UNIT-II

Friction: Types of friction -Limiting friction -Laws of Friction -static and Dynamic Frictions-Motion of Bodies – Wedge & Screw, Screw-jack.

UNIT

2

Principles of Friction

8.1. INTRODUCTION

It has been established since long that all surfaces of the bodies are never perfectly smooth. It has been observed that whenever, even a very smooth surface is viewed under a microscope, it is found to have some roughness and irregularities, which may not be detected by an ordinary touch.

It will be interesting to know that if a block of one substance is placed over the level surface of the same or different material, a certain degree of interlocking of the minutely projecting particles takes place. This does not involve any force, so long as the block does not move or tends to move. But whenever one of the blocks moves or tends to move tangentially with respect to the surface, on which it rests, the interlocking property of the projecting particles opposes the motion. This opposing force, which acts in the opposite direction of the movement of the block, is called *force of friction* or simply *friction.* It is of the following two types:

1. Static friction. **2.** Dynamic friction.

8.2. STATIC FRICTION

It is the friction experienced by a body when it is at rest. Or in other words, it is the friction when the body tends to move.

8.3. DYNAMIC FRICTION

It is the friction experienced by a body when it is in motion. It is also called kinetic friction. The dynamic friction is of the following two types :

1. ***Sliding friction.*** It is the friction, experienced by a body when it slides over another body.
2. ***Rolling friction.*** It is the friction, experienced by a body when it rolls over another body.

8.4. LIMITING FRICTION

It has been observed that when a body, lying over another body, is gently pushed, it does not move because of the frictional force, which prevents the motion. It shows that the force of the hand is being exactly balanced by the force of friction, acting in the opposite direction. If we again push the body, a little harder, it is still found to be in equilibrium. It shows that the force of friction has increased itself so as to become equal and opposite to the applied force. Thus the force of friction has a remarkable property of adjusting its magnitude, so as to become exactly equal and opposite to the applied force, which tends to produce motion.

There is, however, a limit beyond which the force of friction cannot increase. If the applied force exceeds this limit, the force of friction cannot balance it and the body begins to move, in the direction of the applied force. This maximum value of frictional force, which comes into play, when a body just begins to slide over the surface of the other body, is known as limiting friction. It may be noted that when the applied force is less than the limiting friction, the body remains at rest, and the friction is called static friction, which may have any value between zero and limiting friction.

8.5. NORMAL REACTION

It has been experienced that whenever a body, lying on a horizontal or an inclined surface, is in equilibrium, its weight acts vertically downwards through its centre of gravity. The surface, in turn, exerts an upward reaction on the body. This reaction, which is taken to act perpendicular to the plane, is called normal reaction and is, generally, denoted by R. It will be interesting to know that the term 'normal reaction' is very important in the field of friction, as the force of friction is directly propor-tional to it.

8.6. ANGLE OF FRICTION

Consider a body of weight W resting on an inclined plane as shown in Fig. 8.1. We know that the body is in equilibrium under the action of the following forces :

1. Weight (W) of the body, acting vertically downwards,
2. Friction force (F) acting upwards along the plane, and
3. Normal reaction (R) acting at right angles to the plane.

Let the angle of inclination (α) be gradually increased, till the body just starts sliding down the plane. This angle of inclined plane, at which a body just begins to slide down the plane, is called the angle of friction. This is also equal to the angle, which the normal reaction makes with the vertical.

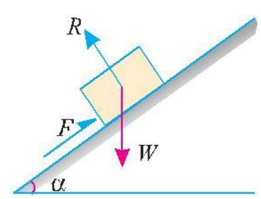


Fig. 8.1. Angle of friction.

8.7. COEFFICIENT OF FRICTION

It is the ratio of limiting friction to the normal reaction, between the two bodies, and is gener-ally denoted by μ.

Mathematically, coefficient of friction,

$$\mu = \frac{F}{R} = \tan \varphi \qquad \text{or} \qquad F = \mu R$$

where φ = Angle of friction,

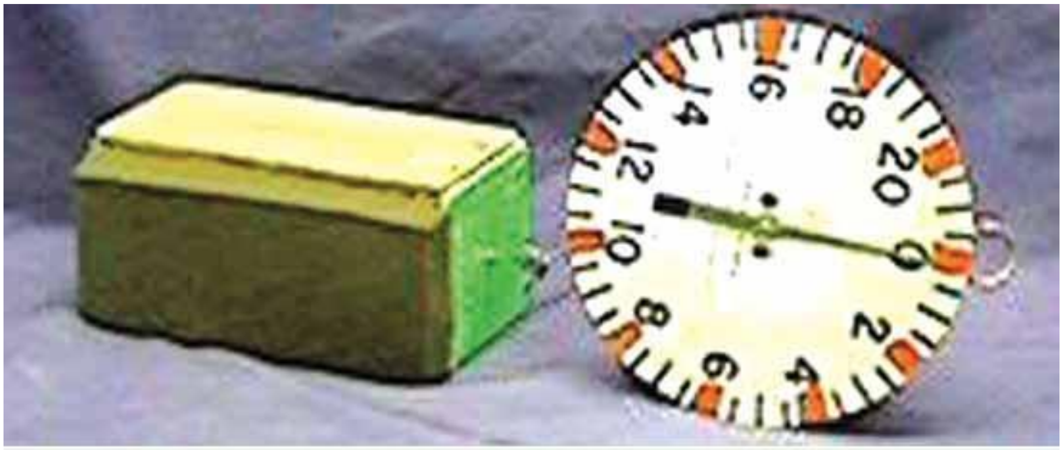
F = Limiting friction, and

R = Normal reaction between the two bodies.

8.8. LAWS OF FRICTION

Prof. Coulomb, after extensive experiments, gave some laws of friction, which may be grouped under the following heads :

1. Laws of static friction, and
2. Laws of kinetic or dynamic friction.



The coefficient of friction of various surfaces, as well as the difference between static and kinetic friction can be illustred by pulling objects with large spring scale.

8.9. LAWS OF STATIC FRICTION

Following are the laws of static friction :

1. The force of friction always acts in a direction, opposite to that in which the body tends to move, if the force of friction would have been absent.
2. The magnitude of the force of friction is exactly equal to the force, which tends to move the body.
3. The magnitude of the limiting friction bears a constant ratio to the normal reaction between the two surfaces. Mathematically :

$$\frac{F}{R} = \text{Constant}$$

where F = Limiting friction, and R = Normal reaction.

4. The force of friction is independent of the area of contact between the two surfaces.
5. The force of friction depends upon the roughness of the surfaces.

8.10. LAWS OF KINETIC OR DYNAMIC FRICTION

Following are the laws of kinetic or dynamic friction :

1. The force of friction always acts in a direction, opposite to that in which the body is moving.
2. The magnitude of kinetic friction bears a constant ratio to the normal reaction between the two surfaces. But this ratio is slightly less than that in case of limiting friction.
3. For moderate speeds, the force of friction remains constant. But it decreases slightly with the increase of speed.

This rock climber uses the static frictional force between her hands and feet and the vertical rock walls.

8.11. EQUILIBRIUM OF A BODY ON A ROUGH HORIZONTAL PLANE

We know that a body, lying on a rough horizontal plane will remain in equilibrium. But whenever a force is applied on it, the body will tend to move in the direction of the force. In such cases, equilibrium of the body is studied first by resolving the forces horizontally and then vertically.

Now the value of the force of friction is obtained from the relation :

$$F = \mu R$$

where μ = Coefficient of friction, and

R = Normal reaction.

Example 8.1. *A body of weight 300 N is lying on a rough horizontal plane having a coefficientof friction as 0.3. Find the magnitude of the force, which can move the body, while acting at an angle of 25° with the horizontal.*

Solution. Given: Weight of the body (W) = 300 N; Coefficient of friction (μ) = 0.3 andangle made by the force with the horizontal (α) = 25°

Let P = Magnitude of the force, which can move the body, and

F = Force of friction.

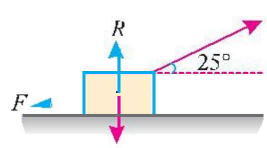


Fig. 8.2.

Resolving the forces horizontally,

$$F = P \cos\alpha = P \cos 25° = P \times 0.9063$$

and now resolving the forces vertically,

$$R = W - P \sin\alpha = 300 - P \sin 25° = 300 - P \times 0.4226$$

We know that the force of friction (F),

$$0.9063\,P = \mu R = 0.3 \times (300 - 0.4226\,P) = 90 - 0.1268\,P$$

or $$90 = 0.9063\,P + 0.1268\,P = 1.0331\,P$$

$$\therefore \quad P = \frac{90}{1.0331} = 87.1 \text{ N} \quad \textbf{Ans.}$$

Example 8.2. *A body, resting on a rough horizontal plane, required a pull of 180 N inclined at 30° to the plane just to move it. It was found that a push of 220 N inclined at 30° to the plane just moved the body. Determine the weight of the body and the coefficient of friction.*

Solution. Given: Pull = 180 N; Push = 220 N and angle at which force is inclined withhorizontal plane (α) = 30°

Let W = Weight of the body R

= Normal reaction, andμ

= Coefficient of friction.

First of all, consider a pull of 180 N acting on the body. We know that in this case, the force of friction (F_1) will act towards left as shown in Fig. 8.3. (*a*).

Resolving the forces horizontally,

$$F_1 = 180 \cos 30° = 180 \times 0.866 = 155.9$$

Nand now resolving the forces vertically,

$$R_1 = W - 180 \sin 30° = W - 180 \times 0.5 = W - 90$$

NWe know that the force of friction (F_1),

$$155.9 = \mu R_1 = \mu (W - 90) \qquad ...(i)$$

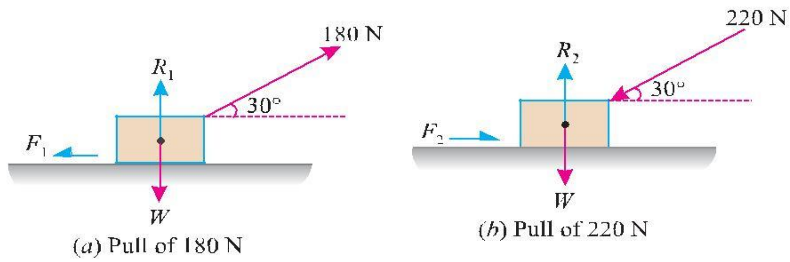


Fig. 8.3.

Now consider a push of 220 N acting on the body. We know that in this case, the force of friction (F_2) will act towards right as shown in Fig. 8.3 (*b*).

Resolving the forces horizontally,

$$F_2 = 220 \cos 30° = 220 \times 0.866 = 190.5 \text{ N}$$

and now resolving the forces horizontally,

$$R_2 = W + 220 \sin 30° = W + 220 \times 0.5 = W + 110 \text{ N}$$

We know that the force of friction (F_2),

$$190.5 = \mu . R_2 = \mu (W + 110) \qquad ...(ii)$$

Dividing equation (*i*) by (*ii*)

$$\frac{155.9}{190.5} = \frac{\mu (W - 90)}{\mu (W + 110)} = \frac{W - 90}{W + 110}$$

$$155.9\, W + 17\,149 = 190.5\, W - 17\,145$$

$$34.6\, W = 34\,294$$

or $$W = \frac{34\,294}{34.6} = 991.2 \text{ N} \quad \textbf{Ans.}$$

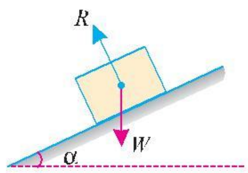
Now substituting the value of W in equation (*i*),

$$155.9 = \mu (991.2 - 90) = 901.2\, \mu$$

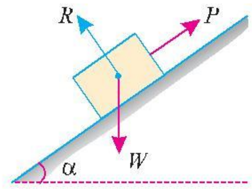
∴ $$\mu = \frac{155.9}{901.2} = 0.173 \quad \textbf{Ans.}$$

8.12. EQUILIBRIUM OF A BODY ON A ROUGH INCLINED PLANE

Consider a body, of weight *W*, lying on a rough plane inclined at an angle α with the horizon-tal as shown in Fig. 8.7 (*a*) and (*b*).



(*a*) Angle of inclination less than the angle of friction



(*b*) Angle of inclination more than the angle of friction

Fig. 8.7.

A little consideration will show, that if the inclination of the plane, with the horizontal, is less the angle of friction, the body will be automatically in equilibrium as shown in Fig. 8.7 (*a*). If in this condition, the body is required to be moved upwards or downwards, a corresponding force is required, for the same. But, if the inclination of the plane is more than the angle of friction, the body will move down. And an upward force (*P*) will be required to resist the body from moving down the plane as shown in Fig. 8.7 (*b*).

Though there are many types of forces, for the movement of the body, yet the following are important from the subject point of view :

1. Force acting along the inclined plane.
2. Force acting horizontally.
3. Force acting at some angle with the inclined plane.

Note. In all the above mentioned three types of forces, we shall discuss the magnitude offorce, which will keep the body in equilibrium, when it is at the point of sliding downwards or upwards.

8.13. EQUILIBRIUM OF A BODY ON A ROUGH INCLINED PLANE SUBJECTED TO A FORCE ACTING ALONG THE INCLINED PLANE

Consider a body lying on a rough inclined plane subjected force acting along the inclined plane, which keeps it in equilibrium as shown in Fig. 8.8. (*a*) and (*b*).

Let W = Weight of the body,

α = Angle, which the inclined plane makes with the horizontal,

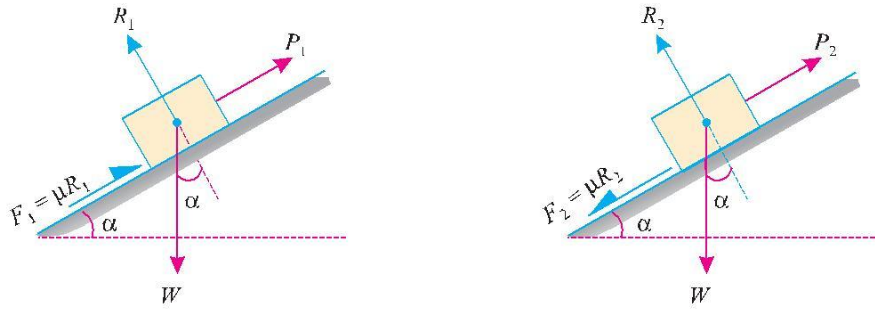
R = Normal reaction,

μ = Coefficient of friction between the body and the inclined plane, and

φ = Angle of friction, such that $\mu = \tan \varphi$.

A little consideration will show that if the force is not there, the body will slide down the plane. Now we shall discuss the following two cases :

1. *Minimum force (P_1) which will keep the body in equilibrium, when it is at the point of slidingdownwards.*



(*a*) Body at the point of sliding downwards

(*b*) Body at the point of sliding upwards

Fig. 8.8.

In this case, the force of friction ($F_1 = \mu . R_1$) will act upwards, as the body is at the point of sliding downwards as shown in Fig. 8.8 (*a*). Now resolving the forces along the plane,

$P_1 = W \sin\alpha - \mu . R_1$ ...(*i*)and now resolving the forces perpendicular to the plane.

$R_1 = W \cos\alpha$ ...(*ii*)Substituting the value of R_1 in equation (*i*),

$P_1 = W \sin\alpha - \mu\, W \cos\alpha = W (\sin\alpha - \mu \cos\alpha)$and

now substituting the value of $\mu = \tan\varphi$ in the above equation,

$P_1 = W (\sin\alpha - \tan\varphi \cos\alpha$

)Multiplying both sides of this equation by $\cos\varphi$,

$P_1 \cos\varphi = W (\sin\alpha \cos\varphi - \sin\varphi \cos\alpha) = W \sin(\alpha - \varphi)$

$$\therefore \qquad P_1 = W \cdot \frac{\sin(\alpha - \varphi)}{\cos\varphi}$$

2. *Maximum force (P_2) which will keep the body in equilibrium*, *when it is at the point of slidingupwards.*

In this case, the force of friction ($F_2 = \mu . R_2$) will act downwards as the body is at the point of sliding upwards as shown in Fig. 8.8 (*b*). Now resolving the forces along the plane,

$P_2 = W \sin\alpha + \mu . R_2$ ...(*i*)and now resolving the forces perpendicular to the plane,

$R_2 = W \cos\alpha$ ...(*ii*)Substituting the value of R_2 in equation (*i*),

$P_2 = W \sin\alpha + \mu\, W \cos\alpha = W (\sin\alpha + \mu \cos\alpha)$and

now substituting the value of $\mu = \tan\varphi$ in the above equation,

$P_2 = W (\sin\alpha + \tan\varphi \cos\alpha$

)Multiplying both sides of this equation by $\cos\varphi$,

$P_2 \cos\varphi = W (\sin\alpha \cos\varphi + \sin\varphi \cos\alpha) = W \sin(\alpha + \varphi)$

$$\therefore \qquad P_2 = W \cdot \frac{\sin(\alpha + \varphi)}{\cos\varphi}$$

8.14. EQUILIBRIUM OF A BODY ON A ROUGH INCLINED PLANE SUBJECTED TO A FORCE ACTING HORIZONTALLY

Consider a body lying on a rough inclined plane subjected to a force acting horizontally, which keeps it in equilibrium as shown in Fig. 8.13. (*a*) and (*b*).

W = Weight of the body,

α = Angle, which the inclined plane makes with the horizontal,

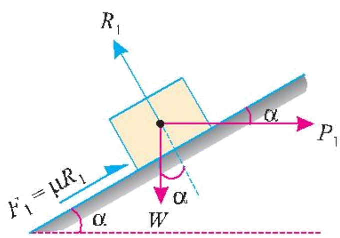
R = Normal reaction,

μ = Coefficient of friction between the body and the inclined plane, and

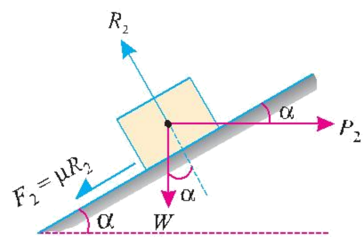
φ = Angle of friction, such that $\mu = \tan \varphi$.

A little consideration will show that if the force is not there, the body will slide down on the plane. Now we shall discuss the following two cases :

1. *Minimum force* (P_1) *which will keep the body in equilibrium, when it is at the point of slidingdownwards.*



(*a*) Body at the point of sliding downwards



(*b*) Body at the point of sliding upwards

Fig. 8.13.

In this case, the force of friction ($F_1 = \mu.R_1$) will act upwards, as the body is at the point of sliding downwards as shown in Fig. 8.13. (*a*). Now resolving the forces along the plane,

$P_1 \cos\alpha = W \sin\alpha - \mu\, R_1$ ...(i)and now resolving the forces perpendicular to the plane,

$R_1 = W \cos\alpha + P_1 \sin\alpha$ ...(ii)Substituting this value of R_1 in equation (i),

$P_1 \cos\alpha = W \sin\alpha - \mu(W \cos\alpha + P_1 \sin\alpha) = W \sin\alpha - \mu W \cos\alpha - \mu P_1 \sin\alpha$

$P_1 \cos\alpha + \mu P_1 \sin\alpha = W \sin\alpha - \mu W \cos\alpha$

$P_1(\cos\alpha+\mu\sin\alpha) = W(\sin\alpha-\mu\cos\alpha)$

$$\therefore \quad P_1 = W \cdot \frac{(\sin\alpha-\mu\cos\alpha)}{(\cos\alpha+\mu\sin\alpha)}$$

Now substituting the value of $\mu = \tan\varphi$ in the above equation,

$$P_1 = W \cdot \frac{(\sin\alpha-\tan\varphi\cos\alpha)}{(\cos\alpha+\tan\varphi\sin\alpha)}$$

Multiplying the numerator and denominator by $\cos\varphi$,

$$P_1 = W \cdot \frac{\sin\alpha\cos\varphi-\sin\varphi\cos\alpha}{\cos\alpha\cos\varphi+\sin\alpha\sin\varphi} = W \cdot \frac{\sin(\alpha-\varphi)}{\cos(\alpha-\varphi)}$$

$$= W\tan(\alpha-\varphi) \quad \text{...(when } \alpha>\varphi)$$

$$= W\tan(\varphi-\alpha) \quad \text{...(when } \varphi>\alpha)$$

2. *Maximum force* (P_2) *which will keep the body in equilibrium*, *when it is at the point of slidingupwards*

In this case, the force of friction ($F_2 = \mu R_2$) will act downwards, as the body is at the point of sliding upwards as shown in Fig.8.12. (*b*). Now resolving the forces along the plane,

$$P_2\cos\alpha = W\sin\alpha+\mu R_2 \quad \text{...}(iii)$$

and now resolving the forces perpendicular to the plane,

$$R_2 = W\cos\alpha + P_2\sin\alpha \quad \text{...}(iv)$$

Substituting this value of R_2 in the equation (*iii*),

$$P_2\cos\alpha = W\sin\alpha + \mu(W\cos\alpha + P_2\sin\alpha)$$

$$= W\sin\alpha + \mu W\cos\alpha + \mu P_2\sin\alpha$$

$P_2\cos\alpha - \mu P_2\sin\alpha = W\sin\alpha + \mu W\cos\alpha$

$P_2(\cos\alpha-\mu\sin\alpha) = W(\sin\alpha+\mu\cos\alpha)$

$$\therefore \quad P_2 = W \cdot \frac{(\sin\alpha+\mu\cos\alpha)}{(\cos\alpha-\mu\sin\alpha)}$$

Now substituting the value of $\mu = \tan\varphi$ in the above equation,

$$P_2 = W \cdot \frac{\sin\alpha+\tan\varphi\cos\alpha}{\cos\alpha-\tan\varphi\sin\alpha}$$

Multiplying the numerator and denominator by $\cos\varphi$,

$$P_2 = W \cdot \frac{\sin\alpha\cos\varphi+\sin\varphi\cos\alpha}{\cos\alpha\cos\varphi-\sin\varphi\sin\alpha} = W \cdot \frac{\sin(\alpha+\varphi)}{\cos(\alpha+\varphi)}$$

$$= W\tan(\alpha+\varphi)$$

8.15. EQUILIBRIUM OF A BODY ON A ROUGH INCLINED PLANE SUBJECTED TO A FORCE ACTING AT SOME ANGLE WITH THE INCLINED PLANE

Consider a body lying on a rough inclined plane subjected to a force acting at some angle with the inclined plane, which keeps it in equilibrium as shown in Fig. 8.17 (*a*) and (*b*).

Let W = Weight of the body,

α = Angle which the inclined plane makes with the horizontal,

θ = Angle which the force makes with the inclined surface,

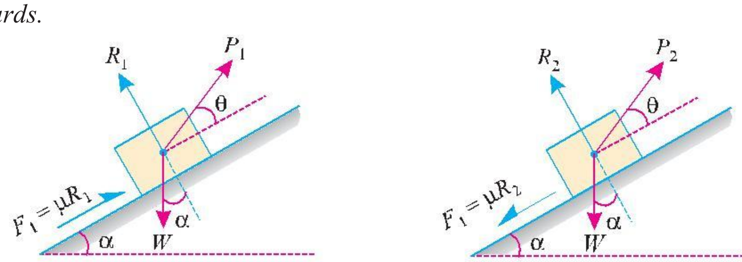
R = Normal reaction,

μ = Coefficient of friction between the body and the inclined plane, and

φ = Angle of friction, such that $\mu = \tan \varphi$.

A little consideration will show that if the force is not there, the body will slide down the plane. Now we shall discuss the following two cases :

1. *Minimum force (P_1) which will keep the body in equilibrium when it is at the point ofsliding downwards.*



(*a*) Body at the point of sliding downwards

(*b*) Body at the point of sliding upwards

Fig. 8.17.

In this case, the force of friction ($F_1 = \mu R_1$) will act upwards, as the body is at the point of sliding downwards as shown in Fig. 8.17 (*a*). Now resolving the forces along the plane,

$P_1 \cos\theta = W \sin\alpha - \mu R_1$ ...(*i*)and now resolving the forces perpendicular to the plane,

$$R_1 = W \cos\alpha - P_1 \sin\theta \qquad ...(ii)$$

Substituting the value of R_1 in equation (*i*),

SCREW FRICTION

The screws, bolts, studs, nuts etc. are widely used in various machines and structures for fastenings. These fastenings have screw threads, which are made by cutting a continuous helical groove on a cylindrical surface. If the threads are cut on the outer surface of a solid rod, these are known as external threads. But if the threads are cut on the internal surface of a hollow rod these are known as internal threads.

$$P_1\cos\theta = W\sin\alpha - \mu(W\cos\alpha - P_1\sin\theta)$$

$$= W\sin\alpha - \mu W\cos\alpha + \mu P_1\sin\theta$$

$$P_1\cos\theta - \mu P_1\sin\theta = W\sin\alpha - \mu W\cos\alpha$$

$$P_1(\cos\theta - \mu\sin\theta) = W(\sin\alpha - \mu\cos\alpha)$$

$$\therefore \quad P_1 = W\cdot\frac{(\sin\alpha - \mu\cos\alpha)}{(\cos\theta - \mu\sin\theta)}$$

and now substituting the value of $\mu = \tan\varphi$ in the above equation,

$$P_1 = W\cdot\frac{(\sin\alpha - \tan\varphi\cos\alpha)}{(\cos\theta - \tan\varphi\sin\theta)}$$

Multiplying the numerator and denominator by $\cos\varphi$,

$$P_1 = W\cdot\frac{(\sin\alpha\cos\varphi - \sin\varphi\cos\alpha)}{(\cos\theta\cos\varphi - \sin\varphi\sin\theta)} = W\cdot\frac{\sin(\alpha-\varphi)}{\cos(\theta+\varphi)}$$

2. *Maximum force* (P_2) *which will keep the body in equilibrium*, *when it is at the point ofsliding upwards.*

In this case, the force of friction ($F_2 = \mu R_2$) will act downwards as the body is at the point of sliding upwards as shown in Fig. 8.17 (*b*). Now resolving the forces along the plane.

$$P_2\cos\theta = W\sin\alpha + \mu R_2 \quad ...(iii)$$

and now resolving the forces perpendicular to the plane,

$$R_2 = W\cos\alpha - P_2\sin\theta \quad ...(iv)$$

Substituting the value of R_2 in equation (*iii*),

$$P_2\cos\theta = W\sin\alpha + \mu(W\cos\alpha - P_2\sin\theta) = W\sin\alpha + \mu W\cos\alpha - \mu P_2\sin\theta$$

$$P_2\cos\theta + \mu P_2\sin\theta = W\sin\alpha + \mu W\cos\alpha$$

$$P_2(\cos\theta + \mu\sin\theta) = W(\sin\alpha + \mu\cos\alpha)$$

$$\therefore \quad P_2 = W\cdot\frac{(\sin\alpha + \mu\cos\alpha)}{(\cos\theta + \mu\sin\theta)}$$

and now substituting the vaue of $\mu = \tan\varphi$ in the above equation,

$$P_2 = W\cdot\frac{(\sin\alpha + \tan\varphi\cos\alpha)}{(\cos\theta + \tan\varphi\sin\theta)}$$

Multiplying the numerator and denominator by $\cos\varphi$,

$$P_2 = W\cdot\frac{(\sin\alpha\cos\varphi + \sin\varphi\cos\alpha)}{(\cos\theta\cos\varphi + \sin\varphi\sin\theta)} = W\cdot\frac{\sin(\alpha+\varphi)}{\cos(\theta-\varphi)}$$

Applications of friction

1. Screw friction.2.wedge friction. 3. Ladder friction

1.screw friction

The screw threads are mainly of two types*viz*. V-threads and square threads. The V-threads are stronger and offer more frictional resistance to motion than square threads. Moreover, the V-threads have an advantage of preventing the nut from slackening. I will be interesting to know that the V-threads are used for the purpose of tightening pieces together (e.g. bolts and nuts etc.). Square threads are used in screw jacks, vice screws etc. which are used for lifting heavy loads. The following terms are important for the study of screws:

1. Helix. It is the curve traced by a particle, while describing a circular path at a uniform speed and advancing in the axial direction at a uniform rate. Or in other words, it is the curve traced by a particle while moving along a screw thread.

2. Pitch. It is the distance from one point of a thread to the corresponding point on the next thread. It is measured parallel to the axis of the screw.

3. Lead. It is the distance through which a screw thread advances axially in one turn.

4. Depth of thread. It is the distance between the top and bottom surfaces of a thread (also known as crest and root of thread).

5. Single-threaded screw. If the lead of a screw is equal to its pitch, it is known as single-threaded screw.

6. Multi-threaded screw. If more than one threads are cut in one lead distance of a screw, it is known as multi-threaded screw e.g. in a double-threaded screw, two threads are cut in one lead length. In such cases, all the threads run independently along the length of the rod. Mathematically,

$$\text{Lead} = \text{Pitch} \times \text{No. of threads.}$$

7. Slope of the thread. It is the inclination of the thread with horizontal. Mathematically,

$$\tan\alpha = \frac{\text{Lead of screw}}{\text{Circumference of screw}}$$

$$= \frac{p}{\pi d} \qquad \text{...(In single-threaded screw)}$$

$$= \frac{np}{\pi d} \qquad \text{...(In multi-threaded screw)}$$

where

α = Angle of inclination of the thread,
p = Pitch of the screw,
d = Mean diameter of the screw, and

n = No. of threads in one lead.

Contents

9.5. RELATION BETWEEN EFFORT AND WEIGHT LIFTED BY A SCREW JACK

The screw jack is a device for lifting heavy loads, by applying a comparatively smaller effort at its handle. The principle, on which a screw jack works, is similar to that of an inclined plane.

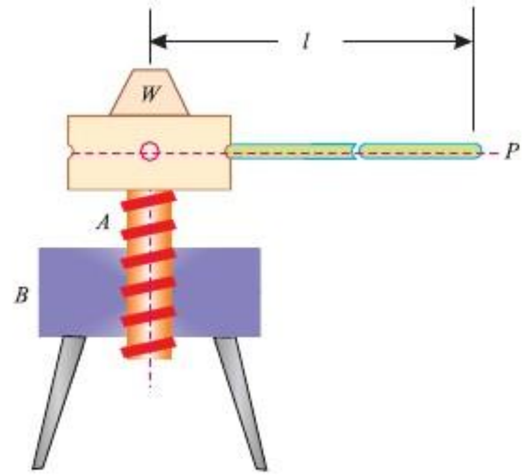


Fig. 9.20. Screw jack

Fig. 9.20 shows common form of a screw jack, which consists of a threaded rod A, called screw rod or simply screw. The screw has square threads, on its outer surface, which fit into the inner threads of the jack B. The load, to be raised or lowered, is placed on the head of the screw, which is rotated by the application of an effort at the end of the lever for lifting or lowering the load.

If one complete turn of a screw thread, be imagined to be unwound, from the body of the screw and developed, it will form an inclined plane as shown in Fig. 9.21

Let p = Pitch of the screw,

d = Mean diameter of the screw

r = Mean radius of the screw, and

α = Helix angle.

From the geometry of the figure, we find that

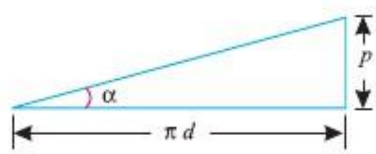


Fig. 9.21. Helix angle

$$\tan \alpha = \frac{p}{\pi d} = \frac{p}{2 \pi r} \quad \text{...(where } d = 2r\text{)}$$

Now let P = Effort applied at the mean radius of the screw jack to lift the load,

W = Weight of the body to be lifted, and

μ = Coefficient of friction, between the screw and nut.

Let ϕ = Angle of friction, such that $\mu = \tan \phi$.

As a matter of fact, the principle, on which a screw jack works, is similar to that of an inclined plane. Thus the force applied on the lever of a screw jack is considered to be horizontal. We have already discussed in Art. 8.14 that the horizontal force required to lift a load on an inclined rough plane

$P = W \tan (\alpha + \phi)$

Example 9.8. A screw jack has mean diameter of 50 mm and pitch 10 mm. If the coefficient of friction between its screw and nut is 0.15, find the effort required at the end of 700 mm long handle to raise a load of 10 kN.

Solution. Given: Mean diameter of screw jack (d) = 50 mm or radius (r) = 25 mm; Pitch of the screw (p) = 10 mm; Coefficient of friction between screw and nut (μ) = 0.15 = tan ϕ or ϕ = 8.5°; Length of the handle (l) = 700 mm and load to be raised (W) = 10 kN.

Let P_1 = Effort required at the end of 700 mm long handle to raise the load,

and α = Helix angle

We know that

$$\tan\alpha = \frac{p}{\pi d} = \frac{10}{\pi \times 50} = 0.0637 \quad \text{or} \quad \alpha = 3.6°$$

and effort required at mean radius of the screw to raise the load,

$$P = W \tan(\alpha + \phi) = W \tan(3.6° + 8.5°)$$
$$= W \tan 12.1° = 10 \times 0.2144 = 2.144 \text{ kN}$$

Now the effort required at the end of the handle may be obtained from the relation.

$$P_1 \times 700 = P \times r = 2.144 \times 25 = 53.6$$

$$\therefore \quad P_1 = \frac{53.6}{700} = 0.0766 \text{ kN} = 76.6 \text{ N} \quad \textbf{Ans.}$$

Example 9.9. The mean radius of the screw of a square threaded screw jack is 25 mm. The pitch of thread is 7.5 mm. If the coefficient of friction is 0.12, what effort applied at the end of lever 60 cm length is needed to raise a weight of 2 kN.

Solution. Given: Mean radius of the screw (r) = 25 mm; Pitch of the thread (p) = 7.5 mm; Coefficient of friction (μ) = 0.12 = tan ϕ; Length of the lever (l) = 60 cm and weight to be raised = 2 kN = 2000 N.

Let P_1 = Effort required at the end of the 60 cm long handle to raise the weight,

and α = Helix angle.

We know that

$$\tan\alpha = \frac{p}{2\pi r} = \frac{0.75}{2\pi \times 2.5} = 0.048$$

and effort required at mean radius of the screw to raise the weight,

$$P = W \tan(\alpha + \phi) = W \times \frac{\tan\alpha + \tan\phi}{1 - \tan\alpha . \tan\phi} = 2000 \times \frac{0.048 + 0.12}{1 - 0.048 \times 0.12}$$

$$= 2000 \times 0.169 = 338 \text{ N}$$

Now the effort applied at the end of the lever, may be found out from the relation,

$$P_1 \times 60 = P \times 2.5 = 338 \times 2.5 = 845$$

$$\therefore \quad P_1 = \frac{845}{60} = 14.1 \text{ N} \quad \textbf{Ans.}$$

Example 9.10. A screw press is used to compress books. The thread is a double thread (square head) with a pitch of 4 mm and a mean radius of 25 mm. The coefficient of the friction (μ) for the contact surface of the thread is 0.3. Find the torque for a pressure of 500 N.

Cont

Solution. Given: No. of threads (n) = 2; Pitch (p) = 4 mm; Mean radius (r) = 25 mm ; Coefficient of friction (μ) = 0.3 = tan ⟩ or ⟩ = 16.7° and pressure (W) = 500 N

Let ⟨ = Helix angle.

We know that tan ⟨ = np2·4 == 0.0509 or ⟨ = 2.9°
2□r 2□ ·25

4 Effort required at the mean radius of the screw to press the books

P = W tan (α + ⟩) = 500 tan (2.9° + 16.7°) N

= 500 tan 19.6° = 500 × 0.356 = 178 N

and* torque required to press the books,

T = P × r = 178 × 25 = 4450 N-mm Ans.

9.6. RELATION BETWEEN EFFORT AND WEIGHT LOWERED BY A SCREW JACK

We have already discussed in the last article that the principle, on which a screw works, is similar to that of an inclined plane. And force applied on the lever of a screw jack is considered to be horizontal. We have also discussed in Art. 8.14 that the horizontal force required to lower a load on an inclined plane,

P = W tan (α – ⟩)...(when ⟨ > ⟩)

= W tan (φ – ⟨)...(when ⟩ > ⟨)

Note. All the notations have the usual values as discussed in the last article.

Example 9.11. A screw Jack has a square thread of 75 mm mean diameter and a pitch of 15 mm. Find the force, which is required at the end of 500 mm long lever to lower a load of 25 kN. Take coefficient of friction between the screw and thread as 0.05.

Solution. Given: Mean diameter of thread (d) = 75 mm or radius (r) = 37.5 mm; Pitch of thread (p) = 15mm; Length of lever (l) = 500 mm; load to be lowered (W) = 25 kN and coefficient of friction between the screw and thread (μ) = 0.05 = tan ⟩ or ⟩ = 2.9°

Let P_1 = Effort required at end of 500 mm long handle to lower the load,

and ⟨ = Helix angle.

p15

tan⟨ === 0.0637 or ⟨ = 3.6° We know that

□d □ ·75

and effort required at the mean radius of the screw to lower the load,

P = W tan (α – ⟩) = W tan (3.6° – 2.9°)

= W tan 0.7° = 25 × 0.0122 = 0.305 kN = 305 N

Now the effort required at the end of the handle may be found out from the relation,

P_1 × 500 = P × r = 305 × 37.5 = 11438

11438

Ans. P_1 == 23 N4

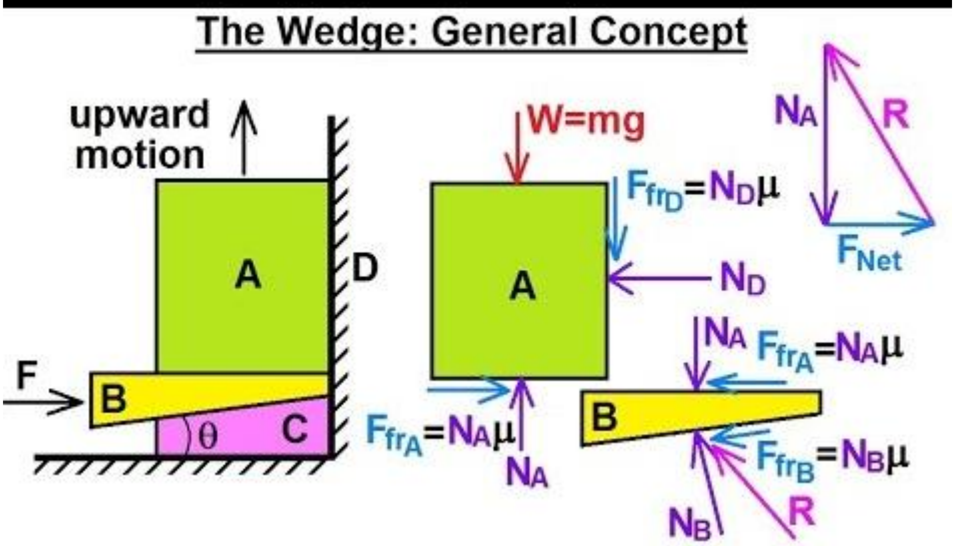
500

Applications of friction

1. Wedge friction.
2. Screw friction.

3. Ladder friction

1. Wedge friction.



A wedge is a piece of metal or wood which is usually of a triangular or trapezoidal in cross-section. It is used for either lifting loads through small vertical distances or used for slight adjustments in the position of a body i.e., for tightening fits or keys for shafts.

When lifting a heavy load the wedge is placed below the load and a horizontal force P is applied as shown in Fig. 3.41. If the force P is just sufficient to lift the load, the wedge will move towards left and load will move up. When the wedge moves towards left, the sliding of the surfaces AC and AB will take place. At the same time load moves up and sliding of the load takes place along GD. Thus for the wedge and load shown in Fig.3.41 sliding takes place along surface AB, AC and DG. Hence there will be three normal reactions at AB, AC and DG.

The problems on wedges are generally the problems of equilibrium on inclined planes. Therefore, these problems are solved by equilibrium method or by applying Lami's Theorem.

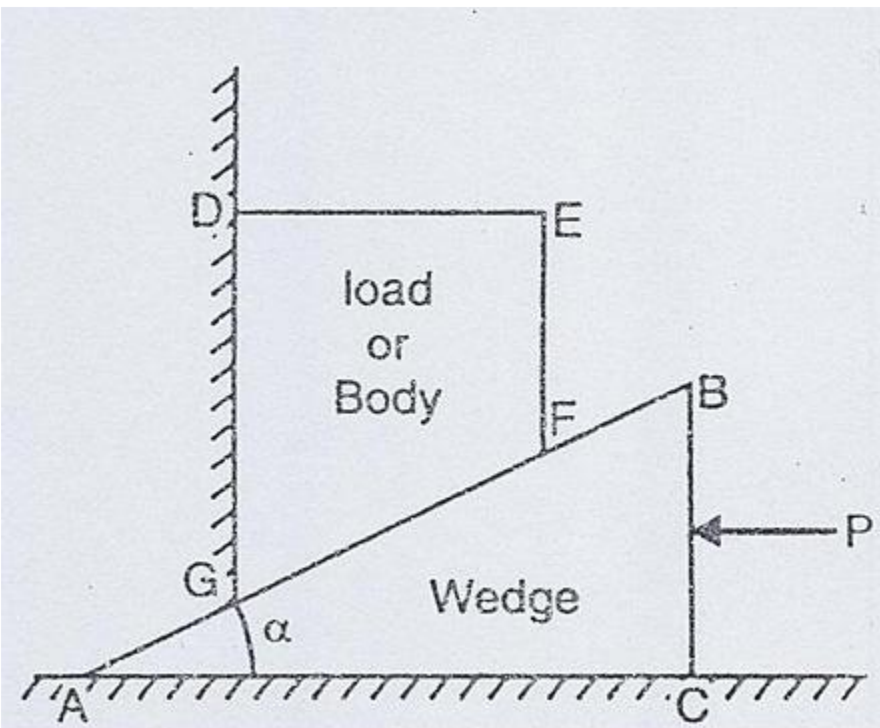


Fig. 3.41

Equilibrium Method. In this method, the equilibrium of the load (or the body placed on the wedge) and the equilibrium of the wedge are considered.

Equilirbium of Wedge

Consider the equilibrium of the wedge. The forces acting on the wedge are shown in Fig. 3.42. They are:

(i) The force P applied horizontally on face BC.

(ii) Reaction R_1 on the face AC (The reaction R_1 is the resultant of the normal reaction N_1 on the rubbing face AB and force of friction on surface AC). The reaction R_1 will be inclined at an angle θ_1 (when θ_1 is angle of friction) with the normal.

(iii) Reaction R_2 on the face AB (The reaction R_2 is the resultant of normal reaction N_2on the rubbing face AB and force of friction on surface AB). The reaction R_2 will be inclied at an angle Φ_2 with the normal.

When the force P is applied on the wedge, the surface CA will be moving towards left and hence force of friction on this surface will be acting towards right. Similarly, the force of friction on face AB will be acting from A to B. These forces are shown in Fig. 3.42.

Resolving the forces horizontally, we get

$$R_1 \sin \phi_1 + R_2 \sin(\phi_2 + \alpha) = P$$

Resolving the forces vertically, we get

$$R_1 \cos \phi_1 = R_2 \cos(\phi_2 + \alpha)$$

By Lami's Theorem

The wedge is in equilibrium under the action of three forces namely R_1, R_2 and P. These forces, when produced, will meet at a point as shown in Fig. 3.43.

Applying Lami's theorem, we get

$$\frac{P}{\sin(180 - \phi_1 - \phi_2 - \alpha)} = \frac{R_1}{\sin(90 + \alpha - \phi_{2)}} = \frac{R_2}{\sin(90 + \phi_{1)}}$$

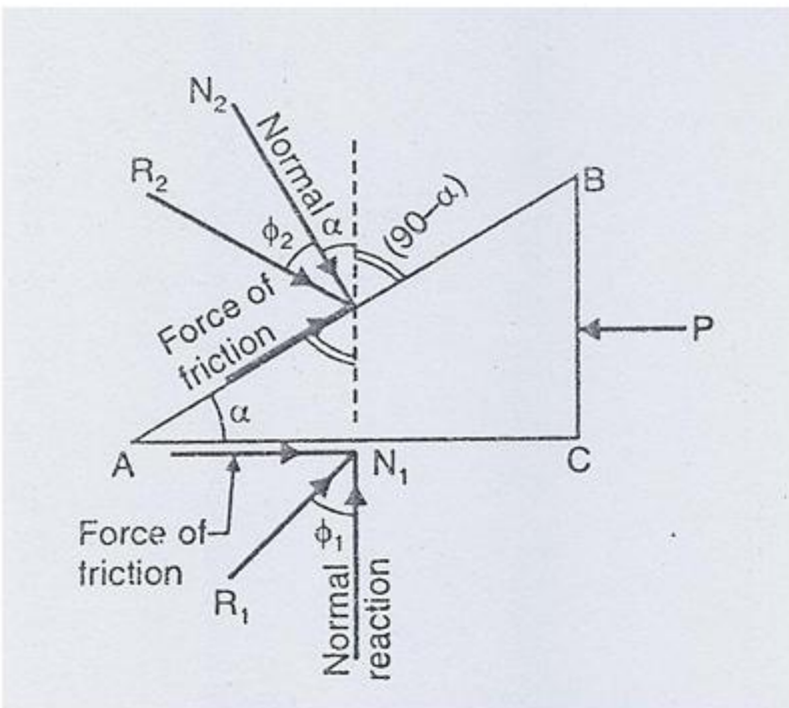
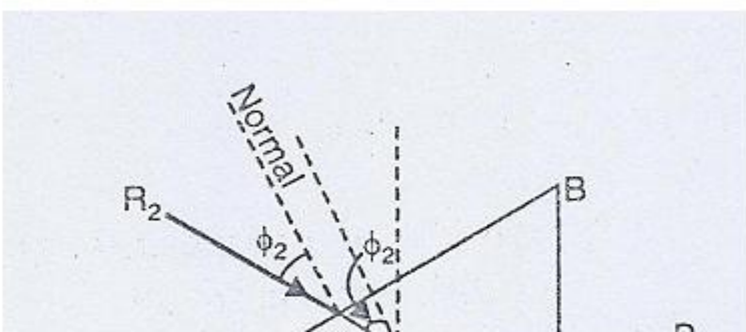


Fig. 3.42 Equilibrium of wedge



Equilibrium of Body placed on the Wedge

The forces acting on the body are shown in Fig.3.44. They are:

(i) The weight W on the body.

(ii)Reaction R_3 on the face GD. (The reaction R_3 is the resultant of the normal reaction N_3 on the rubbing face GD and force of friction on surface GD).

(iii)Reaction R_3 on the face GF (The reaction R_2 is the resultant of the normal reaction N_2 on the rubbing face GF and force of friction on surface GF).

These forces are shown in Fig. 3.44.

Resolving the forces R_2, R_3 and W horizontally, we get

$$R_3 \cos \phi_3 = R_2 \sin(\alpha + \phi_2)$$

Resolving forces vertically,

$$W + R_3 \sin \phi_3 = R_2 \cos(\alpha + \phi_2)$$

By Lami's Theorem

The forces R_3, R_2 and W are produced to meet at a point as shown in Fig. 3.44.

The body is in equilibrium under the action of these forces.

Hence applying Lami's theorem, we get

$$\frac{W}{\sin(90 + \phi_3 + \alpha + \phi_2)} = \frac{R_2}{\sin(90 - \phi_3)} = \frac{R_3}{\sin[180 - (\alpha + \phi_1)]}$$

UNIT-II FRICTION

In Unit-I, generally body surfaces are assumed to be **smooth.** In reality, no body surface is perfectly smooth. As a result, when one body surface slides or tends to slide over another, resistance is always offered to the motion. This resistive force which acts tangential to the contact surfaces is called **friction.**

Friction causes favourable as well as undesirable effects. In machines, it causes wear and tear of material parts. On the other hand, without friction we cannot do things in our daily lives as we do, like we cannot walk or drive a car or hold a pen and write, and so on.

TYPES OF DRY FRICTION

- Sliding friction
- Belt/rope friction
- Wheel friction
- Wedge friction
- Screw friction

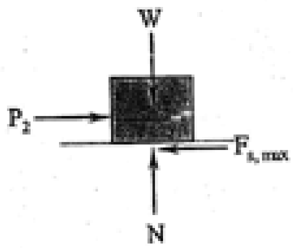
LIMITING FRICTION AND IMPENDING MOTION

When an externally applied force (F) acts tangential to the contact surfaces, we come across three different cases, namely

Case-I: When $F < F_{s,max}$, no motion occurs.

Case-II: When $F = F_{s,max}$, motion impends.

Case-III: When $F > F_{s,max}$, the body is under motion.



Point of impending motion

COULOMB'S LAWS OF DRY FRICTION

1. The frictional force always acts such as to oppose the tendency of one surface to slide relative to the other. It acts tangential to the surfaces in contact.
2. The magnitude of frictional force is exactly equal to the force which tends to move the body till the limiting value is reached.
3. The maximum force of friction is independent of the area of contact between two surfaces and depends on the nature of surfaces in contact.
4. The magnitude of limiting static friction is proportional to the normal reaction between the two surfaces. Mathematically it is expressed as:

$$F_{s,max} \propto N$$

Introducing a constant of proportionality,

$$F_{s,max} = \mu_s N$$

where the constant μ_s is called as **coefficient of static friction.**

5. For low relative velocities between sliding bodies, frictional force is independent of the relative speed with which the surfaces move over each other.
6. The magnitude of kinetic friction is proportional to the normal reaction between the two surfaces. Mathematically it is expressed as:

   $$F_k \propto N$$

   Introducing a constant of proportionality,

   $$F_k = \mu_k N$$

   where the constant μ_k is called as **coefficient of kinetic friction.**

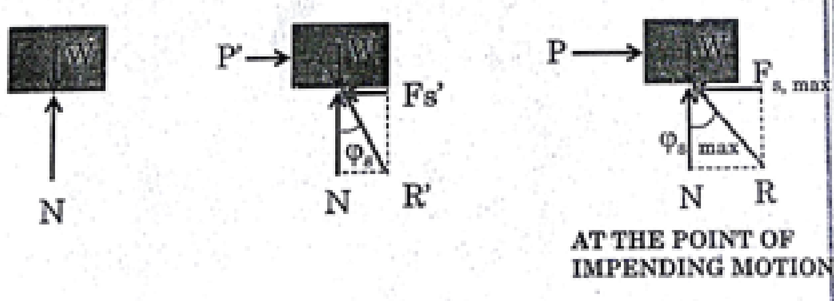
CONTACT SURFACES	μ_s
Wood on wood	0.2-0.5
Wood on leather	0.2-0.5
Metal on metal	0.15-0.25
Metal on wood	0.2-0.6
Metal on stone	0.3-0.7
Metal on leather	0.3-0.5
Stone on stone	0.4-0.7
Earth on earth	0.2-1.0
Rubber on concrete	0.6-0.9
Rope on wood	0.5-0.7

Graphical Representation

In some of the problems, it is better to replace **normal reaction (N)** and the **frictional force (Fs)** by the **resultant (R).**

GRAPHICAL REPRESENTATION

- RESULTANT & ANGLE OF FRICTION

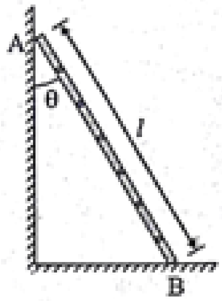


- In some of the problems, it is better to replace **normal reaction (N)** and the **frictional force (Fs)** by the **resultant (R)**

Angle of friction: It is the maximum value of φ, that is the angle between the resultant and the normal reaction.

Cone of friction: It is an imaginary cone formed by revolving the resultant (R) of the normal reaction and maximum force of static friction about the normal reaction for one complete revolution.

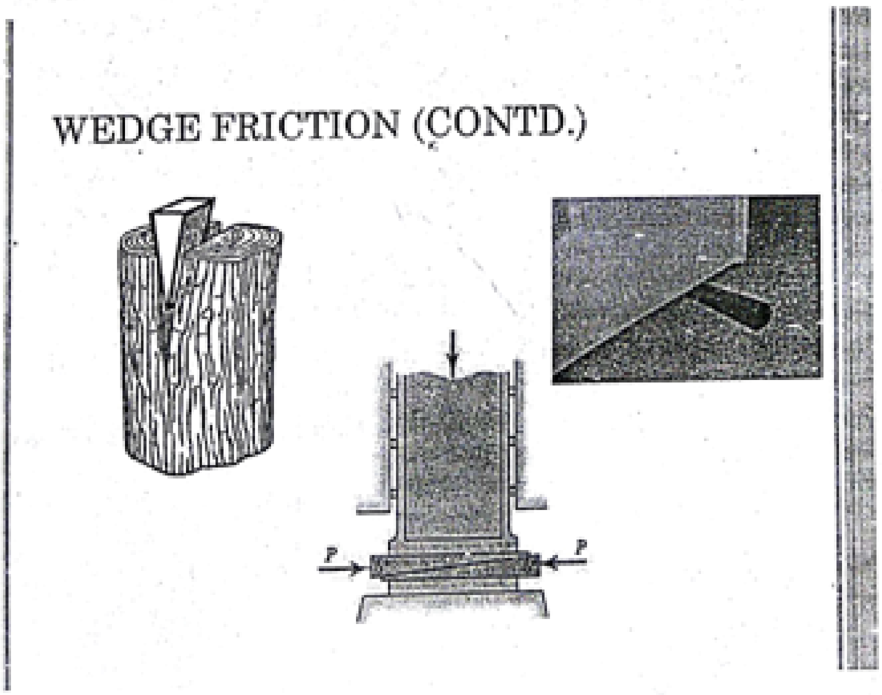
LADDER FRICTION



Unlike previous cases, there are two contact points, one at the wall and the other at the floor. The ladder tends to slide under its own weight or under the weight of a person climbing upon it. Due to friction at the wall and at the floor, resistance is offered to this sliding of ladder. The forces of friction reach maximum values at the point of impending motion.

WEDGE FRICTION

Wedges are triangular or trapezoidal shaped blocks with a very small sloping angle. By the friction developed, they are used to cleave open woods, to do minor alignments in heavy loads before fixing them.



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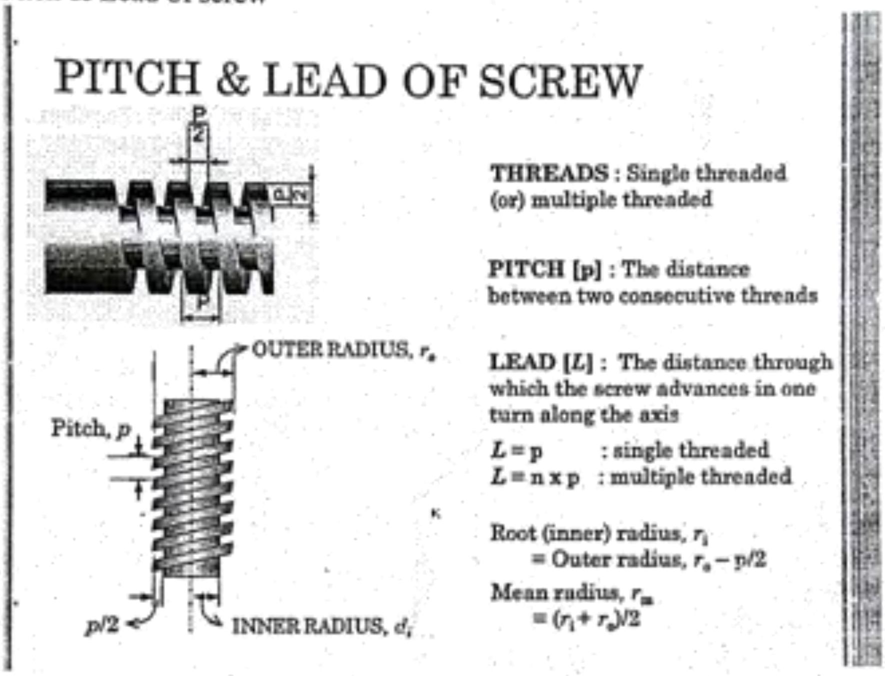
SCREW FRICTION

Threads are cut in a helix on the shank of the screw. When the screw is turned, **friction** is developed between the thread and the surrounding body. This friction is known as **screw friction.** This is utilized for fastening and for lifting of heavy loads.

V-threaded screw: used for fastening purpose
Square threaded screw
  Screw-jack : to lift and hold loads
  Bench vice : To clamp bodies

Pitch & Lead of screw



To raise a load, the effort required to be applied at the handle is:

$$P = \frac{Wr}{a} \tan(\varphi_s + \theta)$$

or the torque required to raise the load is:

$$\tau = P.a = Wr \tan(\varphi_s + \theta)$$

If $\theta > \varphi_s$, then the screw jack will unwind itself once the load is raised.
If $\theta < \varphi_s$, then the screw jack will self-lock and effort will be required to lower the load.

The **efficiency** of screw jack is defined as the ratio of the **effort** required under **frictionless condition** to that of the **actual effort** required to raise a load

$$\eta = \frac{\tan\theta}{\tan(\varphi_s + \theta)}$$

The maximum efficiency is given as:

$$\eta_{max} = \frac{1 - \sin\varphi_s}{1 + \sin\varphi_s}$$

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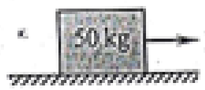
and the condition for maximum efficiency is;

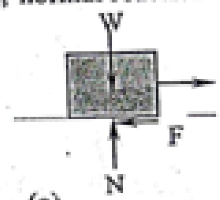
$$\theta = 45 - \varphi_s/2$$

Solved Problems

1. A block of mass 50 kg rests on a rough horizontal plane as shown in figure below. If the coefficients of static and kinetic friction between the block and the plane are respectively 0.2 and 0.15, describe the resulting motion, i) when a horizontal force of 75 N is applied and ii) when a horizontal force of 120 N is applied. Also determine the force required to cause motion to impend.



The forces acting in the free body diagram of the block are its weight W, normal reaction N, force of friction F acting to oppose the motion and the applied force P.



Applying the conditions of equilibrium along X and Y-directions,

$$\Sigma F_y = 0 \Rightarrow$$
$$N - W = 0$$
$$\Rightarrow N = W = 50 \times g = 490.5\,\text{N} \qquad \text{(a)}$$

The force of friction is maximum at the point of impending motion,

$$F_{s,max} = \mu_s N = 0.2 \times 490.5 = 98.1\,\text{N} \qquad \text{(b)}$$

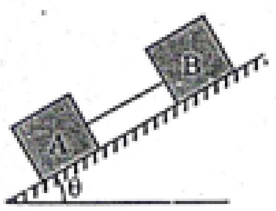
i) When a horizontal force of 75 N is applied:
This is of case-I, in which $P < F_{s,max}$, hence the block is not in motion and the body is in equilibrium. The corresponding force of friction is:

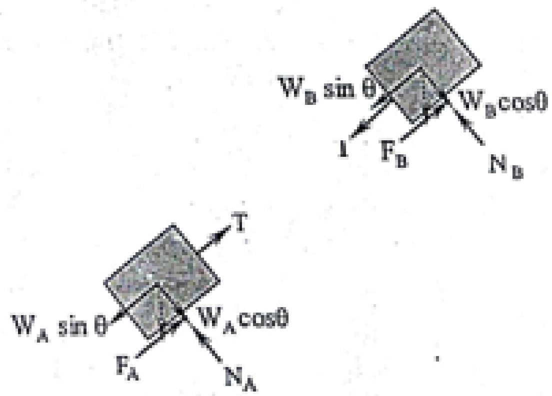
$$F = P = 75\,\text{N}$$

ii) When a horizontal force of 120 N is applied
From equation (b), we can see that the frictional force to maintain equilibrium is equal to 120 N. Since this frictional force is greater than $F_{s,max}$, the block is actually under motion. Therefore, the actual force of friction is equal to the force of kinetic friction, whose magnitude is given as: $F = F_k = \mu_k N = 0.15 \times 490.5 = 73.58\,\text{N}$

Force required to cause motion to impend is equal to the maximum force of static friction, i.e., $P = 98.1\,\text{N}$.

2. Two blocks A and B of weights W_A and W_B respectively rest on a rough inclined plane and are connected by a short piece of string as shown in figure below. If the coefficients of friction between the blocks and the plane are respectively $\mu_A = 0.2$ and $\mu_B = 0.3$, find (a) the angle of inclination of the plane for which sliding will impend and (b) tension in the string. Assume $W_A = W_B = 5\,\text{N}$.





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FBD of Block-A:

Applying the conditions of equilibrium at the point of impending motion,

$$\Sigma F_y = 0 \Rightarrow$$
$$\therefore N_A = W_A \cos\theta$$

At the point of impending motion,

$$F_A = \mu_A N_A = \mu_A W_A \cos\theta$$
$$\Sigma F_x = 0 \Rightarrow$$
$$T + F_A - W_A \sin\theta = 0$$
$$\therefore T = W_A\ (\sin\theta - \mu_A \cos\theta) \qquad \text{(a)}$$

FBD Block-B:

Applying the conditions of equilibrium at the point of impending motion,

$$\Sigma F_y = 0 \Rightarrow$$
$$\therefore N_B = W_B \cos\theta$$

At the point of impending motion,

$$F_B = \mu_B N_B = \mu_B W_B \cos\theta$$
$$\Sigma F_x = 0 \Rightarrow$$
$$-T - W_B \sin\theta + F_B = 0$$
$$\Rightarrow T = F_B - W_B \sin\theta$$
$$= W_B\ (\mu_B \cos\theta - \sin\theta) \qquad \text{(b)}$$

Solving the simultaneous equations (a) and (b), we get

$$W_A (\sin\theta - \mu_A \cos\theta) = W_B (\mu_B \cos\theta - \sin\theta)$$

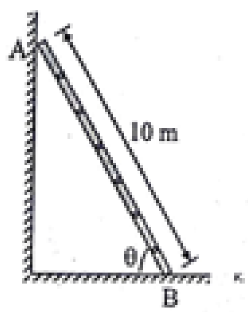
But $W_A = W_B$,

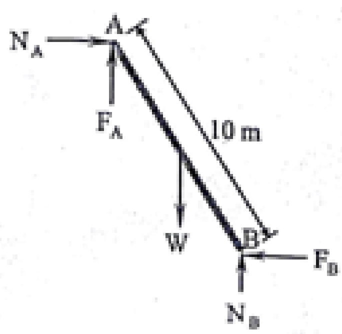
$$2\sin\theta = \cos\theta(\mu_A + \mu_B)$$
$$\tan\theta = \frac{(\mu_A + \mu_B)}{2}$$
$$\theta = 14.04^\circ$$

Substituting the value of θ in either of the equations for T:

$$T = W_A\ (\sin\theta - \mu_A \cos\theta)$$
$$= 5[\sin(14.04) - 0.2\cos(14.04)]$$
$$= 0.243\ N$$

3. A ladder 6 m long rests on a horizontal floor and leans against a vertical wall. If the coefficients of friction between the ladder and, the floor and the wall are respectively μ_f =0.3 μ_w = 0.15, determine the angle of inclination of the ladder with the floor at the point of impending motion.





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At the point of impending motion, we can apply the equations of equilibrium,

$$\Sigma F_x = 0 \Rightarrow$$
$$N_A - F_B = 0$$

But $F_B = \mu_f N_B$,

$$N_A - \mu_f N_B = 0$$
$$N_A - 0.3N_B = 0 \qquad \text{(a)}$$

$$\Sigma F_y = 0 \Rightarrow$$
$$F_A + N_B - W = 0$$

But $F_A = \mu_w N_A$,

$$\mu_w N_A + N_B - W = 0$$
$$0.15N_A + N_B - W = 0 \qquad \text{(b)}$$

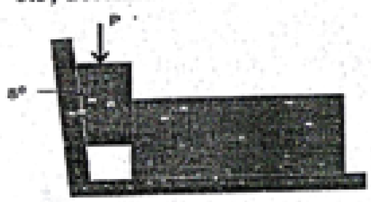
Solving from these two equations, we get

$$W = 0.15N_A + N_A/0.3 \quad = 3.48N_A$$

Taking moment about B and applying the condition of equilibrium,

$$M_B = 0 \Rightarrow$$
$$-[F_A \times 10\cos\theta] - [N_A \times 10\sin\theta] + [W \times 5\cos\theta] = 0$$
$$-[\mu_w N_A \times 10\cos\theta] - [N_A \times 10\sin\theta] + [3.48N_A \times 5\cos\theta] = 0$$
$$-1.5\cos\theta - 10\sin\theta + 17.4\cos\theta = 0$$
$$15.9\cos\theta = 10\sin\theta$$
$$\tan\theta = 1.59 \Rightarrow \theta = 57.83^\circ$$

4. A heavy block of mass 500 kg is to be adjusted horizontally using an 8° wedge by applying a vertical force P. If the coefficient of static friction for both the contact surfaces of the wedge is 0.25 and that between the block and the horizontal surface is 0.5, determine the least force P required to move the block.



5. A single square threaded screw jack has a pitch of 16 mm and a mean radius of 50 mm. Determine the force that must be applied to the end of 60 cm lever to raise a weight of 100 kN and the efficiency of the jack. State whether it is self-locking or not? If yes, determine the force that must be applied to lower the same weight. Assume coefficient of static friction to be 0.2.

Since the screw is *single threaded*, lead (L) = pitch (p) = 16 mm. Therefore,

$$\tan\theta = \frac{L}{2\pi r} = \frac{16}{2\pi(50)} \qquad = 0.051$$
$$\Rightarrow \theta = 2.92^\circ$$

Given, $\tan\varphi_s = \mu_s$

$$\Rightarrow \varphi_s = \tan^{-1}(0.2) = 11.31^\circ$$

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The force required to raise a weight of 100 kN is given as:

$$P = \frac{Wr}{a}\tan(\varphi_s + \theta)$$

$$= \frac{100{,}000 \times 0.05}{0.6}\tan(11.31 + 2.92) \quad = 2113.3\,\text{N}$$

The efficiency of screw jack is given as:

$$\eta = \frac{\tan\theta}{\tan(\varphi_s + \theta)}$$

$$= \frac{\tan(2.92)}{\tan(11.31 + 2.92)} \quad = 0.2011\ (\text{or})\ 20.11\%$$

Since $\varphi_s > \theta$, it is under **self-locking.** Hence, force is required to lower the load. This force is obtained as:

$$P = \frac{Wr}{a}\tan(\varphi_s - \theta)$$

$$= \frac{100{,}000 \times 0.05}{0.6}\tan(11.31 - 2.92) \quad = 1229.07\,\text{N}$$

6. Determine the torque to be applied in a differential screw jack to lift a load of 10 kN. Given that pitch of the outer and inner threads are 10 mm and 6 mm respectively. The mean radius of the outer thread is 30 mm and that of inner thread is 20 mm, coefficient of friction for both threads is 0.1.

We know, $\tan\varphi_s = \mu_s \quad \Rightarrow \varphi_s = \tan^{-1}(0.1) = 5.71^\circ$

Outer screw: Assuming single-threaded screw for outer spindle,

$$L_1 = p_1 = 10\,\text{mm}$$

$$\therefore \theta_1 = \tan^{-1}\left[\frac{L_1}{2\pi r_1}\right] \quad = \tan^{-1}\left[\frac{10}{2\pi(30)}\right] = 3.04^\circ$$

$$\tan(\varphi_s + \theta_1) = 0.154$$

Inner screw: Assuming single-threaded screw for inner spindle,

$$L_2 = p_2 = 6\,\text{mm}$$

$$\therefore \theta_2 = \tan^{-1}\left[\frac{L_2}{2\pi r_2}\right]$$

$$= \tan^{-1}\left[\frac{6}{2\pi(20)}\right] = 2.73^\circ$$

Hence, $\tan(\varphi_s + \theta_2) = 0.148$

Therefore, the torque to be applied to raise a load of 10 kN is given as:

$$\tau = r_1.W\tan(\varphi_1 + \theta_1) + r_2.W\tan(\varphi_2 + \theta_2)$$

$$= W[r_1.\tan(\varphi_1 + \theta_1) + r_2.\tan(\varphi_2 + \theta_2)]$$

$$= 10 \times 10^3[(0.03 \times 0.154) + (0.02 \times 0.148)]$$

$$= 75.8\,\text{N.m}$$

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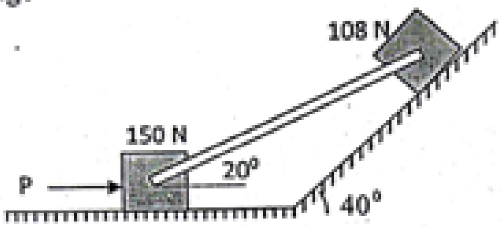
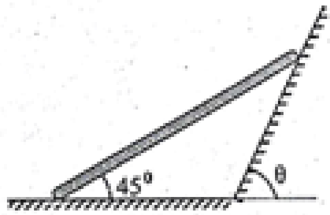
ASSIGNMENT QUESTIONS

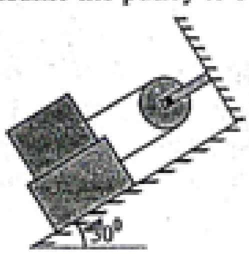
1. A 108 N block is held on a 40^0 incline by a bar attached to a 150 N block on a horizontal plane shown in figure. The bar which is fastened by smooth pins at each end is inclined 200 to the horizontal. The coefficient of friction between each block and its plane is 0.325. For what horizontal force P, applied to 150 N block will motion to the right be impending?



2. A uniform bar of length l rests on a rough horizontal floor and a rough inclined wall as shown in figure. Determine the inclination of the wall with respect to the horizontal for which equilibrium can be maintained. Take coefficient of friction for all contact surfaces to be 0.3.



3. A block weighing 100 N is resting on a rough plane inclined at 20^0 to the horizontal. It is acted upon by a force of 50 N directed upward at angle of 14^0 above the plane. Determine the friction, if the block is about to move up the plane, determine the co-efficient of friction.
4. A ladder 5 m long and of 250 N weight is placed against a vertical wall in a position where its inclination to the vertical is 30^0. A man weighing 800 N climbs the ladder. At what position will he induce slipping? The co-efficient of friction for both the contact surfaces of the ladder viz. with the wall and the floor is 0.2.
5. Determine the weight W of the upper block to prevent downward motion of the lower block of A of mass 300 kg as shown in figure E.6.32. The coefficient of friction between all the contact surfaces is 0.25. Assume the pulley to be frictionless.



6. A body weighing 70 kN rests in equilibrium on a rough plane whose slope is 30^0. The plane is raised to a slope of 45^0. What is the force applied to the body parallel to the plane that will support the body on the plane.
7. Block A weighs 200 N and block B weighs 120 N. They rest on a rough inclined plane and are connected by a short piece of string as shown in figure below. If the coefficients

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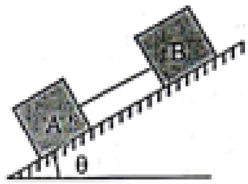
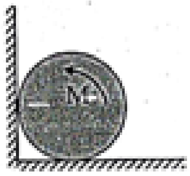
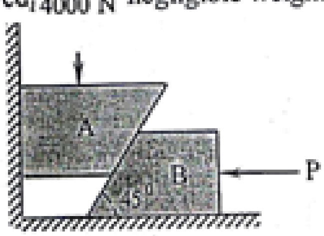
of friction between the blocks and the plane are respectively $\mu_A = 0.25$ and $\mu_B = 0.3$, determine (a) the angle of inclination of the plane for which sliding will impend and (b) tension in the string.



8. A horizontal bar 10 m long and of negligible weight rests on rough inclined plane as shown in figure. If the angle of friction is 15^0, how close to B may the 200N force be applied before motion impends?
9. A block is lying over a 10^0 wedge on a horizontal floor and leaning against a vertical wall and weighing 1500N is to be raised by applying a horizontal force to the wedge. Assuming coefficient of friction between all the forces in contact to be 0.3, determine the minimum horizontal force to be applied to raise the block.
10. Determine the moment required to be applied to the cylinder of weight W shown in figure to cause motion to impend. The coefficient of friction between all contact surfaces is μ.



11. A body weighing 70kN rests in equilibrium on a rough plane whose slope is 30^0. The plane is raised to a slope of 45^0. What is the force applied to the body parallel to the plane that will support the body on the plane?
12. The outer and inner diameters of the spindle in a screw jack are 60 cm and 40 cm respectively. If the screw is single threaded, and the coefficient of friction between screw and nut is 0.2, determine i) the torque required to raise a load of 20 kN and ii) to lower the same load. Also, determine the efficiency of the screw jack.
13. A steam valve of 15 cm diameter has a pressure of 3 MPa acting on it. If it is closed by means of square threaded screw of 60 mm external diameter and pitch of 6 mm. Determine the torque exerted on the handle of the valve. The coefficient of friction is 0.2.
14. The distance between adjacent threads of a double-threaded screw jack is 10 mm; mean radius is 60 mm; coefficient of friction is 0.10. What load can be raised by exerting a moment of 100 N-m?
15. In the figure, block A supports a weight of 4000 N and it is to be prevented from sliding down by applying a horizontal force P on wedge B. If the coefficient of friction at all surfaces of contact is 0.2, determine the smallest force P required to maintain equilibrium. Assume the block and wed[illegible] negligible weight.



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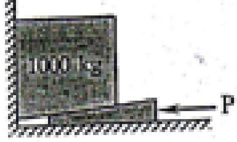
16. A 15° wedge is used to raise a 1000 kg block as shown in figure. Determine the horizontal force P that must be applied on the wedge to raise it. Assume the angle of friction at all contact surfaces to be 0.2.



17. Two equal bodies A and B of weight 'W' each are placed on a rough inclined plane. The bodies are connected by a light string. If $\mu_A = 1/2$ and $\mu_B = 1/3$, show that the bodies will be both at the point of impending motion when the plane is inclined at $\tan^{-1}(5/12)$.
18. If the ratio of the greatest to the least force which acting parallel to a rough inclined plane can support a weight on it is equal to that of the weight to the pressure on the plane, then prove that the coefficient of friction is $\tan\alpha.\tan^2\frac{\alpha}{2}$, where α is the inclination of the plane to the horizontal?
19. The mean radius of the screw in a screw jack is 50 mm and the pitch of the thread is 16 mm. If the coefficient of friction between screw and nut is 0.2, determine the torque required to raise a load of 1kN and the efficiency of the screw jack. Also, determine the torque required to lower the load.
20. A pull of 100 N, inclined at 30 degrees to the horizontal plane, is required just to have a body placed on a rough horizontal plane. But the push required to move the body is 125 N. If the push is inclined at 25 degrees to the horizontal, find the weight of the body and co-efficient of friction.
21. An effort of 500 N is required just to move a certain body up on an inclined plane of angle 20 degrees, the force acting parallel to the plane. If the angle of inclination of the plane is made 30 degrees, the effort required again applied parallel to the plane is found to be 650 N. Find the weight of the body and co-efficient of friction.

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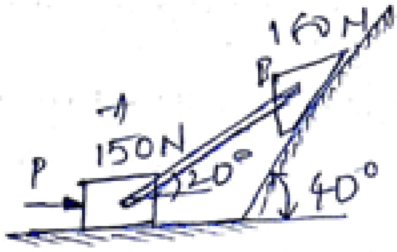
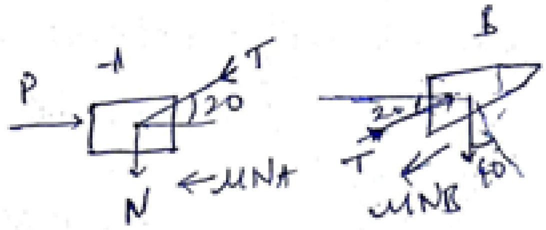
Friction:

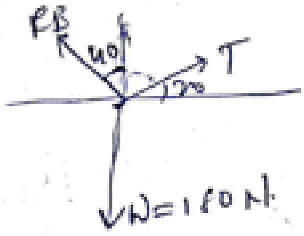
1.



Required to find the value of P which causes the body B to move in the upward direction



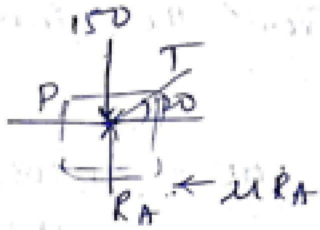
Applying Lami's theorem to B.



$$\frac{T}{\sin(180-40)} = \frac{W}{\sin(110)}$$

$T = 123.12$ N

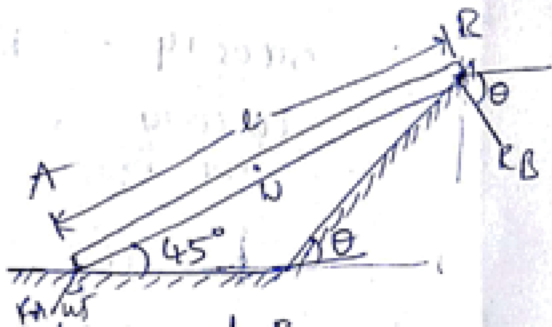
take FBD of A



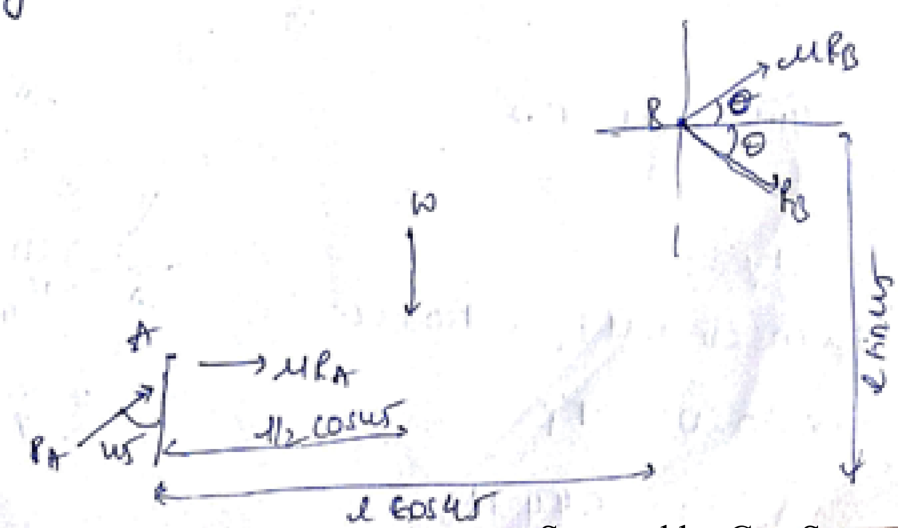
$P = T\cos 20 + \mu\, 150$

$= 164.45$ N

2.



let W be the weight of the ladder and R_A and R_B are the Reactions of point A and B respectively.



1 2

$\Sigma F_y = 0$

$R_A \cos 45 + \mu R_B \sin\theta + R_B \sin\theta = W$

$R_B \cos\theta (0.707) \cos 45 + \mu R_B \sin\theta + R_B \sin\theta = W$

$W = 0.5 \cos\theta \, R_B + 1.3 R_B \sin\theta$ —— (1)

$\Sigma F_y = 0$

$R_A \sin 45 + \mu R_A + \mu R_A \cos\theta = R_B \cos\theta$

$R_A = R_B \cos\theta (0.707)$

$\Sigma M_A = 0$

$W \times \frac{l}{2} \cos 45 + \mu R_B \cos\theta \times l \sin 45 - \mu R_B \sin\theta \times l \cos 45 + R_B \cos\theta \, l \sin 45 + R_B \sin\theta \times l \cos 45$

$W = 2[(\mu+1) R_B \sin\theta + (1-\mu) R_B \cos\theta)$ —— (2)

(1) = (2)

$2(1.3) R_B \sin\theta + 2(0.7) R_B \cos\theta = 0.5 R_B \cos\theta + 1.3 R_B \sin\theta$

$1.3 R_B \sin\theta = 0.5 R_B \cos\theta - 2 \times 0.7 R_B \cos\theta$

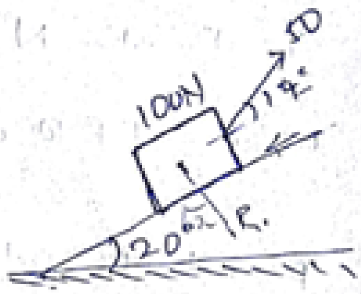
$= -0.9 \, R_B \cos\theta$

$\tan\theta = \frac{-0.9}{1.3}$

$\boxed{\theta = 55.3^\circ}$

3. soln

From the given data



$100 = R \cos 20 + P \sin 14^\circ$

$R = 93.5$ N

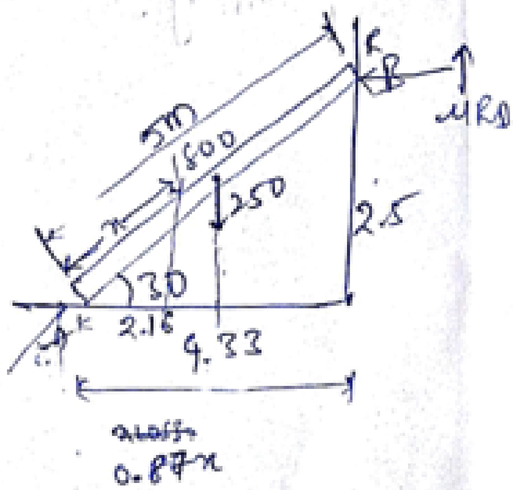
$50 \cos 14 = R \sin 20 + \mu R$

$\frac{50 \cos 14}{93.5} = \sin 20 + \mu$

$\mu = \frac{50 \cos 14}{93.4} - \sin 20$

$\boxed{\mu = 0.1768}$

4 Given $\mu = 0.2$



$\Sigma F_y = 0$

$R_A \cos 30 + \mu R_B = 250 + 800$

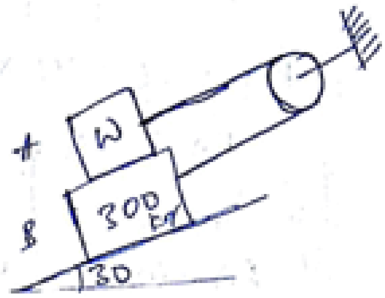
$R_A \sin 30 = R_B$

$\Sigma M_A = 0$

$R_B \times 2.5 + \mu R_B \times 4.33 = 250 \times 2.16 + 800 \times 0.866 x$

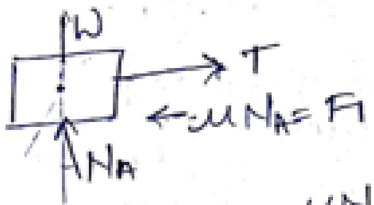
$$\boxed{\therefore x = 1.34\ m}$$

5 Soln

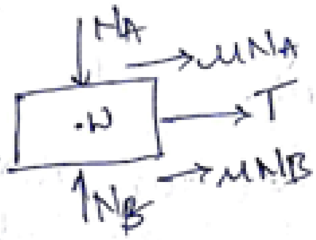


Given $\mu = 0.25$

FBD of A



FBD of B



$T = \mu N_A$

$W \sin 30 = T + \mu N_A + \mu N_B$

$= \mu N_A + \mu N_A + \mu N_B$

$= 2\mu N_A + \mu N_B$

$5886 = 2 N_A + N_B$ — (1)

$N_A + W \cos\theta = N_B$

$N_B - N_A = W \cos 30$

$N_B - N_A = 2548.7$ — (2)

$2N_B - 2N_A = 5097.4$

from (1) and (2)

$N_B + \cancel{2N_A} = 5886$

$2N_B - \cancel{2N_A} = 5097.4$

$3N_B = 10983.4$

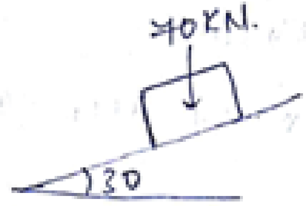
$N_B = 3661.1\ N$

$\therefore$

$N_A = 725.8\ N$

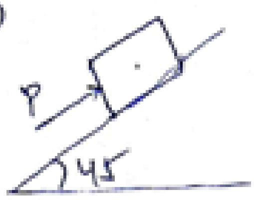
6

Soln



R_A be the reaction of block

μ be the coefficient of friction



$R_A \cos 30 = 70 \times 10^3$

$R_A = 80.82$ kN

$R_A \sin 30 = \mu R_A$

$\mu = 0.5$

IInd case

$70 \times 10^3 = R_A \times \cos 45$

$R_A = 99$ kN

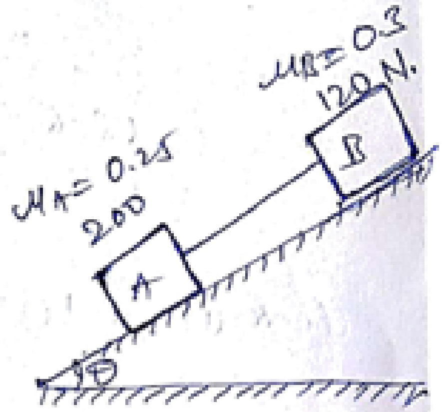
$P + \mu R_A = R_A \sin 45$

$P = R(\sin 45 - \mu)$

$= 20.5$ kN

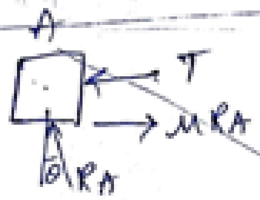
7

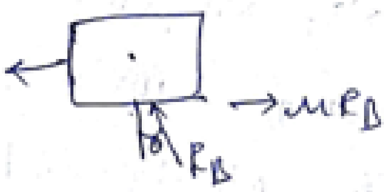
Sol



Required to find θ

FBD of A





$R_A \cos\theta = 200$

$R_A \sin\theta + T = \mu R_A$

$T = \mu R_A - R_A \sin\theta$

=

$R_B \cos\theta = 120$

$T + R_B \sin\theta = \mu R_B$

$T = \mu R_B - R_B \sin\theta$

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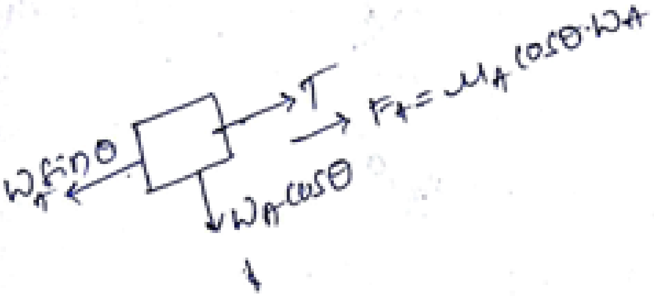
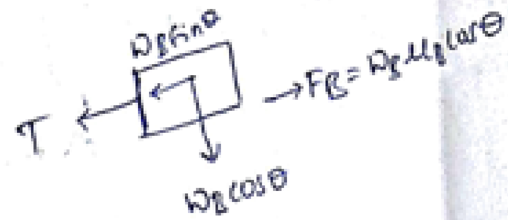
FBD of A 

FBD of B 

$T + \mu_A W_A \cos\theta = W_A \sin\theta$

$T = W_A(\sin\theta - \mu_A \cos\theta)$

$T + W_B \sin\theta = W_B \mu_B \cos\theta$

$T = W_B(\mu_B \cos\theta - \sin\theta)$

$W_A(\sin\theta - \mu_A \cos\theta) = W_B(\mu_B \cos\theta - \sin\theta)$

$\overset{5}{\cancel{200}}(\sin\theta - 0.25\cos\theta) = \overset{3}{\cancel{120}}(0.3\cos\theta - \sin\theta)$

$5\sin\theta - 1.25\cos\theta = 0.9\cos\theta - 3\sin\theta$

$8\sin\theta = \cancel{2.145}\ 2.15\cos\theta$

$\tan\theta = \frac{2.15}{8}$

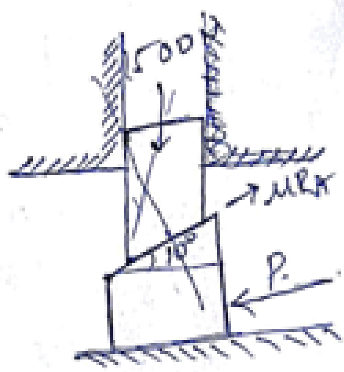
$\theta = \tan^{-1}(0.2687)$

$= 15.04°$

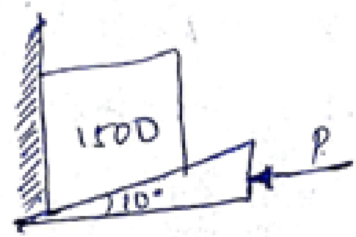
$T = W_A(\sin\theta - \mu_A\cos\theta)$

$= 200(\sin(15.04) - 0.25\cos(15.04))$

$= 3.62$ N.

9. 

Sln: Required to find horizontal force P.



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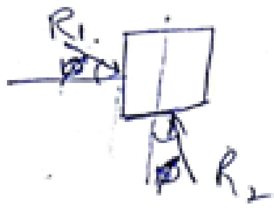
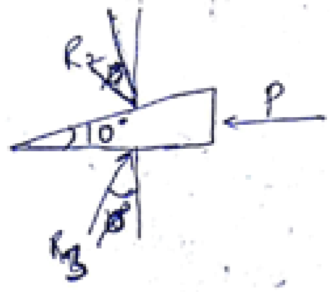
FBD of Body





$\mu = \tan(\phi)$

$\tan^{-1}(\mu) = \phi$

$\phi = 16.7°$

$\Sigma F_x = 0$

$R_1 \cos(16.7) = R_2 \sin(10 + 16.7)$

$= R_2 \sin 26.7$

$R_2 = 2.132 R_1$

$\Sigma F_y = 0$

$R_1 \sin(16.7°) + 1500 = R_2 \cos(10 + 16.7)$

$= R_2(\cos 26.7)$

$= R_2(0.8934)$

$= 2.132 R_1 (0.8934)$

$= 1.905 R_1$

$R_1(1.905 - 0.2874) = 1500$

$\therefore R_1 = \frac{1500}{1.6176} = 927.3\,N$

and $R_2 = 1977\,N$

In wedge condition

$\Sigma F_y = 0$ $\quad R_3 \cos(16.7°) = R_2 \cos(10 + 16.7°)$

$R_3 (0.9578) = R_2 \times 0.8934$

$R_3 = 1844\,N$

$\Sigma F_x = 0$ $\quad P = R_2 \sin(10 + 16.7°) + R_3 \sin 16.7°$

$= 1418.3\,N$

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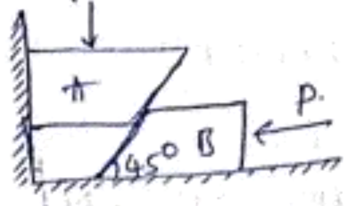
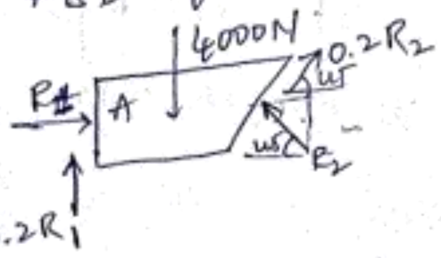
15
COLD

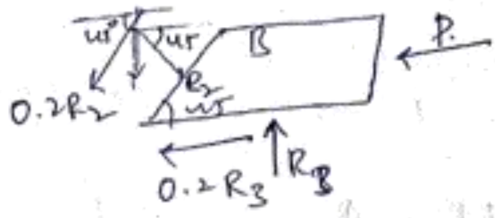
Given $\mu = 0.2$.



Required to find smallest force P to maintain equillibrium. i.e the block is about to move to the left.

FBD of A





Equillibrium of A

$\Sigma F_y = 0$

$0.2R_1 + R_2 \sin 45 + 0.2R_2 \sin 45 = 4000$

$0.2R_1 + R_2(\sin 45 + 0.2 \sin 45) = 4000$

$0.2R_1 + 0.8R_2 = 4000$

$R_1 + 4.24R_2 = 20{,}000$ — ①

$\Sigma F_x = 0$

$R_1 + R_2 \cos 45 = 0.2R_2 \cos 45$

$R_1 + 0.5656 R_2 = 0$ — ②

from ① and ②

$(3.67) R_2 = 20{,}000$

$\boxed{R_2 = 5443.1 \text{ N}}$

Equillibrium of B:

$\Sigma F_y = 0 \Rightarrow R_3 = R_2 \sin 45 + 0.2 R_2 \sin 45$

$R_3 = 4618.7$ N

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$\Sigma F_x = 0$

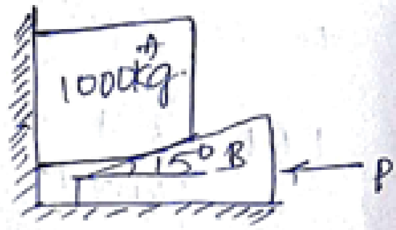
$$P + 0.2R_3 + 0.2R_2\cos 45 = R_2\cos 45$$

$$P = 0.8R_2\cos 45 - 0.2R_3$$

$$\boxed{P = 2155.3N}$$

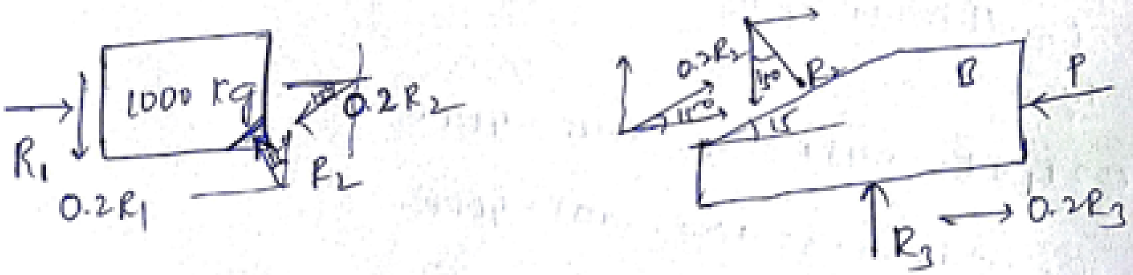
16. Soln

Given wedge angle $15°$ and $\mu = 0.2$



Required to find the horizontal force P is req applied to raise the block of weight 1000kg

FBD of A and wedge B



Equillibrium of A

$\Sigma F_y = 0$

$$1000 + 0.2R_1 + 0.2R_2\sin 15° = R_2\cos 15°$$

$$0.965R_2 - 0.051R_2 - 0.2R_1 = 1000$$

$$0.913R_2 - 0.2R_1 = 1000$$

$$4.566R_2 - R_1 = 5000 \quad \text{--- (1)}$$

$\Sigma F_x = 0$

$$R_1 = 0.2R_2\cos 15° + R_2\sin 15°$$

$$= 0.452R_2 \quad \text{--- (2)}$$

(2) in (1)

$$4.113R_2 = 5000$$

$$\boxed{R_2 = 1215.3\ N}$$

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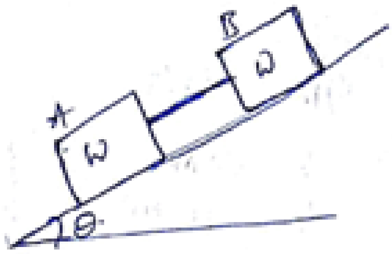
B
Sol

Given

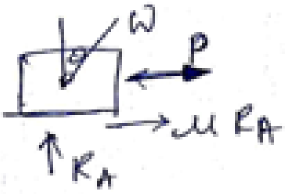


$\mu_A = 0.5$

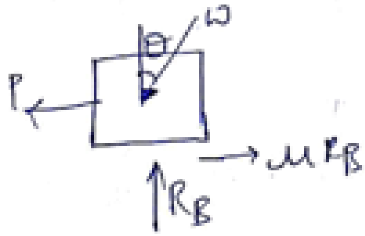
$\mu_B = 0.33.$

Required to show that the angle $\theta = \tan^{-1}(5/12)$ when the bodies are ready to move on inclined plane

FBD of A



FBD of B



$\Sigma F_x = 0$

$P + \mu R_A = W\sin\theta$

$\cancel{= \mu W\cos\theta + W\sin\theta}$

$\cancel{= W(\mu\cos\theta + \sin\theta)}$

$P = W(\sin\theta - \mu_A\cos\theta)$ — (1)

$P = \mu R_B - W\sin\theta$

$= \mu W\cos\theta - W\sin\theta$

$= W(\mu\cos\theta - \sin\theta)$ — (2)

from (1) and (2)

$\cancel{W(\mu_A\cos\theta - \sin\theta) = W(\mu_B\cos\theta - \sin\theta)}$

$\mu_A\cos\theta \cancel{= \mu_B\cos\theta}$

$\cancel{W}(\sin\theta - \mu_A\cos\theta) = (\mu_B\cos\theta - \sin\theta)\cancel{W}$

$2\sin\theta = (\mu_B + \mu_A)\cos\theta$

$\tan\theta = \frac{1}{2}\left(\frac{1}{2} + \frac{1}{3}\right)$

$\tan\theta = \frac{1}{2}\left(\frac{5}{6}\right)$

$\boxed{\theta = \tan^{-1}(5/12)}$

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12
Soln

Given data

$d_o = 60\,mm$ $r_o = 30\,mm = 300\,mm = 0.03\,m$

$d_i = 40\,mm$ $r_i = 20\,mm = \cancel{200\,mm} = 0.2\,m$

$\mu = 0.2$ $\tan\phi = 0.2$ $\phi = 11.309$

$W = 20\,KN.$

Required to find
(i) torque required to raise the load
(ii) torque required to lower the load
(iii) Efficiency of screw jack.

W.K.T

$$\tan\theta = \frac{L}{2\pi r}$$

$L =$ lead $= p$ for single thread

$\cancel{r = \text{mean radius}} = 2(OR - IR)$

$= 2(0.3 - 0.2)$

$= \cancel{0.1\,m} = 0.02\,m$

$r =$ mean radius $= \dfrac{0.3 + 0.2}{2}$

$= 0.025\,m$

$$\tan\theta = \frac{L}{2\pi r} = \frac{0.02}{2 \times \pi \times 0.025} = 0.127$$

$$\theta = \cancel{32}45 = 7.256$$

We also know that

$T_{(raise)} = W r (\tan(\phi + \theta))$

$= 20 \times 10^3 \times 0.025 \tan(7.256 + 11.309)$

$= 167.9\,N.m$

$T_{lower} = 20 \times 10^3 \times 0.025 \tan(11.309 - 7.256)$

$= 35.4\,N.m$

$$\eta = \frac{\tan(7.256)}{\tan(11.309 + 7.256)} = 37.9\,\%$$

18 Soln

$d_r = 15\,cm = 0.15\,m$ $A = 0.017\,m^2$

$P = 3\times10^6\,N/m^2$

$D_o = 60\,mm$ load $= P\times A$

$= \cancel{53}\,N,\ ?\ 51\times10^3$

$p = 6\,mm$

$\mu = 0.2$ $\tan\phi = 0.2$ $\phi = 11.309$

Required to find torque exerted on the handle

W.K.T $T = Wr\tan(\phi+\theta)$

$$\tan\theta = \frac{L}{2\pi r}$$

$p = 0.006\,m$

$ID = OD - P/2$

$= 60 - 6/2$

$= 57\,mm$

$R_o = 30\,mm = 0.030\,m$

$R_i = 28.5\,mm = 0.0285\,m$

$r_{mean} = 0.029$

$$\tan\theta = \frac{0.006}{2\times\pi\times0.029}$$

$\theta = 1.669°$

torque required $T = 51\times10^3\times0.029\times\tan(11.309+1.669)$

$$\boxed{T = 346.29\ N.m}$$

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11

14 soln

Given data

$p = 5 mm$

$r_{mean} = 60 mm$

$\mu = 0.10$ $\qquad \phi = 5.710°$

$T = 100 N.m$

Required to find $W = ?$

$$W = \frac{T}{r \tan(\phi + \theta)}$$

$$\theta = \tan^{-1}\left(\frac{2 \times 5}{2 \times \pi \times 60}\right)$$

$= 1.519°$

$$W = \frac{100}{0.06 \tan(5.710 + 1.519)}$$

$= 10.13 KN$

17 soln

Given

$r_{mean} = 50 mm$

$p = 16 mm$

$\mu = 0.2$ $\qquad \phi = 11.309$

$W = 1 \times 10^3 N$

Required to find $T_{raise}, T_{lower}, \eta$:

$\tan \theta = \tan^{-1}\left(\frac{16}{2 \times \pi \times 50}\right)$

$= 3°$

$\eta = \frac{\tan 3}{\tan(14.309)}$

$= 20.1\%$

$T_{raise} = 10^3 \times 0.05 \times \tan(11.309 + 3)$

$= 12.755 N.m$

$T_{lower} = 10^3 \times 0.05 \times \tan(11.309 - 3)$

$= 7.3 N.m$

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10
Soln

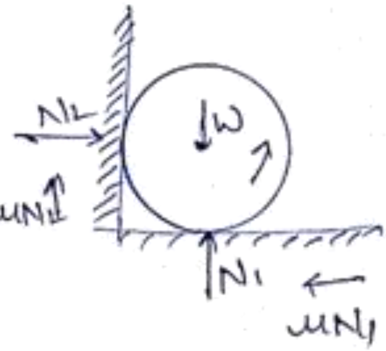
$N_1 + \mu N_2 = W$

$N_2 = \mu N_1$

$N_1 + \mu^2 N_1 = W$

$\Rightarrow N_1 = \frac{W}{1+\mu^2}$

$N_2 = \frac{\mu W}{1+\mu^2}$

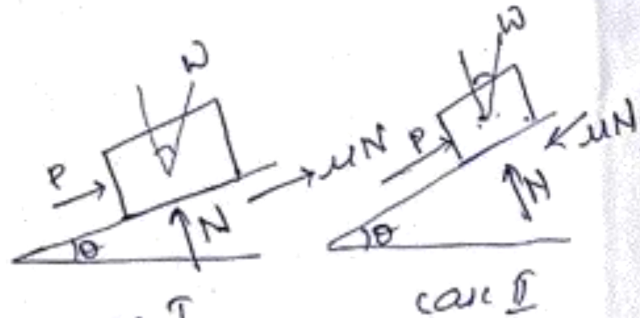


$M = (\mu N_1) r + \mu N_2 (r)$

$= \mu r (N_1 + N_2)$

$= \frac{\mu W r}{1+\mu^2}(1+\mu)$

11
Soln



case I :

$N = W\cos\theta.$

$P + \mu N = W\sin\theta$

$P = W(\sin\theta - \mu\cos\theta)$

case II

$N = W\cos\theta$

$P = W\sin\theta + \mu W\cos\theta$

$= W(\sin\theta + \mu\cos\theta)$

from given condition

$\frac{\cancel{W}(\sin\theta + \mu\cos\theta)}{\cancel{W}(\sin\theta - \mu\cos\theta)} = \frac{W}{W\cos\theta}$

$\sin\theta\cos\theta + \mu\cos^2\theta = \sin\theta - \mu\cos\theta$

$\mu\cos^2\theta + \mu\cos\theta = \sin\theta - \sin\theta\cos\theta$

$\mu\cos\theta(1+\cos\theta) = \sin\theta(1-\cos\theta)$

$\mu = \frac{\sin\theta}{\cos\theta}\left[\frac{1-\cos\theta}{1+\cos\theta}\right] = \tan\theta \tan^2\frac{\theta}{2}$

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Equillibrium of B.

$\Sigma F_y = 0$

$R_3 + 0.2 R_2 \sin 15° = R_2 \cos 15°$

$\boxed{R_3 = 1111.1 \text{ N.}}$

$\Sigma F_x = 0$

$P = 0.2 R_3 + 0.2 R_2 \cos 15 + R_2 \sin 15$

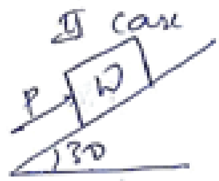
$= 771.5$ kg

$= 7.6$ KN

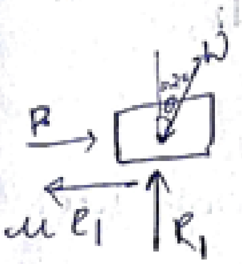
21) Soln

From Given data

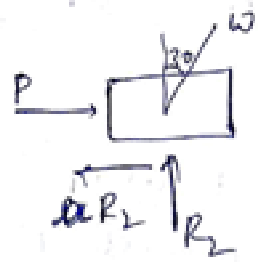
I case	II case
$\theta = 20°$	$\theta = 30$
$P = 500$ N	$P = 650$ N



take case I



take case II



$R_1 = W \cos 20$

$P = W \sin\theta + \mu R_1$

$= W \sin\theta + \mu W \cos 20$

~~$= W(\sin 20 + \mu \cos 20)$~~

$\frac{P - W \sin 20}{W \cos 20} = \mu$

$R_2 = W \cos 30$

$P = W \sin 30 + \mu R_2$

$= W \sin 30 + \mu W \cos 30$

$\frac{P - W \sin 30}{W \cos 30} = \mu$

$\frac{500 - W \sin 20}{\cancel{W} \cos 20} = \frac{650 - W \sin 30}{\cancel{W} \cos 30}$

$(500 - W \sin 20) \cos 30 = (650 - W \sin 30) \cos 20$

$(500 - W \sin 20)\, 0.921 = (650 - W \sin 30)$

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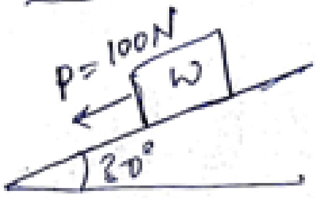
$$W(\sin 30 - 0.921 \sin 20) = 650 - 460.8$$
$$= 189.2$$

$$\boxed{\therefore W = 1022.7\ N}$$

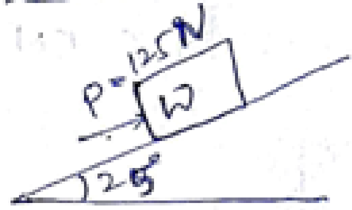
20
Soln

From given data

Case I :-

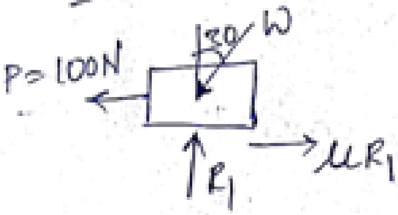


Case II :-

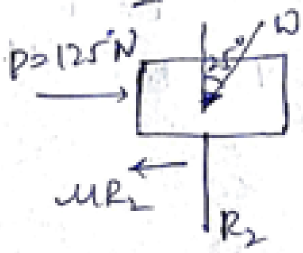


Required to find weigh of body and μ.

I case



II case



$\Sigma F_x = 0$

$P + W \sin 30 = \mu R_1$

~~$P = \mu W \cos 30 - W \sin 30$~~

$$\frac{100 + W \sin 30}{W \cos 30} = \mu \quad \text{——} (1)$$

$P = W \sin 25 + \mu W \cos 25$

$$\frac{125 - W \sin 25}{W \cos 25} = \mu \quad \text{——} (2)$$

from (1) and (2)

$$\frac{100 + W \sin 30}{\cancel{W} \cos 30} = \frac{125 - W \sin 25}{\cancel{W} \cos 25}$$

$$1.04(100 + W \sin 30) = 125 - W \sin 25$$

$$W(\sin 30 + \sin 25) = 25$$

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~~Belt Friction~~

① A rope making $1\frac{1}{4}$ turns around a stationary horizontal drum is used to support a heavy weight. If the $\mu = 0.4$ what weight can be supported by exerting a 50 N force at the other end of the rope?

Soln

$$\frac{T_2}{T_1} = e^{\mu\theta}$$

$$T_2 = e^{0.4 \times 2.5 \times \pi} \times T_1$$

$$= 50 \times e^{0.4 \times 2.5 \times \pi}$$

$$\boxed{T_2 = 1157 N}$$

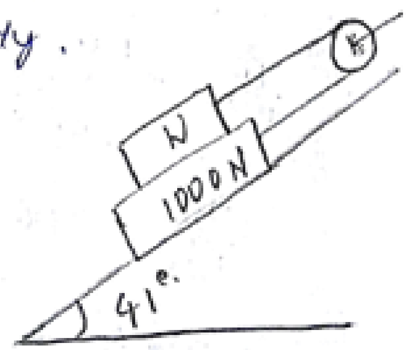
Given $1\frac{1}{4}$ turns

$= 360 + 90$

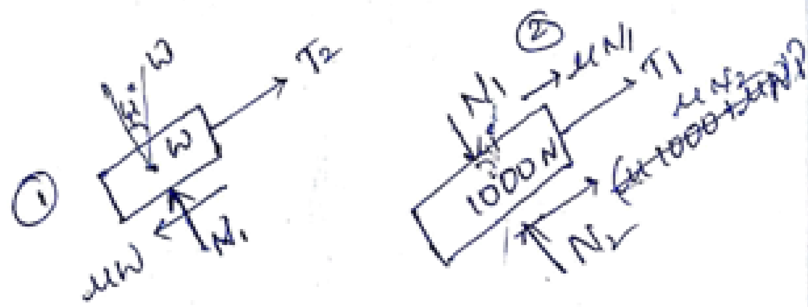
$= 450 \times \frac{\pi}{180}$ radian

$= 2.5 \times \pi$ rad.

2 In figure, the coefficient of friction is 0.30 between the rope and the fixed drum and between all surface in contact. Determine the minimum weight W to prevent down plane motion of the 1000 N. body.



Gi



For block ①

$$T_2 = W\sin\theta + \mu W\cos\theta$$
$$= W(\sin 41 + 0.3\cos 41)$$
$$= W(0.84) \ N$$

For block ②

$$T_1 + \mu(N_1 + N_2) = 1000\sin 41$$

$$T_1 + \mu(W\cos\theta + N_2) = 1000 * \sin 41$$

$$\cancel{W\cos\theta + N_2 = 2000\ N}$$

$$N_2 - N_1 = 1000\cos 41^\circ$$
$$N_2 = 1000\cos 41 + W\cos 41$$
$$= (1000 + W) * 0.8$$

$$T_1 + \mu(W\cos\theta + 1000\cos 41 + W\cos\theta) = 1000\sin 41$$

$$T_1 = 1000\sin 41 - \mu 1000\cos 41 - 2\mu W\cos 41$$
$$= 260.5 - 0.5W \ N$$

$$\frac{T_1}{T_2} = e^{\mu\theta}$$
$$= e^{0.3 \times 180 \times \frac{\pi}{180}}$$
$$\frac{T_1}{T_2} = 2.56$$

$$260.5 - 0.5W = (2.56)(W)(0.84)$$
$$W = \cancel{34}\ 126.\ N$$

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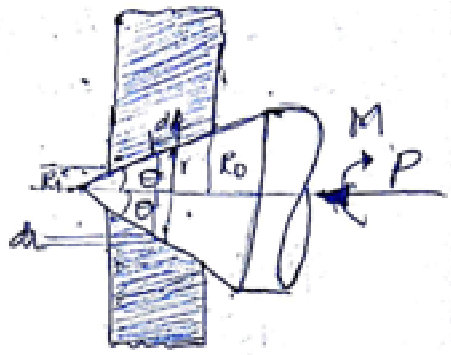
(2) Assuming a uniform pressure and constant f determine the torque M to rotate the pivot bearing shown in figure

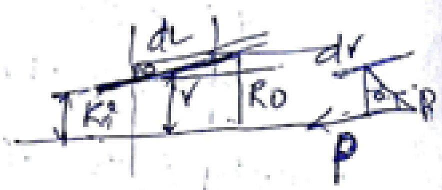


Soln: for uniform pressure

$$\delta P = p \times \text{Area of ring}$$

$$= p \times 2\pi r\, dL$$

$$\delta P = p \times 2\pi r \frac{dr}{\sin\theta}$$



$$\sin\theta = \frac{dr}{dL}$$

$$dL = \frac{dr}{\sin\theta}$$

$$P = \int_{R_i}^{R_o} \delta P \sin\theta$$

$$= \int_{R_i}^{R_o} p \times 2\pi r \frac{dr}{\cancel{\sin\theta}} \times \cancel{\sin\theta}$$

$$= 2\pi p \int_{R_i}^{R_o} r\, dr$$

$$= \cancel{2}\pi p \frac{(R_o^2 - R_i^2)}{\cancel{2}}$$

$$P = \pi p (R_o^2 - R_i^2)$$

$$p = \frac{P}{\pi (R_o^2 - R_i^2)} \quad N/m^2$$

take $\boxed{\mu = f}$

Friction force $F = f \times \delta P$

$$= \mu \times 2\pi p r \frac{dr}{\sin\theta}$$

torque $= F \times r = 2\pi p \mu\, r^2 \frac{dr}{\sin\theta}$

total torque $T = \int_{R_i}^{R_o} 2\pi p \mu \frac{r^2\, dr}{\sin\theta}$

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Disk Friction

(1) An automobile clutch consists of a single annular facing of 1m outside diameter and 0.6m inside diameter. If $f = 0.60$ What axial force on the clutch is required to transmit a torque of 250 N-m?

Soln Given

$d_o = 1\,m$ $\quad r_o = 0.5m$

$d_i = 0.6m$ $\quad r_i = 0.3m$

$f = 0.60$

$M = 250$ N-m

Required to find load/axial force P.

WKT $M = fPR$ for

$$M = fP\frac{2}{3}\frac{r_o^3 - r_i^3}{r_o^2 - r_i^2} \qquad M = f.P\left(\frac{r_o + r_i}{2}\right)$$

(uniform pressure) (uniform wear)

$$250 = 0.60 \times P\left(\frac{2}{3}\right)\left(\frac{0.5^3 - 0.3^3}{0.5^2 - 0.3^2}\right); \quad 250 = 0.60 \times P \times \frac{0.5 + 0.3}{2}$$

$$\frac{250 \times 3}{2} = 0.60 \times P\left(\frac{0.098}{0.16}\right) \quad ; \quad 500 = P \times 0.60 \times 0.8$$

$$\boxed{P = 1020.4\ N} \quad : \quad \boxed{P = 1041.6\ N}$$

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$$= \frac{2\pi\mu p}{\sin\theta} \int_{R_i}^{R_o} r^2 dr$$

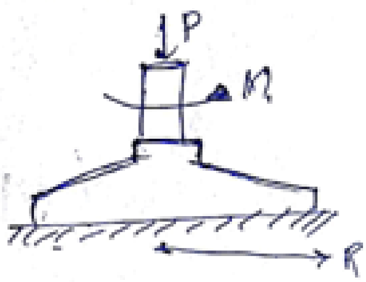
$$= \frac{2\pi\mu p}{\sin\theta} \frac{(R_o^3 - R_i^3)}{3}$$

$$= \frac{2}{3} \frac{\pi\mu}{\sin\theta} \frac{P}{\pi(R_o^2 - R_i^2)} (R_o^3 - R_i^3)$$

$$= \frac{\mu P}{\sin\theta} \cdot \frac{2}{3} \frac{(R_o^3 - R_i^3)}{(R_o^2 - R_i^2)}$$

$$\boxed{M = \frac{2fP}{3\sin\theta} \left(\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}\right)}$$

3. In the flat sanding disk shown in figure Find the Torque M required to rotate the disk at a constant f and at constant speed and the contact pressure varies linearly from $\frac{1}{2}P_o$ at the outside to P_o at the center

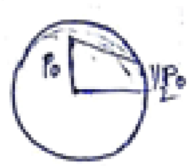


P – i. axial force

p – pressure intensity

Non uniform pressure

and pressure varying linearly



p

$y = mx + c$ —— ①

at $x = 0$ $\quad P_o = c$

at $x = R$ $\quad \frac{1}{2}P_o = mR + P_o$

$$m = \frac{-P_o}{2R}$$

from ① $\quad p = -\frac{P_o x}{2R} + P_o$

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$$\boxed{p = P_0\left(1 - \frac{r}{2R}\right)}$$

p_0 - pressure

P - axial

the total torque equation

$$M = \int 2\pi\mu p r \, r \, dr$$

$$= \int_0^r 2\pi\mu p r^2 dr$$

$$= 2\pi\mu \int_0^R p_0\left(1 - \frac{r}{2R}\right) r^2 dr$$

$$= 2\pi\mu p_0 \int_0^R \left(r^2 - \frac{r^3}{2R}\right) dr$$

$$= \frac{2\cancel{\pi}\mu P}{\cancel{\pi}R^2}\left(\frac{R^3}{3} - \frac{R^{\cancel{4}3}}{2\times\cancel{R}\times 4}\right)$$

$$= \frac{2\mu P}{R^2}\left(\frac{R^3}{3} - \frac{R^3}{8}\right)$$

$$= \frac{2\mu P}{R^2}\left(\frac{8R^3 - 5R^3}{24}\right)$$

$$= \frac{\cancel{2}\mu P}{R^2} \frac{5R^3}{\cancel{24}\,12}$$

$$\boxed{M = \frac{5}{12}\mu P R}$$

Book answer $M = \frac{5}{8}\mu P R$

(Further to be verified)

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Ans

Given that

the pressure varying logarithmically 

$y = mx^2 + c$

at $x = 0$ $\quad p_0 = c$

at $x = R$ $\quad 0 = mR^2 + p_0$ $\qquad$ p – pressure

$m = \frac{-p_0}{R^2}$ $\qquad$ P – axial force

$$p = \frac{-p_0}{R^2} x^2 + p_0$$

$$= p_0\left(1 - \frac{r^2}{R^2}\right)$$

$$= \frac{P}{\pi R^2}\left(1 - \frac{r^2}{R^2}\right)$$

WKT

total Torque developed

$$M = \int_0^R 2\pi\mu r p r \, dr$$

$$= \int_0^R 2\pi\mu r^2 p \, dr$$

$$= 2\pi\mu \int_0^R \frac{P}{\pi R^2}\left(1 - \frac{r^2}{R^2}\right) r^2 \, dr$$

$$= \frac{2\cancel{\pi}\mu P}{\cancel{\pi} R^2} \int_0^R \left(r^2 - \frac{r^4}{R^2}\right) dr$$

$$= \frac{2\mu P}{R^2}\left(\frac{R^3}{3} - \frac{R^5}{5R^2}\right)$$

$$= \frac{2\mu P}{R^2}\left(\frac{R^3}{3} - \frac{R^3}{5}\right)$$

$$= \frac{2\mu P}{R^2} \frac{2R^3}{15}$$

$$\boxed{M = \frac{4}{15}\mu P R}$$

Book Answer $\frac{8}{15}\mu PR$ (To be verified)