omega.\,d\omega$$

by integrating on both sides we get,

$$\int_O^\theta M\,d\theta = \int_{\omega_0}^{\omega} I\omega.\,d\omega$$

$$\Rightarrow \quad M.\theta = I\left(\frac{\omega^2}{2}\right)_{\omega_0}^{\omega}$$

$$\Rightarrow \quad M.\theta = \frac{1}{2} I\omega^2 - \frac{1}{2} I\omega_0{}^2$$

⇒ Work done = change in kinetic energy

This is the work-energy equation in fixed axis rotation (rotatory motion).

10.4 WORK-ENERGY EQUATION FOR PLANE MOTION

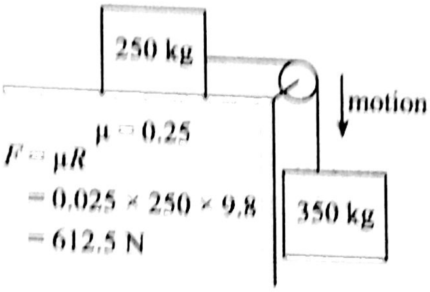
We know that a plane motion is a combination of rectilinear motion and rotatory motion. Hence we can write the "work-Energy" equation in plane motion by combining both rectilinear and rotatory motions.

$\Rightarrow$ Work done $= \left(\frac{1}{2}mv^2 - \frac{1}{2}mu^2\right) + \left(\frac{1}{2}I\omega^2 - \frac{1}{2}I\omega_0^2\right)$

$\Rightarrow$ Work done $= \frac{1}{2}\left[m\,(v^2 - u^2) + I\,(\omega^2 - \omega_0^2)\right]$

Example 1: Two blocks are joined by an inextensible string. If the system is released from rest (Fig. 10.3), determine the velocity of block of 250 kg after it had moved 2 *m* using work energy method ($\mu = 0.25$).

Ans:



$F = \mu R$

$= 0.025 \times 250 \times 9.8$

$= 612.5$ N

Fig. 10.3

We know work-energy equation

Work done = change in kinetic energy

$$F \times s = \frac{1}{2}m\,(v^2 - u^2)$$

$u = 0,\ s = 2\ m$

(i) **for 350 kg**

$m = 350$ kg

$$[(350 \times 9.8) - T]2 = \frac{1}{2} \times 350\,(v^2 - o)$$

$$6860 - 2T = 175\ v^2 \quad \ldots(1)$$

(ii) **for 250 kg**

$$(T - F) \times 2 = \frac{1}{2} \times 250\,(v^2 - o)$$

$$2T - 2F = 125\ v^2$$

$$2T - 1225 = 125\ v^2 \quad \ldots(2)$$

eq (1) + (2)

$$6860 - 2T + 2T - 1225 = 175\ v^2 + 125\ v^2$$

$$5635 = 300\ v^2$$

$$v^2 = 18.78$$

$$v = 4.33\ m/s.$$

Example 2: Find velocity of 150 kg block using work-energy method (Fig. 10.4), after it had moved 2 *m* (given $\mu = 0.2$).

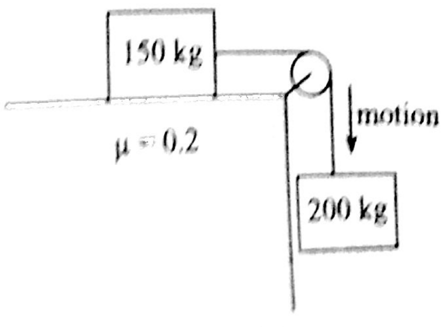


Fig. 10.4

Ans:

(*i*) **for 200 kg**

we know,

$$Fs = \frac{1}{2} m (v^2 - u^2)$$

$$[(200 \times 9.8) - T]2 = \frac{1}{2} 200 (v^2 - 0)$$

$$3920 - 2T = 100 (v^2 - 0)$$

$$3920 - 2T = 100 v^2 \quad \dots (1)$$

(*ii*) **For 150 kg**

$$F = \mu R$$

$$= 0.2 \times 150 \times 9.8$$

$$F = 294 \text{ N}$$

$$(T - F) \times 2 = \frac{1}{2} \times 150 (v^2 - 0)$$

$$2T - 588 = 75 v^2 \dots (2)$$

eq (1) + (2)

$$3920 - 2T + 2T - 588 = 100v^2 + 75v^2$$

$$3332 = 175v^2$$

$$v^2 = 19.04$$

$$v = 4.36 \; m/s$$

Example 3: A body weighting 400 *N* is pushed up by a force of 500 *N* as shown in Fig. 10.5. If the initial velocity of the body is 1.5 m/s and $\mu = 0.2$. What velocity will the body have after moving by using work-energy theorem?

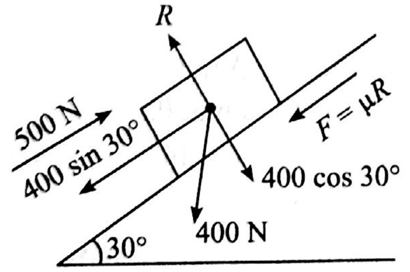


Fig. 10.5

Ans: $R = 400 \cos 30° = 346.41 \; N$

$F = \mu R = 0.2 \times 346.41 = 69.28 \; N$

We know

$$Fs = \frac{1}{2} m (v^2 - u^2)$$

$$(500 - 400 \sin 30° - F) \times 6 = \frac{1}{2} \frac{400}{9.8} (v^2 - (1.5)^2)$$

$$(500 - 200 - 69.28) \times 6 = \frac{200}{9.8} (v^2 - 2.25)$$

$$1384.32 = 20.4 (v^2 - 2.25)$$

$$1384.32 = 20.4 v^2 - 45.9$$

$$1384.32 + 45.9 = 20.4v^2$$

$$v^2 = 70.10$$

$$v = 8.37 \text{ m/s.}$$

Example 4: A block of wood of weight 1000 *N* is placed on a smooth inclined plane as shown in Fig. 10.6. Find the work done in pulling the block up for length of 5 *m*.

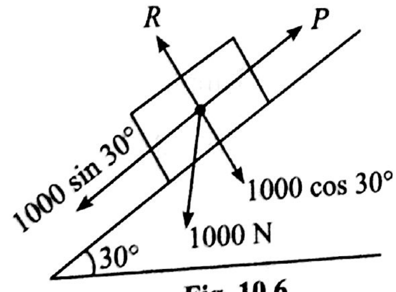


Fig. 10.6

Ans: $P = 1000 \times \sin 30$

$P = 500 \; N$

$W = F \times s$

$= 500 \times 5$

$W = 2,500 \; Nm$

Example 5: A block of wood of weight 1000 N is placed on a smooth inclined plane as shown in Fig. 10.7. Find the work done in pulling the block up for length of 5m, If $\mu = 0.3$.

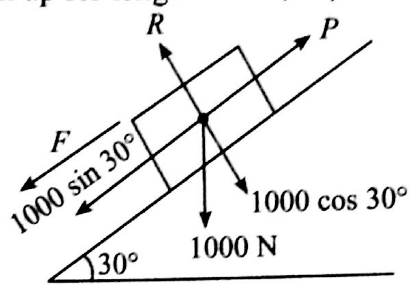


Fig. 10.7

Ans: $P = 1000 \text{ Sin } 30° + \mu R$

$P = 500 + (0.3)(866.02)$

$P = 759.8N$

from Fig., $R = 1000 \cos 30°$

$R = 866.02$

$W = F \times s$

$W = 759.8 \times 5$

$W = 3799\ Nm.$

Example 6: A train of weight 2000 kN is pulled by an engine on a level track at a speed of 36 kmph and with an acceleration of 0.5 m/s^2 on the level track. Find the power of the engine.

Ans: Given, $v = 36$ kmph = 10 m/s, W = 2000 kN

Friction, $F = 8N/kN = 8 \times 2000 = 16{,}000\ N$.

Here, the train is moving with an acceleration of 0.5 m/s^2.

$$P - F = ma$$

$$P - 16{,}000 = \frac{2000 \times 10^3}{9.8} \times 0.5$$

$$P = 102{,}040.81 + 16{,}000$$

$$P = 118040.81\ N$$

Power = force × velocity

$= 118040.81 \times 8$

$P = 944326.48$ watt.

Example 7: A train of weight 2000 kN is pulled by an engine on a level track at a constant speed of 36 kmph. The resistance due to friction is 8 N/kN of the train weight. Find the power of the engine.

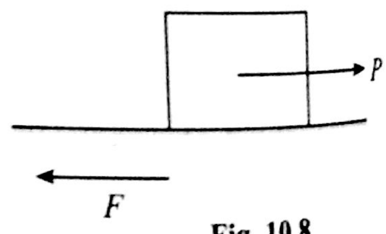


Fig. 10.8

Ans: Given, $W = 2000\ kN$

$$V = 36 \text{ Kmph} = 36 \times \frac{5}{18} = 10 \text{ m/s}$$

Friction, $F = 8\ N/kN = 8 \times 2000\ N$

$F = 16{,}000\ N$

Here the train is moving with constant speed. So, acceleration = 0;

$P - F = 0$

$P = F = 16{,}000$ N

Power = Force × Velocity = 16000 × 8

$P = 1{,}28{,}000$ Nm/s (or) watt.

Example 8: A train of weight 1600 *kN* is ascending a slope of 1 in 100 with a uniform speed of 36 kmph. Find the power exerted by the engine, if the road resistance is 5*N* per *kN* weight of the train.

Ans: Given, $W = 1600\ kN$

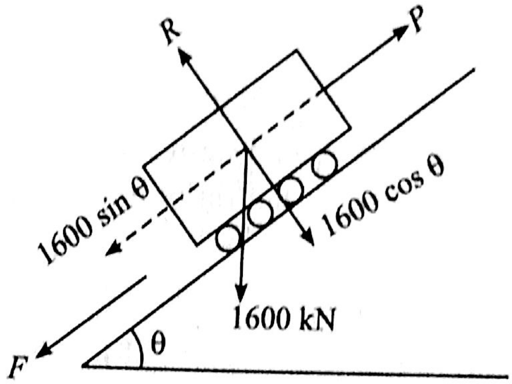


Fig. 10.9

$$\text{Slope} = \tan\theta = \frac{1}{100} = \sin\theta$$

[since θ is very small, $\tan\theta = \theta = \sin\theta$]

Speed of train = 36 kmph

$$36 \times \frac{5}{18} = 10 \text{ m/s}$$

Resistance ⇒ $F = 5N$ per kN

$$= 5 \times 1600 = 8000\ N$$

Now, Net force exerted $= P - F - 1600 \sin\theta\ kN$

$$= P - 8000 - \left(1600 \times 1000 \times \frac{1}{100}\right)$$

$$= P - 8000 - 16000$$

$$= (P - 24000)N$$

Since the train is moving with uniform velocity, acceleration=0

So net force = 0

$$P - 24000 = 0$$

Power = 24000 × 10 = 240000 W

Power = 240 kW

Example 9: Find the power of a locomotive, drawing a train whose weight including that of engine is 450 *kN* up an incline 1 in 120 at a steady speed of 72 kmph, the frictional resistance being 5 N/kN.

Ans: Given, $W = 450$ k*N*.

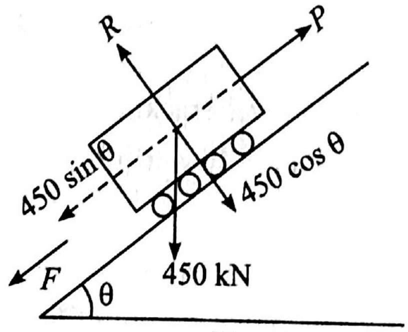


Fig. 10.10

$$\text{Slope} = \tan\theta = \frac{1}{120} = \sin\theta.$$

Speed = 72 kmph

$$= 72 \times \frac{5}{18} = 20 \text{ m/s}.$$

Resistance, $F = 5N/kN$

$$= 5 \times 450 = 2250\ N.$$

Now, Net force exerted $= P - 450 \sin\theta - 2250$

$$= P - \left(450 \times 1000 \times \frac{1}{120}\right) - 2250$$

$$= P - 3750 - 2250.$$

$$= (P - 6000)\ N.$$

Since the train is moving with steady speed,

Example 16: Determine the force that will give system of bodies a velocity of 3*m/s* after moving 4.5 m from rest as shown in Fig. 10.15 take $\mu = 0.3$ (use work energy equations).

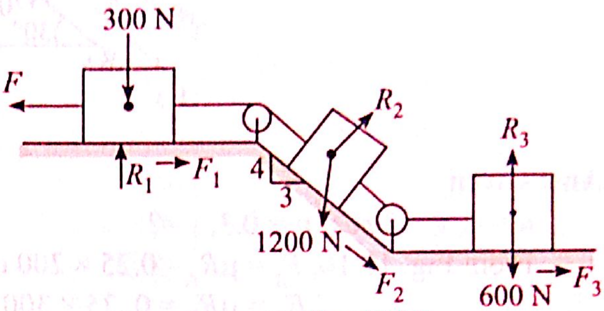


Fig. 10.15

Ans: Given $u = 0$, $v = 3$ m/s and $s = 4.5\ m$

From Fig., $\tan\theta = \frac{4}{3}$, $\theta = 53.13$

$$\text{Work done} = \frac{1}{2} m\,(v^2 - u^2)$$

$$F \times s = \frac{1}{2} m\,(v^2 - u^2)$$

From Fig.,

$$(F - F_1 - F_2 - F_3 - 1200 \sin 53.15) \times 4.5 = \frac{1}{2}(m_1 + m_2 + m_3)\,(3^2 - 0)$$

$F_1 = \mu R_1$

$R_1 = 300\ N$

$\mu = 0.3$

$F_1 = 0.3 \times 300$

$F_1 = 90\ N$

$F_2 = \mu R_2$

$= 0.3 \times 1200 \cos 53.13$

$F_2 = 216\ N$

$F_3 = \mu R_3$

$F_3 = 0.3 \times 600$

$= 180\ N$

$$m_1 = \frac{300}{9.8},\ m_2 = \frac{1200}{9.8},\ m_3 = \frac{600}{9.8}$$

$\therefore\ m_1 + m_2 + m_3 = 214.28$ kg.

From work energy equation

$$\text{Work done} = \frac{1}{2} m\,(v^2 - u^2)$$

$$(F - 486 - 960.2) \times 4.5 = \frac{1}{2}(214.28)\,9$$

$$(F - 1446.24)\,4.5 = \frac{1}{2}(1928.52)$$

$$4.5F - 6508.124 = 964.26$$

$$F = \frac{7472.384}{4.5}$$

$$F = 1660.53\ N.$$