Mass Moment of Inertia

Mass moment of inertia of a body about an axis is defined as the *sum total of product of its elemental* masses and square of their distances from the axis.

Thus, the mass moment of the body shown in Fig. 9.1 about axis AB is given by

$$I_{AB} = \Sigma m_i r_i^2 = \int r^2 \, dm \qquad ...(9.1)$$

where r is the distance of element of mass dm from AB.

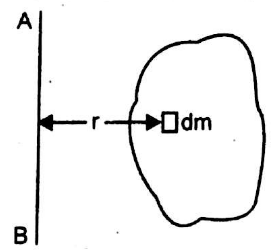

Fig. 9.1

This term which will be very useful in studying rotation of rigid bodies. For mass moment of inertia also, letter I is used, but in a problem where there is a need to differentiate between mass moment of inertia and moment of inertia of area, subscript m may be used with letter I. Since mass moment of inertia is product of mass and square of distance, its unit can be derived as given below.

$$mL^2 = \left(\frac{W}{g}\right) L^2$$

dimensionally, $\dfrac{\text{N}}{\text{m/sec}^2}\text{m}^2$

i.e., N-m-sec^2.

No name has been assigned to this unit. Hence, it may be simply called as unit.

9.1 RADIUS OF GYRATION

Radius of gyration *is that distance which when squared and multiplied with total mass of the body gives the mass moment of inertia of the body.* Thus, if I is moment of inertia of a body of mass M about an axis, then its radius of gyration k about that axis is given by the relation:

$$I = Mk^2 \quad \text{or} \quad k = \sqrt{\frac{I}{M}} \qquad ...(9.2)$$

A physical meaning may be assigned to this term. Radius of gyration is the distance at which the entire mass can be assumed to be concentrated such that the moment of inertia of the actual body and the concentrated mass are the same.

9.2 DETERMINATION OF MASS MOMENT OF INERTIA FROM BASIC PRINCIPLES

Mass moment of inertia of simple bodies can be determined from its definition. The steps to be followed are

(1) Take a general element.

(2) Write the expression for mass of the element, dm, and its distance, r, from the axis.

(3) Integrate the term $r^2\,dm$ between suitable limits such that the entire mass of the body is covered. The producer is illustrated with the examples given below:

Example 9.1. *Determine the mass moment of inertia of a uniform rod of length L about its: (a) centroidal axis normal to rod, and (b) axis at the end of the rod and normal to it.*

Solution. (*a*) *About Centroidal Axis Normal to Rod.*

Consider an elemental length dx at a distance x from centroidal axis y–y as shown in Fig. 9.2. Let the mass of rod be m per unit length. Then mass of the element $dm = mdx$.

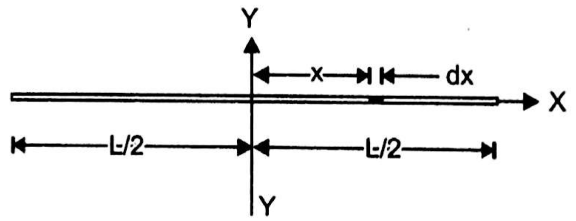

Fig. 9.2

$$\therefore \qquad I = \int_{-\frac{L}{2}}^{\frac{L}{2}} x^2\,dm = \int_{-\frac{L}{2}}^{\frac{L}{2}} x^2 \times mdx = m\left[\frac{x^3}{3}\right]_{-\frac{L}{2}}^{\frac{L}{2}} = \frac{mL^3}{12}$$

i.e.,

$$I = \frac{ML^2}{12} \qquad ...(9.3)$$

where $M = mL$ is total mass of the rod.

(*b*) *About Axis at the end of the Rod and Normal to it:*

Consider an elemental of length dx at a distance x from end as shown in Fig. 9.3.

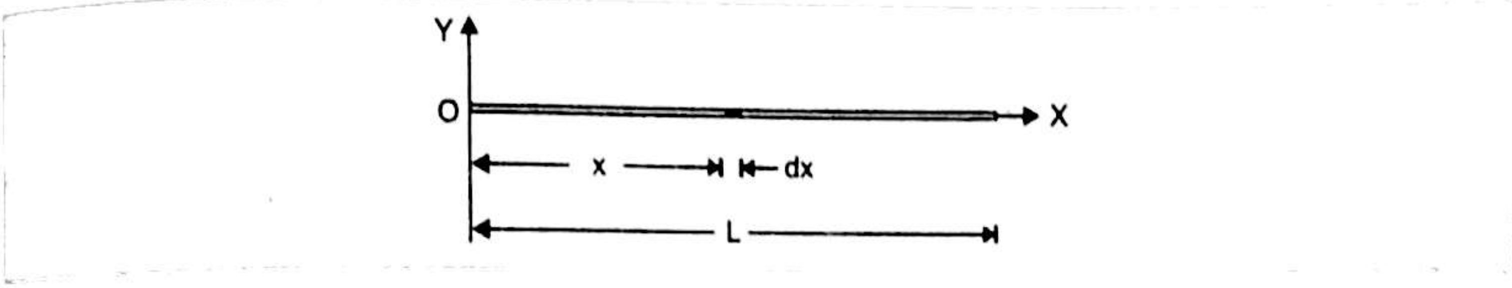

Fig. 9.3

Moment of inertia of the rod about y axis:

$$I = \int_0^L x^2\, m dx = m\left[\frac{x^3}{3}\right]_0^L = \frac{mL^3}{3}$$

$$I = \frac{ML^2}{3} \qquad ...(9.4)$$

Example 9.2. *Determine the moment of inertia of a rectangular plate of size a × b and thickness t about its centroidal axes.*

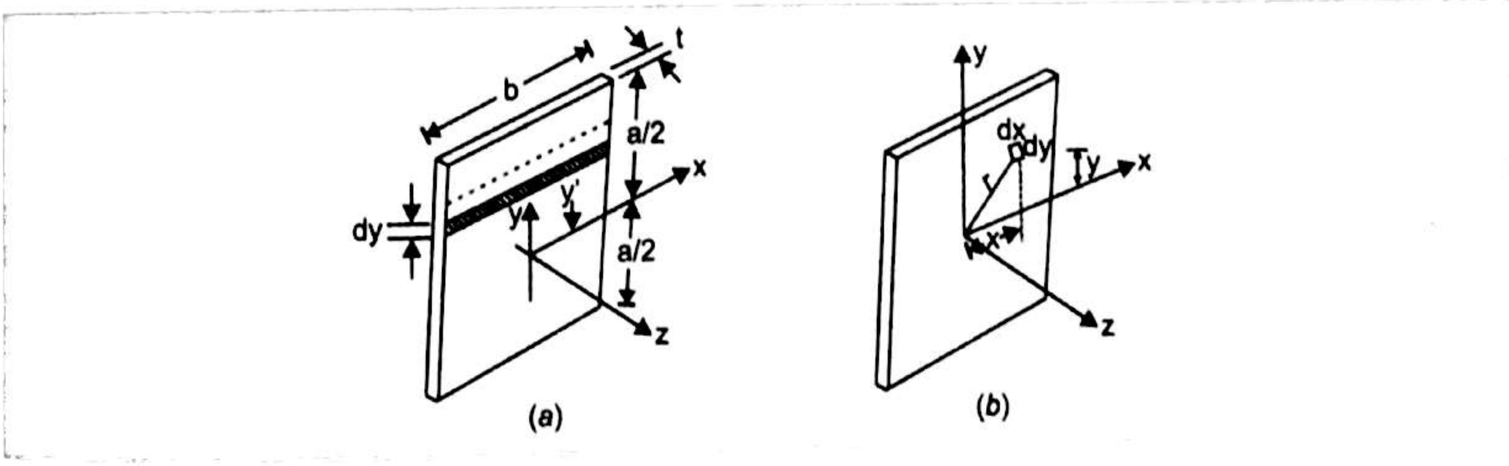

Fig. 9.4

Solution. *To find* I_{xx}

Consider an elemental strip of width dy at a distance y from x axis as shown in Fig. 9.4 (a). Mass of element:

$$dm = \rho b \times t \times dy \quad (\rho\text{–unit mass of the material})$$

$$\therefore \quad I_{xx} = \int_{-\frac{a}{2}}^{a/2} y^2\, dm = \int_{-\frac{a}{2}}^{a/2} y^2 \rho bt\, dy$$

$$= \rho bt\left[y^3/3\right]_{-a/2}^{a/2} = \frac{\rho bt\, a^3}{12}$$

But mass of the plate $\quad M = \rho\, bta$

$$\therefore \quad I_{xx} = \frac{Ma^2}{12} \qquad ...(9.5)$$

To find I_{yy}

Taking an elemental strip parallel to y axis, it can be easily shown that:

$$I_{yy} = \frac{Mb^2}{12} \quad ...(9.6)$$

To find I_{zz}

Consider an element of size $dx\ dy$ and thickness t as shown in Fig. 9.4 (b). Now

$$r^2 = x^2 + y^2$$

$$I_{zz} = \int r^2\, dm = \int (x^2 + y^2)\, dm$$

$$= \int x^2\, dm + \int y^2\, dm = I_{xx} + I_{yy}$$

$$= \frac{Ma^2}{12} + \frac{Mb^2}{12}$$

$$I_{zz} = \frac{1}{12} M(a^2 + b^2) \quad ...(9.7)$$

Example 9.3. *Find the moment of inertia of circular plate of radius R and thickness t about its centroidal axis.*

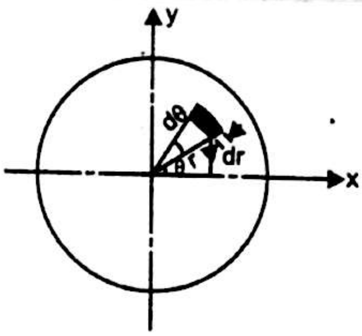

Fig. 9.5

Solution. Consider an elemental area $r\,d\theta\,dr$ and thickness t as shown in Fig. 9.5.

$$dm = \text{mass of the element} = \rho r\, d\theta\, dr t = \rho\, t\, r\, d\theta\, dr$$

Its distance from x axis $= r \sin\theta$

$$\therefore \quad I_{xx} = \oint (r \sin\theta)^2\, dm$$

$$= \int_0^R \int_0^{2\pi} r^2 \sin^2\theta\, \rho t r\, d\theta\, dr = \rho t \int_0^R \int_0^{2\pi} r^3 \left(\frac{1 - \cos 2\theta}{2}\right) dr d\theta$$

$$= \rho t \int_0^R \frac{r^3}{2} \left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi} dr = \rho t \int_0^R \frac{r^3}{2} \times 2\pi\, dr$$

$$= \rho t\, \pi \left[\frac{r^4}{4} \right]_0^R = \rho t\, \frac{\pi R^4}{4}$$

$$\text{Mass of the plate } M = \rho \times \pi R^2 t$$

$$\therefore \quad I_{xx} = \frac{MR^2}{4} \qquad \text{...(9.8)}$$

$$\text{Similarly,} \quad I_{yy} = \frac{MR^2}{4}$$

Actually $I = \dfrac{MR^2}{4}$ is moment of inertia of circular plate about any diametral axis in the plate.

To find I_{zz}, consider the same element.

$$I_{zz} = \oint r^2\, dm = \int_0^R \int_0^{2\pi} r^2\, \rho t\, r\, dr\, d\theta$$

$$= \rho t \int_0^R r^3 \left[\theta\right]_0^{2\pi} dr = \rho t \int_0^R 2\pi r^3\, dr$$

$$= \rho t 2\pi \left[\frac{r^4}{4} \right]_0^R = \rho t 2\pi\, \frac{R^4}{4} = \rho t\, \frac{\pi R^4}{2}$$

$$\text{But total mass } M = \rho t \pi R^2$$

$$\therefore \quad I_{zz} = \frac{MR^2}{2} \qquad \text{...(9.9)}$$

Example 9.4. *Determine the mass moment of inertia of a circular ring of uniform cross-section.*

Solution. Consider uniform ring of radius R as shown in Fig. 9.6. Let its mass per unit length be m.

Hence, total mass $\quad M = 2\pi R m$

Consider an elemental length $ds = R\, d\,\theta$ at an angle θ to the diametral axis x-x. The distance of the element from x axis is $R \sin\theta$ and mass of element is $m\, R d\,\theta$.

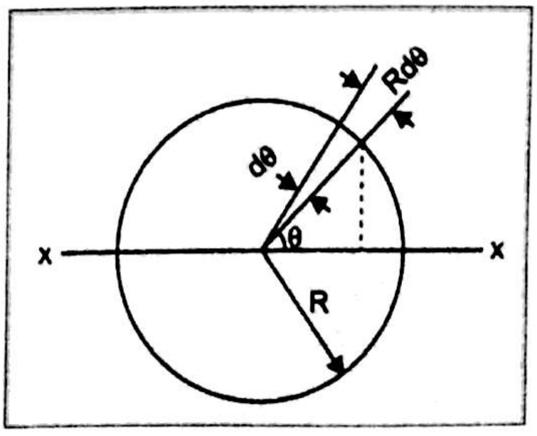

Fig. 9.6

$$\therefore \qquad I = \int_0^{2\pi} (R\sin\theta)^2\, mR d\theta = R^3 m \int_0^{2\pi} \sin^2\theta\, d\theta$$

$$= mR^3 \int_0^{2\pi} \left(\frac{1-\cos 2\theta}{2}\right) d\theta = \frac{mR^3}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi} = mR^3\pi$$

But $\qquad M = 2\pi R m$

$$\therefore \qquad I = \frac{MR^2}{2} \qquad \text{...(9.10)}$$

Example 9.5. *Find the mass moment of inertia of the solid cone of height h and base radius R about:*

(1) its axis of rotation and

(2) an axis through vertex normal to the axis of rotation.

Solution. *Moment of Inertia about its Axis of Rotation:*

Consider an elemental plate at distance x. Let its radius be r and thickness dx.

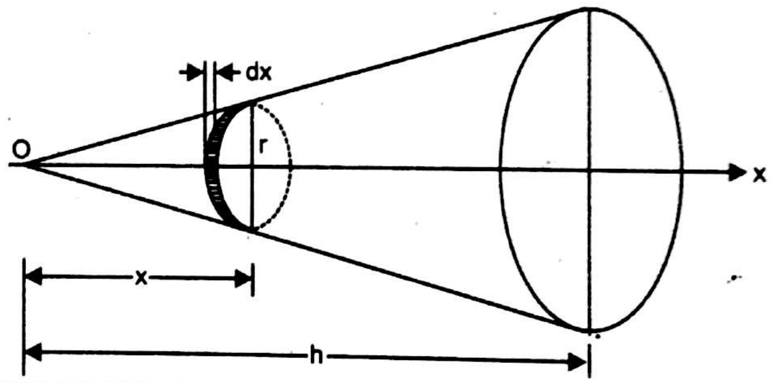

Fig. 9.7

Mass of the elemental plate $= \rho\, \pi r^2\, dx$

But from Eqn. 9.9, the moment of inertia of circular plate about normal axis through its centre is

$$= \frac{1}{2} \times \text{mass} \times \text{square of radius}$$

$$= \frac{1}{2} \times \rho \pi r^2\, dx\, r^2 = \rho \frac{\pi r^4 dx}{2}$$

But now, $\qquad r = \left(\frac{x}{h}\right) R$

$\therefore$ Moment of inertia of the elemental plate

$$\text{about } x \text{ axis} = \rho \frac{\pi}{2} R^4 \frac{x^4}{h^4} dx \qquad \text{...(2)}$$

∴ Moment of inertia of the cone about x axis,

$$I_{xx} = \int_0^h \frac{\rho\pi}{2} R^4 \frac{x^4}{h^4} dx$$

$$= \frac{\rho\pi}{2} \frac{R^4}{h^4} \left[\frac{x^5}{5}\right]_0^h$$

$$I_{xx} = \frac{\pi R^4 h}{10} \quad \text{...(3)}$$

But mass of the cone

$$M = \int_0^h \pi r^2 dx = \int_0^h \pi R^2 \frac{x^2}{h^2} dx$$

$$= \frac{\pi R^2}{h^2} \left[\frac{x^3}{3}\right]_0^h = \frac{\pi R^2 h}{3}$$

From (3) and (4) we get,

$$I_{xx} = \frac{3}{10} MR^2 \quad \text{...(9.11)}$$

Moment of Inertia about an Axis through Vertex and Normal to the Axis of Rotation

Consider an element of size $rd\theta \times dr \times dx$ as shown in Fig. 9.8. Let its coordinates be x, y, z and z_1 be the radius of the plate at distance x from vertex. Now, the mass of this element

$$dm = \rho r d\theta \, dr \, dx$$

and its distance from y axis is given by:

$$l = \sqrt{x^2 + z^2}$$

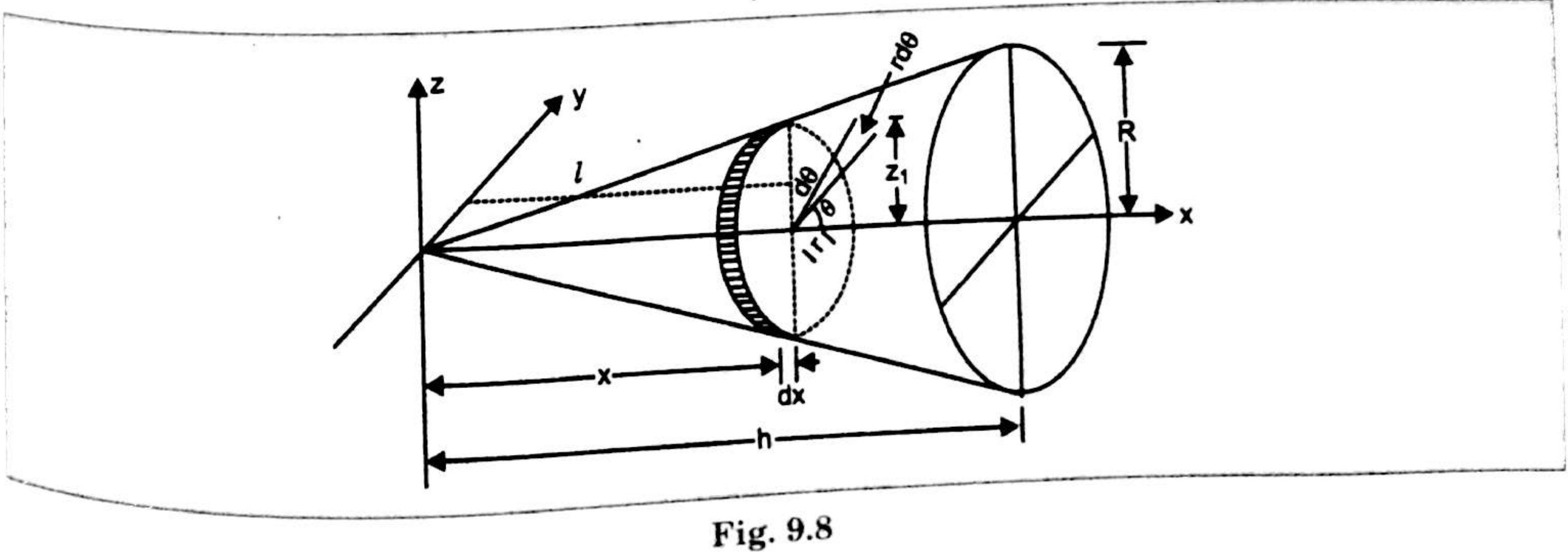

Fig. 9.8

∴ Moment of inertia of the cone about y axis:

$$I_y = \int l^2 dm = \int_0^h \int_0^{z_1} \int_0^{2\pi} (x^2 + z^2)\, \rho\, r d\theta\, dr dx$$

But $$z = r \sin \theta$$

$\therefore$ $$I_{yy} = \int_0^h \int_0^{z_1} \int_0^{2\pi} (x^2 + r^2 \sin^2 \theta)\, \rho r d\theta\, dr dx$$

$$= \int_0^h \int_0^{z_1} \int_0^{2\pi} \rho \left(x^2 r + r^3 \frac{1 - \cos 2\theta}{2} \right) d\theta\, dr dx$$

$$= \int_0^h \int_0^{z_1} \rho \left[x^2 r\theta + \frac{r^3}{2} \left(\theta - \frac{\sin 2\theta}{2} \right) \right]_0^{2\pi} dr dx$$

$$= \int_0^h \int_0^{z_1} \rho \left[2\pi x^2 r + \frac{r^3}{2} 2\pi \right] dr\, dx$$

$$= \int_0^h \rho \left[\pi x^2 r^2 + \pi \frac{r^4}{4} \right]_0^{z_1} dx$$

$$= \int_0^h \rho \left[\pi x^2 z_l^2 + \frac{\pi z_l^4}{4} \right] dx$$

but $$z_l = \frac{x}{h} R$$

$\therefore$ $$I_{yy} = \int_0^h \rho \left[\pi x^2 \frac{x^2}{h^2} R^2 + \frac{\pi}{4} \frac{x^4}{h^4} R^2 \right] dx$$

$$= \rho \left[\pi \frac{x^5}{5h^2} R^2 + \frac{\pi x^5}{20 h^4} R^4 \right]_0^h$$

$$= \rho \left[\frac{\pi}{5} h^3 R^2 + \frac{\pi}{20} h R^4 \right] = \frac{\pi\, \rho\, R^2 h}{5} \left(h^2 + \frac{R^2}{4} \right)$$

But mass of cone $$M = \frac{1}{3} \rho \pi R^2 h$$

$\therefore$ $$I_{yy} = \frac{3M}{5} \left(h^2 + \frac{R^2}{4} \right) \qquad ...(9.12)$$

Example 9.6. *Determine the moment of inertia of a solid sphere of radius R about its diametral axis.*

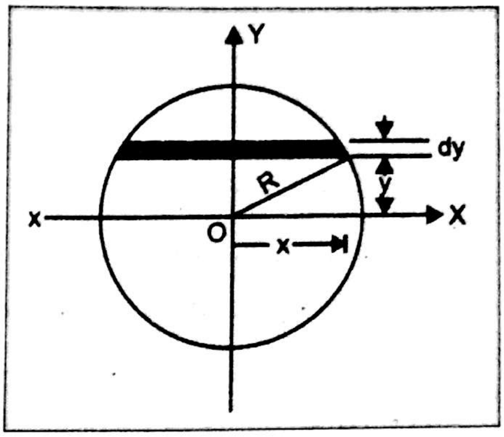

Fig. 9.9

Solution. Consider an elemental plate of thickness dy at distance y from the diametral axis as shown in Fig. 9.9. Radius of this elemental circular plate x is given by the relation:

$$x^2 = R^2 - y^2 \quad ...(1)$$

∴ Mass of the elemental plate

$$dm = \rho\, \pi x^2\, dy$$

$$= \rho\pi(R^2 - y^2)dy \quad ...(2)$$

Moment of inertia of this circular plate element about y axis is given by Eqn. 9.9 as:

$$= \frac{1}{2} \times \text{mass} \times \text{square of radius}$$

$$= \frac{1}{2} \times \rho\pi x^2\, dy \times x^2 = \rho\frac{\pi}{2}x^4 dy$$

$$= \rho\frac{\pi}{2}(R^2 - y^2)^2 dy$$

$$= \rho\frac{\pi}{2}(R^4 - 2R^2y^2 + y^4)dy$$

$$\therefore \quad I_{yy} = 2\int_0^R \rho\frac{\pi}{2}(R^4 - 2R^2y^2 + y^4)dy$$

$$= \rho\pi\left[R^4y - \frac{2R^2y^3}{3} + \frac{y^5}{5}\right]_0^R$$

$$= \rho\pi R^5\left[1 - \frac{2}{3} + \frac{1}{5}\right] = \frac{8}{15}\rho\pi R^5 \quad ...(3)$$

But mass of sphere,

$$M = 2\int_0^R dm = 2\int_0^R \rho\pi x^2 dy = 2\int_0^R \rho\pi(R^2 - y^2)dy$$

$$= 2\rho\pi\left(R^2y - \frac{y^3}{3}\right)_0^R = 2\rho\pi\left[R^3 - \frac{R^3}{3}\right]$$

$$M = \frac{4\pi R^3}{3} \quad ...(4)$$

From (3) and (4) above we get:

$$I_{yy} = \frac{2}{5}MR^2 \quad \text{...(9.13)}$$

Example 9.7. *Using the moment of inertia expression for plates, find the expressions for moment of inertia of*

(a) Parallelepiped and

(b) Circular cylinder about z axis as shown in Fig. 9.10.

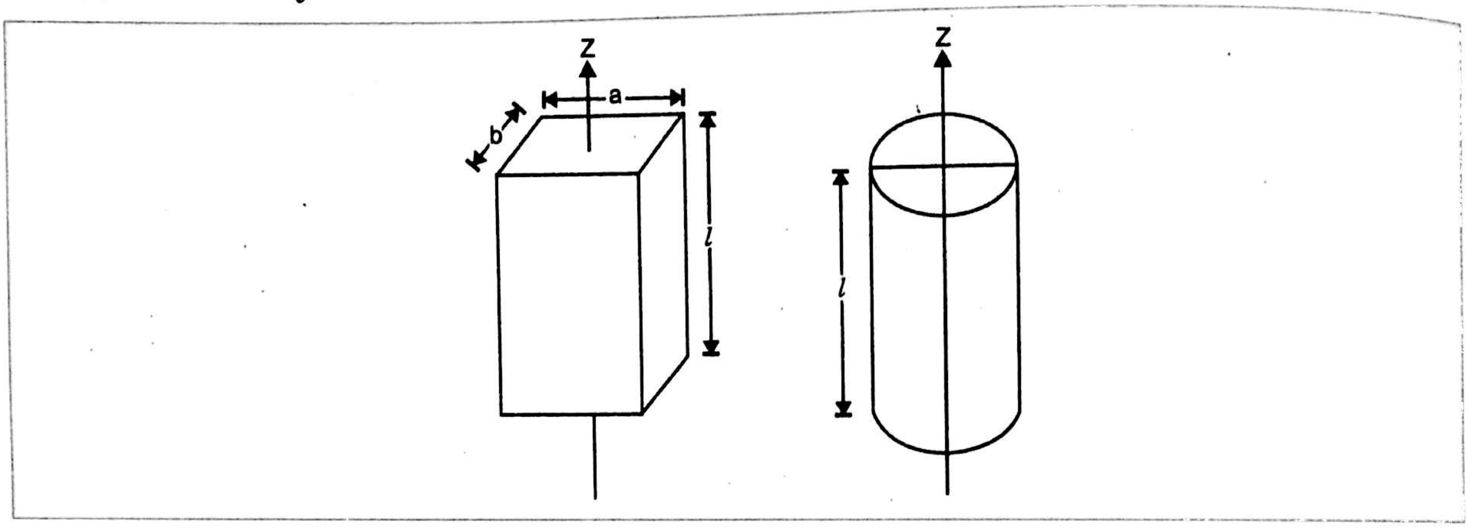

Fig. 9.10

Solution. Parallelepiped can be looked as a rectangular plate of thickness $t = l$, and similarly a solid cylinder as a circular plate of thickness l. Hence the expressions for moment of inertia are:

$\frac{1}{12}M(a^2 + b^2)$ for parallelepiped

$\frac{MR^2}{2}$ for cylinder

where Mass of parallelepiped $= abl\rho$

and that of cylinder $= \pi R^2 l \rho$

9.3 PARALLEL AXIS THEOREM/TRANSFER FORMULA

Parallel axis theorem *i.e.*, transfer formula states *the moment of inertia of a body about an axis at a distance d and parallel to a centroidal axis is equal to sum of moment of inertia about centroidal axis and product of mass and square of distance of parallel axis.* Thus, if I_g is the moment of inertia of a body of mass M about a centroidal axis and I_A is the moment of inertia about a parallel axis through A which is at a distance d from centroidal axis, then

$$I_A = I_g + Md^2$$

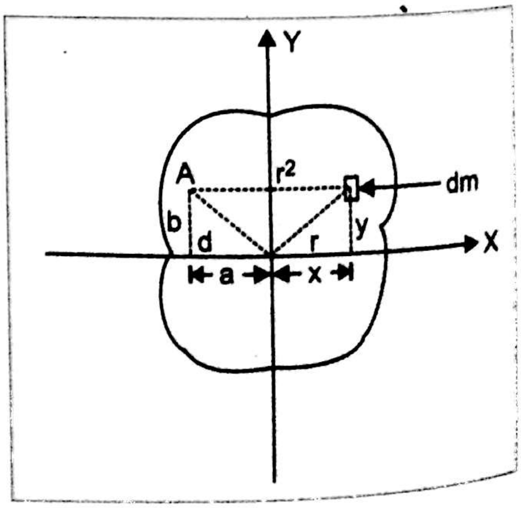

Fig. 9.11

Proof: Let dm be an element at a distance r from centroidal axis z through centre of gravity as shown in Fig. 9.11.

$$I_g = \int r^2\, dm = \int (x^2 + y^2) dm \qquad \text{...(1)}$$

Now, moment of inertia about z axis through A is,

$$I_A = \int r'^2\, dm$$

where r' is the distance of element from A.

$$= \int [(x + a)^2 + (y - b)^2]\, dm$$

$$= \int (x^2 + 2ax + a^2 + y^2 - 2yb + b^2) dm$$

$$= \int (x^2 + y^2) dm + \int (a^2 + b^2) dm + \int 2ax dm - \int 2yb\, dm$$

$$= \int r^2\, dm + \int d^2\, dm + \int 2ax dm - \int 2b y dm$$

Since $x^2 + y^2 = r^2$

and $a^2 + b^2 = d^2$

but, $\int r^2\, dm = I_g$

$$\int d^2\, dm = d^2 \int dm = Md^2$$

$$\int 2axdm = 2a \int x\, dm = 2a M \bar{x}$$

and $\int 2b y dm = 2b \int y dm = 2b\, M \bar{y}$

where $\bar{x}$ and $\bar{y}$ are distance of centre of gravity from the reference axis. In this case $\bar{x}$ and $\bar{y}$ are zero since reference axis contains centroid. Thus $\int 2ax\, dm = \int 2b y dm = 0$.

Hence $I_A = I_g + Md^2$ ...(9.14)

9.4 MOMENT OF INERTIA OF COMPOSITE BODIES

In order to determine the moment of inertia, the composite body is divided into a set of simple bodies. The centre of gravity and moment of inertia expressions for such simple bodies are known. Moment of inertia of simple bodies about their centroidal axis are calculated and then using parallel axis theorem, moment of inertia of each simple body is found about the required axis. Summing up of the moment of inertia of each simple body about the required axis, gives the moment of inertia of the composite body. The procedure is illustrated with the examples 9.8 and 9.9.

Example 9.8. *Determine the radius of gyration of the body shown in Fig. 9.12 about the centroidal x axis. The grooves are semicircular with radius 40 mm.*

All dimensions shown are in mm.

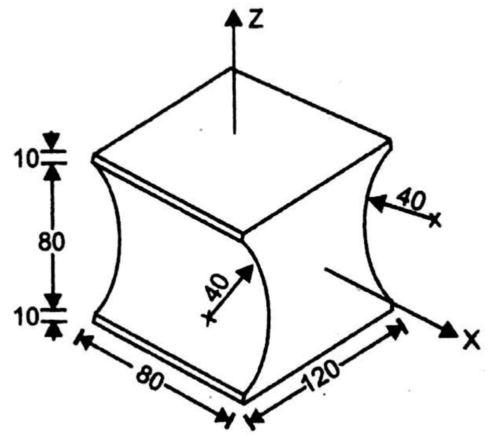

Fig. 9.12

Solution. The composite body may be divided into

(1) A solid block of size 80 × 120 × 100 mm. and,

(2) Two semicircular grooves each of radius 40 mm and length 80 mm.

Mass of solid block,

$$M_1 = 80 \times 120 \times 100\ \rho$$
$$= 960000\ \rho$$

where, ρ is mass of 1 mm^3 of material.

Its moment of inertia about x axis

$$Ix_1 = M_1 \frac{(100^2 + 120^2)}{12} = 960 \times 10^3\ \rho \frac{(100^2 + 120^2)}{12}$$
$$= 1.952 \times 10^9 \rho$$

Semicircular groove:

Mass,

$$M_2 = \frac{1}{2}\pi r^2\ l\ \rho = \frac{1}{2}\pi 40^2 \times 80\rho = 201061.93\ \rho$$

Moment of inertia about the axis parallel to x axis through diametral axis of semicircle:

$$= \frac{1}{2} \text{ of that of cylinder}$$
$$= \frac{1}{2} \times 2M_2 \times \frac{r^2}{2} = \frac{M_2 r^2}{2}$$

Centre of gravity of semicircular groove from this axes is at a distance $d = \frac{4r}{3\pi}$.

$$= \frac{4 \times 40}{3 \times \pi} = 16.9765$$

∴ Moment of inertia about the axis through centre of gravity I_g, is given by:

$$\frac{M_2 r^2}{2} = I_g + M_2 d^2$$

$$\therefore \quad I_g = M_2\left(\frac{r^2}{2} - d^2\right)$$

$$= 201061.93\,\rho\left(\frac{40^2}{2} - 16.9765^2\right)$$

$$= 1.029 \times 10^8\,\rho$$

The distance of this centroid from x axis is

$$d' = 60 - 16.9765 = 43.0235$$

$$\therefore \quad Ix_2 = I_g + M_2 d'^2$$

$$= 1.1561 \times 10^8\,\rho + 201061.93\,\rho \times 43.0235^2$$

$$= 4.7507 \times 10^8\,\rho$$

$$\therefore \quad I_{xx} \text{ of composite body} = Ix_1 - 2Ix_2$$

(since there are two semicircular grooves placed symmetrically with respect to x–x axis)

$$I_{xx} = 1.952 \times 10^9\,\rho - 2 \times 4.7507 \times 10^8\,\rho$$

$$= 1000186 \times 10^8\,\rho$$

$$\text{Total mass } M = M_1 - 2M_2$$

$$= 960000\,\rho - 2 \times 201061.93\,\rho$$

$$= 557876.14 \text{ units}$$

$$k = \sqrt{\frac{I}{M}} = \sqrt{\frac{10.0186 \times 10^8}{552876.14}}$$

i.e., $$k = \mathbf{42.97\ mm}$$

Example 9.9. *A cast iron fly wheel has the following dimensions:*

Diameter = *1.5 m*

Rim width = *300 mm*

Thickness of rim = *50 mm*

Hub length = *200 mm*

Outer diameter of hub = *250 mm*

Inner diameter of hub = *100 mm*

Arms: 6 equally spaced uniform slender rods of length of 0.575 m

Cross-sectional area of each arm = *8000 mm*^{2}

Determine the moment of inertia of the wheel about the axis of rotation. Take mass of cast iron as 7200 kg/m^3.

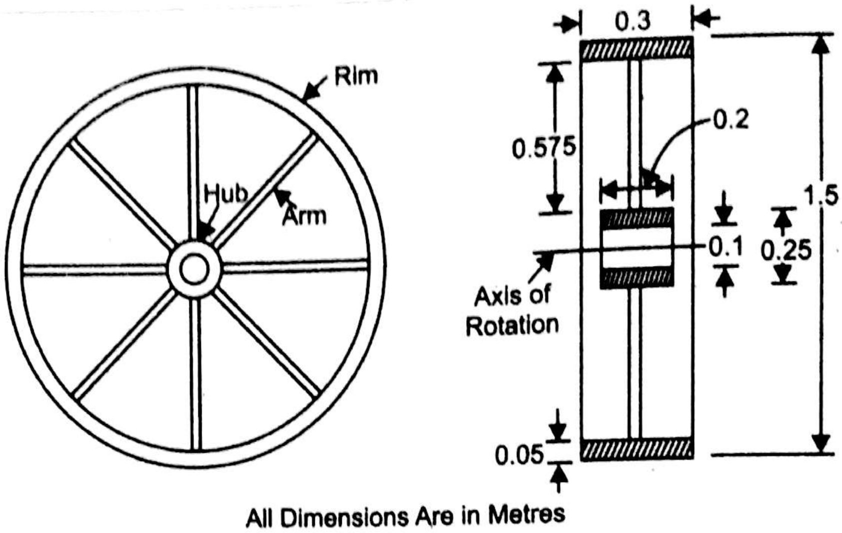

Fig. 9.13

Solution. *Moment of inertia of rim:*

Outer diameter = 1.5 m

thickness = 0.05 m

∴ Inner diameter = 1.5 − 2 × 0.05 = 1.4 m

Treating it as a solid circular plate of 1.5 m diameter in which a circular plate of diameter 1.4 m is cut and noting that for a plate moment of inertia about the axis is:

$$\frac{\text{Mass} \times \text{Square of radius}}{2}$$

The moment of inertia of rim is

$$I_1 = \frac{M_o R_o^2 - M_i R_i^2}{2}$$

$$= \frac{1}{2}\left[\left(\pi R_o^2\ t\ \rho\ R_o^2 - \pi R_i^2\ t\ \rho\ R_i^2\right)\right]$$

Where,

$$R_o = \text{outer radius} = \frac{1.5}{2} = 0.75$$

$$R_i = \text{inner radius} = \frac{1.4}{2} = 0.7$$

M_o and M_i are the masses of circular plates of radii R_o and R_i, respectively

∴

$$t = \text{width} = 0.30$$

$$\rho = \text{mass per cubic meter} = 7200 \text{ kg/m}^3$$

i.e.

$$I_1 = \left(\frac{1}{2}\right)\pi t\left[R_o^4 - R_i^4\right]$$

∴

$$I_1 = \left(\frac{1}{2}\right)\pi \times 0.3 \times 7200\ [0.75^4 - 0.7^4]$$

$$= 258.9010 \text{ units.}$$

Moment of inertia of hub:

This hollow cylinder may be considered as a circular plate with circular cut out and thickness equal to length of cylinder

In this case, outer radius $R_o = \frac{0.25}{2} = 0.125$ m, and

Inner radius $R_i = \frac{0.1}{2} = 0.05$ m

Length of cylinder $t = 0.2$

As in the above case, the moment of inertia can be worked out as the moment of inertia of solid plate minus the moment of inertia of hollow plate

$$I_2 = \frac{1}{2} \times \pi \times 0.2 \times 7200\,(0.125^4 - 0.05^4) = 0.5381 \text{ units.}$$

Moment of Inertia of Arms:

Moment of inertia of arm about its centre of gravity is $= \frac{Ml^2}{12}$

and when it is shifted to axis of rotation it will be equal to $\frac{Ml^2}{12} + Md^2$

Now,

$$A = 8000 \text{ mm}^2 = 8000 \times 10^{-9} \text{ m}^2$$

$$l = 0.575 \text{ m}$$

$$d = \frac{0.575}{2} + 0.125 = 0.4125 \text{ m}$$

$$M = l\,A\,\rho = 0.575 \times 8000 \times 10^{-9} \times 7200$$

$$= 0.03312 \text{ kg}$$

As there are six such arms,

$$I_3 = 6 \times \left(\frac{Ml^2}{2}\right) \times d^2$$

$$= 6 \times 0.03312 \left(\frac{0.575^2}{12}\right) \times 0.4125^2$$

$$= 0.0393 \text{ units}$$

∴ Moment of inertia of flywheel

$$I = I_1 + I_2 + I_3$$

$$= 258.90 + 0.5381 + 0.0393 = 259.4774 \text{ units.}$$

Important Definitions and Formulae

1. Mass moment of inertia of a body about an axis is defined as the sum total of product of its elemental masses and square of their distances from the axis

$$I_{AB} = \Sigma m_i r_i^2 = \int r^2 dm$$

2. Radius of gyration is that distance which when squared and multiplied with total mass of the body gives the mass moment of inertia of the body.

$$I = Mk^2 \text{ or } k = \sqrt{M / I}$$

A physical meaning may be assigned to the term radius of gyration. It is the distance at which entire mass can be assumed to be concentrated such that the moment of inertia of the actual body and that of the concentrated mass are the same.

3. The expressions for moment of inertia of various standard shaped bodies are presented in Eqns. 9.3 to 9.13.

4. Parallel axis theorem *i.e.,* transfer formulae: The moment of inertia of a body about an axis at a distance '*d*' and parallel to a centroidal axis is equal to sum of moment of inertia about the centroidal axis and product of mass and square of the distance of parallel axis.

$$I_A = I_g + Md^2$$

PROBLEM FOR EXERCISE

9.1. Determine the moment of inertia of the link shown in Fig. 9.14.

[**Ans.** 17.23×10^8 ρ units]