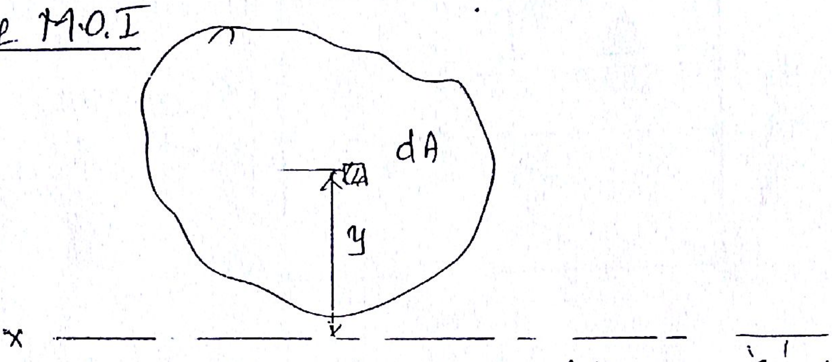
UNIT – III

MOMENT OF INERTIA:

Area Moment of Inertia:

In determining the strength of structural objects like beams, one comes across evaluation of mathematical expressions like $\int y^2 dA$ where 'y' is the distance of an elemental area from a reference axis in the plane of the area. This is called the 'second moment' or 'Area Moment of Inertia'.

Defination of M.O.I of an Area



If 'dA' is an elemental area at a distance 'y' from the axis x–x,

$\int_A y^2 dA$ is called the M.O.I. of A about X–X.

Obviously, the moment of ~~are~~ inertia depends both upon the area and the axis. Therefore, while defining M.O.I, the axis must be specified.

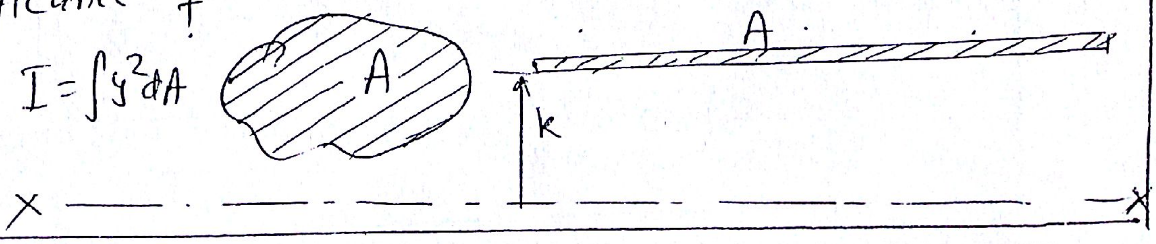
Radius of Gyration:

If $I = \int y^2 dA = k^2 A$ where 'A' is the area, the number 'k' is called the radius of gyration and has units of length.

Units of M.I. are $(\text{length})^4$ i.e. mm^4, m^4 etc.

Significance of R.O.G.

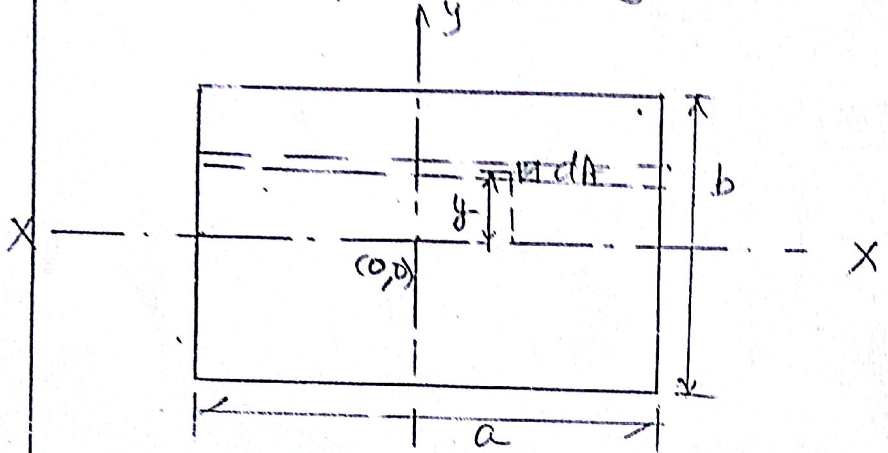
$I = \int y^2 dA$



If we imagine the given area 'A' of M.O.I 'I' to be remoulded into a thin strip of area 'A' parallel to X-X axis such that its M.I. is equal to I, then the distance at which the strip must be placed so that this happens, is called the radius of gyration.

Moment of Inertia of Simple Shapes.

I) M.I. of a rectangle



Let us consider a rectangular area $dA = dx\,dy$ at a distance of 'y' from X-X axis.

M.I. of this element $= dA.y^2 = dx.dy.y^2$

If we now take a long strip of area of length 'a' and at a distance y from x-x axis containing the element dx dy, the M.I of this strip area

$=$ Area of the strip $\times\ y^2$ ($\because$ 'y' is same for all dxdy elements)

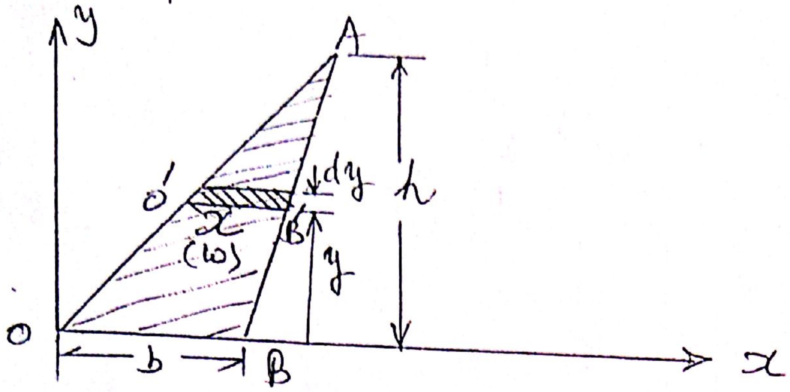
i.e. $dI = \int_0^a y^2 dy.dx = \int y^2.dA$ (where $dA = a \times dy$)

$\therefore\ I_{xy} = \int_{-b/2}^{b/2} y^2 dA = \int_{-b/2}^{b/2} y^2.a\,dy = a\left[\frac{y^3}{3}\right]_{-b/2}^{b/2}$

or, $I_{xx} = \frac{a}{3}\left[\frac{b^3}{8} - \left(-\frac{b^3}{8}\right)\right] = \frac{1}{12}ab^3$

Similarly, for a central 'Y' axis, $I_{YY} = \frac{1}{12}ba^3$.

II) M.I. of triangle about its base:



Let the M.I. be required about an axis through the base X-X (coinciding with x axis for convenience).

Then, by defination, $I_x = \int_A y^2 dA$.

Let us consider an area strip parallel to the base (x-axis) at a distance of 'y' and of length 'x' and heig[ht] 'dy'.

Then, $I = \int_A y^2 . x . dy$

Now in Δ's OAB and O'AB'.

$\angle A$ is common, and OB' & OB are parallel,

Hence, Δ's OAB and O'AB' are similar,

Therefore, $\frac{x}{b} = \frac{h-y}{h}$, $\Rightarrow$ $x = \frac{b}{h}(h-y)$

$\therefore$, $I = \int_A \frac{b}{h}(h-y).y^2.dy = \frac{b}{h}\int_0^h (hy^2 - y^3)dy$

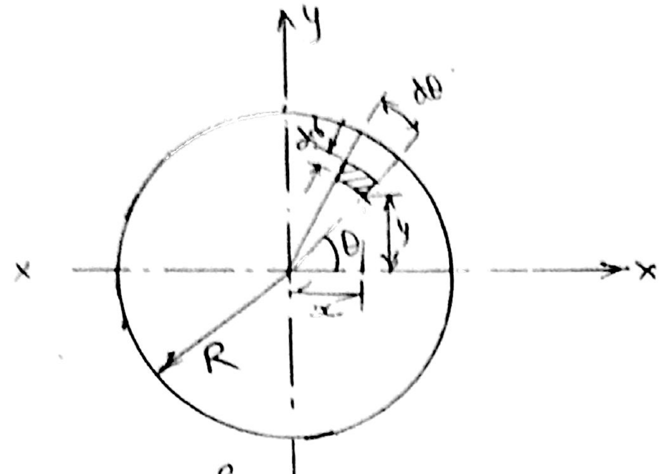
or, $I = \frac{b}{h}\left[\frac{hy^3}{3} - \frac{y^4}{4}\right]_0^h$

$= \frac{b}{h}\left[\left(\frac{h^4}{3} - \frac{h^4}{4}\right)\right] = \frac{1}{12} bh^3$

III) M.I. of a Circular area about Diameteral Axi[s]

Let us consider an area element $r dr d\theta$ in polar coordinates.

Then, $I_{xx} = \int_0^R \int_0^{2\pi} r^2 \sin^2\theta \; r . d\theta . dr$



$x = r\cos\theta$
$y = r\sin\theta$

$$\therefore,\quad I = \int_0^R \int_0^{2\pi} r^2 \sin^2\theta \cdot dr \cdot r\,d\theta = \int_0^R\int_0^{2\pi} (r^3\,dr)\sin^2\theta\,d\theta$$

$$= \int_0^{2\pi} \left[\frac{r^4}{4}\right]_0^R \sin^2\theta\,d\theta = \frac{R^4}{4}\int_0^{2\pi} \sin^2\theta\,d\theta$$

$$= \frac{R^4}{4}\int_0^{2\pi}\left(\frac{1-\cos 2\theta}{2}\right)d\theta = \frac{R^4}{4}\left[\frac{\theta}{2} - \sin 2\theta\right]_0^{2\pi}$$

$$\text{or,}\quad I = \frac{R^4}{4}\left[\left(\frac{2\pi}{2} - 0\right) - (0-0)\right]$$

$$= \frac{\pi R^4}{4} \quad \text{--------------} \quad (1)$$

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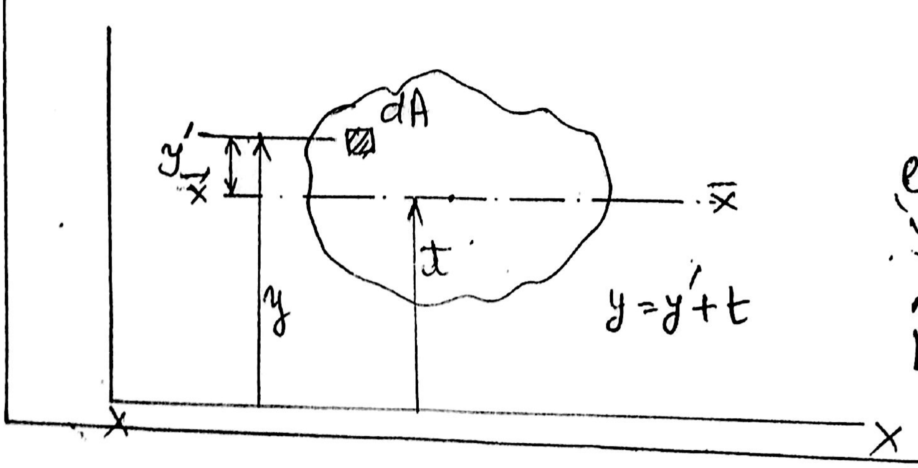
Parallel and Perpendicular Axes theorems:

Parallel Axis (or Transfer Axis) Theorem.

The moment of Inertia of an area A about an axis parallel to a centroidal axis in the same plane is equal to the M.I. ~~area~~ of the same area about its centroidal axis + Area × d^2 where 'd' is the distance between the centroid and the required axis



Let us consider an area element 'dA' at a distance 'y' from the required axis. Let 'd' be the distance between the reqd. axis and centroidal axis

M.I of the elemental area 'dA' about X X

$$dI = y^2 \cdot dA = (y' + t)^2 \, dA$$

or, $$dI = (y'^2 + 2y' \cdot t + t^2) \cdot dA.$$

Hence, $$I = \int_A (y'^2 + 2y't + t^2) \, dA$$

$$= \int_A y'^2 dA + 2\int y' \cdot t \cdot dA + \int t^2 dA.$$

But, We know, $2\int y' \cdot t \, dA = 2t \int y' \cdot dA = \bar{y} \cdot A$ where $\bar{y}$ is the coordinate of c.G. of the area w.r.to $x'-x' = 0$ (which is zero) --(1)

$$\therefore \quad t\int y' \, dA = 0.$$

and, since 't' is a constant,

$$\int t^2 \cdot dA = t^2 \int dA = t^2 \cdot A. \quad \text{---(2)}$$

Hence, $I = \int y'^2 dA + t^2 A.$ or, $I = \bar{I}_x + A t^2$

= M.I. about parallel centroidal axis + $t^2 A$.

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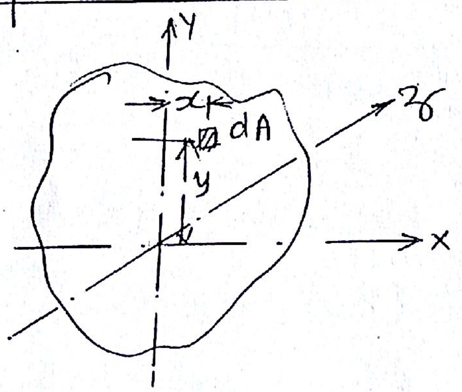
Perpendicular Axis Theorem;



Let 'dA' be an area element in the X-Y plane as shown

Then,

$$dI_x = y^2 \, dA \,, \text{ and}$$

$$dI_y = x^2 \, dA$$

Hence, $$dI_x + dI_y = y^2 dA + x^2 dA$$

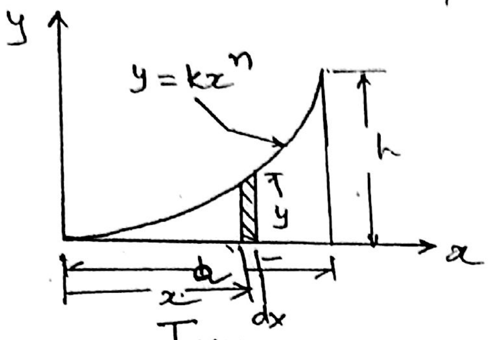
$$= (x^2 + y^2) \, dA.$$

Or ~~But~~, $$I_x + I_y = \int x^2 dA + \int y^2 dA$$

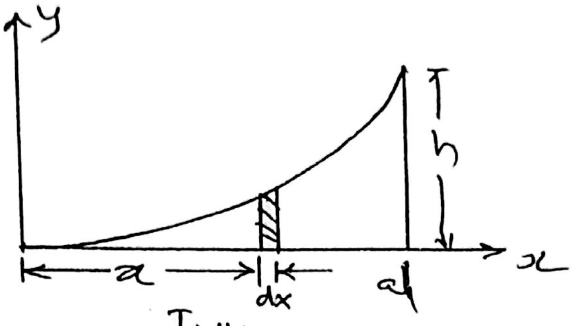
$$= \int_A (x^2 + y^2) \, dA = \int_A z^2 dA = I_z$$

Thus, the M.I. about 'z' axis passing thr. the intersect of X and Y axis is the sum of M.I about X & Y axes.

To find out M.I. of a spandrel about coordinate axes.



I_{XX}



I_{YY}

1) I_{XX} :- $dI_{XX} = \frac{1}{3} y^3 dx. = \frac{1}{3}(kx^n)^3 . dx.$

$$\therefore \quad I_{XX} = \int_0^a \frac{1}{3}(kx^n)^3 dx = \frac{k^3}{3}\int_0^a x^{3n}.dx$$

$$= \frac{k^3}{3}\left(\frac{x^{3n+1}}{3n+1}\right)_0^a = \frac{k^3 . a^{3n+1}}{3(3n+1)}$$

$$\text{or,} \quad I_{XX} = (k^3.a^{3n}). a \times \frac{1}{3(3n+1)} = \frac{ah^3}{3(3n+1)} \quad \text{- - -}$$

$$\text{Also,} \quad \bar{I}_{XX} = I_{XX} - \frac{ah}{n+1}.\left(\frac{n+1}{4n+2}.h\right)^2$$

$$= \frac{ah^3}{3(3n+1)} - \frac{ah^3(n+1)}{(4n+2)^2} = \frac{ah^3}{3} \frac{(7n^2+4n+1)}{(3n+1)(4n+2)^2}$$

2) I_{YY} :

$$dI_{YY} = x^2 dA = x^2 y.dx.$$

$$\text{or,} \quad dI_{YY} = x^2.(kx^n)dx = kx^{n+2}.dx$$

$$\rightarrow \quad I_{YY} = k.\int_0^a x^{n+2}.dx = k\left[\frac{x^{n+3}}{n+3}\right]_0^a$$

$$= \frac{ka^{n+3}}{n+3} = \frac{a^3 h}{n+3} \quad \text{- - - - - - -}$$

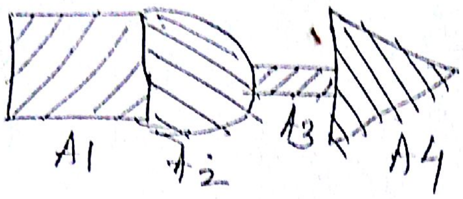
$$\text{Also,} \quad \bar{I}_{YY} = \frac{a^3 h}{(n+3)} - \frac{ah}{(n+1)}.\left(\frac{n+1}{n+2}.a\right)^2$$

$$= \frac{a^3 h}{(n+2)^2.(n+3)} \quad \text{- - - - - - - -}$$

MOMENT OF INERTIA OF COMPOSITE SECTIONS:

The moment of inertia of a composite section about a prescribed axis is found in the following steps.

(I) The area is divided into geometrically simple shapes by adding (or subtracting) which the required area is formed.

A =


(II) The centroid of the areas is found using a convenient coordinate system if M.I. is required about centroidal axis

(III) For each of the part area, the M.I. is calculated about its own centroidal axis using appropriate formula.

(IV) The M.I. of the areas for each area is transferred to the required axis using parallel axis theorem.

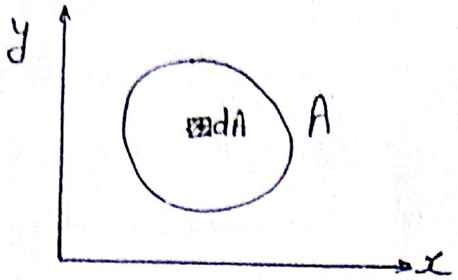
(V) All the part M.I's about the required axis are
(VI) added up to give the net M.I. about that axis.
(*) The whole operation is recorded in a Tabular format.

Important Points

i) M.I. of two areas can be added / subtracted only if it is about the same axis. Otherwise not.

ii) Parallel axis theorem always requires identification of the centroidal axis of the concerned area.

PRODUCT OF INERTIA

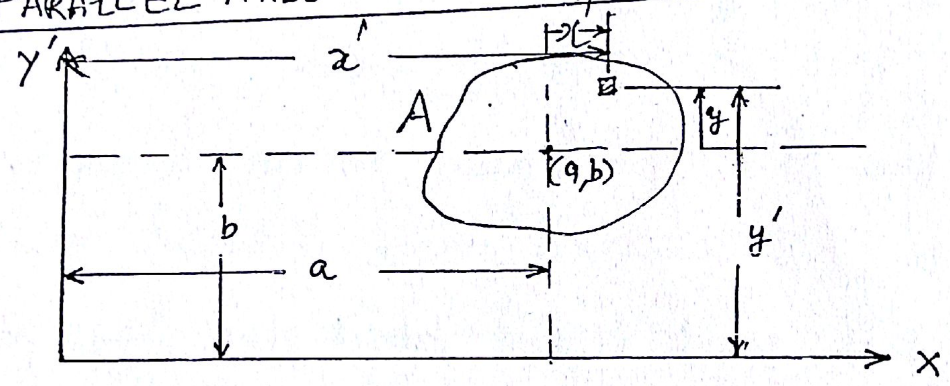


$$I_{xy} = \Sigma\, x \cdot y \cdot dA = \int xy\, dA$$

The product of Inertia of an area w.r. to a coordinate axis system x-y, containing the area is defined as the sum $\Sigma\, xy\, dA$ over the area A. In the limit as the sub areas dA's in which A is divided tends to zero, $\Sigma\, xy\, dA$ tends to the integral $\int x \cdot y\, dA$ and is termed I_{xy}.

Unlike I_{xx} or I_{yy} which are always +ve, I_{xy} can be positive or negative.

PARALLEL AXIS THEOREM of P.O.I.



Let the P.O.I. of an area A w.r. to an axes system $x'y'$ be $P_{x'y'}$.

$$P_{x'y'} = \int x'y'\, dA = \int x'y'\, dx'\, dy'$$

$$P_{x'y'} = \int (x+a)(y+b)\, dx\, dy, \quad \{(a, b) \text{ being coordinates of C.G of } A\}$$

$$= \int_A (xy + xb + ay + ab)\, dx\, dy$$

$$= \int_A xy\, dA + b\int_A x\, \overset{dx}{dy} + a\int_A y\, dx\, dy + ab\int_A dx\, dy$$

But, by definition, $\int x\, dA = 0$, $\int y\, dA = 0$ (when taken about centroidal axis).

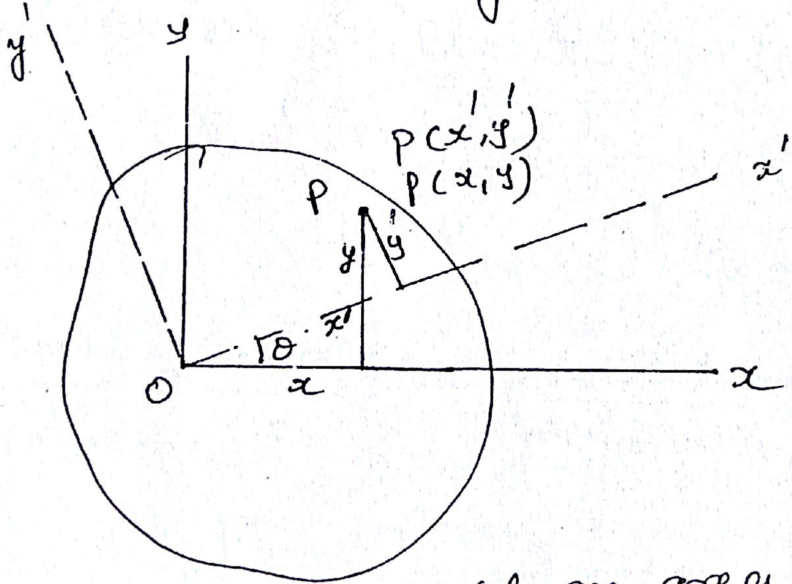
$$\therefore \quad I_{x'y'} = \int xy\, dA + ab \cdot A.$$

$$= \bar{I}_{xy} + ab \cdot A.$$

Thus, the product of inertia about a set of axes is equal to the sum of product of inertia about a parallel set of axes through the C.G. of the area and the area times the product of coordinates of C.G.

PRINCIPAL AXES OF INERTIA:

There exist two axes ... (X' and Y') such that the P.I. about them is zero. These two perp. axes are ones about which M.I. is maximum and minimum. They are called the principal axes.



Let the P.I. of an area about an axis set xy be P_{xy}. Let $x'y'$ be another set through the same origin at an angle θ to x, y. Then

$$x' = (x\cos\theta + y\sin\theta), \quad y' = (y\cos\theta - x\sin\theta).$$

$$\therefore P_{x'y'} = \int (x\cos\theta + y\sin\theta)(y\cos\theta - x\sin\theta)\, dx\, dy$$

or, $P_{x'y'} = \iint \{ xy\cos^2\theta - xy\sin^2\theta + y^2\sin\theta\cos\theta - x^2\sin\theta\cos\theta \} dx\,dy$

or, $P_{x'y'} = (\cos^2\theta - \sin^2\theta)\int xy\,dx\,dy + \left(\int (y^2 - x^2)\,dx\,dy\right) \times \frac{\sin 2\theta}{2}$

$$= \cos 2\theta \; P_{xy} + \frac{\sin 2\theta}{2}\left(I_{xx} - I_{yy}\right) \quad \text{---} (1)$$

If $P_{x'y'} = 0$, $\quad 2\cos 2\theta \times P_{xy} = -\sin 2\theta\,(I_{xx} - I_{yy})$

$$= \sin 2\theta\,(I_{yy} - I_{xx})$$

$$\therefore \tan 2\theta = \frac{2P_{xy}}{(I_{yy} - I_{xx})} \quad \text{------} (2)$$

Since, P_{xy} and I_{yy}, I_{xx} are are real & finite, there always exists an angle θ such that $\tan 2\theta$ is $\left(\frac{2P_{xy}}{I_{yy} - I_{xx}}\right)$. For this angle, $P_{x'y'} = 0$

Also, by differentiating $P_{x'y'}$ w.r. to θ and equating to zero, $\frac{d}{d\theta}(P_{x'y'}) = -2\sin 2\theta' . P_{xy} + \cos 2\theta'(I_{xx} - I_{yy}) = 0$

$$\therefore \tan 2\theta' = (I_{xx} - I_{yy}) / 2P_{xy} \quad \text{---} (3)$$

Substituting (3) in (1), we get,

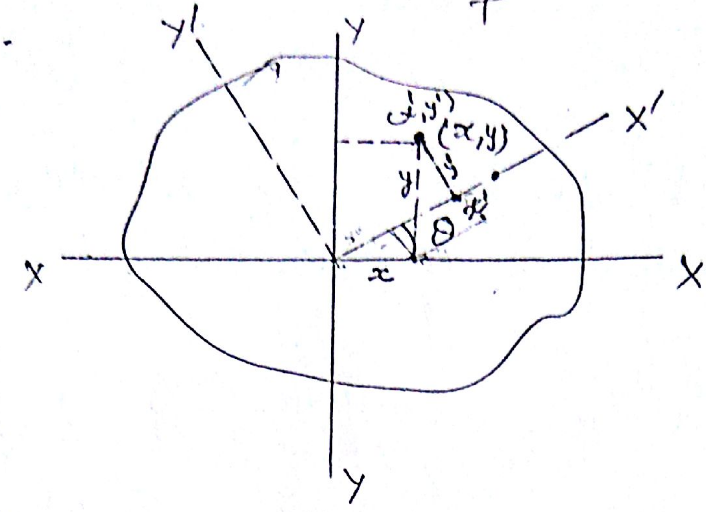
$$P_{x'y'max} = P_{xy} . \frac{2P_{xy}}{\sqrt{(I_{xx} - I_{yy})^2 + 4P_{xy}^2}} + \frac{(I_{xx} - I_{yy}) . (I_{xx} - I_{yy})}{2\sqrt{(I_{xx} - I_{yy})^2 + 4P_{xy}^2}}$$

$$= \frac{1}{2}\left[\frac{4P_{xy}^2 + (I_{xx} - I_{yy})^2}{\sqrt{(I_{xx} - I_{yy})^2 + 4P_{xy}^2}}\right]$$

$$\therefore P_{x'y'max} = \frac{1}{2}\sqrt{(I_{xx} - I_{yy})^2 + 4P_{xy}^2} \quad \text{---} (4)$$

(3) is the max. value of P_{xy} for two values of θ' which are at rt. angles. For other two values of θ, $P_{x'y'} = 0$ $\quad\theta$ and θ' are at $45°$ to each other.

Moment and Product of Inertia about Inclined Axes.



Let the coordinate axes be rotated through 'θ'.
We know, from elementary transformations,

$$x' = x\cos\theta + y\sin\theta, \quad \text{and} \quad y' = (y\cos\theta - x\sin\theta)$$

Hence, $I_{x'x'} = \int (y\cos\theta - x\sin\theta)^2\, dA$

or, $I_{x'x'} = \int (y^2\cos^2\theta + x^2\sin^2\theta - 2xy\sin\theta\cos\theta)\, dA$

$$= \int \cos^2\theta\, y^2\, dA + \int x^2 \sin^2\theta\, dA - \int xy\sin 2\theta\, dA$$

$$= \cos^2\theta \int y^2\, dA + \left(\int x^2\, dA\right)\sin^2\theta - \sin 2\theta \int xy\, dA$$

$$= \cos^2\theta\, I_{xx} + \sin^2\theta\, I_{yy} - \sin 2\theta\, P_{xy} \quad \text{---(1)}$$

(by defination)

Also, substituting $x' = (x\cos\theta + y\sin\theta)$, or alternatively, putting $\theta = (\theta + 90^\circ)$

$$I_{y'y'} = \cos^2(90+\theta)\, I_{yy} + \sin^2(90+\theta)\, I_{xx} - \sin 2(90+\theta)\, P_{xy}$$

$$= \sin^2\theta\, I_{yy} + \cos^2\theta\, I_{xx} + \sin 2\theta\, P_{xy} \quad \text{---(2)}$$

Adding (1) and (2), we get

$$I_{x'x'} + I_{y'y'} = I_{xx} + I_{yy} \quad (\text{First invarient})$$

Also, $I_{x'x'} = \left(\frac{1+\cos 2\theta}{2}\right) I_{xx} + \left(\frac{1-\cos 2\theta}{2}\right) I_{yy} - \sin 2\theta \, P_{xy}$

$$= \frac{(I_{yy}+I_{xx})}{2} + \frac{(I_{xx}-I_{yy})}{2}\cos 2\theta - P_{xy}\sin 2\theta \quad \text{---}(3)$$

For max. $I_{x'x'}$, differentiating (3) w.r.to 'θ' and equating to zero, we get

$$I_{x'x'\,\substack{max\\min}} = \left(\frac{I_{xx}+I_{yy}}{2}\right) \pm \sqrt{\left(\frac{I_{yy}-I_{xx}}{2}\right)^2 + P_{xy}^2} \quad \text{---}(4)$$

at $\theta = \frac{1}{2}\left[\tan^{-1}\left(\frac{+2P_{xy}}{I_{yy}-I_{xx}}\right)\right]$ ---(5)

Also, $P_{x'y'} = \int (x\cos\theta + y\sin\theta)(y\cos\theta - x\sin\theta)\, dA$

$$= \int \{xy\cos^2\theta - xy\sin^2\theta + y^2\sin\theta\cos\theta - x^2\sin\theta\cos\theta\}\, dA$$

$$= \cos 2\theta \int xy\, dA - \frac{\sin 2\theta}{2}\left\{\int x^2 dA - \int y^2 dA\right\}$$

$$= \cos 2\theta \; P_{xy} - \frac{(I_{yy} - I_{xx})}{2}\sin 2\theta.$$

$P_{x'y'}$ is 0 at a value of θ given by

$$\tan 2\theta = \frac{2P_{xy}}{(I_{yy}-I_{xx})} \quad \text{---}(6)$$

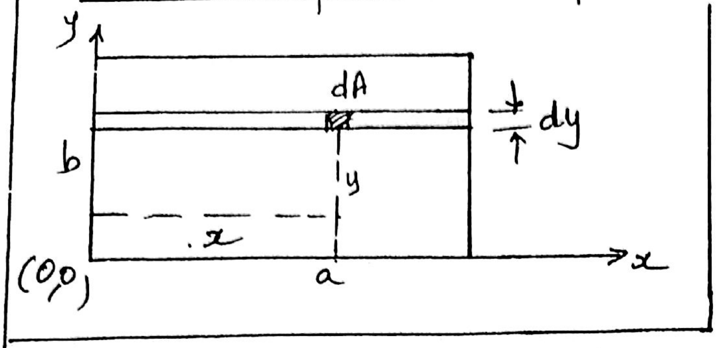
which is the value of θ at which I is max./min.

$\therefore$ The two values of $I_{x'x'}$ (Max. & min.) are called 'Principal' values. All other values of I_{xx} lie between these two extreme values.

The principal values of P_{xy} are given by

$$P_{xy\,max} = \pm \frac{1}{2}\sqrt{(I_{yy}-I_{xx})^2 + 4P_{xy}^2}$$

Product of Inertia of a Rectangle:-

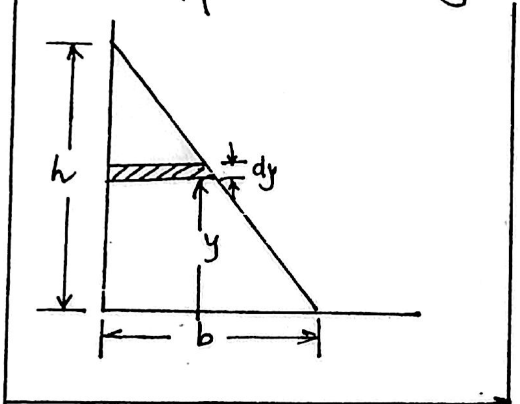


For the infinitesimal strip = to x axis between y and y + dy

$$dP = \int xy \, dA = \int y.dy \int_0^a x\,dx$$
$$= \left[\int y dy . \left(\frac{a^2}{2}\right)\right]$$

Or, $P = \frac{a^2}{2}\int_0^b y\,dy = \frac{a^2}{2}\cdot\frac{b^2}{2} = \frac{a^2b^2}{4}$ — (1)

P.I of a triangular area (right angled)



For a strip || to x axis at y

$$dP = \int xy\,dA = y\int x\,dA.$$
$$= y.\left[\frac{x}{2}\, x.dy\right] = \frac{x^2}{2} y\,dy$$

But, $\frac{x}{b} = \frac{h-y}{h}$, $\Rightarrow x = \frac{b}{h}(h-y)$

$$\therefore \; dP = \frac{b^2}{2h^2}(h-y)^2 . y\,dy = \frac{b^2}{2h^2}\left\{h^2y - 2hy^2 + y^3\right\}dy$$

$$\therefore \; P = \frac{b^2}{2h^2}\int_0^h \left\{h^2y - 2hy^2 + y^3\right\}dy = \frac{b^2}{2h^2}\left[\frac{h^2y^2}{2} - \frac{2hy^3}{3} + \frac{y^4}{4}\right]_0^h$$

or, $P = \frac{b^2}{2h^2}\left[\frac{h^4}{2} - \frac{2h^4}{3} + \frac{h^4}{4}\right] = \frac{b^2h^2}{24}$ — (2)

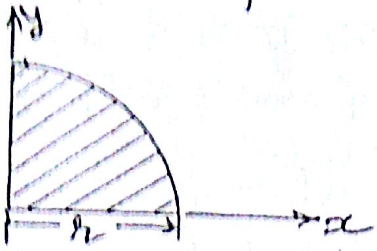
$$\bar{P}_{(c.g)} = \frac{b^2h^2}{24} - \left(\frac{h}{3}\cdot\frac{b}{3}\right)\cdot\frac{b.h}{2} = b^2h^2\left\{\frac{1}{24} - \frac{1}{18}\right\}$$

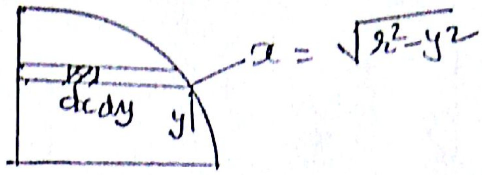
$$= -\frac{1}{72}(b^2h^2)$$ — (3)

. I about an axis of symmetry is zero.

$P_{xy} = 0.$

Find the Product of Inertia of a quarter of a circular area in the first quadrant about its edges.





$$dI = x.y.dx.dy.$$

For a strip of constant y, $dI_{strip} = \int_0^x xy\,dx\,dy$

$$dP_{strip} = (y\,dy)\int_0^x x\,dx = \left(\frac{x^2}{2}\right) y\,dy$$

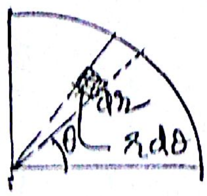
$$P_{strip} = \int_0^r \frac{x^2}{2} y\,dy = \int_0^r \left(\frac{r^2-y^2}{2}\right) y\,dy$$

or, $P = \frac{1}{2}\int_0^r (r^2 y - y^3)\,dy = \frac{1}{2}\left[\frac{r^2 y^2}{2} - \frac{y^4}{4}\right]_0^r$

$$= \underline{\underline{\frac{1}{8} r^4}} \text{ ---------}$$

Alternatively. $dP = (dr)(r\,d\theta)\,x.y$

$$= r\,dr\,d\theta.\,r\cos\theta\, r\sin\theta$$



$$\therefore P = \int_0^{\pi/2}\int_0^r r^3.dr\,\frac{\sin 2\theta}{2}\,d\theta$$

$$= \frac{1}{2}\left[\frac{r^4}{4}\right]_0^r \left[-\frac{\cos 2\theta}{2}\right]_0^{\pi/2} = \frac{1}{2}.\frac{r^4}{4}.\frac{1}{2}-\left[(-1)-(1)\right]$$

$$= \underline{\underline{\frac{r^4}{8}}} \text{ ---------}$$

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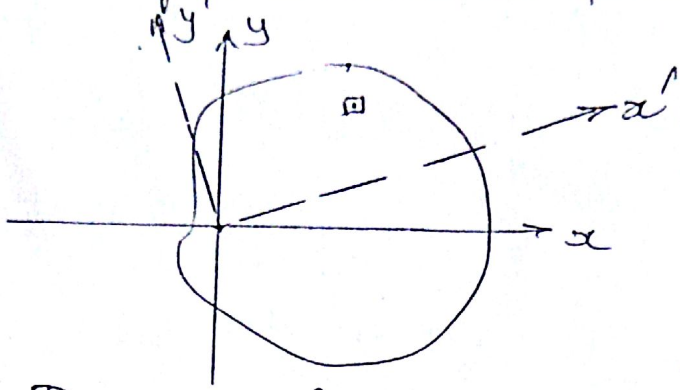

$$x' = x\cos\theta + y\sin\theta$$
$$y' = y\cos\theta - x\sin\theta$$

$$P_{x'y'} = \int (x\cos\theta + y\sin\theta)(y\cos\theta - x\sin\theta)\,dA$$

$$= \int xy(\cos^2\theta - \sin^2\theta)\,dA + \int \underbrace{\cos\theta \sin\theta}_{(\sin 2\theta/2)} y^2\,dA - \int \underbrace{\sin\theta\cos\theta}_{(\sin 2\theta/2)} x^2\,dA$$

$$= \cos 2\theta \int xy\,dA + \frac{\sin 2\theta}{2}\left(\int y^2\,dA - \int x^2\,dA\right) = 0$$

or, $$P_{xy}\cos 2\theta + (I_{xx} - I_{yy})\frac{\sin 2\theta}{2} = 0$$

When $P_{x'y'}$ is zero

$$\tan 2\theta = \left(2P_{xy} / (I_{yy} - I_{xx})\right)$$

$$\therefore \theta = \frac{1}{2}\tan^{-1}\left(\frac{2P_{xy}}{I_{yy} - I_{xx}}\right) \quad \text{--- (1)}$$

Also,
$$I_{x'x'} = \int (x\cos\theta + y\sin\theta)^2\,dA$$

$$= \int (x^2\cos^2\theta + y^2\sin^2\theta + 2xy\sin\theta\cos\theta)\,dA$$

$$= \cos^2\theta \int x^2\,dA + \sin^2\theta \int y^2\,dA + \sin 2\theta \int xy\,dA$$

or, $$I_{x'x'} = \cos^2\theta\, I_{yy} + \sin^2\theta\, I_{xx} + \sin 2\theta\, P_{xy}$$

But, $$\cos^2\theta = \frac{1+\cos 2\theta}{2}, \quad \& \quad \sin^2\theta = \left(\frac{1-\cos 2\theta}{2}\right)$$

$$\therefore I_{x'x'} = \frac{1}{2}(I_{xx} + I_{yy}) + \frac{(I_{xx} - I_{yy})}{2}\cos 2\theta + \sin 2\theta\, P_{xy}$$

and,
$$I_{y'y'} = \frac{1}{2}(I_{xx} + I_{yy}) - \left(\frac{I_{xx} - I_{yy}}{2}\right)\cos 2\theta - \sin 2\theta\, P_{xy} \quad \text{--- (2)}$$

Hence, $I_{x'x'} + I_{y'y'} = I_{xx} + I_{yy}$

(1st invarient of M.O.I.) — (2)

For Max $I_{x'x'}$, $dI_{x'x'}/d\theta = 0$

$$\therefore \left(\frac{I_{xx} - I_{yy}}{2}\right)(-2\sin 2\theta) + 2\cos 2\theta\, P_{xy} = 0$$

or, $(I_{yy} - I_{xx})\sin 2\theta = 2P_{xy}\cos 2\theta$

or, $\tan 2\theta = \dfrac{2P_{xy}}{(I_{yy} - I_{xx})}$

i.e. $\theta = \frac{1}{2}\tan^{-1}\left(\dfrac{2P_{xy}}{I_{yy} - I_{xx}}\right)$

which is the same value of θ for which $P_{x'y'} = 0$

$\therefore$ Principal axes are those for which the product of inertia is zero. and M.I. is either a maximum or minimum.

PROBLEMS

CENTRE OF GRAVITY / CENTROID: (*)

6-5. 5, 6, 7, 8, 9

6-6. 4, 5, 8, 9, 13, 14, 15, 16, 18, 19, 20, 21

6-7. 3, 4, 5, 6, 8, 9, 10, 13.

6-8. 3, 4, 5, 6, 9, 11

MOMENT OF INERTIA / PRODUCT OF INERTIA

7-5. 4, 5, 6, 7, 8, 10, 12

7-6. 7, 8, 9, 10, 12, 14

7-9. 2, 6, 11, 12

7-12. 5, 6.

(*) Singers' Engineering Mechanics
Statics and Dynamics,
K.V.K. Reddy, J. Suresh Kumar.