UNIT-II

Friction: Types of friction -Limiting friction -Laws of Friction -static and Dynamic Frictions-Motion of Bodies – Wedge & Screw, Screw-jack.

UNIT

2

Principles of Friction

8.1. INTRODUCTION

It has been established since long that all surfaces of the bodies are never perfectly smooth. It has been observed that whenever, even a very smooth surface is viewed under a microscope, it is found to have some roughness and irregularities, which may not be detected by an ordinary touch.

It will be interesting to know that if a block of one substance is placed over the level surface of the same or different material, a certain degree of interlocking of the minutely projecting particles takes place. This does not involve any force, so long as the block does not move or tends to move. But whenever one of the blocks moves or tends to move tangentially with respect to the surface, on which it rests, the

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interlocking property of the projecting particles opposes the motion. This opposing force, which acts in the opposite direction of the movement of the block, is called *force of friction* or simply *friction.* It is of the following two types:

1. Static friction. **2.** Dynamic friction.

8.2. STATIC FRICTION

It is the friction experienced by a body when it is at rest. Or in other words, it is the friction when the body tends to move.

8.3. DYNAMIC FRICTION

It is the friction experienced by a body when it is in motion. It is also called kinetic friction. The dynamic friction is of the following two types :

1. ***Sliding friction.*** It is the friction, experienced by a body when it slides over another body.
2. ***Rolling friction.*** It is the friction, experienced by a body when it rolls over another body.

8.4. LIMITING FRICTION

It has been observed that when a body, lying over another body, is gently pushed, it does not move because of the frictional force, which prevents the motion. It shows that the force of the hand is being exactly balanced by the force of friction, acting in the opposite direction. If we again push the body, a little harder, it is still found to be in equilibrium. It shows that the force of friction has increased itself so as to become equal and opposite to the applied force. Thus the force of friction has a remarkable property of adjusting its magnitude, so as to become exactly equal and opposite to the applied force, which tends to produce motion.

There is, however, a limit beyond which the force of friction cannot increase. If the applied force exceeds this limit, the force of friction cannot balance it and the body begins to move, in the direction of the applied force. This maximum value of frictional force, which comes into play, when a body just begins to slide over the surface of the other body, is known as limiting friction. It may be noted that when the applied force is less than the limiting friction, the body remains at rest, and the friction is called static friction, which may have any value between zero and limiting friction.

8.5. NORMAL REACTION

It has been experienced that whenever a body, lying on a horizontal or an inclined surface, is in equilibrium, its weight acts vertically downwards through its centre of gravity. The surface, in turn, exerts an upward reaction on the body. This reaction, which is taken to act perpendicular to the plane, is called normal reaction and is, generally, denoted by R. It will be interesting to know that the term 'normal reaction' is very important in the field of friction, as the force of friction is directly propor-tional to it.

8.6. ANGLE OF FRICTION

Consider a body of weight W resting on an inclined plane as shown in Fig. 8.1. We know that the body is in equilibrium under the action of the following forces :

1. Weight (W) of the body, acting vertically downwards,
2. Friction force (F) acting upwards along the plane, and
3. Normal reaction (R) acting at right angles to the plane.

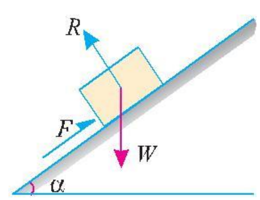


Fig. 8.1. Angle of friction.

Let the angle of inclination (α) be gradually increased, till the body just starts sliding down the plane. This angle of inclined plane, at which a body just begins to slide down the plane, is called the angle of friction. This is also equal to the angle, which the normal reaction makes with the vertical.

8.7. COEFFICIENT OF FRICTION

It is the ratio of limiting friction to the normal reaction, between the two bodies, and is gener-ally denoted by μ.

Mathematically, coefficient of friction,

$$\mu = \frac{F}{R} = \tan \varphi \qquad \text{or} \qquad F = \mu R$$

where φ = Angle of friction,

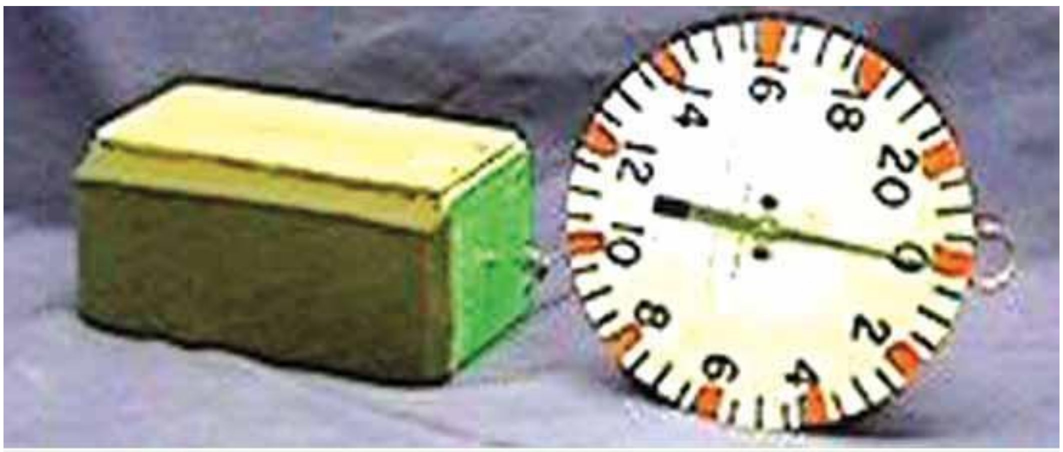
F = Limiting friction, and

R = Normal reaction between the two bodies.

8.8. LAWS OF FRICTION

Prof. Coulomb, after extensive experiments, gave some laws of friction, which may be grouped under the following heads :

1. Laws of static friction, and
2. Laws of kinetic or dynamic friction.



The coefficient of friction of various surfaces, as well as the difference between static and kinetic friction can be illustred by pulling objects with large spring scale.

8.9. LAWS OF STATIC FRICTION

Following are the laws of static friction :

1. The force of friction always acts in a direction, opposite to that in which the body tends to move, if the force of friction would have been absent.
2. The magnitude of the force of friction is exactly equal to the force, which tends to move the body.
3. The magnitude of the limiting friction bears a constant ratio to the normal reaction between the two surfaces. Mathematically :

$$\frac{F}{R} = \text{Constant}$$

where F = Limiting friction, and R = Normal reaction.

4. The force of friction is independent of the area of contact between the two surfaces.
5. The force of friction depends upon the roughness of the surfaces.

8.10. LAWS OF KINETIC OR DYNAMIC FRICTION

Following are the laws of kinetic or dynamic friction :

1. The force of friction always acts in a direction, opposite to that in which the body is moving.
2. The magnitude of kinetic friction bears a constant ratio to the normal reaction between the two surfaces. But this ratio is slightly less than that in case of limiting friction.
3. For moderate speeds, the force of friction remains constant. But it decreases slightly with the increase of speed.

This rock climber uses the static frictional force between her hands and feet and the vertical rock walls.

8.11. EQUILIBRIUM OF A BODY ON A ROUGH HORIZONTAL PLANE

We know that a body, lying on a rough horizontal plane will remain in equilibrium. But whenever a force is applied on it, the body will tend to move in the direction of the force. In such cases, equilibrium of the body is studied first by resolving the forces horizontally and then vertically.

Now the value of the force of friction is obtained from the relation :

$$F = \mu R$$

where μ = Coefficient of friction, and

R = Normal reaction.

Example 8.1. *A body of weight 300 N is lying on a rough horizontal plane having a coefficientof friction as 0.3. Find the magnitude of the force, which can move the body, while acting at an angle of 25° with the horizontal.*

Solution. Given: Weight of the body (W) = 300 N; Coefficient of friction (μ) = 0.3 andangle made by the force with the horizontal (α) = 25°

Let P = Magnitude of the force, which can move the body, and

F = Force of friction.

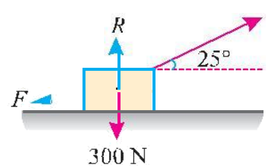


Fig. 8.2.

Resolving the forces horizontally,

$$F = P\cos\alpha = P\cos 25° = P \times 0.9063$$

and now resolving the forces vertically,

$$R = W - P\sin\alpha = 300 - P\sin 25° = 300 - P \times 0.4226$$

We know that the force of friction (F),

$$0.9063\,P = \mu R = 0.3 \times (300 - 0.4226\,P) = 90 - 0.1268\,P$$

or $$90 = 0.9063\,P + 0.1268\,P = 1.0331\,P$$

$\therefore \quad P = \frac{90}{\ldots} = 87.1$ N **Ans.**

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Example 8.2. *A body, resting on a rough horizontal plane, required a pull of 180 N inclined at 30° to the plane just to move it. It was found that a push of 220 N inclined at 30° to the plane just moved the body. Determine the weight of the body and the coefficient of friction.*

Solution. Given: Pull = 180 N; Push = 220 N and angle at which force is inclined withhorizontal plane (α) = 30°

Let W = Weight of the body R = Normal reaction, and μ = Coefficient of friction.

First of all, consider a pull of 180 N acting on the body. We know that in this case, the force of friction (F_1) will act towards left as shown in Fig. 8.3. (*a*).

Resolving the forces horizontally,

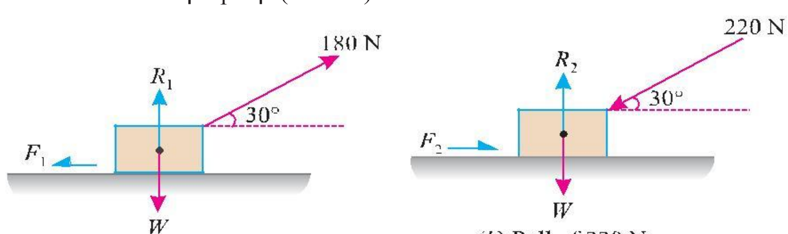
$$F_1 = 180 \cos 30° = 180 \times 0.866 = 155.9$$

Nand now resolving the forces vertically,

$$R_1 = W - 180 \sin 30° = W - 180 \times 0.5 = W - 90$$

NWe know that the force of friction (F_1),

$$155.9 = \mu R_1 = \mu (W - 90) \quad ...(i)$$



(*a*) Pull of 180 N (*b*) Pull of 220 N

Fig. 8.3.

Now consider a push of 220 N acting on the body. We know that in this case, the force of friction (F_2) will act towards right as shown in Fig. 8.3 (*b*).

Resolving the forces horizontally,

$$F_2 = 220 \cos 30° = 220 \times 0.866 = 190.5 \text{ N}$$

and now resolving the forces horizontally,

$$R_2 = W + 220 \sin 30° = W + 220 \times 0.5 = W + 110 \text{ N}$$

We know that the force of friction (F_2),

$$190.5 = \mu . R_2 = \mu (W + 110) \quad ...(ii)$$

Dividing equation (*i*) by (*ii*)

$$\frac{155.9}{190.5} = \frac{\mu (W - 90)}{\mu (W + 110)} = \frac{W - 90}{W + 110}$$

$$155.9\ W + 17\ 149 = 190.5\ W - 17\ 145$$

$$34.6\ W = 34\ 294$$

or $$W = \frac{34\ 294}{34.6} = 991.2 \text{ N} \quad \textbf{Ans.}$$

Now substituting the value of W in equation (*i*),

$$155.9 = \mu (991.2 - 90) = 901.2\ \mu$$

$\therefore$ $$\mu = \frac{155.9}{901.2} = 0.173 \quad \textbf{Ans.}$$

Example 8.3. *Two blocks A and B of weights 1 kN and 2 kN respectively are in equilibriumposition as shown in Fig. 8.4.*

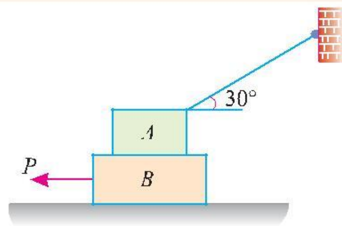
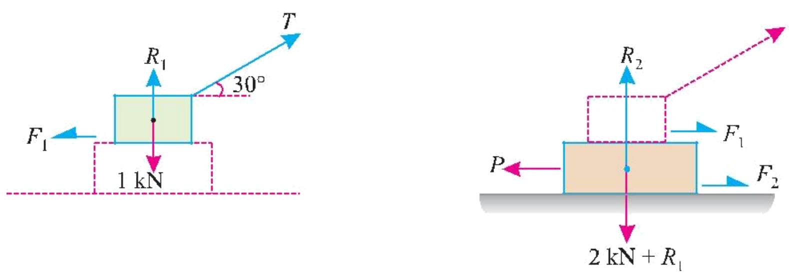


Fig. 8.4.

If the coefficient of friction between the two blocks as well as the block B and the floor is 0.3, find the force (P) required to move the block B.

Solution. Given: Weight of block $A (W_A)$ = 1 kN; Weight of block $B (W_B)$ = 2 kN andcoefficient of friction (μ) = 0.3.



(*a*) Block *A* (*b*) Block *B*

Fig. 8.5.

The forces acting on the two blocks *A* and *B* are shown in Fig. 8.5 (*a*) and (*b*) respectively. First of all, consider the forms acitng in the block *A*.

Resolving the forces vertically,

$$R_1 + T \sin 30° = 1 \text{ kN}$$

or $T \sin 30° = 1 - R_1$ ...(*i*) and now resolving the forces horizontally,

$$T \cos 30° = F_1 = \mu R_1 = 0.3\, R_1 \quad ...(ii)$$

Dividing equation (*i*) by (*ii*)

$$\frac{T \sin 30°}{T \cos 30°} = \frac{1 - R_1}{0.3\, R_1} \quad \text{or} \quad \tan 30° = \frac{1 - R_1}{0.3\, R_1}$$

$$\therefore \quad 0.5774 = \frac{1 - R_1}{0.3\, R_1} \quad \text{or} \quad 0.5774 \times 0.3\, R_1 = 1 - R_1$$

$$\text{or} \quad 0.173\, R_1 = 1 - R_1 \quad \text{or} \quad 1.173\, R_1 = 1$$

$$\text{or} \quad R_1 = \frac{1}{1.173} = 0.85 \text{ kN}$$

and $F_1 = \mu . R_1 = 0.3 \times 0.85 = 0.255$ kN ...(*iii*)

Now consider the block *B*. A little consideration will show that the downward force of the block *A* (equal to R_1) will also act alongwith the weight of the block *B*.

Resolving the forces vertically,

$$R_2 = 2 + R_1 = 2 + 0.85 = 2.85 \text{ kN}$$

$$F_2 = \mu R_2 = 0.3 \times 2.85 = 0.855 \text{ kN} \qquad ...(iv)$$

and now resolving the forces horizontally,

$$P = F_1 + F_2 = 0.255 + 0.855 = 1.11 \text{ kN} \quad \textbf{Ans.}$$

Example 8.4. *What is the maximum load (W) which a force P equal to 5 kN will hold up, ifthe coefficient of friction at C is 0.2 in the arrangement shown in Fig. 8.6. Neglect other friction and weight of the member.*

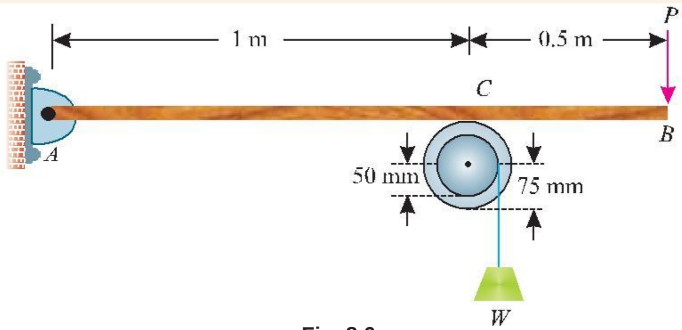


Fig. 8.6.

If W = 3 kN and P = 4.5 kN, what are the normal and tangential forces transmitted at C ?

Solution. Given: Force (P) = 5 kN and coefficient of friction at C (μ) = 0.2

Maximum load W

Let R = Normal reaction of the pulley on the beam at C.

First of all, consider the equilibrium of the beam AB. Taking moments about the hinge A and equating the same,

$$R \times 1 = 5 \times 1.5 = 7.5 \qquad \text{or} \qquad R = 7.5 \text{ kN}$$

Now consider the equilibrium of the pulley. It is subjected to a normal reaction of 7.5 kN (as calculated above). The load (W) tends to rotate it. A little consideration will show that the rotation of the pulley is prevented by the frictional force between the pulley and beam at C. We know that maximum force of friction at C

$$= \mu . R = 0.2 \times 7.5 = 1.5 \text{ kN}$$

Now taking moments about the centre of the pulley and equating the same,

$$W \times 50 = 1.5 \times 75 = 112.5$$

or $$W = \frac{112.5}{50} = 2.25 \text{ kN} \quad \textbf{Ans.}$$

Normal and tangential forces transmitted at C

Now consider a weight W equal to 3 kN suspended from the pulley and a force P equal to 4.5 kN applied at B.

Let R_1 = Normal force or normal reaction at C, and

F_1 = Tangential force at C.

Again consider equilibrium of the beam. Taking moments about the hinge A and equating the same,

$$R_1 \times 1 = 4.5 \times 1.5 = 6.75 \text{ or}$$

$R_1 = 6.75$ kN **Ans.**

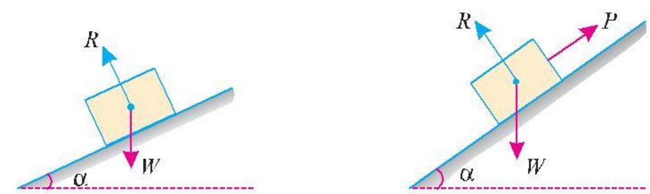
We know that the tangential force at *C* will be the frictional force between the pulley and beam. Again taking moments about the centre of the pulley and equating the same,

$$F_1 \times 75 = W \times 50 = 3 \times 50 = 150$$

or $$F_1 = \frac{150}{75} = 2 \text{ kN} \quad \textbf{Ans.}$$

8.12. EQUILIBRIUM OF A BODY ON A ROUGH INCLINED PLANE

Consider a body, of weight *W*, lying on a rough plane inclined at an angle α with the horizon-tal as shown in Fig. 8.7 (*a*) and (*b*).



(*a*) Angle of inclination less than the angle of friction

(*b*) Angle of inclination more than the angle of friction

Fig. 8.7.

A little consideration will show, that if the inclination of the plane, with the horizontal, is less the angle of friction, the body will be automatically in equilibrium as shown in Fig. 8.7 (*a*). If in this condition, the body is required to be moved upwards or downwards, a corresponding force is required, for the same. But, if the inclination of the plane is more than the angle of friction, the body will move down. And an upward force (*P*) will be required to resist the body from moving down the plane as shown in Fig. 8.7 (*b*).

Though there are many types of forces, for the movement of the body, yet the following are important from the subject point of view :

1. Force acting along the inclined plane.
2. Force acting horizontally.
3. Force acting at some angle with the inclined plane.

Note. In all the above mentioned three types of forces, we shall discuss the magnitude offorce, which will keep the body in equilibrium, when it is at the point of sliding downwards or upwards.

8.13. EQUILIBRIUM OF A BODY ON A ROUGH INCLINED PLANE SUBJECTED TO A FORCE ACTING ALONG THE INCLINED PLANE

Consider a body lying on a rough inclined plane subjected force acting along the inclined plane, which keeps it in equilibrium as shown in Fig. 8.8. (*a*) and (*b*).

Let W = Weight of the body,

α = Angle, which the inclined plane makes with the horizontal,

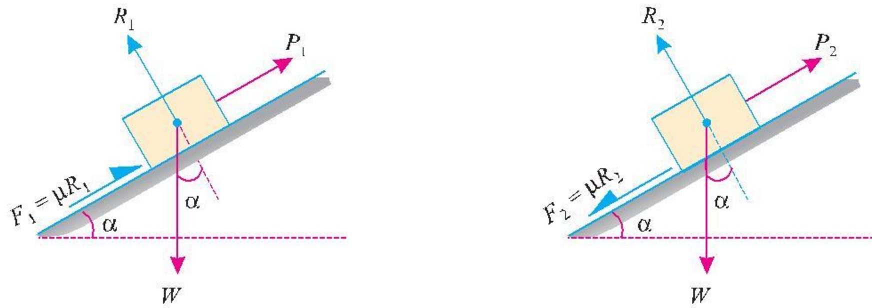
R = Normal reaction,

μ = Coefficient of friction between the body and the inclined plane, and

φ = Angle of friction, such that $\mu = \tan \varphi$.

A little consideration will show that if the force is not there, the body will slide down the plane. Now we shall discuss the following two cases :

1. *Minimum force (P_1) which will keep the body in equilibrium, when it is at the point of slidingdownwards.*



(*a*) Body at the point of sliding downwards

(*b*) Body at the point of sliding upwards

Fig. 8.8.

In this case, the force of friction ($F_1 = \mu.R_1$) will act upwards, as the body is at the point of sliding downwards as shown in Fig. 8.8 (*a*). Now resolving the forces along the plane,

$P_1 = W \sin\alpha - \mu.R_1$ ...(*i*) and now resolving the forces perpendicular to the plane.

$R_1 = W \cos\alpha$ ...(*ii*) Substituting the value of R_1 in equation (*i*),

$P_1 = W \sin\alpha - \mu\, W \cos\alpha = W (\sin\alpha - \mu \cos\alpha)$ and

now substituting the value of $\mu = \tan\varphi$ in the above equation,

$P_1 = W (\sin\alpha - \tan\varphi \cos\alpha$

) Multiplying both sides of this equation by $\cos\varphi$,

$$P_1 \cos\varphi = W (\sin\alpha \cos\varphi - \sin\varphi \cos\alpha) = W \sin(\alpha - \varphi)$$

$$\therefore \qquad P_1 = W \cdot \frac{\sin(\alpha - \varphi)}{\cos\varphi}$$

2. *Maximum force (P_2) which will keep the body in equilibrium, when it is at the point of slidingupwards.*

In this case, the force of friction ($F_2 = \mu.R_2$) will act downwards as the body is at the point of sliding upwards as shown in Fig. 8.8 (*b*). Now resolving the forces along the plane,

$P_2 = W \sin\alpha + \mu.R_2$ ...(*i*) and now resolving the forces perpendicular to the plane,

$R_2 = W \cos\alpha$ ...(*ii*) Substituting the value of R_2 in equation (*i*),

$P_2 = W \sin\alpha + \mu\, W \cos\alpha = W (\sin\alpha + \mu \cos\alpha)$ and

now substituting the value of $\mu = \tan\varphi$ in the above equation,

$P_2 = W (\sin\alpha + \tan\varphi \cos\alpha$

) Multiplying both sides of this equation by $\cos\varphi$,

$$P_2 \cos\varphi = W (\sin\alpha \cos\varphi + \sin\varphi \cos\alpha) = W \sin(\alpha + \varphi)$$

$$\therefore \qquad P_2 = W \cdot \frac{\sin(\alpha + \varphi)}{\cos\varphi}$$

Example 8.5. *A body of weight 500 N is lying on a rough plane inclined at an angle of 25° with the horizontal. It is supported by an effort (P) parallel to the plane as shown in Fig. 8.9.*

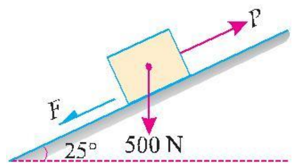


Fig. 8.9.

Determine the minimum and maximum values of P, for which the equilibrium can exist, if the angle of friction is 20°.

Solution. Given: Weight of the body (*W*) = 500 N ; Angle at which plane is inclined (α) = 25° and angle of friction (φ) = 20°.

Minimum value of P

We know that for the minimum value of *P*, the body is at the point of sliding downwards. We also know that when the body is at the point of sliding downwards, then the force

$$P_1 = W \cdot \frac{\sin(\alpha - \varphi)}{\cos\varphi} = 500 \cdot \frac{\sin(25° - 20°)}{\cos 20°} \text{ N}$$

$$= 500 \cdot \frac{\sin 5°}{\cos 20°} = 500 \cdot \frac{0.0872}{0.9397} = 46.4 \text{ N}$$ **Ans.**

Maximum value of P

We know that for the maximum value of *P*, the body is at the point of sliding upwards. We also know that when the body is at the point of sliding upwards, then the force

$$P_2 = W \cdot \frac{\sin(\alpha + \varphi)}{\cos\varphi} = 500 \cdot \frac{\sin(25° + 20°)}{\cos 20°} \text{ N}$$

$$= 500 \cdot \frac{\sin 45°}{\cos 20°} = 500 \cdot \frac{0.7071}{0.9397} = 376.2 \text{ N}$$ **Ans.**

Example 8.6. *An inclined plane as shown in Fig. 8.10. is used to unload slowly a body weighing 400 N from a truck 1.2 m high into the ground.*

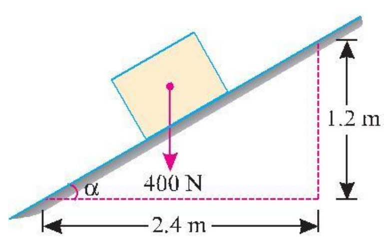


Fig. 8.10.

The coefficient of friction between the underside of the body and the plank is 0.3. State whether it is necessary to push the body down the plane or hold it back from sliding down. What minimum force is required parallel to the plane for this purpose ?

Solution. Given: Weight of the body (W) = 400 N and coefficient of friction (μ) = 0.3.

Whether it is necessary to push the body down the plane or hold it back from sliding down.

We know that

$$\tan \alpha = \frac{1.2}{2.4} = 0.5 \quad \text{or} \quad \alpha = 26.5°$$

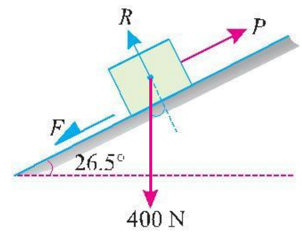


Fig. 8.11.

and normal reaction, $R = W \cos \alpha = 400 \cos 26.5°$ N

$= 400 \times 0.8949 = 357.9$ N

∴ Force of friction,

$$F = \mu R = 0.3 \times 357.9 = 107.3 \text{ N} \quad ...(i)$$

Now resolving the 400 N force along the plane

$= 400 \sin \alpha = 400 \times \sin 26.5°$ N

$= 400 \times 0.4462 = 178.5$ N ...(*ii*)

We know that as the force along the plane (which is responsible for sliding the body) is more than the force of friction, therefore the body will slide down. Or in other words, it is not necessary to push the body down the plane, rather it is necessary to hold it back from sliding down. **Ans.**

Minimum force required parallel to the plane

We know that the minimum force required parallel to the plane to hold the body back,

$$P = 178.5 - 107.3 = 71.2 \text{ N} \quad \textbf{Ans.}$$

Example 8.7. *An effort of 200 N is required just to move a certain body up an inclinedplane of angle 15° the force acting parallel to the plane. If the angle of inclination of the plane is made 20° the effort required, again applied parallel to the plane, is found to be 230 N. Find the weight of the body and the coefficient of friction.*

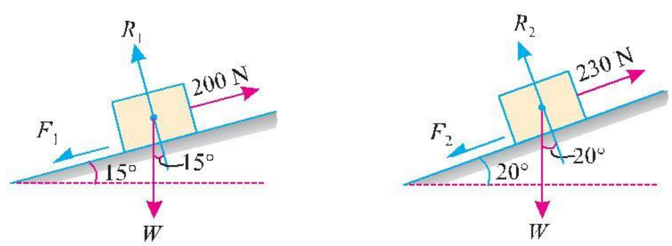
Solution. Given: First case : When effort (P_1) = 200 N, then angle of inclination (α_1) = 15° and second case : When effort (P_2) = 230 N, then angle of inclination (α_2) = 20°.

Let μ = Coefficient of friction,

W = Weight of the body,

R = Normal reaction, and

F = Force of friction.



(*a*) Body lying at 15° (*b*) Body lying at 20°

Fig. 8.12.

First of all, consider the body lying on a plane inclined at an angle of 15° with the horizontal and subjected to an effort of 200 N as shown in Fig. 8.12 (*a*).

Resolving the forces at right angles to the plane,

$$R_1 = W \cos 15° \quad ...(i)$$

and now resolving the forces along the plane,

$$200 = F_1 + W \sin 15° = \mu.R_1 + W \sin 15° \qquad ...(\because F = \mu.R)$$

$$= \mu W \cos 15° + W \sin 15° \qquad ...(\because R_1 = W\cos 15°)$$

$$= W(\mu\cos 15° + \sin 15°) \qquad ...(ii)$$

Now consider the body lying on a plane inclined at an angle of 20° with the horizontal and subjected to an effort of 230 N shown in Fig. 8.12 (*b*).

Resolving the forces at right angles to the plane,

$$R_2 = W\cos 20° \qquad ...(iii)$$

and now resolving the forces along the plane,

$$230 = F_2 + W\sin 20° = \mu R_2 + W \sin 20° \qquad ...(\because F = \mu.R)$$

$$= \mu W\cos 20° + W\sin 20° \qquad ...(\because R_2 = W\cos 20°)$$

$$= W(\mu\cos 20° + \sin 20°) \qquad ...(iv)$$

Coefficient of friction

Dividing equation (*iv*) by (*ii*),

$$\frac{230}{200} = \frac{W(\mu\cos 20° + \sin 20°)}{W(\mu\cos 15° + \sin 20°)}$$

$$230\,\mu\cos 15° + 230\sin 15° = 200\,\mu\cos 20° + 200\sin 20°$$

$$230\,\mu\cos 15° - 200\,\mu\cos 20° = 200\sin 20° - 230\sin 15°$$

$$\mu(230\cos 15° - 200\cos 20°) = 200\sin 20° - 230\sin 15°$$

$$\therefore \mu = \frac{200\sin 20° - 230\sin 15°}{230\cos 15° - 200\cos 20°} = \frac{(200 \cdot 0.3420) - (230 \cdot 0.2588)}{(230 \cdot 0.9659) - (200 \cdot 0.9397)} = 0.259 \qquad \textbf{Ans.}$$

Weight of the body

Substituting the value of μ in equation (*ii*),

$$200 = W(0.259\cos 15° + \sin 15°)$$

$$= W(0.259 \times 0.9659 + 0.2588) = 0.509\,W$$

$$\therefore \quad W = \frac{200}{0.509} = 392.9 \text{ N} \qquad \textbf{Ans.}$$

EXERCISE 8.1

1. Find the horizontal force required to drag a body of weight 100 N along a horizontal plane. If the plane, when gradually raised up to 15°, the body will begin to slide. [**Ans.** 26.79 N]

   Hint. $\varphi = 15°$ or $\mu = \tan\varphi = \tan 15° = 0.2679$

2. A body of weight 50 N is hauled along a rough horizontal plane by a pull of 18 N acting at an angle of 14° with the horizontal. Find the coefficient of friction. [**Ans.** 0.383]

3. A man is walking over a dome of 10 m radius. How far he can descend from the top of the dome without slipping ? Take coefficient of friction between the surface of the dome and shoes of the man as 0.6. [**Ans.** 1.413 m]

Hint. Let the man start from the top of the dome (*A*) and reach a point (*B*), beyond whichhe can not walk. Now let (*W*) be the weight of the man and (θ) and angle subtended by the arc *AB* at the centre of the dome.

∴ Normal reaction at $B = W\cos\theta$

and force of friction, $F = W\sin\theta$

∴ $W\sin\theta = \mu.W\cos\theta$

$\tan\theta = \mu = 0.6$ or $\theta = 31°$

4. A force of 250 N pulls a body of weight 500 N up an inclined plane, the force being applied parallel to the plane. If the inclination of the plane to the horizontal is 15°, find the coefficient of friction. [**Ans.** 0.25]

8.14. EQUILIBRIUM OF A BODY ON A ROUGH INCLINED PLANE SUBJECTED TO A FORCE ACTING HORIZONTALLY

Consider a body lying on a rough inclined plane subjected to a force acting horizontally, which keeps it in equilibrium as shown in Fig. 8.13. (*a*) and (*b*).

W = Weight of the body,

α = Angle, which the inclined plane makes with the horizontal,

R = Normal reaction,

μ = Coefficient of friction between the body and the inclined plane, and

φ = Angle of friction, such that $\mu = \tan\varphi$.

A little consideration will show that if the force is not there, the body will slide down on the plane. Now we shall discuss the following two cases :

1. *Minimum force* (P_1) *which will keep the body in equilibrium, when it is at the point of slidingdownwards.*

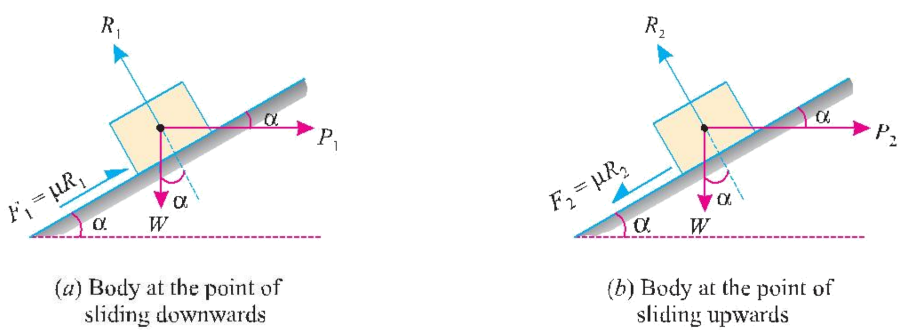


Fig. 8.13.

In this case, the force of friction ($F_1 = \mu.R_1$) will act upwards, as the body is at the point of sliding downwards as shown in Fig. 8.13. (*a*). Now resolving the forces along the plane,

$P_1\cos\alpha = W\sin\alpha - \mu R_1$ ...(*i*)and now resolving the forces perpendicular to the plane,

$R_1 = W\cos\alpha + P_1\sin\alpha$ ...(*ii*)Substituting this value of R_1 in equation (*i*),

$P_1\cos\alpha = W\sin\alpha - \mu(W\cos\alpha + P_1\sin\alpha) = W\sin\alpha - \mu W\cos\alpha - \mu P_1\sin\alpha$

$P_1\cos\alpha + \mu P_1\sin\alpha = W\sin\alpha - \mu W\cos\alpha$

$P_1(\cos\alpha + \mu \sin\alpha) = W(\sin\alpha - \mu\cos\alpha)$

$$\therefore \quad P_1 = W \cdot \frac{(\sin\alpha - \mu\cos\alpha)}{(\cos\alpha + \mu \sin\alpha)}$$

Now substituting the value of $\mu = \tan\varphi$ in the above equation,

$$P_1 = W \cdot \frac{(\sin\alpha - \tan\varphi\cos\alpha)}{(\cos\alpha + \tan\varphi \sin\alpha)}$$

Multiplying the numerator and denominator by $\cos\varphi$,

$$P_1 = W \cdot \frac{\sin\alpha\cos\varphi - \sin\varphi\cos\alpha}{\cos\alpha\cos\varphi + \sin\alpha\sin\varphi} = W \cdot \frac{\sin(\alpha - \varphi)}{\cos(\alpha - \varphi)}$$

$= W \tan(\alpha - \varphi)$ ...(when $\alpha > \varphi$)

$= W \tan(\varphi - \alpha)$ ...(when $\varphi > \alpha$)

2. *Maximum force* (P_2) *which will keep the body in equilibrium, when it is at the point of slidingupwards*

In this case, the force of friction ($F_2 = \mu R_2$) will act downwards, as the body is at the point of sliding upwards as shown in Fig.8.12. (*b*). Now resolving the forces along the plane,

$$P_2\cos\alpha = W \sin\alpha + \mu R_2 \qquad ...(iii)$$

and now resolving the forces perpendicular to the plane,

$$R_2 = W\cos\alpha + P_2\sin\alpha \qquad ...(iv)$$

Substituting this value of R_2 in the equation (*iii*),

$P_2\cos\alpha = W\sin\alpha + \mu(W\cos\alpha + P_2\sin\alpha)$

$= W\sin\alpha + \mu W\cos\alpha + \mu P_2\sin\alpha$

$P_2\cos\alpha - \mu P_2\sin\alpha = W\sin\alpha + \mu W\cos\alpha$

$P_2(\cos\alpha - \mu\sin\alpha) = W(\sin\alpha + \mu\cos\alpha)$

$$\therefore \quad P_2 = W \cdot \frac{(\sin\alpha + \mu\cos\alpha)}{(\cos\alpha - \mu\sin\alpha)}$$

Now substituting the value of $\mu = \tan\varphi$ in the above equation,

$$P_2 = W \cdot \frac{\sin\alpha + \tan\varphi\cos\alpha}{\cos\alpha - \tan\varphi\sin\alpha}$$

Multiplying the numerator and denominator by $\cos\varphi$,

$$P_2 = W \cdot \frac{\sin\alpha\cos\varphi + \sin\varphi\cos\alpha}{\cos\alpha\cos\varphi - \sin\varphi\sin\alpha} = W \cdot \frac{\sin(\alpha + \varphi)}{\cos(\alpha + \varphi)}$$

$= W \tan(\alpha + \varphi)$

Example 8.8. *An object of weight 100 N is kept in position on a plane inclined 30° to thehorizontal by a horizontally applied force* (*F*). *If the coefficient of friction of the surface of the inclined plane is 0.25, determine the minimum magnitude of the force* (*F*).

Solution. Given: Weight of the object (W) = 100 N; Angle at which plane is inclined(α) = 30° and coefficient of friction (μ) = 0.25 = $\tan\varphi$ or φ = 14°.

We know that the minimum magnitude of the force to kept the object in position (when it is at the point of sliding downwards),

$F = W\tan(\alpha - \varphi) = 100\tan(30° - 14°) = 100\tan 16° =$

$100 \times 0.2867 = 28.67$ N **Ans.**

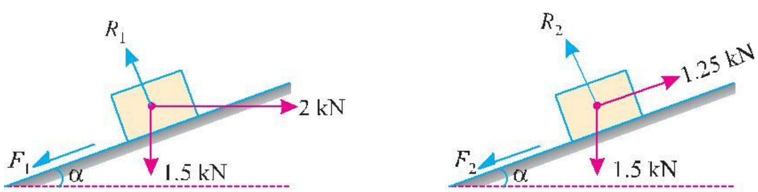
Example 8.9. *A load of 1.5 kN, resting on an inclined rough plane, can be moved up theplane by a force of 2 kN applied horizontally or by a force 1.25 kN applied parallel to the plane. Find the inclination of the plane and the coefficient of friction.*

Solution. Given: Load (W) = 1.5 kN; Horizontal effort (P_1) = 2 kN and effort parallel tothe inclined plane (P_2) = 1.25 kN.

Inclination of the plane

Let α = Inclination of the plane, and

φ = Angle of friction.



(*a*) Horizontal force (*b*) Force parallel to the plane

Fig. 8.14.

First of all, consider the load of 1.5 kN subjected to a horizontal force of 2 kN as shown in Fig. 8.14 (*a*). We know that when the force is applied horizontally, then the magnitude of the force, which can move the load up the plane.

$$P = W \tan(\alpha + \varphi)$$

or $$2 = 1.5 \tan(\alpha + \varphi)$$

$$\therefore \quad \tan(\alpha + \varphi) = \frac{2}{1.5} = 1.333 \quad \text{or} \quad (\alpha + \varphi) = 53.1°$$

Now consider the load of 1.5 kN subjected to a force of 1.25 kN along the plane as shown in Fig. 8.14 (*b*). We Know that when the force is applied parallel to the plane, then the magnitude of the force, which can move the load up the plane,

$$P = W \cdot \frac{\sin(\alpha + \varphi)}{\cos\varphi}$$

or $$1.25 = 1.5 \cdot \frac{\sin 53.1°}{\cos\varphi} = 1.5 \cdot \frac{0.8}{\cos\varphi} = \frac{1.2}{\cos\varphi}$$

$$\therefore \quad \cos\varphi = \frac{1.2}{1.25} = 0.96 \quad \text{or} \quad \varphi = 16.3°$$

and $$\alpha = 53.1° - 16.3° = 36.8° \quad \textbf{Ans.}$$

Coefficient of friction

We know that the coefficient of friction,

$$\mu = \tan\varphi = \tan 16.3° = 0.292 \quad \textbf{Ans.}$$

Contents

Example 8.10. *Two blocks A and B, connected by a horizontal rod and frictionless hingesare supported on two rough planes as shown in Fig. 8.15.*

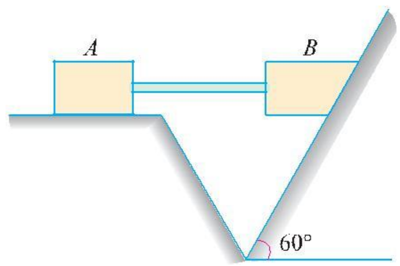


Fig. 8.15.

The coefficients of friction are 0.3 between block A and the horizontal surface, and 0.4 between block B and the inclined surface. If the block B weighs 100 N, what is the smallest weight of block A, that will hold the system in equilibrium?

Solution. Given: Coefficient of friction between block A and horizontal surface (μ_A) = 0.3;Coefficient of friction between block *B* and inclined surface (μ_B) = 0.4 and weight of block *B* (W_B) = 100 N.

Let W_A = Smallest weight of block *A*.

We know that force of friction of block *A*, which is acting horizontally on the block *B*,

$$P = \mu_A W_A = 0.3 \times W_A = 0.3$$

W_A and angle of friction of block *B*

$$\tan\phi = \mu_B = 0.4 \quad \text{or} \quad \phi = 21.8°$$

We also know that the smallest force, which will hold the system in equilibrium (or will prevent the block *B* from sliding downwards),

$$P = W_B \tan(\alpha - \phi) = 100 \tan(60° - 21.8°)$$

or $$0.3\, W_A = 100 \tan 38.2° = 100 \times 0.7869 = 78.69$$

$\therefore$ $$W_A = \frac{78.69}{0.3} = 262.3 \text{ N} \quad \textbf{Ans.}$$

Alternative method

Consider the equilibrium of block *B*. We know that it is in equilibrium under the action of the following four forces as shown in Fig. 8.16.

1. Its own weight 100 N
2. Normal reaction *R*,
3. Force of friction of block *A* (acting horizontally on *B*),

$$F_A = \mu_A \times W_A = 0.3 \times W_A = 0.3\, W_A$$

4. Force of friction between the block *B* and inclined surface,

$$F = \mu_B \times R = 0.4\, R$$

Resolving the forces along the plane,

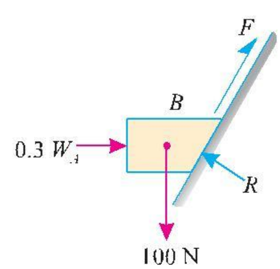


Fig. 8.16.

$$F = 100 \cos 30° - 0.3\, W_A \cos 60°$$

$$= 100 \times 0.866 - 0.3\, W_A \times 0.5$$

or $$0.4\, R = 86.6 - 0.15\, W_A \quad \ldots(i)$$

and now resolving the forces at right angles to the plane, $R =$

$$0.3\ W_A \cos 30° + 100 \sin 30°$$

$$= 0.3\ W_A \times 0.866 + 100 \times 0.5$$

$$= 0.26\ W_A + 50 \quad ...(ii)$$ Substituting the value of R in equation (i)

$$0.4\ (0.26\ W_A + 50) = 86.6 - 0.15\ W_A$$

$$0.104\ W_A + 20 = 86.6 - 0.15\ W_A$$

$$0.254\ W_A = 86.6 - 20 = 66.6$$

$$\therefore \quad W_A = \frac{66.6}{0.254} = 262.2 \text{ N}$$ **Ans.**

8.15. EQUILIBRIUM OF A BODY ON A ROUGH INCLINED PLANE SUBJECTED TO A FORCE ACTING AT SOME ANGLE WITH THE INCLINED PLANE

Consider a body lying on a rough inclined plane subjected to a force acting at some angle with the inclined plane, which keeps it in equilibrium as shown in Fig. 8.17 (a) and (b).

Let W = Weight of the body,

α = Angle which the inclined plane makes with the horizontal,

θ = Angle which the force makes with the inclined surface,

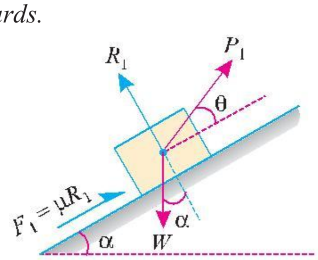
R = Normal reaction,

μ = Coefficient of friction between the body and the inclined plane, and

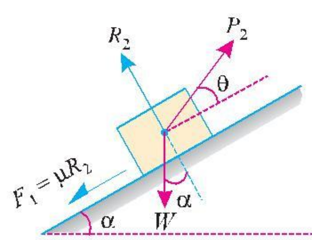
φ = Angle of friction, such that $\mu = \tan \varphi$.

A little consideration will show that if the force is not there, the body will slide down the plane. Now we shall discuss the following two cases :

1. *Minimum force* (P_1) *which will keep the body in equilibrium when it is at the point ofsliding downwards.*



(a) Body at the point of sliding downwards



(b) Body at the point of sliding upwards

Fig. 8.17.

In this case, the force of friction ($F_1 = \mu R_1$) will act upwards, as the body is at the point of sliding downwards as shown in Fig. 8.17 (a). Now resolving the forces along the plane,

$$P_1 \cos\theta = W \sin\alpha - \mu R_1 \quad ...(i)$$

and now resolving the forces perpendicular to the plane,

$$R_1 = W \cos\alpha - P_1 \sin\theta \quad ...(ii)$$

Substituting the value of R_1 in equation (*i*),

$$P_1 \cos\theta = W \sin\alpha - \mu(W \cos\alpha - P_1 \sin\theta)$$

$$= W \sin\alpha - \mu W \cos\alpha + \mu P_1 \sin\theta$$

$$P_1 \cos\theta - \mu P_1 \sin\theta = W \sin\alpha - \mu W \cos\alpha$$

$$P_1(\cos\theta - \mu \sin\theta) = W(\sin\alpha - \mu\cos\alpha)$$

$$\therefore \quad P_1 = W \cdot \frac{(\sin\alpha - \mu\cos\alpha)}{(\cos\theta - \mu \sin\theta)}$$

and now substituting the value of $\mu = \tan\varphi$ in the above equation,

$$P_1 = W \cdot \frac{(\sin\alpha - \tan\varphi \cos\alpha)}{(\cos\theta - \tan\varphi \sin\theta)}$$

Multiplying the numerator and denominator by $\cos\varphi$,

$$P_1 = W \cdot \frac{(\sin\alpha\cos\varphi - \sin\varphi\cos\alpha)}{(\cos\theta\cos\varphi - \sin\varphi\sin\theta)} = W \cdot \frac{\sin(\alpha - \varphi)}{\cos(\theta + \varphi)}$$

2. *Maximum force* (P_2) *which will keep the body in equilibrium, when it is at the point ofsliding upwards.*

In this case, the force of friction ($F_2 = \mu R_2$) will act downwards as the body is at the point of sliding upwards as shown in Fig. 8.17 (*b*). Now resolving the forces along the plane.

$$P_2 \cos\theta = W \sin\alpha + \mu R_2 \quad \ldots(iii)$$

and now resolving the forces perpendicular to the plane,

$$R_2 = W \cos\alpha - P_2 \sin\theta \quad \ldots(iv)$$

Substituting the value of R_2 in equation (*iii*),

$$P_2 \cos\theta = W \sin\alpha + \mu(W \cos\alpha - P_2 \sin\theta) = W \sin\alpha + \mu W \cos\alpha - \mu P_2 \sin\theta$$

$$P_2 \cos\theta + \mu P_2 \sin\theta = W \sin\alpha + \mu W \cos\alpha$$

$$P_2(\cos\theta + \mu\sin\theta) = W(\sin\alpha + \mu\cos\alpha)$$

$$\therefore \quad P_2 = W \cdot \frac{(\sin\alpha + \mu\cos\alpha)}{(\cos\theta + \mu \sin\theta)}$$

and now substituting the vaue of $\mu = \tan\varphi$ in the above equation,

$$P_2 = W \cdot \frac{(\sin\alpha + \tan\varphi\cos\alpha)}{(\cos\theta + \tan\varphi \sin\theta)}$$

Multiplying the numerator and denominator by $\cos\varphi$,

$$P_2 = W \cdot \frac{(\sin\alpha\cos\varphi + \sin\varphi\cos\alpha)}{(\cos\theta\cos\varphi + \sin\varphi\sin\theta)} = W \cdot \frac{\sin(\alpha + \varphi)}{\cos(\theta - \varphi)}$$

Example 8.11. *Find the force required to move a load of 300 N up a rough plane,the forcebeing applied parallel to the plane. The inclination of the plane is such that when the same load is kept on a perfectly smooth plane inclined at the same angle, a force of 60 N applied at an inclination of 30° to the plane, keeps the same load in equilibrium.*

Assume coefficient of friction between the rough plane and the load to be equal to 0.3.

Solution. Given: Load (W) = 300 N; Force (P_1) = 60 N and angle at which force is inclined (θ) = 30°,

Let α = Angle of inclination of the plane.

First of all, consider the load lying on a smooth plane inclined at an angle (α) with the horizontal and subjected to a force of 60 N acting at an angle 30° with the plane as shown in Fig. 8.18 (*a*).

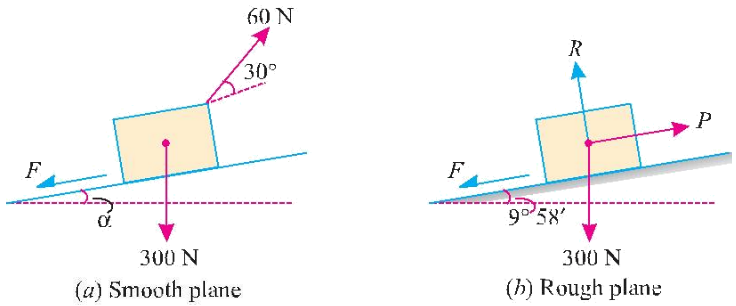


Fig. 8.18.

We know that in this case, because of the smooth plane $\mu = 0$ or $\varphi = 0$. We also know that the force required, when the load is at the point of sliding upwards (P),

$$60 = W \cdot \frac{\sin(\alpha + \varphi)}{\cos(\theta - \varphi)} = 300 \cdot \frac{\sin \alpha}{\cos 30°} = 300 \cdot \frac{\sin \alpha}{0.866} = 346.4 \sin \alpha$$

...($\because \varphi = 0$)

or $\sin \alpha = \dfrac{60}{346.4} = 0.1732$ or $\alpha = 10°$

Now consider the load lying on the rough plane inclined at an angle of 10° with the horizontal as shown in Fig. 8.18. (*b*). We know that in this case, $\mu = 0.3 = \tan \varphi$ or $\varphi = 16.7°$.

We also know that force required to move the load up the plane,

$$P = W \cdot \frac{\sin(\alpha + \varphi)}{\cos \varphi} = 300 \cdot \frac{\sin(10° + 16.7°)}{\cos 16.7°} \text{ N}$$

$$= 300 \cdot \frac{\sin 26.7°}{\cos 16.7°} = 300 \cdot \frac{0.4493}{0.9578} = 140.7 \text{ N} \quad \textbf{Ans.}$$

Alternative method

Ist case

Given: In this case load (P) = 60 N; Angle (θ) = 30° and force of friction $F = 0$ (because of smooth plane). Resolving the forces along the inclined plane,

$$60 \cos 30° = 300 \sin \alpha$$

$$\therefore \quad \sin \alpha = \frac{60 \cos 30°}{300} = \frac{60 \cdot 0.866}{300} = 0.1732 \quad \text{or} \quad \alpha = 10°$$

2nd case

Given: In this case, coefficient of friction (μ) = 0.3 = $\tan \varphi$ or $\varphi = 16.7°$

Let P = Force required to move the load up the plane,

R = Normal reaction, and

F = Force of friction equal to 0.3 R.

Resolving the forces along the plane,

$$P = F + 300 \sin 10° = 0.3\,R + (300 \times 0.1732) = 0.3\,R + 51.96 \text{ N} \quad ...(i)$$

and now resolving the forces at right angles to the plane,

$R = 300 \cos 10° = 300 \times 0.9849 = 295.5$ N ...(*ii*)Substituting the value of R in equation (*i*),

$P = (0.3 \times 295.5) + 51.96 = 140.7$ N **Ans.**

Example 8.12. *The upper half of an inclined having inclination θ with the horizontal issmooth, while the lower half in rough as shown in Fig 8.19.*

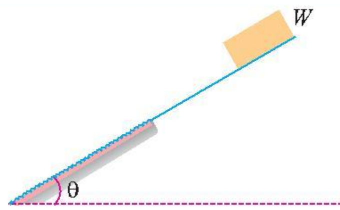


Fig. 8.19.

If a body of weight W slides down from rest at the top, again comes to rest at the bottom of the plane,then determine the value of coefficient of friction for the lower half of the plane.

Solution. Given: Angle of inclination = θ and weight of the body = W.

Let μ = Coefficient of friction for the lower half of the inclined alone.

We know that acceleration on the smooth surface of the plane

$= g \sin \theta$ ...(*i*)

and retardation on the rough surface of the plane

$= - g (\sin \theta - \mu \cos\theta)$ ...(Minus sign due to retardation)

$= g (\mu\cos\theta - \sin \theta)$ ...(*ii*)

Since body starts from rest at the top comes to rest at the bottom of therefore acceleration on the smooth surface is equal to retardation on the rough surface.

∴ $g \sin\theta = g (\mu\cos\theta - \sin\theta)$ or $\sin \theta = \mu\cos\theta - \sin \theta$

or $\mu\cos\theta = 2 \sin \theta$ or $\mu = 2 \tan \theta$ **Ans.**

Example 8.13. *Two loads, W_1 (equal to 1 kN) and W_2 resting on two inclined roughplanes OA and OB are connected by a horizontal link PQ as shown in Fig. 8.20.*

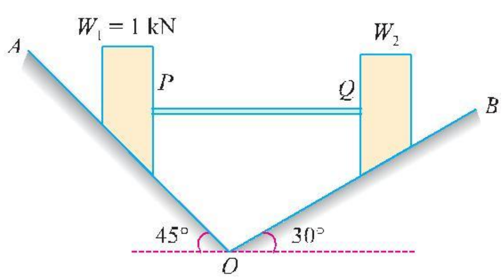


Fig. 8.20.

Find the maximum and minimum values of W_2 for which the equilibrium can exist. Take angle of friction for both the planes as 20°.

Solution. Given: First load (W_1) = 1 kN ; Angle made by inclined plane *OA* with the horizontal (α_1) 45°; Angle made by inclined plane *OB* with the horizontal (α_2) = 30° and Angle of friction for both the planes (φ) = 20°.

Maximum value of W_2

We know that for maximum value of W_2, the load W_2 will be at the point of sliding downwards whereas the load W_1 will be at the point of sliding upwards. We also know that when the load W_1 is at the point of sliding upwards on the plane *OA*, the horizontal thrust in the link *PQ*,

$$P = W_1 \tan(\alpha_1 + \varphi) = 1 \times \tan(45° + 20°) \text{ kN}$$

$$= 1 \tan 65° = 1 \times 2.1445 = 2.1445 \text{ kN} \quad ...(i)$$

and when the load W_2 is at the point of sliding downwards on the plane *OB*, the horizontal thrust in the link *PQ*

$$P = W_2 \tan(\alpha_2 - \varphi) = W_2 \tan(30° - 20°) \text{ kN}$$

$$= W_2 \tan 10° = W_2 \times 0.1763 \text{ kN} \quad ...(ii)$$

Since the values of the horizontal thrusts in the link *PQ*, obtained in both the above equations is the same, therefore equating equations (*i*) and (*ii*),

$$2.1445 = W_2 \times 0.1763$$

$$\therefore \quad W_2 = \frac{2.1445}{0.1763} = 12.16 \text{ kN} \quad \textbf{Ans.}$$

Minimum value of W_2

We know that for maximum value of W_2, the load W_2 will be at the point of sliding upwards whereas the load W_1 will be at the point of sliding downwards. We also know that when the load W_1 is at the point of sliding downwards on the plane *OA*, the horizontal thrust in the link *PQ*,

$$P = W_1 \tan(\alpha_1 - \varphi) = 1 \times \tan(45° - 20°) \text{ kN}$$

$$= 1 \times \tan 25° = 1 \times 0.4663 = 0.4663 \text{ kN} \quad ...(iii)$$

and when the load W_2 is at the point of sliding upwards on the plane *OB*, the horizontal thrust in the link *PQ*,

$$P = W_2 \tan(\alpha_2 + \varphi) = W_2 (30° + 20°) \text{ kN}$$

$$= W_2 \tan 50° = W_2 \times 1.1918 \text{ kN} \quad ...(iv)$$

Since the values of the horizontal thrust in the link *PQ*, obtained in the above equations is the same, therefore, equating (*iii*) and (*iv*),

$$0.4663 = W_2 \times 1.1918$$

$$\therefore \quad W_2 = \frac{0.4663}{1.1918} = 0.391 \text{ kN} = 391 \text{ N} \quad \textbf{Ans.}$$

Example 8.14. *A block (A) weighing 1 kN rests on a rough inclined plane whose inclination to the horizontal is 45°. This block is connected to another block (B) weighing 3 kN rests on a rough horizontal plane by a weightless rigid bar inclined at an angle of 30° to the horizontal as shown in Fig. 8.21.*

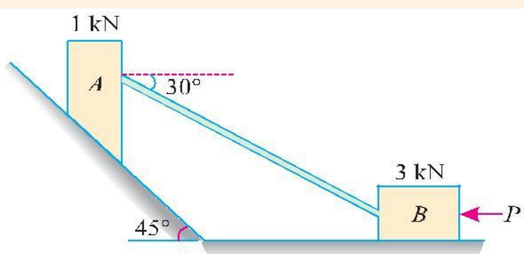


Fig. 8.21.

Find horizontal force (P) required to be applied to the block (B) just to move the block (A) in upward direction. Assume angle of limiting friction as 15° at all surface where there is sliding.

Solution. Given: Weight of block A (W_A) = 1 kN; Weight of block B (W_B) = 3 kN; Angle of inclination of plane with horizontal (α) = 45° or coefficient of friction (μ) = tan φ = tan 15° = 0.2679 or angle between rod and inclined plane (θ) = 45° – 30° = 15° and angle of limiting friction (φ) = 15°.

First of all, consider the equilibrium of the block (A), which is subjected to the forces as shown in Fig. 8.22 (a).

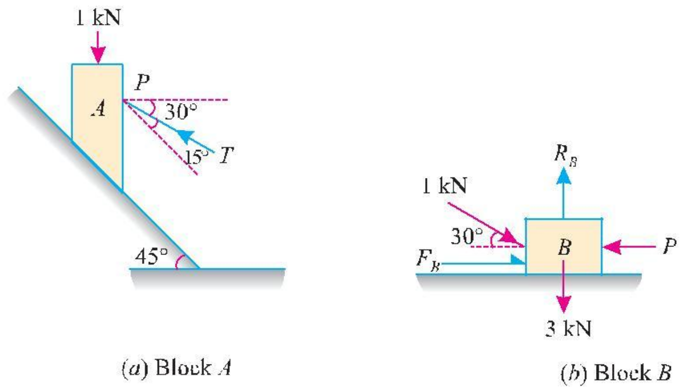


Fig. 8.22.

We know that the thrust or force in the rigid bar just to move the block (A) in the upward direction,

$$T = W_A \cdot \frac{\sin(\alpha + \varphi)}{\cos(\theta - \varphi)} = 1 \cdot \frac{\sin(45° + 15°)}{\cos(-15° - 15°)} \text{ kN}$$

...(The value of θ is taken as negative)

$$= 1 \cdot \frac{\sin 60°}{\cos(-30°)} = 1 \cdot \frac{\sin 60°}{\cos 30°} = 1 \cdot \frac{0.866}{0.866} = 1 \text{ kN} \quad ...[\because \cos(-\theta) = \cos\theta]$$

Now consider the equilibrium of the block (B), which is subjected to the forces as shown in Fig. 8.22 (b). We know that as the block is at the point of sliding towards left, therefore, the force of friction ($F_B = \mu R_B$) will act towards right as shown in the figure. Now resolving the forces vertically,

$$R_B = 3 + 1 \sin 30° = 3 + (1 \times 0.5) = 3.5$$

kN and now resolving the forces horizontally,

$$P = 1 \cos 30° + F_B = (1 \times 0.866) + (0.2679 \times 3.5) = 1.8 \text{ kN} \quad \textbf{Ans.}$$

Example 8.15. *A solid body is formed by joining the base of a right circular cone of height 12 cm to the equal base of a right circular cylinder of height 3 cm. The solid is placed with its face on a rough inclined plane, and the inclination to the horizontal of the plane is gradually increased. Show that if radius r of the base is 2 cm, and the coefficient of friction μ = 0.5, the body will topple over before it begins to slide.*

If the heights are so chosen that the centre of mass of the solid is at the centre of the common base, show that if $\mu H < r\sqrt{6}$, the solid will slide before it topples.

Solution. Given: Height of right circular cone (H) = 12 cm; Height of right circular cylinder(h) = 3 cm; Radius of the base (r) = 2 cm and coefficient of friction (μ) = 0.5.

We have already found out in example 6.6 that the centre of gravity of the body is at a height of 4.07 cm from the base. A little consideration will show, that when the body is at the point of toppling, the weight of the body will pass through the extreme point B as shown in Fig. 8.23. Thus in the limiting case, the angle (θ_1) atwhich the body is inclined with the horizontal is given by:

$$\tan \theta_1 = \frac{BD}{DG} = \frac{2}{4.07} = 0.49$$

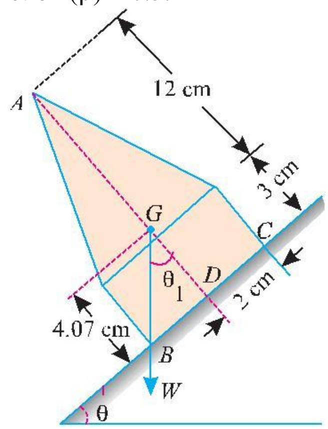


Fig. 8.23.

Thus we see that the angle (θ_1) is less than the angle (θ). It is so, as the value of $\tan \theta_1$ (equal to 0.49) is less than $\tan \theta$ (equal to 0.5). Or in other words, the angle of inclination is less than the angle of friction. Thus the body will topple over before it begins to slide. **Ans.**

Now in the second case, let us first find out the relation between h and H from the given condition that the centre of the body is at the centre of the common base (G) as shown in Fig. 8.24.

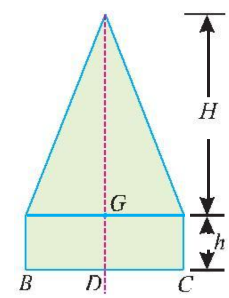


Fig. 8.24.

(*i*) *Cylinder*

$$v_1 = \pi r^2 h$$

and $$y_1 = \frac{h}{2}$$

(*ii*) *Cone*

$$v_2 = \frac{1}{3} \pi r^2 H$$

and $$y_2 = \frac{H}{4}$$

We know that the distance between centre of gravity of the body from the base of the cylinder ($\bar{y}$)

$$h = \frac{v_1 y_1 + v_2 y_2}{v_1 + v_2} = \frac{\pi r^2 h \cdot \dfrac{h}{2} + \dfrac{1}{3}\pi r^2 H\left(h + \dfrac{H}{4}\right)}{\pi r^2 h + \dfrac{1}{3}\pi r^2 H}$$

$$\pi r^2 h^2 + \frac{\pi r^2 H h}{3} = \frac{\pi r^2 h^2}{2} + \frac{\pi r^2 H h}{3} + \frac{\pi r^2 H^2}{12}$$

$$\frac{\pi r^2 h^2}{2} = \frac{\pi r^2 H^2}{12} \quad \text{or} \quad h^2 = \frac{H^2}{6} \quad \text{or} \quad h = \frac{H}{\sqrt{6}}$$

In this case, when the body is at the point of toppling, the angle (θ_2) at which it is inclined with the horizontal is given by :

$$\tan \theta_2 = \frac{r}{h} = \frac{r}{H/\sqrt{6}} = \frac{r\sqrt{6}}{H}$$

Now if the body is to slide before toppling, the angle of friction should be less than the angle of friction. Or in other words, tan θ should be less than tan θ_2, Mathematically,

$$\mu < \frac{r\sqrt{6}}{H} \quad \text{or} \quad \mu H < r\sqrt{6} \quad \textbf{Ans.}$$

EXERCISE 8.2

1. A load of 500 N is lying on an inclined plane, whose inclination with the horizontal is 30°. If the coefficient of friction between the load and the plane is 0.4, find the minimum and maximum horizontal force, which will keep the load in equilibrium.

[**Ans.** 72.05 N, 635.4 N]

2. Two blocks *A* and *B* of weight 100 N and 300 N respectively are resting on a rough inclined plane as shown in Fig. 8.25.

Find the value of the angle (θ) when the block *B* is about to slide. Take coefficient of friction between the two blocks as well as block *B* and the inclined plane as 0.25.

[**Ans.** 22.6°]

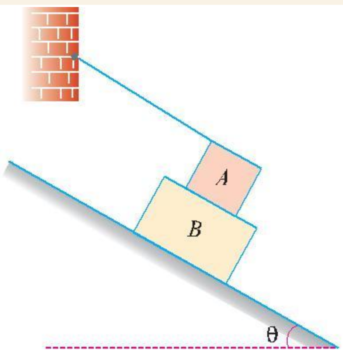


Fig. 8.25.

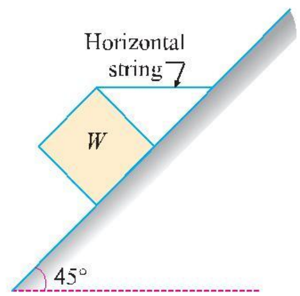


Fig. 8.26.

3. A rectangular prism (*W*) weighing 150 N, is lying on an inclined plane whose inclination with the horizontal is shown in Fig. 8.26.
The block is tied up by a horizontal string, which has a tension of 50 N. From fundamentals find (*i*) the frictional force on the block (*ii*) the normal reaction of the inclined plane, (*iii*) the coefficient of friction between the surface of contact.

[**Ans.** 70.7 N ; 141.4 N ; 0.5 ;]

QUESTIONS

1. What do you understand by the term friction ? Explain clearly why it comes into play ?
2. How will you distinguish between static friction and dynamic friction ?
3. State the laws of friction.
4. Explain the term angle of friction.
5. Define coefficient of friction and limiting friction.

OBJECTIVE TYPE QUESTIONS

1. The force of friction between two bodies in contact
(*a*) Depends upon the area of their contact
(*b*) Depends upon the relative velocity between them
(*c*) Is always normal to the surface of their contact (*d*)
All of the above

2. The magnitude of the force of friction between two bodies, one lying above the other, depends upon the roughness of the
   (*a*) Upper body (*b*) Lower body
   (*c*) Both the bodies (*d*) The body having more roughness
3. The force of friction always acts in a direction opposite to that
   (*a*) In which the body tends to move
   (*b*) In which the body is moving
   (*c*) Both (*a*) and (*b*)
   (*d*) None of the two
4. Which of the following statement is correct ?
   (*a*) The force of friction does not depend upon the area of contact
   (*b*) The magnitude of limiting friction bears a constant ratio to the normal reaction between the two surfaces
   (*c*) The static friction is slightly less than the limiting friction.
   (*d*) All (*a*), (b) and (*c*)

ANSWERS

1. (*c*)	**2.** (*c*)	**3.** (*c*)	**4.** (*d*)

Top

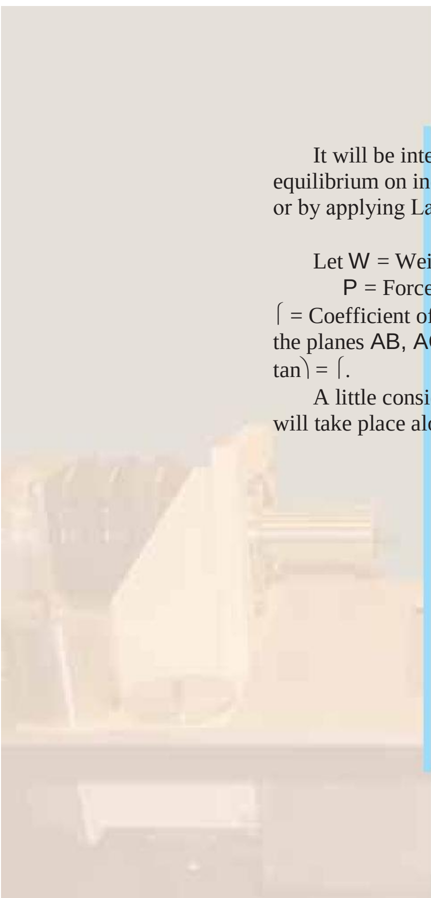
CHAPTER

9

Applications of Friction

WEDGE FRICTION

A wedge is, usually, of a triangular or trapezoidal in cross-section. It is, generally, used for slightadjustement in the position of a body i.e. for tightening fits or keys for shafts. Sometimes, a wedge is also used for lifting heavy weights as shown in Fig. 9.10.



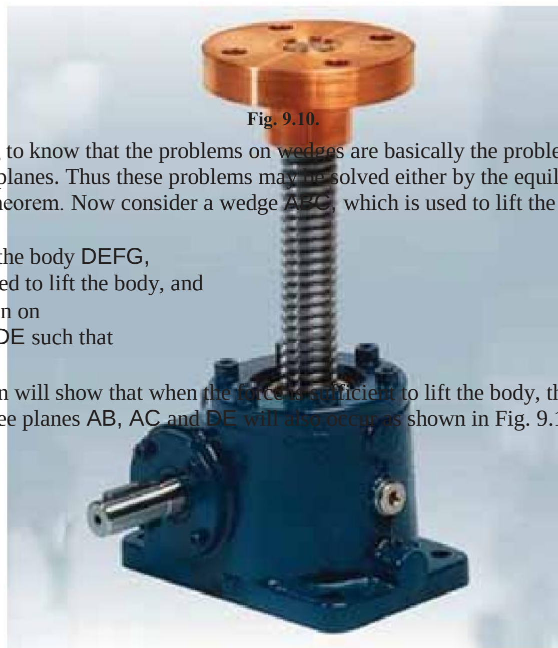


Fig. 9.10.

It will be interesting to know that the problems on wedges are basically the problems of equilibrium on inclined planes. Thus these problems may be solved either by the equilibrium method or by applying Lami's theorem. Now consider a wedge ABC, which is used to lift the body DEFG.

Let W = Weight fo the body DEFG,

P = Force required to lift the body, and

μ = Coefficient of friction on the planes AB, AC and DE such that $\tan \phi = \mu$.

A little consideration will show that when the force is sufficient to lift the body, the sliding will take place along three planes AB, AC and DE will also occur as shown in Fig. 9.11 (a) and (b).

Fig. 9.11.

The three reactions and the horizontal force (P) may now be found out either by graphical method or analytical method as discussed below:

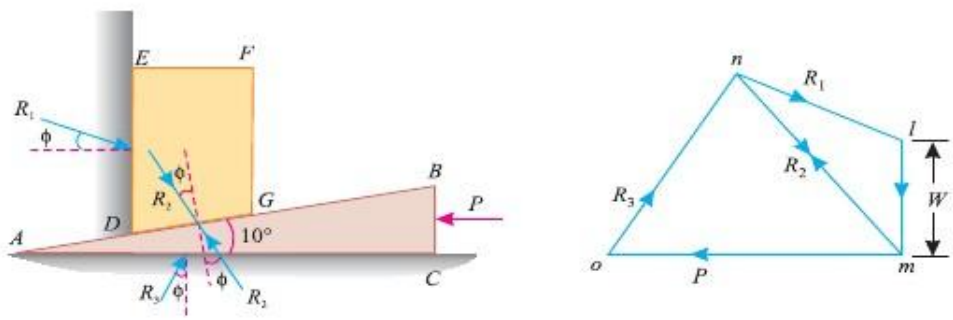
Graphical method

1. First of all, draw the space diagram for the body DEFG and the wedge ABC as shown in Fig. 9.12 (a). Now draw the reactions R_1, R_2 and R_3 at angle f with normal to the faces DE, AB and AC respectively (such that $\tan \phi = \mu$).

2. Now consider the equilibrium of the body DEFG. We know that the body is in equilibrium under the action of

(a) Its own weight (W) acting downwards
(b) Reaction R_1 on the face DE, and
(c) Reaction R_2 on the face AB.

Now, in order to draw the vector diagram for the above mentioned three forces, take some suitable point l and draw a vertical line lm parallel to the line of action of the weight (W) and cut off lm equal to the weight of the body to some suitable scale. Through l draw a line parallel to the reaction R_1. Similarly, through m draw a line parallel to the reaction R_2, meeting the first line at n as shown in Fig. 9.12 (b).



(a) Space diagram (b) Vector diagram

Fig. 9.12.

3. Now consider the equilibrium of the wedge ABC. We know that it is equilibrium under the action of

(a) Force acting on the wedge (P),
(b) Reaction R_2 on the face AB, and
(c) Reaction R_3 on the face AC.

Now, in order to draw the vector diagram for the above mentioned three forces, through m draw a horizontal line parallel to the force (P) acting on the wedge. Similarly, through n draw a line parallel to the reaction R_3 meeting the first line at O as shown in Fig. 9.12 (b).

4. Now the force (P) required on the wedge to raise the load will be given by mo to the scale.

Analytical method

1. First of all, consider the equilibrium of the body DEFG. And resolve the forces W, R_1 and R_2 horizontally as well as vertically.

2. Now consider the equilibrium of the wedge ABC. And resolve the forces P, R_2 and R_3 horizontally as well as vertically.

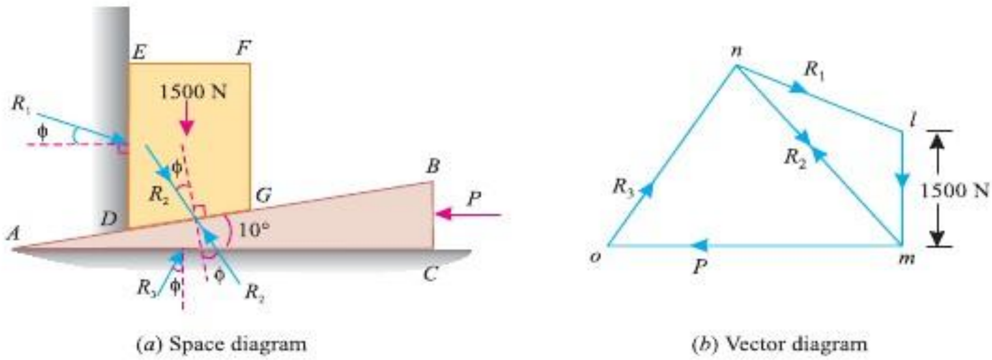
Example 9.6. A block weighing 1500 N, overlying a 10° wedge on a horizontal floor and leaning against a vertical wall, is to be raised by applying a horizontal force to the wedge.

Assuming the coefficient of friction between all the surface in contact to be 0.3, determine the minimum horizontal force required to raise the block.

Solution. Given: Weight of the block (W) = 1500 N; Angle of the wedge (α) = 10° and coefficient of friction between all the four surfaces of contact (μ) = 0.3 = $\tan \phi$ or ϕ = 16.7°.

Let P = Minimum horizontal force required to raise the block.

The example may be solved graphically or analytically. But we shall solve it by both the methods.



(a) Space diagram (b) Vector diagram

Fig. 9.13.

Graphical method

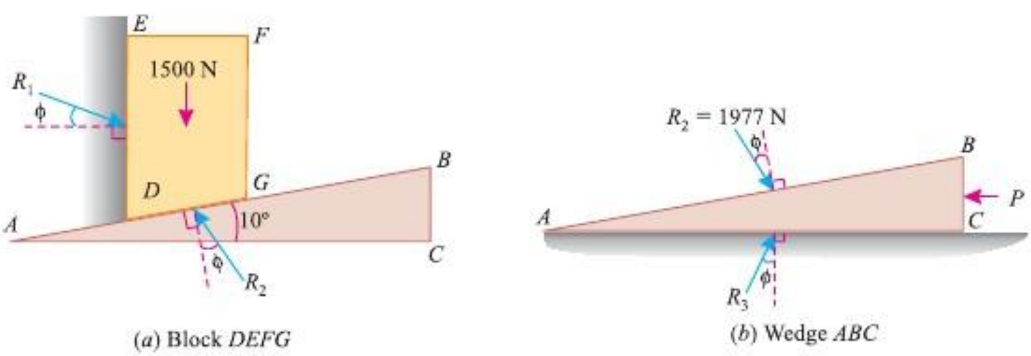
1. First of all, draw the space diagram for the block DEFG and the wedge ABC as shown in Fig. 9.13 (a). Now draw reactions R_1, R_2 and R_3 at angles of φ (i.e. 16.7° with normal to the faces DE, AB and AC respectively.

2. Take some suitable point l, and draw vertical line lm equal to 1500 N to some suitable scale (representing the weight of the block). Through l, draw a line parallel to the reactionR_1. Similarly, through m draw another line parallel to the reaction R_2 meeting the first line at n.

3. Now through m, draw a horizontal line (representing the horizontal force P). Similarly,throughn draw a line parallel to the reaction R_3 meeting the first line at O as shown in Fig. 9.13(b).

4. Now measuring moto the scale, we find that the required horizontal force P = 1420 N. Ans.

Analytical method



(a) Block DEFG (b) Wedge ABC

Fig. 9.14.

First of all, consider the equilibrium of the block. We know that it is in equilibrium under the action of the following forces as shown in Fig. 9.14 (a).

1. Its own weight 1500 N acting downwards.
2. Reaction R_1 on the face DE.
3. Reaction R_2 on the face DG of the block.

Resolving the forces horizontally,

$$R_1 \cos (16.7°) = R_2 \sin (10 + 16.7°) = R_2 \sin 26.7°$$

$$R_1 \times 0.9578 = R_2 \times 0.4493$$

or $$R_2 = 2.132\, R_1$$

and now resolving the forces vertically,

$$R_1 \times \sin (16.7°) + 1500 = R_2 \cos (10° + 16.7°) = R_2 \cos 26.7°$$

$$R_1 \times 0.2874 + 1500 = R_2 \times 0.8934 = (2.132\, R_1)0.8934$$

$$= 1.905\, R_1 \qquad ...(R_2 = 2.132\, R_1)$$

$$R_1(1.905 - 0.2874) = 1500$$

4 $$R_1 = \frac{1500}{1.6176} = 927.3 \text{ N}$$

and $$R_2 = 2.132\, R_1 = 2.132 \times 927.3 = 1977 \text{ N}$$

Now consider the equilibrium of the wedge. We know that it is in equilibrium under the action of the following forces as shown in Fig. 9.14 (b).

1. Reaction R_2 of the block on the wedge.
2. Force (P) acting horizontally, and
3. Reaction R_3 on the face AC of the wedge.

Resolving the forces vertically,

$$R_3 \cos 16.7° = R_2 \cos (10° + 16.7°) = R_2 \cos 26.7°$$

$$R_3 \times 0.9578 = R_2 \times 0.8934 = 1977 \times 0.8934 = 1766.2$$

4 $$R_3 = \frac{1766.2}{0.9578} = 1844 \text{ N}$$

and now resolving the forces horizontally,

$$P = R_2 \sin (10° + 16.7°) + R_3 \sin 16.7° = 1977 \sin 26.7° + 1844 \sin 16.7° \text{ N}$$

$$= (1977 \times 0.4493) + (1844 \times 0.2874) = 1418.3 \text{ N}$$ **Ans.**

Example 9.7. A 15° wedge (A) has to be driven for tightening a body (B) loaded with 1000 N weight as shown in Fig. 9.15.

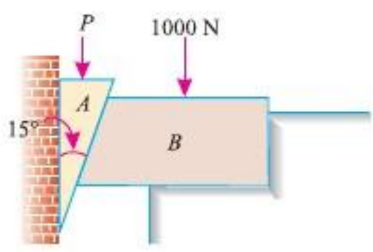
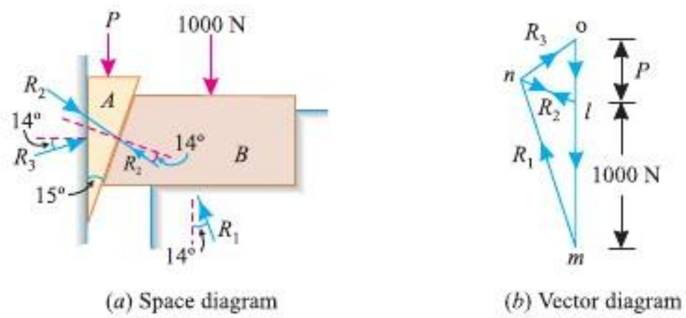


Fig. 9.15.

If the angle of friction for all the surfaces is 14°, find graphically the force (P), which should be applied to the wedge. Also check the answer analytically.

Solution. Given: Angle of the Wedge (α) = 15°; Weight acting on the body (W) = 1000 N and angle of friction for all the surfaces of contact (φ) = 14°.

Graphical solution



(a) Space diagram (b) Vector diagram

Fig. 9.16.

1. First of all, draw the space diagram for the body (B) and wedge (A) as shown in Fig. 9.16 (a). Now draw the reactions R_1, R_2 and R_3 at angles of 14° with normal to the faces.

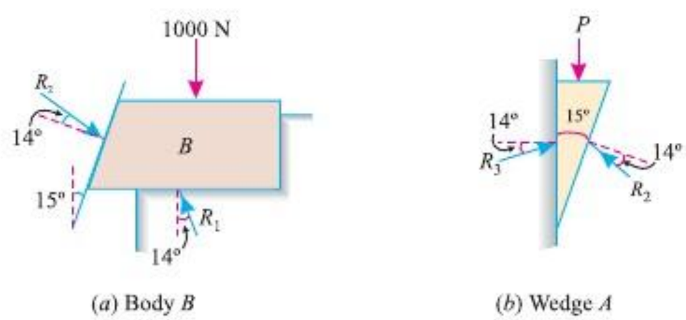
2. Take some suitable point l and draw a vertical line lm equal to 1000 N to some suitable scale, representing the weight of the body. Through l draw a line parallel to the reaction R_2. Similarly, through m draw another line parallel to the reaction R_1 meeting first line at n.

3. Now through l draw a vertical line representing the vertical force (P). Similarly, through n draw a line parallel to the reaction R_3 meeting the first line at O as shown in Fig. 9.16 (b).

4. Now measuring ol to the scale, we find that the required vertical force, P = 232 N **Ans.**

Analytical check

First of all, consider equilibrium of the body. We know that it is in equilibrium under the action of the following forces as shown in Fig. 9.17 (a).



(a) Body B (b) Wedge A

Fig. 9.17.

1. Its own weight 1000 N acting downwards
2. Reaction R_1 acting on the floor, and
3. Reaction R_2 of the wedge on the body.

Resolving the forces horizontally,

$R_1 \sin 14° = R_2 \cos (15° + 14°) = R_2 \cos 29°$

$$R_1 \times 0.2419 = R_2 \times 0.8746$$

$$\therefore \quad R_2 = \frac{0.8746}{0.2419} R_2 = 3.616\, R_2$$

and now resolving the forces vertically,

$$R_2 \sin (15° + 14°) + 1000 = R_1 \cos 14°$$

$$R_2 \times 0.4848 + 1000 = R_1 \times 0.9703 = (3.616\ R_2)\ 0.9703 = 3.51\ R_2 \qquad ...(\because R_1 = 3.616\ R_2)$$

or $$1000 = R_2 (3.51 – 0.4848) = 3.0252\ R_2$$

$$\therefore \quad R_2 = \frac{1000}{3.0252} = 330.6 \text{ N}$$

Now consider equilibrium of the wedge. We know that it is in equilibrium under the action of the following forces as shown in Fig. 9.17. (b) :

1. Reaction R_2 of the body on the wedge,
2. Force (P) acting vertically downwards, and
3. Reaction R_3 on the vertical surface.

Resolving the forces horizontally,

$$R_3 \cos 14° = R_2 \cos (14° + 15°) = R_2 \cos 29°$$

$$R_3 \times 0.9703 = R_2 \times 0.8746 = 330.6 \times 0.8746 = 289.1$$

$$\therefore \quad R_3 = \frac{289.1}{0.9703} = 297.9 \text{ N}$$

and now resolving the forces vertically,

$$P = R_3 \sin 14° + R_2 \sin (14° + 15°)$$

$$= (297.9 \times 0.2419) + (330.6 \times 0.4848) = 232.3 \text{ N} \quad \textbf{Ans.}$$

EXERCISE 9.2

1. A block (A) of weight 5 kN is to be raised by means of a 20° wedge (B) by the application of a horizontal force (P) as shown in Fig. 9.18. The block A is constrained to move vertically by the application of a horizontal force (S). Find the magnitude of the forces F and S, when the coefficient of friction at the contact surfaces is 0.25.

[Ans. 4.62 kN; 3.77 kN]

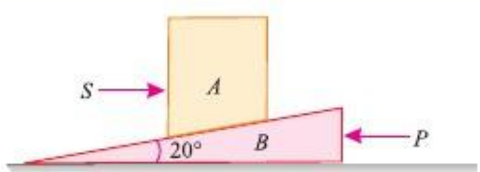


Fig. 9.18.

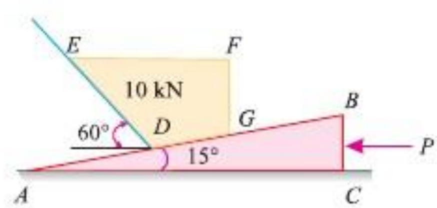


Fig. 9.19.

2. A block weighing 10 kN is to be raised against a surface, which is inclined at 60° with the horizontal by means of a 15° wedge as shown in Fig. 9.19.

Find graphically the horizontal force (P) which will just start the block to move, if the coefficient of friction between all the surfaces of contact be 0.2. Also check the answer analytically. [Ans. 6 kN]

SCREW FRICTION

The screws, bolts, studs, nuts etc. are widely used in various machines and structures for fastenings. These fastenings have screw threads, which are made by cutting a continuous helical groove on a cylindrical surface. If the threads are cut on the outer surface of a solid rod, these are known as external threads. But if the threads are cut on the internal surface of a hollow rod these are known as internal threads.

The screw threads are mainly of two types viz. V-threads and square threads. The V-threads are stronger and offer more frictional resistance to motion than square threads. Moreover, the V-threads have an advantage of preventing the nut from slackening. I will be interesting to know that the V-threads are used for the purpose of tightening pieces together (e.g. bolts and nuts etc.). Square threads are used in screw jacks, vice screws etc. which are used for lifting heavy loads. The following terms are important for the study of screws:

1. Helix. It is the curve traced by a particle, while describing a circular path at a uniform speed and advancing in the axial direction at a uniform rate. Or in other words, it is the curve traced by a particle while moving along a screw thread.

2. Pitch. It is the distance from one point of a thread to the corresponding point on the next thread. It is measured parallel to the axis of the screw.

3. Lead. It is the distance through which a screw thread advances axially in one turn.

4. Depth of thread. It is the distance between the top and bottom surfaces of a thread (also known as crest and root of thread).

5. Single-threaded screw. If the lead of a screw is equal to its pitch, it is known as single-threaded screw.

6. Multi-threaded screw. If more than one threads are cut in one lead distance of a screw, it is known as multi-threaded screw e.g. in a double-threaded screw, two threads are cut in one lead length. In such cases, all the threads run independently along the length of the rod. Mathematically,

$$\text{Lead} = \text{Pitch} \times \text{No. of threads.}$$

7. Slope of the thread. It is the inclination of the thread with horizontal. Mathematically,

$$\tan\alpha = \frac{\text{Lead of screw}}{\text{Circumference of screw}}$$

$$= \frac{p}{\pi d} \qquad \text{...(In single-threaded screw)}$$

$$= \frac{np}{\pi d} \qquad \text{...(In multi-threaded screw)}$$

where α = Angle of inclination of the thread,

p = Pitch of the screw,

d = Mean diameter of the screw, and

n = No. of threads in one lead.

9.5. RELATION BETWEEN EFFORT AND WEIGHT LIFTED BY A SCREW JACK

The screw jack is a device for lifting heavy loads, by applying a comparatively smaller effort at its handle. The principle, on which a screw jack works, is similar to that of an inclined plane.

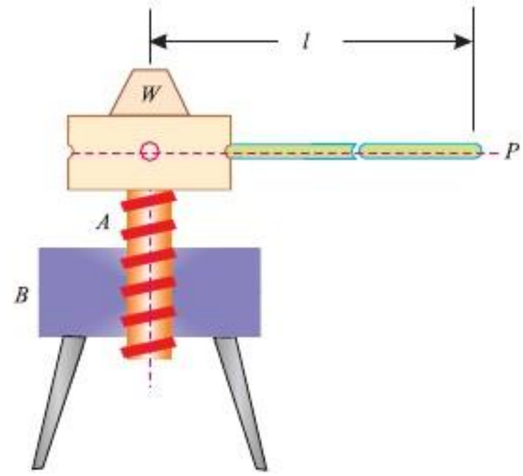


Fig. 9.20. Screw jack

Fig. 9.20 shows common form of a screw jack, which consists of a threaded rod A, called screw rod or simply screw. The screw has square threads, on its outer surface, which fit into the inner threads of the jack B. The load, to be raised or lowered, is placed on the head of the screw, which is rotated by the application of an effort at the end of the lever for lifting or lowering the load.

If one complete turn of a screw thread, be imagined to be unwound, from the body of the screw and developed, it will form an inclined plane as shown in Fig. 9.21

Let p = Pitch of the screw,

d = Mean diameter of the screw

r = Mean radius of the screw, and

α = Helix angle.

From the geometry of the figure, we find that

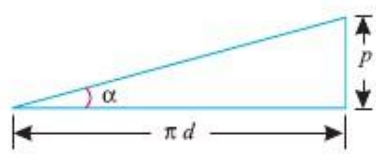


Fig. 9.21. Helix angle

$$\tan \alpha = \frac{p}{\pi d} = \frac{p}{2\pi r} \quad \text{...(where } d = 2r\text{)}$$

Now let P = Effort applied at the mean radius of the screw jack to lift the load,

W = Weight of the body to be lifted, and

μ = Coefficient of friction, between the screw and nut.

Let ϕ = Angle of friction, such that $\mu = \tan \phi$.

As a matter of fact, the principle, on which a screw jack works, is similar to that of an inclined plane. Thus the force applied on the lever of a screw jack is considered to be horizontal. We have already discussed in Art. 8.14 that the horizontal force required to lift a load on an inclined rough plane

$P = W \tan (\alpha + \phi)$

Example 9.8. A screw jack has mean diameter of 50 mm and pitch 10 mm. If the coefficient of friction between its screw and nut is 0.15, find the effort required at the end of 700 mm long handle to raise a load of 10 kN.

Contents

Solution. Given: Mean diameter of screw jack (d) = 50 mm or radius (r) = 25 mm; Pitch of the screw (p) = 10 mm; Coefficient of friction between screw and nut (μ) = 0.15 = tan ϕ or ϕ = 8.5°; Length of the handle (l) = 700 mm and load to be raised (W) = 10 kN.

Let P_1 = Effort required at the end of 700 mm long handle to raise the load,

and α = Helix angle

We know that

$$\tan\alpha = \frac{p}{\pi d} = \frac{10}{\pi \times 50} = 0.0637 \quad \text{or} \quad \alpha = 3.6°$$

and effort required at mean radius of the screw to raise the load,

$$P = W \tan(\alpha + \phi) = W \tan(3.6° + 8.5°)$$
$$= W \tan 12.1° = 10 \times 0.2144 = 2.144 \text{ kN}$$

Now the effort required at the end of the handle may be obtained from the relation.

$$P_1 \times 700 = P \times r = 2.144 \times 25 = 53.6$$

$$\therefore \quad P_1 = \frac{53.6}{700} = 0.0766 \text{ kN} = 76.6 \text{ N} \quad \textbf{Ans.}$$

Example 9.9. The mean radius of the screw of a square threaded screw jack is 25 mm. The pitch of thread is 7.5 mm. If the coefficient of friction is 0.12, what effort applied at the end of lever 60 cm length is needed to raise a weight of 2 kN.

Solution. Given: Mean radius of the screw (r) = 25 mm; Pitch of the thread (p) = 7.5 mm; Coefficient of friction (μ) = 0.12 = tan ϕ; Length of the lever (l) = 60 cm and weight to be raised = 2 kN = 2000 N.

Let P_1 = Effort required at the end of the 60 cm long handle to raise the weight,

and α = Helix angle.

We know that

$$\tan\alpha = \frac{p}{2\pi r} = \frac{0.75}{2\pi \times 2.5} = 0.048$$

and effort required at mean radius of the screw to raise the weight,

$$P = W \tan(\alpha + \phi) = W \times \frac{\tan\alpha + \tan\phi}{1 - \tan\alpha . \tan\phi} = 2000 \times \frac{0.048 + 0.12}{1 - 0.048 \times 0.12}$$

$$= 2000 \times 0.169 = 338 \text{ N}$$

Now the effort applied at the end of the lever, may be found out from the relation,

$$P_1 \times 60 = P \times 2.5 = 338 \times 2.5 = 845$$

$$\therefore \quad P_1 = \frac{845}{60} = 14.1 \text{ N} \quad \textbf{Ans.}$$

Example 9.10. A screw press is used to compress books. The thread is a double thread (square head) with a pitch of 4 mm and a mean radius of 25 mm. The coefficient of the friction (μ) for the contact surface of the thread is 0.3. Find the torque for a pressure of 500 N.

Solution. Given: No. of threads $(n) = 2$; Pitch $(p) = 4$ mm; Mean radius $(r) = 25$ mm ;
Coefficient of friction $(\mu) = 0.3 = \tan \phi$ or $\phi = 16.7°$ and pressure $(W) = 500$ N

Let α = Helix angle.

We know that $\tan \alpha = \dfrac{np}{2\pi r} = \dfrac{2 \times 4}{2\pi \times 25} = 0.0509$ or $\alpha = 2.9°$

∴ Effort required at the mean radius of the screw to press the books

$P = W \tan (\alpha + \phi) = 500 \tan (2.9° + 16.7°)$ N

$= 500 \tan 19.6° = 500 \times 0.356 = 178$ N

and* torque required to press the books,

$T = P \times r = 178 \times 25 = 4450$ N-mm **Ans.**

9.6. RELATION BETWEEN EFFORT AND WEIGHT LOWERED BY A SCREW JACK

We have already discussed in the last article that the principle, on which a screw works, is similar to that of an inclined plane. And force applied on the lever of a screw jack is considered to be horizontal. We have also discussed in Art. 8.14 that the horizontal force required to lower a load on an inclined plane,

$P = W \tan (\alpha - \phi)$ ...(when $\alpha > \phi$)

$= W \tan (\phi - \alpha)$ ...(when $\phi > \alpha$)

Note. All the notations have the usual values as discussed in the last article.

Example 9.11. A screw Jack has a square thread of 75 mm mean diameter and a pitch of 15 mm. Find the force, which is required at the end of 500 mm long lever to lower a load of 25 kN. Take coefficient of friction between the screw and thread as 0.05.

Solution. Given: Mean diameter of thread $(d) = 75$ mm or radius $(r) = 37.5$ mm; Pitch of thread $(p) = 15$mm; Length of lever $(l) = 500$ mm; load to be lowered $(W) = 25$ kN and coefficient of friction between the screw and thread $(\mu) = 0.05 = \tan \phi$ or $\phi = 2.9°$

Let P_1 = Effort required at end of 500 mm long handle to lower the load,

and α = Helix angle.

We know that $\tan \alpha = \dfrac{p}{\pi d} = \dfrac{15}{\pi \times 75} = 0.0637$ or $\alpha = 3.6°$

and effort required at the mean radius of the screw to lower the load,

$P = W \tan (\alpha - \phi) = W \tan (3.6° - 2.9°)$

$= W \tan 0.7° = 25 \times 0.0122 = 0.305$ kN $= 305$ N

Now the effort required at the end of the handle may be found out from the relation,

$P_1 \times 500 = P \times r = 305 \times 37.5 = 11438$

$P_1 = \dfrac{11438}{500} = 23$ N **Ans.**

Example 9.12. The screw of a jack is square threaded with two threads in a centimeter. The outer diameter of the screw is 5 cm. If the coefficient of friction is 0.1, calculate the force required to be applied at the end of the lever, which is 70 cm long (a) to lift a load of 4 kN, and (ii) to lower it.

Solution. Given: Outer diameter of the screw $(D) = 5$ cm; Coefficient of friction $(\mu) = 0.1 = \tan \phi$; Length of the lever $(l) = 70$ cm and load to be lifted $(W) = 4$ kN $= 4000$ N.

* Torque = Force × Radius

We know that as there are two threads in a cm, (i.e. $n = 2$) therefore pitch of the screw,

$p = 1/2 = 0.5$ cm

and internal diameter of the screw,

$= 5 - (2 \times 0.5) = 4$ cm

∴ Mean diameter of the screw,

$$d = \frac{5 + 4}{2} = 4.5 \text{ cm}$$

Let α = Helix angle.

We know that $\tan\alpha = \dfrac{p}{\pi d} = \dfrac{0.5}{\pi \times 4.5} = 0.0353$

(i) Force required at the end of 70 cm long lever to lift the load

Let P_1 = Force required at the end of the lever to lift the load.

We know that the force required to be applied at the mean radius to lift the load,

$$P = W \tan(\alpha + \phi) = W \times \frac{\tan\alpha + \tan\phi}{1 - \tan\alpha \cdot \tan\phi}$$

$$= 4000 \times \frac{0.0353 + 0.1}{1 - 0.0353 \times 0.1} = 543.1 \text{ N}$$

Now the force required at the end of the lever may be found out from the relation,

$$P_1 \times 70 = P \times \frac{d}{2} = 543.1 \times \frac{4.5}{2} = 1222$$

$$\therefore P_1 = \frac{1222}{70} = 17.5 \text{ N} \quad \textbf{Ans.}$$

(ii) Force required at the end of 70 cm long lever to lower the load

Let P_2 = Force required at the end of the lever to lower the load.

We know that the force required at the mean radius to lower the load,

$$P = W \tan(\phi - \alpha) = 4000 \times \frac{\tan\phi - \tan\alpha}{1 + \tan\phi \tan\alpha}$$

$$= 4000 \times \frac{0.1 - 0.0353}{1 + 0.1 \times 0.0353} = 257.9 \text{ N}$$

Now the force required at the end of the lever may be found out from the relation:

$$P_2 \times 70 = P \times \frac{d}{2} = 257.9 \times \frac{4.5}{2} = 580.3$$

$$\therefore P_2 = \frac{580.3}{70} = 8.3 \text{ N} \quad \textbf{Ans.}$$

9.7. EFFICIENCY OF A SCREW JACK

We have seen in Art. 9.6 that the effort (P) required at the mean radius of a jack to lift the load (W),

$P = W \tan(\alpha + \phi)$ ...(i)

where α = Helix angle, and

μ = Coefficient of friction between the screw and the nut,

$= \tan\phi$ ...(where ϕ = Angle of friction)

If there would have been no friction between the screw and the nut, then ϕ will be zero. In such a case, the value of effort (P_0) necessary to raise the same load, will be given by the equation :

$$P_0 = W \tan\alpha \qquad \text{...[substituting } \phi = 0 \text{ in equation (i)]}$$

$$\therefore \quad \text{Efficiency } (\eta) = \frac{\text{Ideal effort}}{\text{Actual effort}} = \frac{P_0}{P} = \frac{W \tan\alpha}{W \tan(\alpha + \phi)} = \frac{\tan\alpha}{\tan(\alpha + \phi)}$$

It shows that the efficiency of a screw jack is independent of the weight lifted or effort applied. The above equation for the efficiency of a screw jack may also be written as:

$$\eta = \frac{\dfrac{\sin\alpha}{\cos\alpha}}{\dfrac{\sin(\alpha + \phi)}{\cos(\alpha + \phi)}} = \frac{\sin\alpha \cdot \cos(\alpha + \phi)}{\cos\alpha \cdot \sin(\alpha + \phi)}$$

or

$$1 - \eta = 1 - \frac{\sin\alpha \cdot \cos(\alpha + \phi)}{\cos\alpha \cdot \sin(\alpha + \phi)} = \frac{\cos\alpha \sin(\alpha + \phi) - \sin\alpha \cos(\alpha + \phi)}{\cos\alpha \cdot \sin(\alpha + \phi)}$$

$$= \frac{\sin\phi}{\cos\alpha \sin(\alpha + \phi)} \qquad [\because \sin(A - B) = \sin A \cos B - \cos A \sin B]$$

$$1 - \eta = \frac{2 \sin\phi}{2 \cos\alpha \sin(\alpha + \phi)} \qquad \text{[Multiplying and dividing by 2]}$$

$$= \frac{2 \sin\phi}{\sin(2\alpha + \phi) - \sin\phi}$$

...$[\because 2 \cos A \sin B = \sin(A + B) + \sin(A - B)]$

Now for the efficiency to be maximum, the term $(1 - \eta)$ should be the least. Or in other words, the value of $\sin(2\alpha + \phi)$ should be the greatest. This is only possible, when

$$2\alpha + \phi = 90° \qquad \text{or} \qquad 2\alpha = 90° - \phi$$

$$\therefore \quad \alpha = 45° - \frac{\phi}{2}$$

It shows that the maximum efficiency of a screw jack is also independent of the weight lifted or effort applied.

Example 9.13. A load of 2.5 kN is to be raised by a screw jack with mean diameter of 75 mm and pitch of 12 mm. Find the efficiency of the screw jack, if the coefficient of friction between the screw and nut is 0.075.

Solution. Given: Load (W) = 2.5 kN ; Mean diameter of the screw (d) = 75 mm; Pitch of the screen (p) = 12 mm and coefficient of friction between the screw and nut (μ) = 0.075 = $\tan\phi$.

We know that

$$\tan\alpha = \frac{p}{\pi d} = \frac{12}{\pi \times 75} = 0.051$$

and efficiency of the screw jack,

$$\eta = \frac{\tan\alpha}{\tan(\alpha + \phi)} = \frac{\tan\alpha}{\dfrac{\tan\alpha + \tan\phi}{1 - \tan\alpha \tan\phi}} = \frac{0.051}{\dfrac{0.051 + 0.075}{1 - (0.051 \times 0.075)}} = \frac{0.051}{0.1265}$$

$$= 0.403 = 40.3\% \qquad \textbf{Ans.}$$