UNIT-II FRICTION

In Unit-I, generally body surfaces are assumed to be **smooth.** In reality, no body surface is perfectly smooth. As a result, when one body surface slides or tends to slide over another, resistance is always offered to the motion. This resistive force which acts tangential to the contact surfaces is called **friction.**

Friction causes favourable as well as undesirable effects. In machines, it causes wear and tear of material parts. On the other hand, without friction we cannot do things in our daily lives as we do, like we cannot walk or drive a car or hold a pen and write, and so on.

TYPES OF DRY FRICTION

- Sliding friction
- Belt/rope friction
- Wheel friction
- Wedge friction
- Screw friction

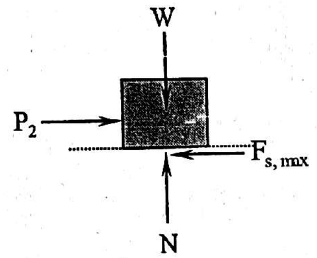
LIMITING FRICTION AND IMPENDING MOTION

When an externally applied force (F) acts tangential to the contact surfaces, we come across three different cases, namely

Case-I: When $F < F_{s,max}$, no motion occurs.

Case-II: When $F = F_{s,max}$, motion impends.

Case-III: When $F > F_{s,max}$, the body is under motion.



Point of impending motion

COULOMB'S LAWS OF DRY FRICTION

1. The frictional force always acts such as to oppose the tendency of one surface to slide relative to the other. It acts tangential to the surfaces in contact.
2. The magnitude of frictional force is exactly equal to the force which tends to move the body till the limiting value is reached.
3. The maximum force of friction is independent of the area of contact between two surfaces and depends on the nature of surfaces in contact.
4. The magnitude of limiting static friction is proportional to the normal reaction between the two surfaces. Mathematically it is expressed as:

   $$F_{s,max} \propto N$$

   Introducing a constant of proportionality,

   $$F_{s,max} = \mu_s N$$

   where the constant μ_s is called as **coefficient of static friction.**

5. For low relative velocities between sliding bodies, frictional force is independent of the relative speed with which the surfaces move over each other.
6. The magnitude of kinetic friction is proportional to the normal reaction between the two surfaces. Mathematically it is expressed as:

$$F_k \propto N$$

Introducing a constant of proportionality,

$$F_k = \mu_k N$$

where the constant μ_k is called as **coefficient of kinetic friction.**

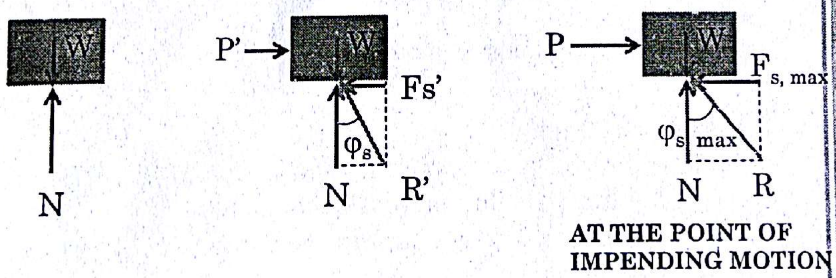
CONTACT SURFACES	μ_s
Wood on wood	0.2-0.5
Wood on leather	0.2-0.5
Metal on metal	0.15-0.25
Metal on wood	0.2-0.6
Metal on stone	0.3-0.7
Metal on leather	0.3-0.5
Stone on stone	0.4-0.7
Earth on earth	0.2-1.0
Rubber on concrete	0.6-0.9
Rope on wood	0.5-0.7

Graphical Representation

In some of the problems, it is better to replace **normal reaction** (N) and the **frictional force** (Fs) by the **resultant (R).**

GRAPHICAL REPRESENTATION

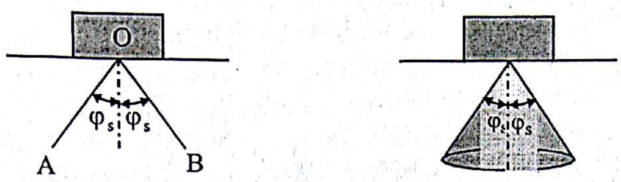
- RESULTANT & ANGLE OF FRICTION



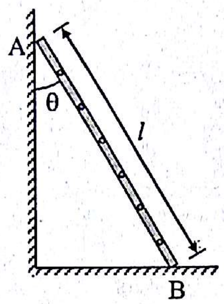
- In some of the problems, it is better to replace **normal reaction** (N) and the **frictional force** (Fs) by the **resultant (R)**

Angle of friction: It is the maximum value of φ, that is the angle between the resultant and the normal reaction.

Cone of friction: It is an imaginary cone formed by revolving the resultant (R) of the normal reaction and maximum force of static friction about the normal reaction for one complete revolution.



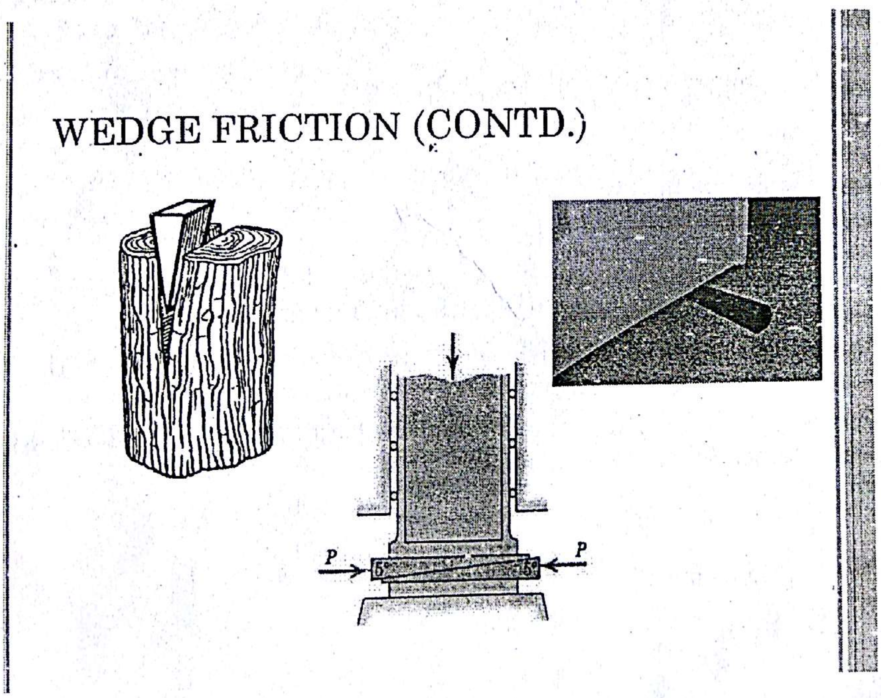
LADDER FRICTION



Unlike previous cases, there are **two** contact points, one at the **wall** and the other at the **floor.** The ladder tends to slide under **its own weight** or under the **weight of a person** climbing upon it. Due to friction at the wall and at the floor, resistance is offered to this sliding of ladder. The forces of friction reach maximum values at the point of impending motion.

WEDGE FRICTION

Wedges are triangular or trapezoidal shaped blocks with a very small sloping angle. By the friction developed, they are used to cleave open woods, to do minor alignments in heavy loads before fixing them.



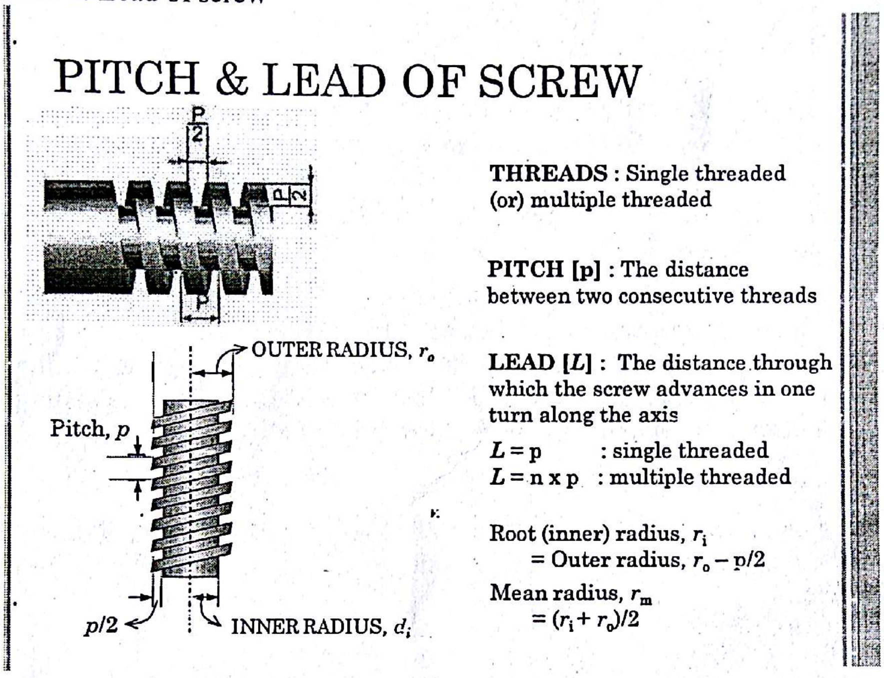
SCREW FRICTION

Threads are cut in a helix on the shank of the screw. When the screw is turned, **friction** is developed between the thread and the surrounding body. This friction is known as **screw friction.** This is utilized for fastening and for lifting of heavy loads.

V-threaded screw: used for fastening purpose
Square threaded screw

- Screw-jack : to lift and hold loads
- Bench vice : To clamp bodies

Pitch & Lead of screw



To raise a load, the effort required to be applied at the handle is:

$$P = \frac{Wr}{a}\tan(\varphi_s + \theta)$$

or the torque required to raise the load is:

$$\tau = P.a = Wr\tan(\varphi_s + \theta)$$

If $\theta > \varphi_s$, then the screw jack will unwind itself once the load is raised.
If $\theta < \varphi_s$, then the screw jack will self-lock and effort will be required to lower the load.

The **efficiency** of screw jack is defined as the ratio of the **effort** required under **frictionless condition** to that of the **actual effort** required to raise a load

$$\eta = \frac{\tan\theta}{\tan(\varphi_s + \theta)}$$

The maximum efficiency is given as:

$$\eta_{max} = \frac{1 - \sin\varphi_s}{1 + \sin\varphi_s}$$

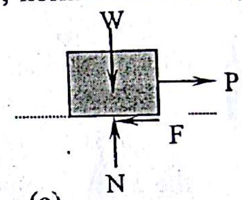
and the condition for maximum efficiency is;

$$\theta = 45 - \varphi_s/2$$

Solved Problems

1. A block of mass 50 kg rests on a rough horizontal plane as shown in figure below. If the coefficients of static and kinetic friction between the block and the plane are respectively 0.2 and 0.15, describe the resulting motion, i) when a horizontal force of 75 N is applied and ii) when a horizontal force of 120 N is applied. Also determine the force required to cause motion to impend.

The forces acting in the free body diagram of the block are its weight W, normal reaction N, force of friction F acting to oppose the motion and the applied force P.



Applying the conditions of equilibrium along X and Y-directions,

$$\Sigma F_y = 0 \Rightarrow$$
$$N - W = 0$$
$$\Rightarrow N = W = 50 \times g = 490.5\,N \qquad (a)$$

The force of friction is maximum at the point of impending motion,

$$F_{s,max} = \mu_s N = 0.2 \times 490.5 = 98.1\,N \qquad (b)$$

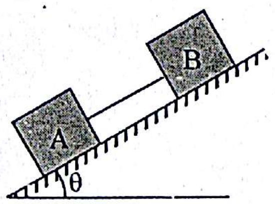
i) When a horizontal force of 75 N is applied:
This is of case-I, in which $P < F_{s,max}$, hence the block is not in motion and the body is in equilibrium. The corresponding force of friction is:

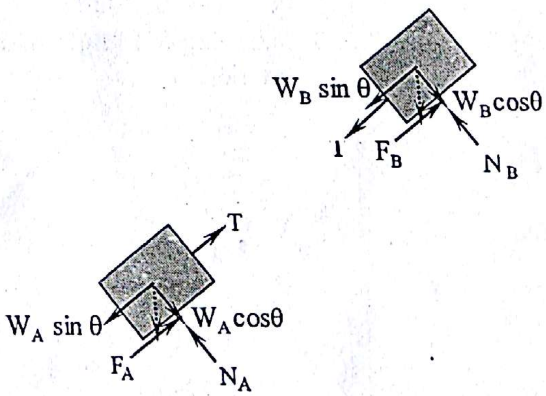
$$F = P = 75\,N$$

ii) When a horizontal force of 120 N is applied
From equation (b), we can see that the frictional force to maintain equilibrium is equal to 120 N. Since this frictional force is greater than $F_{s,max}$, the block is actually under motion. Therefore, the actual force of friction is equal to the force of kinetic friction, whose magnitude is given as: $F = F_k = \mu_k N = 0.15 \times 490.5 = 73.58\,N$

Force required to cause motion to impend is equal to the maximum force of static friction, i.e., $P = 98.1\,N$.

2. Two blocks A and B of weights W_A and W_B respectively rest on a rough inclined plane and are connected by a short piece of string as shown in figure below. If the coefficients of friction between the blocks and the plane are respectively $\mu_A = 0.2$ and $\mu_B = 0.3$, find (a) the angle of inclination of the plane for which sliding will impend and (b) tension in the string. Assume $W_A = W_B = 5\,N$.





FBD of Block-A:

Applying the conditions of equilibrium at the point of impending motion,

$$\Sigma F_y = 0 \Rightarrow$$

$$\therefore N_A = W_A \cos\theta$$

At the point of impending motion,

$$F_A = \mu_A N_A = \mu_A W_A \cos\theta$$

$$\Sigma F_x = 0 \Rightarrow$$

$$T + F_A - W_A \sin\theta = 0$$

$$\therefore T = W_A\ (\sin\theta - \mu_A \cos\theta) \qquad \text{(a)}$$

FBD Block-B:

Applying the conditions of equilibrium at the point of impending motion,

$$\Sigma F_y = 0 \Rightarrow$$

$$\therefore N_B = W_B \cos\theta$$

At the point of impending motion,

$$F_B = \mu_B N_B = \mu_B W_B \cos\theta$$

$$\Sigma F_x = 0 \Rightarrow$$

$$-T - W_B \sin\theta + F_B = 0$$

$$\Rightarrow T = F_B - W_B \sin\theta$$

$$= W_B\ (\mu_B \cos\theta - \sin\theta) \qquad \text{(b)}$$

Solving the simultaneous equations (a) and (b), we get

$$W_A (\sin\theta - \mu_A \cos\theta) = W_B (\mu_B \cos\theta - \sin\theta)$$

But $W_A = W_B$,

$$2\sin\theta = \cos\theta(\mu_A + \mu_B)$$

$$\tan\theta = \frac{(\mu_A + \mu_B)}{2}$$

$$\theta = 14.04^0$$

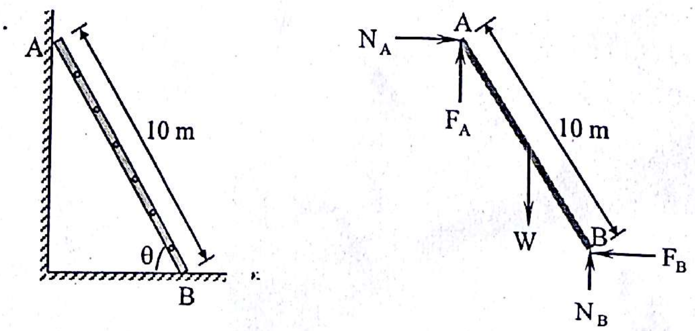
Substituting the value of θ in either of the equations for T:

$$T = W_A\ (\sin\theta - \mu_A \cos\theta)$$

$$= 5[\sin(14.04) - 0.2\cos(14.04)]$$

$$= 0.243\ N$$

3. A ladder 6 m long rests on a horizontal floor and leans against a vertical wall. If the coefficients of friction between the ladder and, the floor and the wall are respectively μ_f =0.3 μ_w = 0.15, determine the angle of inclination of the ladder with the floor at the point of impending motion.



At the point of impending motion, we can apply the equations of equilibrium,

$$\Sigma F_x = 0 \Rightarrow$$
$$N_A - F_B = 0$$

But $F_B = \mu_f N_B$,

$$N_A - \mu_f N_B = 0$$
$$N_A - 0.3N_B = 0 \qquad \text{(a)}$$

$$\Sigma F_y = 0 \Rightarrow$$
$$F_A + N_B - W = 0$$

But $F_A = \mu_w N_A$,

$$\mu_w N_A + N_B - W = 0$$
$$0.15N_A + N_B - W = 0 \qquad \text{(b)}$$

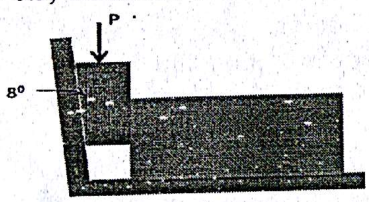
Solving from these two equations, we get

$$W = 0.15N_A + N_A/0.3 \quad = 3.48N_A$$

Taking moment about B and applying the condition of equilibrium,

$$M_B = 0 \Rightarrow$$
$$-[F_A \times 10\cos\theta] - [N_A \times 10\sin\theta] + [W \times 5\cos\theta] = 0$$
$$-[\mu_w N_A \times 10\cos\theta] - [N_A \times 10\sin\theta] + [3.48N_A \times 5\cos\theta] = 0$$
$$-1.5\cos\theta - 10\sin\theta + 17.4\cos\theta = 0$$
$$15.9\cos\theta = 10\sin\theta$$
$$\tan\theta = 1.59 \Rightarrow \theta = 57.83^0$$

4. A heavy block of mass 500 kg is to be adjusted horizontally using an 8° wedge by applying a vertical force P. If the coefficient of static friction for both the contact surfaces of the wedge is 0.25 and that between the block and the horizontal surface is 0.5, determine the least force P required to move the block.



5. A single square threaded screw jack has a pitch of 16 mm and a mean radius of 50 mm. Determine the force that must be applied to the end of 60 cm lever to raise a weight of 100 kN and the efficiency of the jack. State whether it is self-locking or not? If yes, determine the force that must be applied to lower the same weight. Assume coefficient of static friction to be 0.2.

Since the screw is *single threaded*, lead (L) = pitch (p) = 16 mm. Therefore,

$$\tan\theta = \frac{L}{2\pi r} = \frac{16}{2\pi(50)} = 0.051$$
$$\Rightarrow \theta = 2.92^0$$

Given, $\tan\varphi_s = \mu_s$

$$\Rightarrow \varphi_s = \tan^{-1}(0.2) = 11.31^0$$

The force required to raise a weight of 100 kN is given as:

$$P = \frac{Wr}{a}\tan(\varphi_s + \theta)$$

$$= \frac{100{,}000 \times 0.05}{0.6}\tan(11.31 + 2.92) \quad = 2113.3\,\text{N}$$

The efficiency of screw jack is given as:

$$\eta = \frac{\tan\theta}{\tan(\varphi_s + \theta)}$$

$$= \frac{\tan(2.92)}{\tan(11.31 + 2.92)} \quad = 0.2011 \text{ (or) } 20.11\%$$

Since $\varphi_s > \theta$, it is under **self-locking.** Hence, force is required to lower the load. This force is obtained as:

$$P = \frac{Wr}{a}\tan(\varphi_s - \theta)$$

$$= \frac{100{,}000 \times 0.05}{0.6}\tan(11.31 - 2.92) \quad = 1229.07\,\text{N}$$

6. Determine the torque to be applied in a differential screw jack to lift a load of 10 kN. Given that pitch of the outer and inner threads are 10 mm and 6 mm respectively. The mean radius of the outer thread is 30 mm and that of inner thread is 20 mm, coefficient of friction for both threads is 0.1.

We know, $\tan\varphi_s = \mu_s \quad \Rightarrow \varphi_s = \tan^{-1}(0.1) = 5.71^0$

Outer screw: Assuming single-threaded screw for outer spindle,

$$L_1 = p_1 = 10\,\text{mm}$$

$$\therefore \theta_1 = \tan^{-1}\left[\frac{L_1}{2\pi r_1}\right] \quad = \tan^{-1}\left[\frac{10}{2\pi(30)}\right] = 3.04^0$$

$$\tan(\varphi_s + \theta_1) = 0.154$$

Inner screw: Assuming single-threaded screw for inner spindle,

$$L_2 = p_2 = 6\,\text{mm}$$

$$\therefore \theta_2 = \tan^{-1}\left[\frac{L_2}{2\pi r_2}\right]$$

$$= \tan^{-1}\left[\frac{6}{2\pi(20)}\right] = 2.73^0$$

Hence, $\tan(\varphi_s + \theta_2) = 0.148$

Therefore, the torque to be applied to raise a load of 10 kN is given as:

$$\tau = r_1.W\tan(\varphi_1 + \theta_1) + r_2.W\tan(\varphi_2 + \theta_2)$$

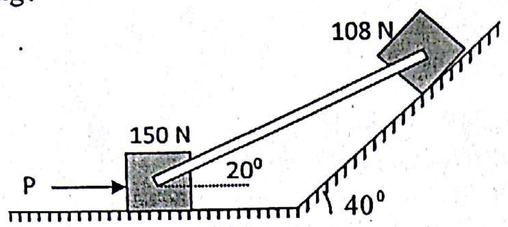
$$= W[r_1.\tan(\varphi_1 + \theta_1) + r_2.\tan(\varphi_2 + \theta_2)]$$

$$= 10 \times 10^3[(0.03 \times 0.154) + (0.02 \times 0.148)]$$

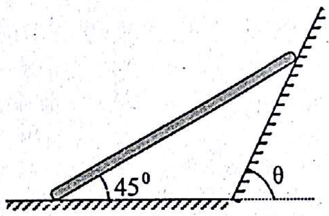
$$= 75.8\,\text{N.m}$$

ASSIGNMENT QUESTIONS

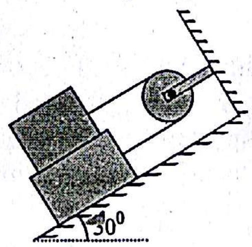
1. A 108 N block is held on a 40^0 incline by a bar attached to a 150 N block on a horizontal plane shown in figure. The bar which is fastened by smooth pins at each end is inclined 200 to the horizontal. The coefficient of friction between each block and its plane is 0.325. For what horizontal force P, applied to 150 N block will motion to the right be impending?



2. A uniform bar of length *l* rests on a rough horizontal floor and a rough inclined wall as shown in figure. Determine the inclination of the wall with respect to the horizontal for which equilibrium can be maintained. Take coefficient of friction for all contact surfaces to be 0.3.

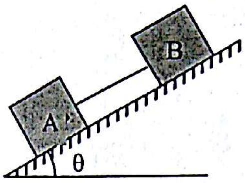


3. A block weighing 100 N is resting on a rough plane inclined at 20^0 to the horizontal. It is acted upon by a force of 50 N directed upward at angle of 14^0 above the plane. Determine the friction, if the block is about to move up the plane, determine the co-efficient of friction.
4. A ladder 5 m long and of 250 N weight is placed against a vertical wall in a position where its inclination to the vertical is 30^0. A man weighing 800 N climbs the ladder. At what position will he induce slipping? The co-efficient of friction for both the contact surfaces of the ladder viz. with the wall and the floor is 0.2.
5. Determine the weight W of the upper block to prevent downward motion of the lower block of A of mass 300 kg as shown in figure E.6.32. The coefficient of friction between all the contact surfaces is 0.25. Assume the pulley to be frictionless.

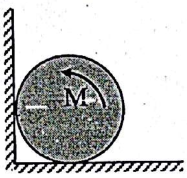


6. A body weighing 70 kN rests in equilibrium on a rough plane whose slope is 30^0. The plane is raised to a slope of 45^0. What is the force applied to the body parallel to the plane that will support the body on the plane.
7. Block A weighs 200 N and block B weighs 120 N. They rest on a rough inclined plane and are connected by a short piece of string as shown in figure below. If the coefficients

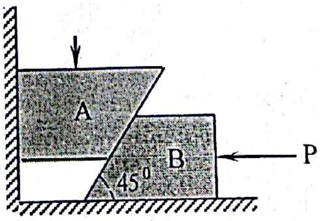
of friction between the blocks and the plane are respectively $\mu_A = 0.25$ and $\mu_B = 0.3$, determine (a) the angle of inclination of the plane for which sliding will impend and (b) tension in the string.



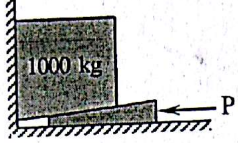
8. A horizontal bar 10 m long and of negligible weight rests on rough inclined plane as shown in figure. If the angle of friction is 15^0, how close to B may the 200N force be applied before motion impends?
9. A block is lying over a 10^0 wedge on a horizontal floor and leaning against a vertical wall and weighing 1500N is to be raised by applying a horizontal force to the wedge. Assuming coefficient of friction between all the forces in contact to be 0.3, determine the minimum horizontal force to be applied to raise the block.
10. Determine the moment required to be applied to the cylinder of weight W shown in figure to cause motion to impend. The coefficient of friction between all contact surfaces is μ.



11. A body weighing 70kN rests in equilibrium on a rough plane whose slope is 30^0. The plane is raised to a slope of 45^0. What is the force applied to the body parallel to the plane that will support the body on the plane?
12. The outer and inner diameters of the spindle in a screw jack are 60 cm and 40 cm respectively. If the screw is single threaded, and the coefficient of friction between screw and nut is 0.2, determine i) the torque required to raise a load of 20 kN and ii) to lower the same load. Also, determine the efficiency of the screw jack.
13. A steam valve of 15 cm diameter has a pressure of 3 MPa acting on it. If it is closed by means of square threaded screw of 60 mm external diameter and pitch of 6 mm. Determine the torque exerted on the handle of the valve. The coefficient of friction is 0.2.
14. The distance between adjacent threads of a double-threaded screw jack is 10 mm; mean radius is 60 mm; coefficient of friction is 0.10. What load can be raised by exerting a moment of 100 N-m?
15. In the figure, block A supports a weight of 4000 N and it is to be prevented from sliding down by applying a horizontal force P on wedge B. If the coefficient of friction at all surfaces of contact is 0.2, determine the smallest force P required to maintain equilibrium. Assume the block and wed; 4000 N negligible weight.



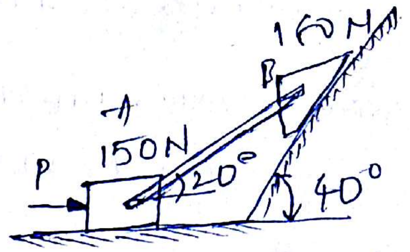
16. A 15^0 wedge is used to raise a 1000 kg block as shown in figure. Determine the horizontal force P that must be applied on the wedge to raise it. Assume the angle of friction at all contact surfaces to be 0.2.



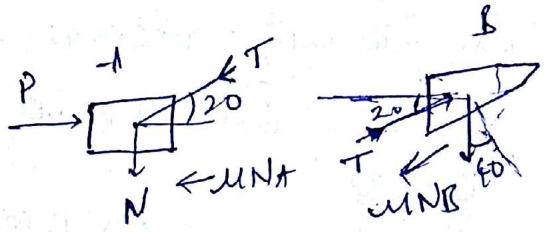
17. Two equal bodies A and B of weight 'W' each are placed on a rough inclined plane. The bodies are connected by a light string. If $\mu_A = 1/2$ and $\mu_B = 1/3$, show that the bodies will be both at the point of impending motion when the plane is inclined at $\tan^{-1}(5/12)$.
18. If the ratio of the greatest to the least force which acting parallel to a rough inclined plane can support a weight on it is equal to that of the weight to the pressure on the plane, then prove that the coefficient of friction is $\tan\alpha.\tan^2\frac{\alpha}{2}$, where α is the inclination of the plane to the horizontal?
19. The mean radius of the screw in a screw jack is 50 mm and the pitch of the thread is 16 mm. If the coefficient of friction between screw and nut is 0.2, determine the torque required to raise a load of 1kN and the efficiency of the screw jack. Also, determine the torque required to lower the load.
20. A pull of 100 N, inclined at 30 degrees to the horizontal plane, is required just to have a body placed on a rough horizontal plane. But the push required to move the body is 125 N. If the push is inclined at 25 degrees to the horizontal, find the weight of the body and co-efficient of friction.
21. An effort of 500 N is required just to move a certain body up on an inclined plane of angle 20 degrees, the force acting parallel to the plane. If the angle of inclination of the plane is made 30 degrees, the effort required again applied parallel to the plane is found to be 650 N. Find the weight of the body and co-efficient of friction.

Friction:

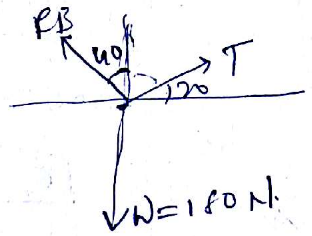
1\.



Required to find the value of P which causes the body B to move in the upward direction



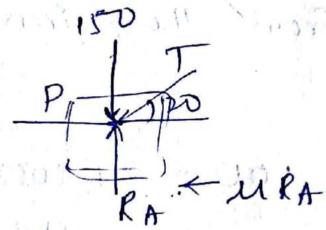
Applying Lamis theorem to B.



$$\frac{T}{\sin(180-40)} = \frac{W}{\sin(110)}$$

$T = 123.12\ N$

take FBD of A

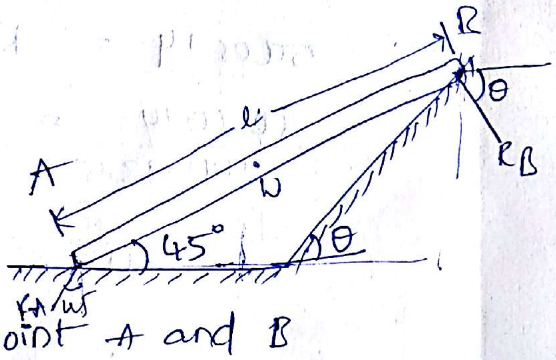


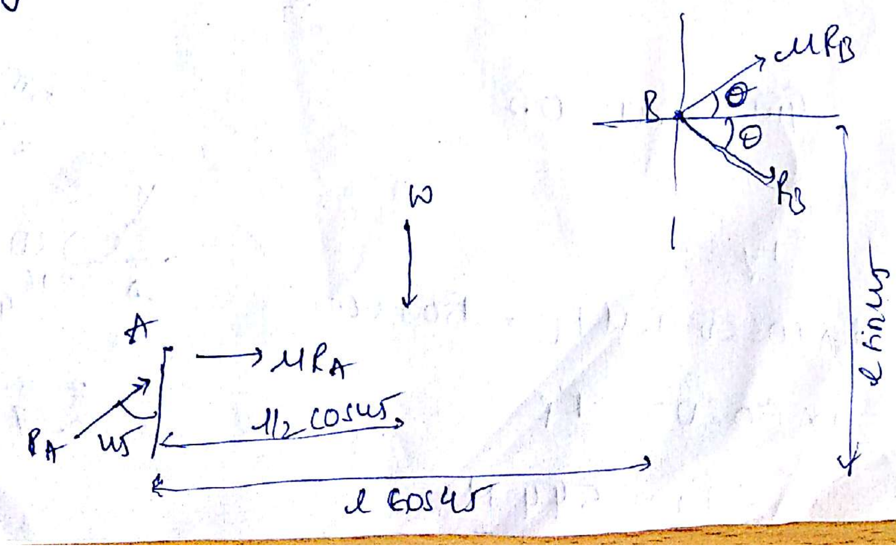
$P = T\cos 20 + \mu\, 150$

$= 164.45\ N$

2\.

let W be the weight of the ladder and R_A and R_B are the Reactions of point A and B respectively.





2

$\Sigma F_y = 0$

$R_A \cos 45 + \mu R_B \sin\theta + R_B \sin\theta = W$

$R_B \cos\theta (0.707) \cos 45 + \mu R_B \sin\theta + R_B \sin\theta = W$

$W = 0.5 \cos\theta\, R_B + 1.3 R_B \sin\theta$ —— ①

$\Sigma F_y = 0$

$R_A \sin 45 + \mu R_A + \mu R_A \cos\theta = R_B \cos\theta$

$R_A = R_B \cos\theta (0.707)$

$\Sigma M_A = 0$

$W \times \frac{l}{2} \cos 45 + \mu R_B \cos\theta \times l \sin 45 = \mu R_B \sin\theta \times l \cos 45 + R_B \cos\theta\, l \sin 45 + R_B \sin\theta \times l \cos 45$

$W = 2\{(\mu+1) R_B \sin\theta + (-\mu) R_B \cos\theta)$ —— ②

① = ②

$2(1.3) R_B \sin\theta + 2(0.7) R_B \cos\theta = 0.5 R_B \cos\theta + 1.3 R_B \sin\theta$

$1.3 R_B \sin\theta = 0.5 R_B \cos\theta - 2 \times 0.7 R_B \cos\theta$

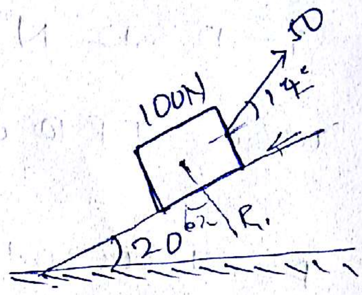
$= -0.9 R_B \cos\theta$

$\tan\theta = \frac{-0.9}{1.3}$

$\boxed{\theta = 55.3^\circ}$

3\. soln

From the given data



$100 = R\cos 20 + P \sin 14°$

$R = 93.5$ N

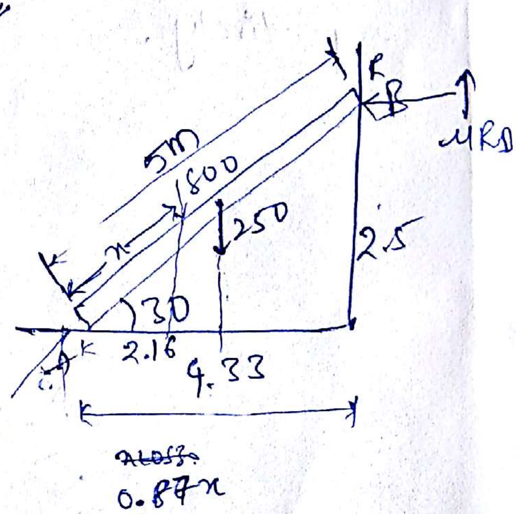
$50 \cos 14 = R \sin 20 + \mu R$

$\frac{50 \cos 14}{\cancel{100}\ 93.5} = \sin 20 + \mu$

$\mu = \frac{50 \cos 14}{93.4} - \sin 20$

$\boxed{\mu = 0.1768}$

4\. Given $\mu = 0.2$



$\Sigma F_y = 0$

$R_A \cos 30 + \mu R_B = 250 + 800$

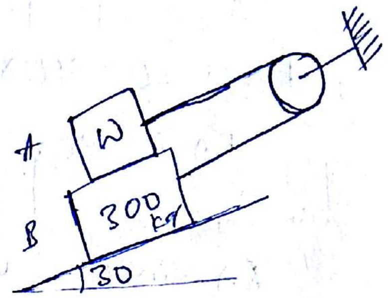
$R_A \sin 30 = R_B$

$\Sigma M_A = 0$

$R_B \times 2.5 + \mu R_B \times 4.33 = 250 \times 2.16 + 800 \times 0.866 x$

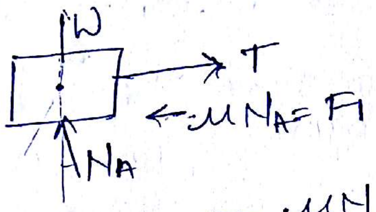
$\boxed{\therefore x = 1.34 \text{ m}}$

5 Soln



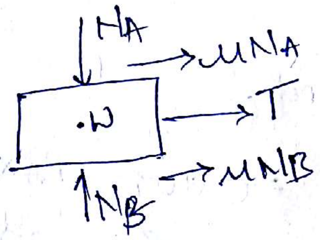
Given $\mu = 0.25$

FBD of A



$T = \mu N_A$

FBD of B



$$W \sin 30 = T + \mu N_A + \mu N_B$$
$$= \mu N_A + \mu N_A + \mu N_B$$
$$= 2\mu N_A + \mu N_B$$

$$5886 = 2 N_A + N_B \quad \text{--- (1)}$$

$$N_A + W \cos\theta = N_B$$
$$N_B - N_A = W \cos 30$$
$$N_B - N_A = 2548.7 \quad \text{--- (2)}$$
$$2N_B - 2N_A = 5097.4$$

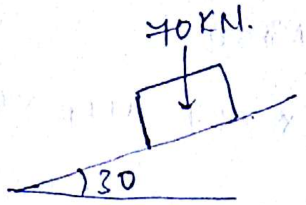
from (1) and (2)

$$N_B + \cancel{2N_A} = 5886$$
$$2N_B - \cancel{2N_A} = 5097.4$$
$$3N_B = 10983.4$$
$$N_B = 3661.1 \text{ N}$$

$$N_A = 725.8 \text{ N}$$

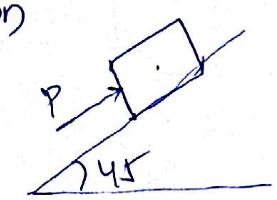
6

Soln



R_A be the reaction of block

μ be the coefficient of friction



$R_A \cos 30 = 70 \times 10^3$

$R_A = 80.82$ KN

$R_A \sin 30 = \mu R_A$

$\mu = 0.5$

II nd case.

$70 \times 10^3 = R_A \times \cos 45$

$R_A = 99$ KN.

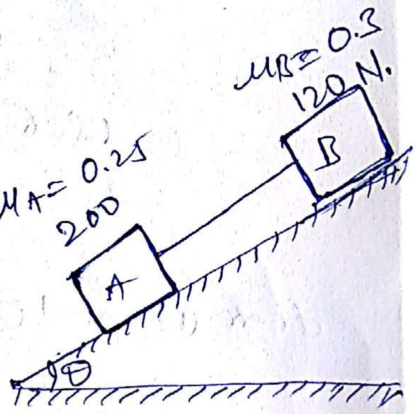
$P + \mu R_A = R_A \sin 45$

$P = R(\sin 45 - \mu)$

$= 20.5$ KN

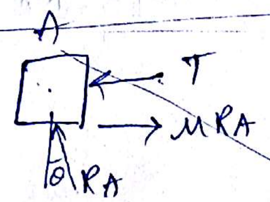
7

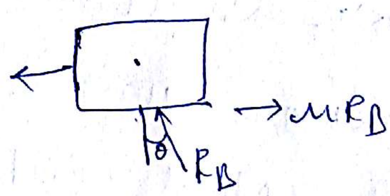
Sol



Required to find θ

FBD of A





$R_A \cos\theta = 200$

$R_A \sin\theta + T = \mu R_A$

$T = \mu R_A - R_A \sin\theta$

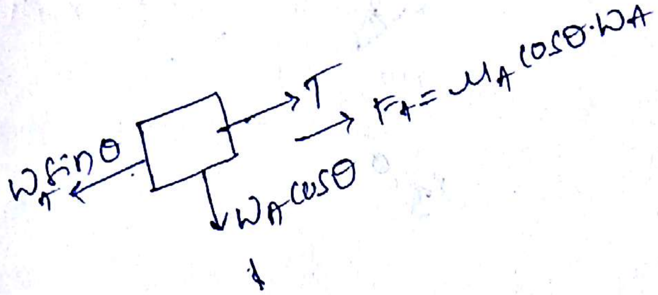
$=$

$R_B \cos\theta = 120$

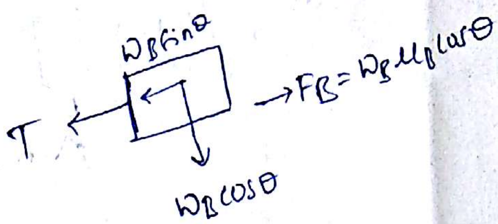
$T + R_B \sin\theta = \mu R_B$

$T = \mu R_B - R_B \sin\theta$

FBD of A



FBD of B



$T + \mu_A W_A \cos\theta = W_A \sin\theta$

$T = W_A(\sin\theta - \mu_A \cos\theta)$

$T + W_B \sin\theta = W_B \mu_B \cos\theta$

$T = W_B(\mu_B \cos\theta - \sin\theta)$

$W_A(\sin\theta - \mu_A \cos\theta) = W_B(\mu_B \cos\theta - \sin\theta)$

$\overset{5}{2\cancel{00}}(\sin\theta - 0.25\cos\theta) = \overset{3}{\cancel{150}}(0.3\cos\theta - \sin\theta)$

$5\sin\theta - 1.25\cos\theta = 0.9\cos\theta - 3\sin\theta$

$8\sin\theta = \cancel{2.125}\ 2.15\cos\theta$

$\tan\theta = \frac{2.15}{8}$

$\theta = \tan^{-1}(0.2687)$

$= 15.04°$

$T = W_A(\sin\theta - \mu_A \cos\theta)$

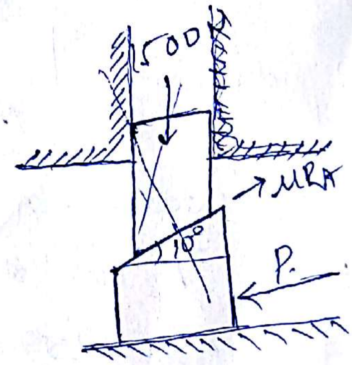
$= 200(\sin(15.04) - 0.25\cos(15.04))$

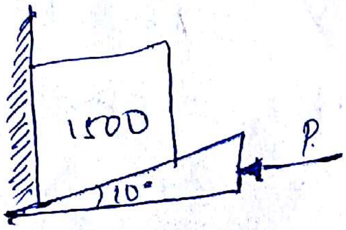
$= 3.62$ N.

9.

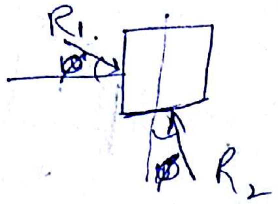
Sol:

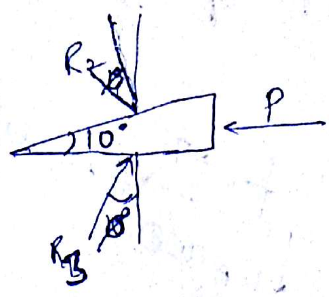
Required to find horizontal force P.





FBD of Body





$\mu = \tan(\phi)$

$\tan^{-1}(\mu) = \phi$

$\phi = 16.7°$

$\Sigma F_x = 0$

$R_1 \cos(16.7) = R_2 \sin(10+16.7)$

$= R_2 \sin 26.7$

$R_2 = 2.132 R_1$

$\Sigma F_y = 0$

$R_1 \sin(16.7°) + 1500 = R_2 \cos(10+16.7)$

$= R_2(\cos 26.7)$

$= R_2(0.8934)$

$= 2.132 R_1 (0.8934)$

$= 1.905 R_1$

$R_1(1.905 - 0.2874) = 1500$

$\therefore R_1 = \frac{1500}{1.6176} = 927.3\,N$

and $R_2 = 1977\,N$

In wedge condition

$\Sigma F_y = 0 \quad R_3 \cos(16.7°) = R_2 \cos(10+16.7°)$

$R_3(0.9578) = R_2 \times 0.8934$

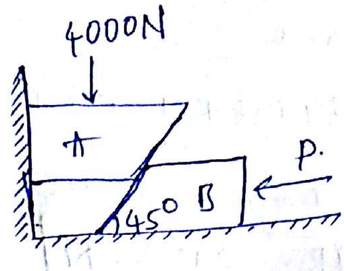
$R_3 = 1844\,N$

$\Sigma F_x = 0 \quad P = R_2 \sin(10+16.7°) + R_3 \sin 16.7°$

$= 1418.3\,N$

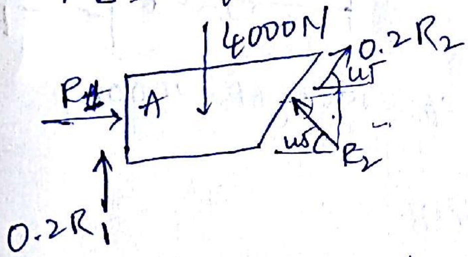
15
Soln

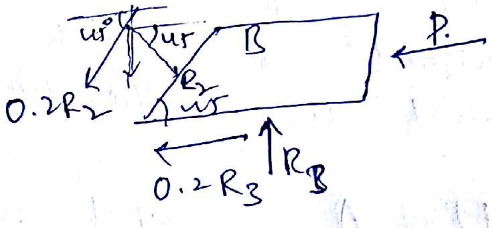
Given $\mu = 0.2$.



Required to find smallest force P to maintain equillibrium. i.e the block is about to move to the left.

FBD of A





Equillibrium of A

$\Sigma F_y = 0$

$0.2R_1 + R_2 \sin 45 + 0.2R_2 \sin 45 = 4000$

$0.2R_1 + R_2(\sin 45 + 0.2 \sin 45) = 4000$

$0.2R_1 + 0.8R_2 = 4000$

$R_1 + 4.24R_2 = 20{,}000$ —①

$\Sigma F_x = 0$

$R_1 + R_2 \cos 45 = 0.2R_2 \cos 45$

$R_1 + 0.5656 R_2 = 0$ —②

from ① and ②

$(3.67) R_2 = 20{,}000$

$\boxed{R_2 = 5443.1 \text{ N}}$

Equillibrium of B:

$\Sigma F_y = 0 \Rightarrow R_3 = R_2 \sin 45 + 0.2 R_2 \sin 45$

$R_3 = 4618.7$ N

$\Sigma F_x = 0$

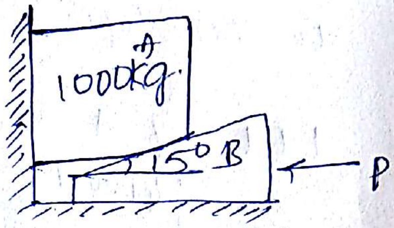
$P + 0.2R_3 + 0.2R_2\cos 45 = R_2\cos 45$

$P = 0.8R_2\cos 45 - 0.2R_3$

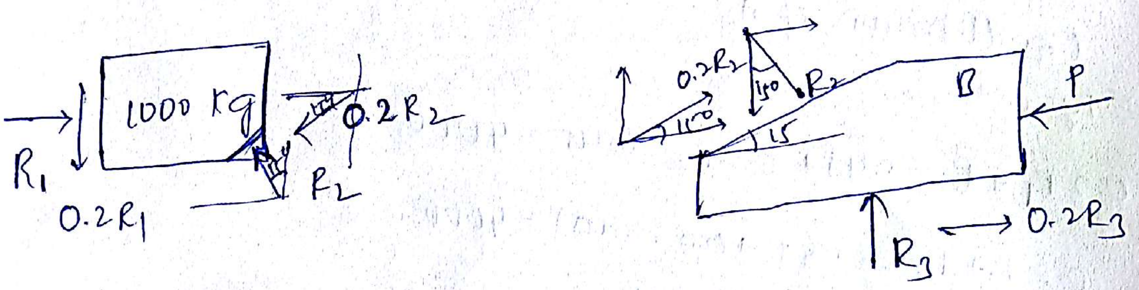
$\boxed{P = 2155.3N}$

16. Soln

Given wedge angle $15°$
and $\mu = 0.2$
Required to find the horizontal force P is req applied to rise the block of weight 1000kg



FBD of A and wedge B



Equillibrium of A

$\Sigma F_y = 0$

$1000 + 0.2R_1 + 0.2R_2\sin 15° = R_2\cos 15°$

$0.965R_2 - 0.051R_2 - 0.2R_1 = 1000$

$0.913R_2 - 0.2R_1 = 1000$

$4.566R_2 - R_1 = 5000 \quad —(1)$

$\Sigma F_x = 0$

$R_1 = 0.2R_2\cos 15° + R_2\sin 15°$

$= 0.452R_2 \quad —(2)$

(2) in (1)

$4.113R_2 = 5000$

$\boxed{R_2 = 1215.3\ N}$

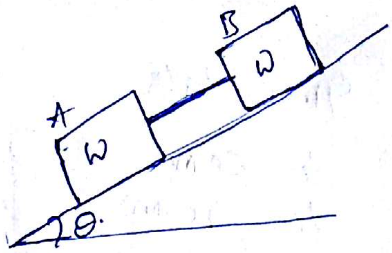
12

Sub

Given

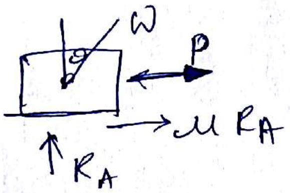
$\mu_A = 0.5$

$\mu_B = 0.33.$

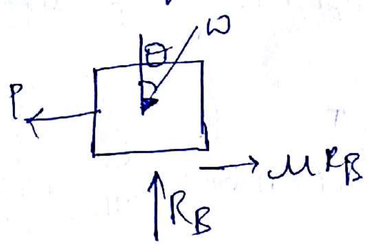


Required to show that the angle $\theta = \tan^{-1}(5/12)$ when the bodies are ready to move on inclined plane

FBD of A



FBD of B



$\Sigma F_x = 0$

$P + \mu R_A = W\sin\theta$

$\cancel{= \mu W\cos\theta + W\sin\theta}$

$\cancel{= W(\mu\cos\theta + \sin\theta)}$

$P = W(\sin\theta - \mu_A\cos\theta)$ — (1)

$P = \mu R_B - W\sin\theta$

$= \mu W\cos\theta - W\sin\theta$

$= W(\mu\cos\theta - \sin\theta)$ — (2)

from (1) and (2)

$\cancel{W(\mu_A\cos\theta - \sin\theta)} = \cancel{W(\mu_B\cos\theta - \sin\theta)}$

$\mu_A\cos\theta \cancel{= \mu_B\cos\theta}$

$\cancel{W}(\sin\theta - \mu_A\cos\theta) = (\mu_B\cos\theta - \sin\theta)\cancel{W}$

$2\sin\theta = (\mu_B + \mu_A)\cos\theta$

$\tan\theta = \frac{1}{2}\left(\frac{1}{2} + \frac{1}{3}\right)$

$\tan\theta = \frac{1}{2}\left(\frac{5}{6}\right)$

$\boxed{\theta = \tan^{-1}(5/12)}$

12 Soln

Given data

$d_o = 60$ mm $r_o = 30$ mm $=$ ~~300 mm~~ $= 0.03$ m

$d_i = 40$ mm $r_i = 20$ mm $=$ ~~200 mm~~ $= 0.02$ m

$\mu = 0.2$ $\tan\phi = 0.2$ $\phi = 11.309$

$W = 20$ KN.

Required to find

(i) torque required to raise the load

(ii) torque required to lower the load

(iii) Efficiency of screw jack.

W.K.T

$$\tan\theta = \frac{l}{2\pi r}$$

$l =$ lead $= p$ for single thread

~~r = mean radius~~ $= 2(OR - IR)$

$= 2(0.3 - 0.2)$

$=$ ~~0.1 m~~ $= 0.02$ m

$r =$ mean radius $= \frac{0.03 + 0.02}{2}$

$= 0.025$ m

$$\tan\theta = \frac{l}{2\pi r} = \frac{0.02}{2 \times \pi \times 0.025} = 0.127$$

$\theta =$ ~~32.48~~ $= 7.256$

We also know that

$T_{(raise)} = W r (\tan\phi + \theta)$

$= 20 \times 10^3 \times 0.025 \tan(7.256 + 11.309)$

$= 167.9$ N.m

$T_{lower} = 20 \times 10^3 \times 0.025 \tan(11.309 - 7.256)$

$= 35.4$ N.m

$$\eta = \frac{\tan(7.256)}{\tan(11.309 + 7.256)} = 37.9\%$$

18

Soln

$d_r = 15 cm = 0.15 m$ $A = 0.017 m^2$

$p = 3 \times 10^6 N/m^2$

$Do = 60 mm$ $load = P \times A$

$= \cancel{53 N} = 51 \times 10^3$

$p = 6 mm$

$\mu = 0.2$ $\tan\phi = 0.2$ $\phi = 11.309$

Required to find torque exerted on the handle

W.K.T $T = Wr \tan(\phi + \theta)$

$\tan\theta = \frac{L}{2\pi r}$

$p = 0.006 m$

$ID = OD - P/2$

$= 60 - 6/2$

$= 57 mm$

$R_o = 30 mm = 0.03 m$

$R_i = 28.5 mm = 0.0285 m$

$r_{mean} = 0.029$

$\tan\theta = \frac{0.006}{2 \times \pi \times 0.029}$

$\theta = 1.869°$

torque required $T = 51 \times 10^3 \times 0.029 \times \tan(11.309 + 1.869)$

$\boxed{T = 346.29 \ N \cdot m}$

14 soln

Given data

$p = 5\,mm$

$r_{mean} = 60\,mm$

$\mu = 0.10$ $\phi = 5.710^\circ$

$T = 100\,N.m$

Required to find $W = ?$

$$W = \frac{T}{r\tan(\phi+\theta)}$$

$$\theta = \tan^{-1}\left(\frac{2\times5}{2\times\pi\times60}\right)$$

$$= 1.519^\circ$$

$$W = \frac{100}{0.06\tan(5.710+1.519)}$$

$$= 10.13\,KN$$

19 soln

Given

$r_{mean} = 50\,mm$

$p = 16\,mm$

$\mu = 0.2$ $\phi = 11.309$

$W = 1\times10^3\,N$

Required to find $T_{raise}, T_{lower}, \eta$:

$$\theta = \tan^{-1}\left(\frac{16}{2\times\pi\times50}\right)$$

$$= 3^\circ$$

$$\eta = \frac{\tan 3}{\tan(14.309)}$$

$$= 20.1\,\%$$

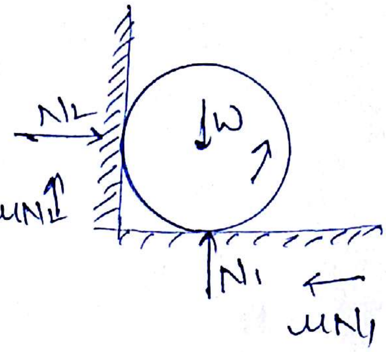
$$T_{raise} = 10^3\times0.05\times\tan(11.309+3)$$

$$= 12.755\,N.m$$

$$T_{lower} = 10^3\times0.05\times\tan(11.309-3)$$

$$= 7.3\,N.m$$

10
Soln



$$N_1 + \mu N_2 = W$$

$$N_2 = \mu N_1$$

$$N_1 + \mu^2 N_1 = W$$

$$\Rightarrow N_1 = \frac{W}{1+\mu^2}$$

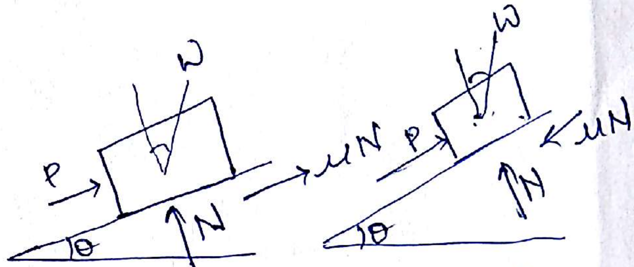
$$N_2 = \frac{\mu W}{1+\mu^2}$$

$$M = (\mu N_1) r + \mu N_2 (r)$$

$$= \mu r (N_1 + N_2)$$

$$= \frac{\mu W r}{1+\mu^2} (1+\mu)$$

18
Soln



case I   case II

case I :

$$N = W\cos\theta.$$

$$P + \mu N = W\sin\theta$$

$$P = W(\sin\theta - \mu\cos\theta)$$

case II

$$N = W\cos\theta$$

$$P = W\sin\theta + \mu W\cos\theta$$

$$= W(\sin\theta + \mu\cos\theta)$$

from given condition

$$\frac{\cancel{W}(\sin\theta + \mu\cos\theta)}{\cancel{W}(\sin\theta - \mu\cos\theta)} = \frac{\cancel{W}}{W\cos\theta}$$

$$\sin\theta\cos\theta + \mu\cos^2\theta = \sin\theta - \mu\cos\theta$$

$$\mu\cos^2\theta + \mu\cos\theta = \sin\theta - \sin\theta\cos\theta$$

$$\mu\cos\theta(1+\cos\theta) = \sin\theta(1-\cos\theta)$$

$$\mu = \frac{\sin\theta}{\cos\theta}\left[\frac{1-\cos\theta}{1+\cos\theta}\right] = \tan\theta\tan^2\theta/2$$

Equillibrium of B.

$\Sigma F_y = 0$

$R_3 + 0.2 R_2 \sin 15° = R_2 \cos 15°$

$\boxed{R_3 = 1111.1 \text{ N.}}$

$\Sigma F_x = 0$

$P = 0.2 R_3 + 0.2 R_2 \cos 15 + R_2 \sin 15°$

$= \cancel{771.5} \text{ kg}$

$= 7.6 \text{ KN}$

21) Soln

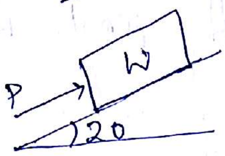
From Given data

I case — II case

$\theta = 20°$ — $\theta = 30$

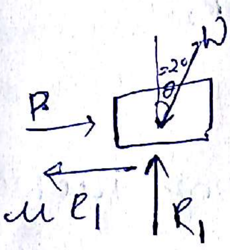
$P = 500 N$ — $P = 650 N$

I case

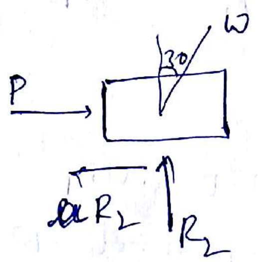


II case

take case i



take case II



$R_1 = W \cos 20$

$P = W \sin\theta + \mu R_1$

$= W \sin 20 + \mu W \cos 20$

$\cancel{= W(\sin 20 + \mu \cos 20)}$

$\frac{P - W \sin 20}{W \cos 20} = \mu$

$R_2 = W \cos 30$

$P = W \sin 30 + \mu R_2$

$= W \sin 30 + \mu W \cos 30$

$\frac{P - W \sin 30}{W \cos 30} = \mu$

$\frac{500 - W \sin 20}{\cancel{W} \cos 20} = \frac{650 - W \sin 30}{\cancel{W} \cos 30}$

$(500 - W \sin 20) \cos 30 = (650 - W \sin 30) \cos 20$

$(500 - W \sin 20)\, 0.921 = (650 - W \sin 30)$

$W(\sin 30 - 0.921 \sin 20) = 650 - 460.8$

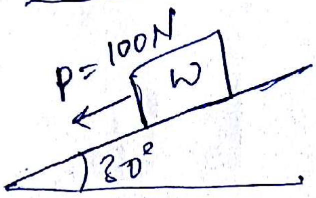
$= 189.2$

$\boxed{\therefore W = 1022.7\ N}$

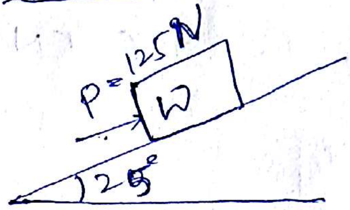
20
Soln

From given data

Case I :-

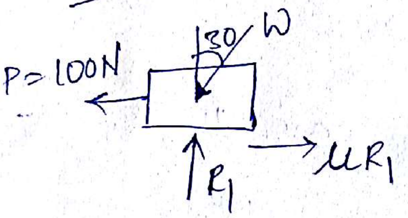


Case II :-

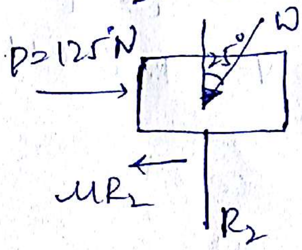


Required to find weigh of body and μ.

I case



II case



$\Sigma F_x = 0$

$P + W \sin 30 = \mu R_1$

~~$P = \mu W \cos 30 - W \sin 30$~~

$$\frac{100 + W \sin 30}{W \cos 30} = \mu \quad \text{——} (1)$$

$P = W \sin 25 + \mu W \cos 25$

$$\frac{125 - W \sin 25}{W \cos 25} = \mu \quad \text{——} (2)$$

from (1) and (2)

$$\frac{100 + W \sin 30}{\cancel{W} \cos 30} = \frac{125 - W \sin 25}{\cancel{W} \cos 25}$$

$1.04(100 + W \sin 30) = 125 - W \sin 25$

$W(\sin 30 + \sin 25) = 25$

~~Belt Friction~~

① A rope making $1\frac{1}{4}$ turns around a stationary horizantal drum is used to support a heavy weight. If ~~the~~ $\mu = 0.4$ what weight can be supported by exerting a 50 N force at the other end of the rope?

Sol)

$$\frac{T_2}{T_1} = e^{\mu\theta}$$

$$T_2 = e^{0.4 \times 2.5 \times \pi} \times T_1$$

$$= 50 \times e^{0.4 \times 2.5 \times \pi}$$

$$\boxed{T_2 = 1157 N}$$

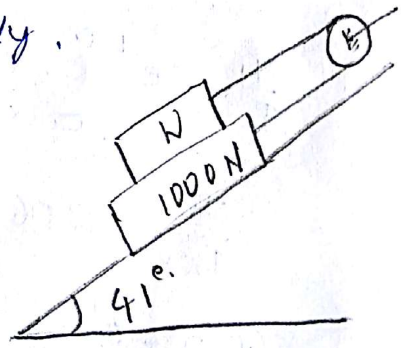
Given $1\frac{1}{4}$ turns

$= 360 + 90$

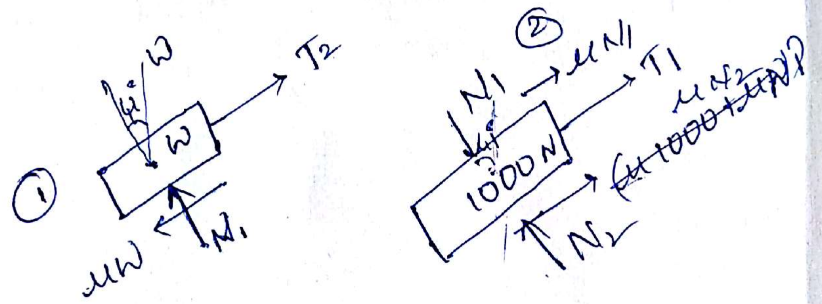
$= 450 \times \frac{\pi}{180}$ radians

$= 2.5 \times \pi$ rad.

2 In figure, the coefficient of friction is 0.30 between the rope and the fixed drum and between all surfaces in contact. Determine the minimum weight W to prevent down plane motion of the 1000 N. body.



Giv



For block ①

$T_2 = W\sin\theta + \mu W\cos\theta$

$= W(\sin 41 + 0.3\cos 41)$

$= W(0.84)$ N

For block ②

$T_1 + \mu(N_1 + N_2) = 1000\sin 41$

$T_1 + \mu(W\cos\theta + N_2) = 1000 \cancel{*} \sin 41$

$\cancel{W\cos\theta + N_2 = -2000}$ N

$N_2 - N_1 = 1000\cos 41°$

$N_2 = 1000\cos 41 + W\cos 41$

$= (1000 + W) \cancel{*} 0.8$

$T_1 + \mu(W\cos\theta + 1000\cos 41 + W\cos\theta) = 1000\sin 41$

$T_1 = 1000\sin 41 - \mu 1000\cos 41 - 2\mu W\cos 41$

$= 360.5 - 0.5W$ N

$\frac{T_1}{T_2} = e^{\mu\theta}$

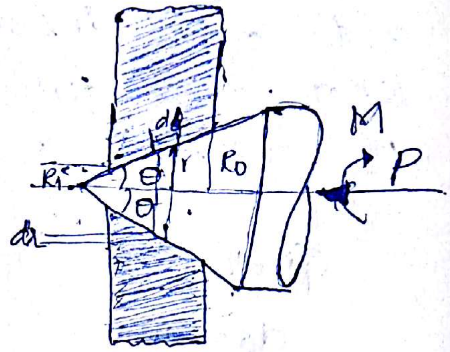
$= e^{0.3 \times 180 \times \frac{\pi}{180}}$

$\frac{T_1}{T_2} = 2.56$

$360.5 - 0.5W = (2.56)(W)(0.84)$

$W = \cancel{34}\ 136.$ N

(2) Assuming a uniform pressure and constant f determine the torque M to rotate the pivot bearing shown in figure

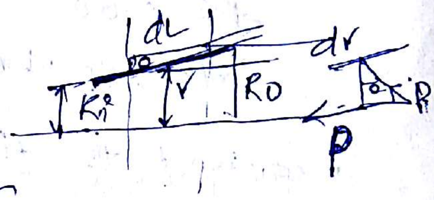


Soln. For uniform pressure

$\delta P = p \times$ Area of ring

$= p \times 2\pi r \, dL$

$\delta P = p \times 2\pi r \dfrac{dr}{\sin\theta}$



$P = \int_{Ri}^{Ro} \delta P \sin\theta$

$= \int_{Ri}^{Ro} p \times 2\pi r \dfrac{dr}{\cancel{\sin\theta}} \times \cancel{\sin\theta}$

$\sin\theta = \dfrac{dr}{dL}$

$dL = \dfrac{dr}{\sin\theta}$

$= 2\pi p \int_{Ri}^{Ro} r \, dr$

$= \cancel{2}\pi p \dfrac{(Ro^2 - Ri^2)}{\cancel{2}}$

$P = \pi p (Ro^2 - Ri^2)$

take $\boxed{\mu = f}$

$p = \dfrac{P}{\pi (Ro^2 - Ri^2)}$ N/m²

Friction force $F = f \times \delta P$

$= \mu \times 2\pi p r \dfrac{dr}{\sin\theta}$

Torque $= F \times r = 2\pi p \mu \, r^2 \dfrac{dr}{\sin\theta}$

total torque $T = \int_{Ri}^{Ro} 2\pi p \mu \dfrac{r^2 \, dr}{\sin\theta}$

Disk Friction

① An automobile clutch consists of a single annular facing of 1m outside diameter and 0.6m inside diameter. If $f = 0.60$ What axial force on the clutch is required to transmit a torque of 250 N-m?

Soln

Given.

$d_o = 1\ m$ $\qquad r_o = 0.5m$

$d_i = 0.6m$ $\qquad r_i = 0.3m$

$f = 0.60$

$M = 250\ N\text{-}m$

Required to find load/axial force P.

WKT $\quad M = fPR \quad$ ~~for~~

$$M = fP\,\frac{2}{3}\,\frac{r_o^3 - r_i^3}{r_o^2 - r_i^2} \qquad\qquad M = f.P\left(\frac{r_o + r_i}{2}\right)$$

(uniform pressure) $\qquad\qquad$ (uniform wear)

$$250 = 0.60 \times P\left(\frac{2}{3}\right)\left(\frac{0.5^3 - 0.3^3}{0.5^2 - 0.3^2}\right); \quad 250 = 0.60 \times P \times \frac{0.5 + 0.3}{2}$$

$$\frac{250 \times 3}{2} = 0.60 \times P\left(\frac{0.098}{0.16}\right); \quad 500 = P \times 0.60 \times 0.8$$

$$\boxed{P = 1020.4\ N} \qquad ; \qquad \boxed{P = 1041.6\ N}$$

$$= \frac{2\pi\mu p}{\sin\theta}\int_{Ri}^{Ro} r^2 dr$$

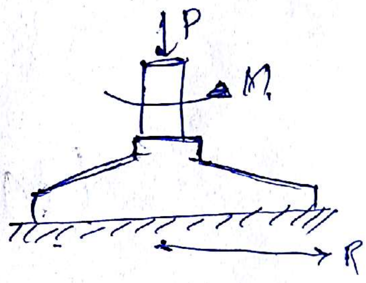
$$= \frac{2\pi\mu p}{\sin\theta}\frac{(Ro^3 - Ri^3)}{3}$$

$$= \frac{2}{3}\frac{\cancel{\pi}\mu}{\sin\theta}\frac{P}{\cancel{\pi}(Ro^2 - Ri^2)}(Ro^3 - Ri^3)$$

$$= \frac{\mu P}{\sin\theta}\cdot\frac{2}{3}\frac{(Ro^3 - Ri^3)}{(Ro^2 - Ri^2)}$$

$$\boxed{M = \frac{2\mu P}{3\sin\theta}\left(\frac{Ro^3 - Ri^3}{Ro^2 - Ri^2}\right)}$$

3. In the flat sanding disk shown in figure Find the Torque M required to rotate the disk at a constant f and at constant speed and the contact pressure varies linearly from $\frac{1}{2}P_o$ at the outside to P_o at the center

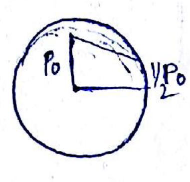


P – axial force

p – pressure intensity

Non uniform pressure

and pressure varying linearly



p

$y = mx + c$ —— (1)

at $x = 0$ $P_o = c$

at $x = R$ $\frac{1}{2}P_o = mR + P_o$

$$m = \frac{-P_o}{2R}$$

from (1) $p = \frac{-P_o x}{2R} + P_o$

$$\boxed{p = P_0\left(1 - \frac{r}{2R}\right)}$$

P_0 - pressure

P - axial

the total torque equation

$$M = \int 2\pi\mu p r \, r \, dr$$

$$= \int_0^R 2\pi\mu p r^2 dr$$

$$= 2\pi\mu \int_0^R p_0\left(1 - \frac{r}{2R}\right) r^2 dr$$

$$= 2\pi\mu p_0 \int_0^R \left(r^2 - \frac{r^3}{2R}\right) dr$$

$$= 2\cancel{\pi}\mu \frac{P}{\cancel{\pi} R^2} \left(\frac{R^3}{3} - \frac{R^{4\,3}}{2 \times \cancel{R} \times 4}\right)$$

$$= \frac{2\mu P}{R^2}\left(\frac{R^3}{3} - \frac{R^3}{8}\right)$$

$$= \frac{2\mu P}{R^2}\left(\frac{8R^3 - 3R^3}{24}\right)$$

$$= \frac{\cancel{2}\mu P}{\cancel{R^2}} \frac{5R^3}{\cancel{24}\,12}$$

$$\boxed{M = \frac{5}{12}\mu P R}$$

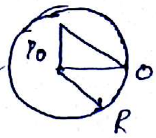
Book answer $M = \frac{5}{8}\mu P R$

(Further to be verified)

Ans

Given that

the pressure varying logarithmically



$y = mx^2 + c$

at $x=0$ $\quad p_0 = c$

at $x=R$ $\quad 0 = mR^2 + p_0$

$\quad m = \frac{-p_0}{R^2}$

p – pressure

P – axial force

$$p = \frac{-p_0}{R^2}x^2 + p_0$$

$$= p_0\left(1 - \frac{r^2}{R^2}\right)$$

$$= \frac{P}{\pi R^2}\left(1 - \frac{r^2}{R^2}\right)$$

WKT

total Torque developed

$$M = \int_0^R 2\pi\mu r p \, r\,dr$$

$$= \int_0^R 2\pi\mu r^2 p\,dr$$

$$= 2\pi\mu \int_0^R \frac{P}{\pi R^2}\left(1 - \frac{r^2}{R^2}\right) r^2\,dr$$

$$= \frac{2\cancel{\pi}\mu P}{\cancel{\pi} R^2}\int_0^R \left(r^2 - \frac{r^4}{R^2}\right)dr$$

$$= \frac{2\mu P}{R^2}\left(\frac{R^3}{3} - \frac{R^5}{5R^2}\right)$$

$$= \frac{2\mu P}{R^2}\left(\frac{R^3}{3} - \frac{R^3}{5}\right)$$

$$= \frac{2\mu P}{\cancel{R^2}}\,\frac{2R^{\cancel{3}}}{15}$$

$$\boxed{M = \frac{4}{15}\mu P R}$$

Book answer $\frac{8}{15}\mu P R$ (To be verified)