ENGINEERING MECHANICS COURSE MATERIAL

PREPARED BY

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UNIT-I

Introduction to Mechanics: Basic Concepts, system of Forces Coplanar Concurrent Forces -Components in Space Resultant - Moment of Forces and its Application - Couples and Resultant of Force Systems. Equilibrium of system of Forces: Free body diagrams, Equations of Equilibrium of Coplanar Systems.

UNIT 1

INTRODUCTION

.1. SCIENCE

In this modern age, the word 'science' has got different meanings for different people. An ordinary man takes it as 'something' beyond his understanding, whereas others may take it as 'mysteries of research' which are understood only by a few persons working amidst complicated apparatus in a laboratory. A non-scientist feels that it is a 'subject' whose endeavour is aimed to improve the man's life on the earth. A business executive has the idea that it is 'something' which solves our day to day manufacturing and quality control problems, so that the nation's economic prosperity keeps on improving.

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this sense, the subject of science does not, necessarily, has to contribute something to the welfare of the human life, although the man has received many benefits from the scientific investigations.

1.2. APPLIED SCIENCE

Strictly speaking, the world of science is so vast that the present day scientists and technologists have to group the various spheres of scientific activities according to some common characteristics to facilitate their training and research programmes. All these branches of science, still have the common principle of employing observation and experimentation. The branch of science, which co-ordinates the research work, for practical utility and services of the mankind, is known as Applied Science.

1.3. ENGINEERING MECHANICS

The subject of Engineering Mechanics is that branch of Applied Science, which deals with the laws and principles of Mechanics, alongwith their applications to engineering problems. As a matter of fact, knowledge of Engineering Mechanics is very essential for an engineer in planning, designing and construction of his various types of structures and machines. In order to take up his job more skilfully, an engineer must persue the study of Engineering Mechanics in a most systematic and scientific manner.

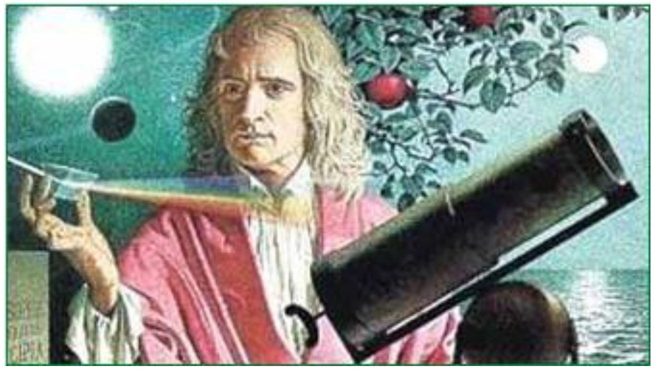
1.4. BEGINNING AND DEVELOPMENT OF ENGINEERING MECHANICS

It will be interesting to know, as to how the early man had been curious to know about the different processes going on the earth. In fact, he used to content himself, by holding gods responsible for all the processes. For a long time, the man had been trying to improve his ways of working. The first step, in this direction, was the discovery of a circular wheel, which led to the use of animal driven carts. The study of ancient civilization of Babylonians, Egyptians, Greeks and Roman reveal the use of water wheels and wind mills even during the pre-historic days.

It is believed that the word 'Mechanics' was coined by a Greek philosopher **Aristotle** (384–322 BC). He used this word for the problems of lever and the concept of centre of gravity. At that time, it included a few ideas, which were odd,

Sir Issac Newton (1643–1727)

unsystematic and based mostly on observations containing incomplete information. The first mathematical concept of this subject was developed by Archimedes (287–212 BC). The story, for the discovery of First Law of Hydrostatics, is very popular even today in the history of the development of Engineering Mechanics. In the normal course, Hieron king of Syracuse got a golden crown made for his use. He suspected that the crown has been made with an adultrated gold. The king liked the design of the crown so much that he did not want it to be melted,in order to check its purity. It is said that the king announced a huge reward for a person, who can check the purity of the crown gold without melting it. The legend goes that Archimedes, a pure mathematician, one day sitting in his bath room tub realised that if a body is immersed in water, its apparent weight is reduced. He thought that the apparent loss of weight of the immersed body is equal to the weight of the liquid displaced. It is believed that without further thought, **Archimedes**jumeped out of the bath tub and ran naked down the street shouting 'Eureka, eureka !' *i.e*. I have found it, I have found it !'



The subject did not receive any concrete contribution for nearly 1600 years. In 1325, Jean Buridan of Paris University proposed an idea that a body in motion possessed a certain impetus *i.e*. motion. In the period 1325–1350, a group of scientists led by the Thomas Bradwardene ofOxford University did lot of work on plane motion of bodies. Leonarodo Da Vinci (1452–1519), a great engineer and painter, gave many ideas in the study of mechanism, friction and motion of bodies on inclined planes. Galileo (1564–1642) established the theory of projectiles and gave a rudimentary idea of inertia. Huyghens (1629–1695) developed the analysis of motion of a pendulum.

As a matter of fact, scientific history of Engineering Mechanics starts with **Sir Issac Newton** (1643–1727). He introduced the concept of force and mass, and gave Laws of Motion in 1686. James Watt introduced the term horse power for comparing performance of his engines. John Bernoulli (1667–1748) enunciated the priciple of virtual work. In eighteenth century, the subject of Mechanics was termed as Newtonian Mechanics. A further development of the subject led to a controversy between those scientists who felt that the proper measure of force should be change in kinetic energy produced by it and those who preferred the change in momentum. In the nineteenth century, many scientists worked tirelessly and gave a no. of priciples, which enriched the scientific history of the subject.

In the early twentieth century, a new technique of research was pumped in all activities of science. It was based on the fact that progress in one branch of science, enriched most of the bordering branches of the same science or other sciences. Similarly with the passage of time, the concept of Engineering Mechanics aided by Mathematics and other physical sciences, started contributing and development of this subject gained new momentum in the second half of this century. Today, knowledge of Engineering Mechanics, coupled with the knowledge of other specialised subjects *e.g*. Calculus, Vector Algebra, Strength of Materials, Theory of Machines etc. has touched its present height. The knowledge of this subject is very essential for an engineer to enable him in designing his all types of structures and machines.

1.5. DIVISIONS OF ENGINEERING MECHANICS

The subject of Engineering Mechanics may be divided into the following two main groups:
1. Statics, and 2. Dynamics.

1.6. STATICS

It is that branch of Engineering Mechanics, which deals with the forces and their effects, while acting upon the bodies at rest.

1.7. DYNAMICS

It is that branch of Engineering Mechanics, which deals with the forces and their effects, while acting upon the bodies in motion. The subject of Dynamics may be further sub-divided into the following two branches :

1. Kinetics, and 2. Kinematics.

1.8. KINETICS

It is the branch of Dynamics, which deals with the bodies in motion due to the application of forces.

1.9. KINEMATICS

It is that branch of Dynamics, which deals with the bodies in motion, without any reference to the forces which are responsible for the motion.

1.10. FUNDAMENTAL UNITS

The measurement of physical quantities is one of the most important operations in engineering. Every quantity is measured in terms of some arbitrary, but internationally accepted units, called *fundamental units.*

All the physical quantities, met with in Engineering Mechanics, are expressed in terms of three fundamental quantities, *i.e.*

1. length, 2. mass and 3. time.

1.11. DERIVED UNITS

Sometimes, the units are also expressed in other units (which are derived from fundamental units) known as *derived units e.g.* units of area, velocity, acceleration, pressure etc.

1.12. SYSTEMS OF UNITS

There are only four systems of units, which are commonly used and universally recognised. These are known as :

1. C.G.S. units, **2.** F.P.S. units, **3.** M.K.S. units and **4.** S.I. units.

In this book, we shall use only the S.I. system of units, as the future courses of studies are conduced in this system of units only.

1.13. S.I. UNITS (INTERNATIONAL SYSTEM OF UNITS)

The eleventh General Conference* of Weights and Measures has recommended a unified and systematically constituted system of fundamental and derived units for international use. This system of units is now being used in many countries.

In India, the Standards of Weights and Measures Act of 1956 (vide which we switched over to M.K.S. units) has been revised to recognise all the S.I. units in industry and commerce.

In this system of units, the †fundamental units are metre (m), kilogram (kg) and second (s) respectively. But there is a slight variation in their derived units. The following derived units will be used in this book :

Density (Mass density)	kg / m^3
Force	N (Newton)
Pressure	N/mm^2 or N/m^2
Work done (in joules)	J = N-m
Power in watts	W = J/s

International metre, kilogram and second are discussed here.

† The other fundamental units are electric current, ampere (A), thermodynamic temperature, kelvin (K) and luminous intensity, candela (cd). These three units will not be used in this book.

1.14. METRE

The international metre may be defined as the shortest distance (at 0°C) between two parallel lines engraved upon the polished surface of the Platinum-Iridium bar, kept at the International Bureau of Weights and Measures at Sevres near Paris.

A bar of platinum - iridium metre kept at a temperature of 0º C.

The standard platinum - kilogram is kept at the International Bureau of Weights and Measures at Serves in France.

1.15. KILOGRAM

The international kilogram may be defined as the mass of the Platinum-Iridium cylinder, which is also kept at the International Bureau of Weights and Measures at Sevres near Paris.

1.16. SECOND

The fundamental unit of time for all the four systems is second, which is 1/(24 × 60 × 60) = 1/86 400th of the mean solar day. A solar day may be defined as the interval of time between the instants at which the sun crosses the meridian on two consecutive days. This value varies throughout the year. The average of all the solar days, of one year, is called the mean solar day.

1.17. PRESENTATION OF UNITS AND THEIR VALUES

The frequent changes in the present day life are facililtated by an international body known as International Standard Organisation (ISO). The main function of this body is to make recommendations regarding international procedures. The implementation of ISO recommendations in a country is assisted by an organisation appointed for the purpose. In India, Bureau of Indian Standard formerly known as Indian Standards Institution (ISI) has been created for this purpose.

We have already discussed in the previous articles the units of length, mass and time. It is always necessary to express all lengths in metres, all masses in kilograms and all time in seconds. According to convenience, we also use larger multiples or smaller fractions of these units. As a typical example, although metre is the unit of length; yet a smaller length equal to one-thousandth of a metre proves to be more convenient unit especially in the dimensioning of drawings. Such convenient units are formed by using a prefix in front of the basic units to indicate the multiplier.

1.1COMPOSITION OF RESOLUTION OF FORCES

2.1. INTRODUCTION

The force is an important factor in the field of Mechanics, which may be broadly *defined as an agent which produces or tends to produce, destroys or tends to destroy motion. *e.g.*, a horse applies force to pull a cart and to set it in motion. Force is also required to work on a bicycle pump. In this case, the force is supplied by the muscular power of our arms and shoulders.

Sometimes, the applied force may not be sufficient to move a body, *e.g.*, if we try to lift a stone weighing 2 or 3 quintals, we fail to do so. In this case we exert a force, no doubt, but no motion is produced. This shows that a force may not necessarily produce a motion in a body ; but it may, simply, tend to do so. In a tug-of-war the two parties, when balanced, neutralize each other's force. But the moment one party gets weaker, the other party pulls off, in spite of first party's best effort to destroy motion.

2.2. EFFECTS OF A FORCE

A force may produce the following effects in a body, on which it acts :

1. It may change the motion of a body. *i.e.* if a body is at rest, the force may set it in motion. And if the body is already in motion, the force may accelerate it.
2. It may retard the motion of a body.
3. It may retard the forces, already acting on a body, thus bringing it to rest or in equilibrium. We shall study this effect in chapter 5 of this book.
4. It may give rise to the internal stresses in the body, on which it acts. We shall study this effect in the chapters 'Analysis of Perfect Frames' of this book.

2.3. CHARACTERISTICS OF A FORCE

In order to determine the effects of a force, acting on a body, we must know the following characteristics of a force :

1. Magnitude of the force (*i.e.*, 100 N, 50 N, 20 kN, 5 kN, etc.)
2. The direction of the line, along which the force acts (*i.e.*, along *OX*, *OY*, at 30° North of East etc.). It is also known as line of action of the force.
3. Nature of the force (*i.e.*, whether the force is push or pull). This is denoted by placing an arrow head on the line of action of the force.
4. The point at which (or through which) the force acts on the body.

2.4. PRINCIPLE OF PHYSICAL INDEPENDENCE OF FORCES

It states, "*If a number of forces are simultaneously acting on a*particle*, *then the resultant ofthese forces will have the same effect as produced by all the forces.* "

2.5. PRINCIPLE OF TRANSMISSIBILITY OF FORCES

It states, "*If a force acts at any point on a †rigid body*, *it may also be considered to act at anyother point on its line of action*, *provided this point is rigidly connected with the body.*"

2.6. SYSTEM OF FORCES

When two or more forces act on a body, they are called to form a *system of forces*.
Following systems of forces are important from the subject point of view :

1. ***Coplanar forces.*** The forces, whose lines of action lie on the same plane, are known ascoplanar forces.
2. ***Collinear forces.*** The forces, whose lines of action lie on the same line, are known ascollinear forces.

* A particle may be defined as a body of infinitely small volume and is considered to be concentrated point.

† A rigid body may be defined as a body

3. ***Concurrent forces.*** The forces, which meet at one point, are known as concurrent forces.The concurrent forces may or may not be collinear.
4. ***Coplanar concurrent forces.*** The forces, which meet at one point and their lines of actionalso lie on the same plane, are known as coplanar concurrent forces.
5. ***Coplanar non-concurrent forces.*** The forces, which do not meet at one point, but theirlines of action lie on the same plane, are known as coplanar non-concurrent forces.
6. ***Non-coplanar concurrent forces.*** The forces, which meet at one point, but their lines ofaction do not lie on the same plane, are known as non-coplanar concurrent forces.
7. ***Non-coplanar non-concurrent forces.*** The forces, which do not meet at one point and theirlines of action do not lie on the same plane, are called non-coplanar non-concurrent forces.

2.7. RESULTANT FORCE

If a number of forces, P, Q, R ... etc. are acting simultaneously on a particle, then it is possible to find out a single force which could replace them *i.e.*, which would produce the same effect as produced by all the given forces. This single force is called *resultant force* and the given forces R ... etc. are called component forces.

2.8. COMPOSITION OF FORCES

The process of finding out the resultant force, of a number of given forces, is called *compositionof forces* or compounding of forces.

2.9. METHODS FOR THE RESULTANT FORCE

Though there are many methods for finding out the resultant force of a number of given forces, yet the following are important from the subject point of view :

1. Analytical method. 2. Method of resolution.

2.10. ANALYTICAL METHOD FOR RESULTANT FORCE

The resultant force, of a given system of forces, may be found out analytically by the following methods :

1. Parallelogram law of forces. 2. Method of resolution.

2.11. PARALLELOGRAM LAW OF FORCES

It states, "*If two forces, acting simultaneously on a particle, be represented in magnitude anddirection by the two adjacent sides of a parallelogram ; their resultant may be represented in magnitude and direction by the diagonal of the parallelogram, which passes through their point of intersection.*"

Mathematically, resultant force,

$$R = \sqrt{F_1^2 + F_2^2 + 2 F_1 F_2 \cos\theta}$$

and

$$\tan\alpha = \frac{F_2 \sin\theta}{F_1 + F_2 \cos\theta}$$

where F_1 and F_2 = Forces whose resultant is required to be found out,

θ = Angle between the forces F_1 and F_2, and

α = Angle which the resultant force makes with one of the forces (say F_1).

Note. It the angle (α) which the resultant force makes with the other force F_2,

then $$\tan\alpha = \frac{F_1 \sin\theta}{F_2 + F_1\cos\theta}$$

Cor.

1. If θ = 0 *i.e.*, when the forces act along the same line, then

$$R = F_1 + F_2 \quad \text{...(Since } \cos 0° = 1)$$

2. If θ = 90° *i.e.*, when the forces act at right angle, then

$$\theta = R = \sqrt{F_1^2 + F_2^2} \quad \text{...(Since } \cos 90° = 0)$$

3. If θ = 180° *i.e.*, when the forces act along the same straight line but in opposite directions, then $R = F_1 - F_2$ ...(Since cos 180° = – 1)

In this case, the resultant force will act in the direction of the greater force.

4. If the two forces are equal *i.e.*, when $F_1 = F_2 = F$ then

$$R = \sqrt{F^2 + F^2 + 2F^2\cos\theta} = \sqrt{2F^2(1+\cos\theta)}$$

$$= \sqrt{2F^2 \cdot 2\cos^2\frac{\theta}{2}} \quad \text{...} \because 1 + \cos\theta = 2\cos^2\frac{\theta}{2}$$

$$= \sqrt{4F^2\cos^2\frac{\theta}{2}} = 2F\cos\frac{\theta}{2}$$

Example 2.1. *Two forces of 100 N and 150 N are acting simultaneously at a point. What is the resultant of these two forces, if the angle between them is 45°?*

Solution. Given : First force (F_1) = 100 N; Second force (F_2) = 150 N and angle between F_1 and F_2 (θ) = 45°.

We know that the resultant force,

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$= \sqrt{(100)^2 + (150)^2 + 2 \cdot 100 \cdot 150 \cos 45°} \text{ N}$$

$$= \sqrt{10\,000 + 22\,500 + (30\,000 \cdot 0.707)}$$

N = 232 N **Ans.**

Example 2.2. *Two forces act at an angle of 120°. The bigger force is of 40 N and the resultant is perpendicular to the smaller one. Find the smaller force.*

Solution. Given : Angle between the forces ∠AOC = 120°, Bigger force (F_1) = 40 N and angle between the resultant and F_2 (∠BOC) = 90° ;

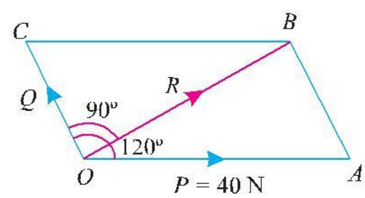


Fig. 2.1.

Let F_2 = Smaller force in N

From the geometry of the figure, we find that ∠AOB,

$$\alpha = 120° - 90° = 30°$$

We know that

$$\tan\alpha = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta}$$

$$\tan 30° = \frac{F_2\sin 120°}{40 + F_2\cos 120°} = \frac{F_2\sin 60°}{40 + F_2(-\cos 60°)}$$

$$\therefore \quad 0.577 = \frac{F_2 \cdot 0.866}{40 - F_2 \cdot 0.5} = \frac{0.866\, F_2}{40 - 0.5\, F_2}$$

$$40 - 0.5\, F_2 = \frac{0.866\, F_2}{0.577} = 1.5\, F_2$$

$$\therefore \quad 2F_2 = 40 \quad \text{or} \quad F_2 = 20 \quad \textbf{Ans.}$$

Example 2.3. *Find the magnitude of the two forces, such that if they act at right angles, their resultant is* $\sqrt{10}$ *N. But if they Act at 60°, their resultant is* $\sqrt{13}$ *N.*

Solution. Given : Two forces $= F_1$ and F_2.

First of all, consider the two forces acting at right angles. We know that when the angle between the two given forces is 90°, then the resultant force (R)

$$\sqrt{10} = \sqrt{F_1^2 + F_2^2}$$

Or $$10 = F_1^2 + F_2^2 \quad \text{...(Squaring both sides)}$$

Similarly, when the angle between the two forces is 60°, then the resultant force (R)

$$\sqrt{13} = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 60°}$$

$$\therefore \quad 13 = F_1^2 + F_2^2 + 2F_1F_2 \cdot 0.5 \quad \text{...(Squaring both sides)}$$

Or $$F_1F_2 = 13 - 10 = 3 \quad \text{...(Substituting } F_1^2 + F_2^2 = 10)$$

We know that $(F_1 + F_2)^2 = F_1^2 + F_2^2 + 2F_1F_2 = 10 + 6 = 16$

$$\therefore \quad F_1 + F_2 = \sqrt{16} = 4 \quad \text{...}(i)$$

Similarly $(F_1 - F_2)^2 = F_1^2 + F_2^2 - 2F_1F_2 = 10 - 6 = 4$

$$\therefore \quad F_1 - F_2 = \sqrt{4} = 2 \quad \text{...}(ii)$$

Solving equations (i) and (ii),

$$F_1 = 3 \text{ N} \quad \text{and} \quad F_2 = 1 \text{ N} \quad \textbf{Ans.}$$

2.12. RESOLUTION OF A FORCE

The process of splitting up the given force into a number of components, without changing its effect on the body is called resolution of a force. A force is, generally, resolved along two mutually perpendicular directions. In fact, the resolution of a force is the reverse action of the addition of the component vectors.

2.13. PRINCIPLE OF RESOLUTION

It states, "*The algebraic sum of the resolved parts of a no. of forces, in a given direction, isequal to the resolved part of their resultant in the same direction.*"

Note : In general, the forces are resolved in the vertical and horizontal directions.

Example 2.4. *A machine component 1.5 m long and weight 1000 N is supported by tworopes AB and CD as shown in Fig. 2.2 given below.*

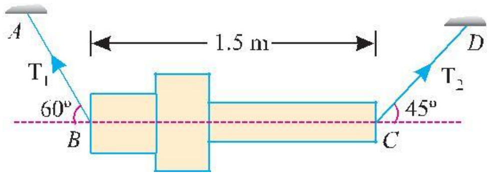


Fig. 2.2.

Calculate the tensions T_1 and T_2 in the ropes AB and CD.

Solution. Given : Weight of the component = 1000 N

Resolving the forces horizontally (*i.e.*, along *BC*) and equating the same,

$$T_1 \cos 60° = T_2 \cos 45°$$

$$\therefore \quad T_1 = \frac{\cos 45°}{\cos 60°} \cdot T_2 = \frac{0.707}{0.5} \cdot T_2 = 1.414\, T_2 \qquad ...(i)$$

and now resolving the forces vertically,

$$T_1 \sin 60° + T_2 \sin 45° = 1000$$

$$(1.414\, T_2)\, 0.866 + T_2 \times 0.707 = 1000$$

$$1.93\, T_2 = 1000$$

$$\therefore \quad T_2 = \frac{1000}{1.93} = 518.1 \text{ N} \qquad \textbf{Ans.}$$

And $\quad T_1 = 1.414 \times 518.1 = 732.6$ N **Ans.**

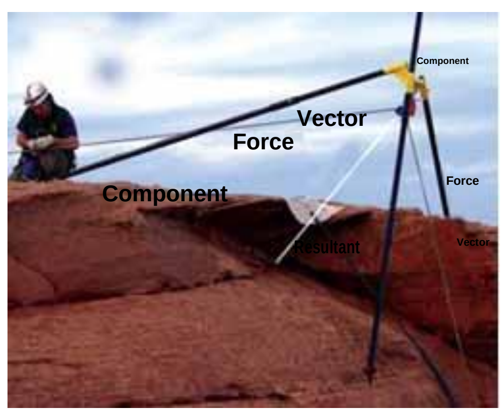
2.14. METHOD OF RESOLUTION FOR THE RESULTANT FORCE

1. Resolve all the forces horizontally and find the algebraic sum of all the horizontal components (*i.e.*, $\sum H$).
2. Resolve all the forces vertically and find the algebraic sum of all the vertical components (*i.e.*, $\sum V$).
3. The resultant *R* of the given forces will be given by the equation :

$$R = \sqrt{(\sum H)^2 + (\sum V)^2}$$

4. The resultant force will be inclined at an angle θ, with the horizontal, such that

$$\tan\theta = \frac{\sum V}{\sum H}$$



Notes : The value of the angle θ will vary depending upon the values of $\sum V$ and $\sum H$ as discussed below :

1. When $\sum V$ is +ve, the resultant makes an angle between 0° and 180°. But when $\sum V$ is –ve, the resultant makes an angle between 180° and 360°.
2. When $\sum H$ is +ve, the resultant makes an angle between 0° to 90° or 270° to 360°. But when $\sum H$ is –ve, the resultant makes an angle between 90° to 270°.

Example 2.5. *A triangle ABC has its side AB=40 mm along positive x-axis and side BC = 30 mm along positive y-axis. Three forces of 40 N, 50 N and 30 N act along the sides AB, BC and CA respectively. Determine magnitude of the resultant of such a system of forces.*

Solution. The system of given forces is shown in Fig. 2.3. From the geometry of the figure, we find that the triangle *ABC* is a right angled triangle, in which the *side *AC* = 50 mm. Therefore

$$\sin\theta = \frac{30}{50} = 0.6$$

And $$\cos\theta = \frac{40}{50} = 0.8$$

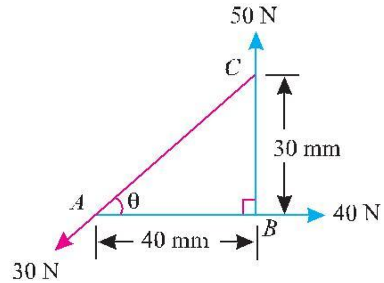


Fig. 2.3.

Resolving all the forces horizontally (*i.e.*, along *AB*), $\sum H = 40 - 30\cos\theta$

$= 40 - (30 \times 0.8) = 16$ N and

now resolving all the forces vertically (*i.e.*, along *BC*)

$$\sum V = 50 - 30\sin\theta$$

$$= 50 - (30 \times 0.6) = 32 \text{ N}$$

We know that magnitude of the resultant force,

$$R = \sqrt{(\sum H)^2 + (\sum V)^2} = \sqrt{(16)^2 + (32)^2} = 35.8 \text{ N} \quad \textbf{Ans.}$$

Example 2.6. *A system of forces are acting at the corners of a rectangular block as shown in Fig.* 2.4.

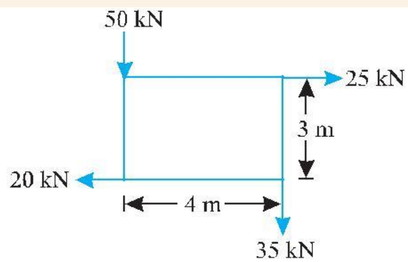


Fig. 2.4.

Determine the magnitude and direction of the resultant force.

Solution. Given : System of forces

Magnitude of the resultant force

Resolving forces horizontally,

$$\sum H = 25 - 20 = 5 \text{ kN}$$

and now resolving the forces vertically

$$\sum V = (-50) + (-35) = -85 \text{ kN}$$

∴ Magnitude of the resultant force

$$R = \sqrt{(\sum H)^2 + (\sum V)^2} = \sqrt{(5)^2 + (-85)^2} = 85.15 \text{ kN} \quad \textbf{Ans.}$$

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* Since the side *AB* is along *x*-axis, and the side *BC* is along *y*-axis, there fore it is a right-angled triangle. Now in triangle *ABC*,

$$AC = \sqrt{AB^2 + BC^2} = \sqrt{(40)^2 + (30)^2} = 50 \text{ mm}$$

Direction of the resultant force

Let θ = Angle which the resultant force makes with the horizontal.

We know that

$$\tan\theta = \frac{\sum V}{\sum H} = \frac{-85}{5} = -17 \quad \text{or} \quad \theta = 86.6°$$

Since $\sum H$ is positive and $\sum V$ is negative, therefore resultant lies between 270° and 360°. Thus actual angle of the resultant force

= 360° – 86.6° = 273.4° **Ans.**

Example 2.7. *The forces 20 N, 30 N, 40 N, 50 N and 60 N are acting at one of the angularpoints of a regular hexagon, towards the other five angular points, taken in order. Find the magnitude and direction of the resultant force.*

Solution. The system of given forces is shown in Fig. 2.5

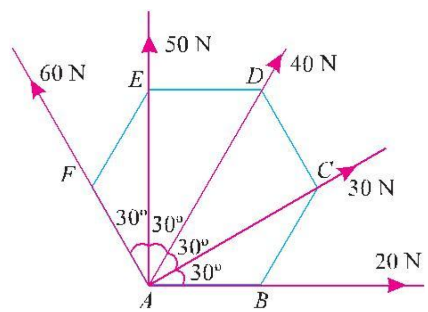


Fig. 2.5.

Magnitude of the resultant force

Resolving all the forces horizontally (*i.e.*, along *AB*),

$$\sum H = 20 \cos 0° + 30 \cos 30° + 40 \cos 60° + 50 \cos 90° + 60 \cos 120° \text{ N}$$
$$= (20 \times 1) + (30 \times 0.866) + (40 \times 0.5) + (50 \times 0) + 60\,(-0.5) \text{ N}$$
$$= 36.0 \text{ N} \qquad \ldots(i)$$

and now resolving the all forces vertically (*i.e.*, at right angles to *AB*),

$$\sum V = 20 \sin 0° + 30 \sin 30° + 40 \sin 60° + 50 \sin 90° + 60 \sin 120° \text{ N}$$
$$= (20 \times 0) + (30 \times 0.5) + (40 \times 0.866) + (50 \times 1) + (60 \times 0.866) \text{ N}$$
$$= 151.6 \text{ N} \qquad \ldots(ii)$$

We know that magnitude of the resultant force,

$$R = \sqrt{(\sum H)^2 + (\sum V)^2} = \sqrt{(36.0)^2 + (151.6)^2} = 155.8 \text{ N} \quad \textbf{Ans.}$$

Direction of the resultant force

Let θ = Angle, which the resultant force makes with the horizontal (*i.e.*, *AB*).

We know that

$$\tan\theta = \frac{\sum V}{\sum H} = \frac{151.6}{36.0} = 4.211 \quad \text{or} \quad \theta = 76.6° \quad \textbf{Ans.}$$

Note. Since both the values of $\sum H$ and $\sum V$ are positive, therefore actual angle of resultantforce lies between 0° and 90°.

Example 2.8. *The following forces act at a point:* (*i*) *20 N inclined at 30° towards North of East,*
(*ii*) *25 N towards North,*
(*iii*) *30 N towards North West, and*
(*iv*) *35 N inclined at 40° towards South of West.*
Find the magnitude and direction of the resultant force.

Solution. The system of given forces is shown in Fig. 2.6.

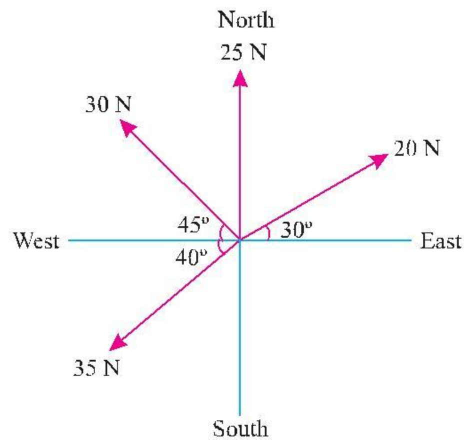


Fig. 2.6.

Magnitude of the resultant force

Resolving all the forces horizontally *i.e.*, along East-West line,

$$\sum H = 20 \cos 30° + 25 \cos 90° + 30 \cos 135° + 35 \cos 220° \text{ N}$$
$$= (20 \times 0.866) + (25 \times 0) + 30\,(-0.707) + 35\,(-0.766) \text{ N}$$
$$= -30.7 \text{ N} \quad ...(i)$$

and now resolving all the forces vertically *i.e.*, along North-South line,

$$\sum V = 20 \sin 30° + 25 \sin 90° + 30 \sin 135° + 35 \sin 220° \text{ N}$$
$$= (20 \times 0.5) + (25 \times 1.0) + (30 \times 0.707) + 35\,(-0.6428) \text{ N}$$
$$= 33.7 \text{ N} \quad ...(ii)$$

We know that magnitude of the resultant force,

$$R = \sqrt{(\sum H)^2 + (\sum V)^2} = \sqrt{(-30.7)^2 + (33.7)^2} = 45.6 \text{ N} \quad \textbf{Ans.}$$

Direction of the resultant force

Let θ = Angle, which the resultant force makes with the East.

We know that

$$\tan\theta = \frac{\sum V}{\sum H} = \frac{33.7}{-30.7} = -1.098 \quad \text{or} \quad \theta = 47.7°$$

Since $\sum H$ is negative and $\sum V$ is positive, therefore resultant lies between 90° and 180°. Thus actual angle of the resultant = 180° – 47.7° = 132.3° **Ans.**

Example 2.9. *A horizontal line PQRS is 12 m long, where PQ=QR=RS=4 m. Forces of 1000 N, 1500 N, 1000 N and 500 N act at P, Q, R and S respectively with downward direction. The lines of action of these forces make angles of 90°, 60°, 45° and 30° respectively with PS. Find the magnitude, direction and position of the resultant force.*

Solution. The system of the given forces is shown in Fig. 2.7

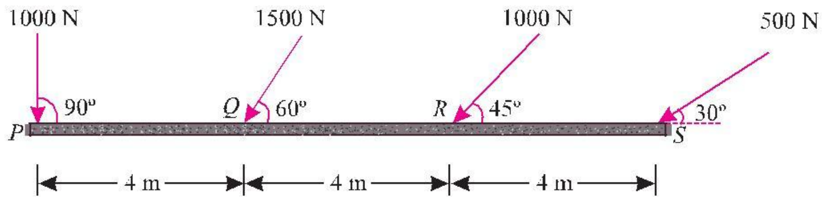


Fig. 2.7.

Magnitude of the resultant force

Resolving all the forces horizontally,

$$\Sigma H = 1000 \cos 90° + 1500 \cos 60° + 1000 \cos 45° + 500 \cos 30° \text{ N}$$
$$= (1000 \times 0) + (1500 \times 0.5) + (1000 \times 0.707) + (500 \times 0.866) \text{ N}$$
$$= 1890 \text{ N} \quad ...(i)$$

and now resolving all the forces vertically,

$$\Sigma V = 1000 \sin 90° + 1500 \sin 60° + 1000 \sin 45° + 500 \sin 30° \text{ N}$$
$$= (1000 \times 1.0) + (1500 \times 0.866) + (1000 \times 0.707) + (500 \times 0.5) \text{ N}$$
$$= 3256 \text{ N} \quad ...(ii)$$

We know that magnitude of the resultant force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(1890)^2 + (3256)^2} = 3765 \text{ N } \textbf{Ans.}$$

Direction of the resultant force

Let θ = Angle, which the resultant force makes with *PS*.

$\therefore \quad \tan \theta = \dfrac{\Sigma V}{\Sigma H} = \dfrac{3256}{1890} = 1.722$ or $\theta = 59.8°$ **Ans.**

Note. Since both the values of ΣH and ΣV are +ve. therefore resultant lies between 0° and 90°.

Position of the resultant force

Let x = Distance between *P* and the line of action of the resultant force.

Now taking moments* of the vertical components of the forces and the resultant force about *P*, and equating the same,

$$3256\, x = (1000 \times 0) + (1500 \times 0.866)\, 4 + (1000 \times 0.707) 8 + (500 \times 0.5) 12$$
$$= 13\,852$$

$\therefore \quad x = \dfrac{13\,852}{3256} = 4.25 \text{ m}$ **Ans.**

* This point will be discussed in more details in the chapter on 'Moments and Their Applications'.

EXERCISE 2.1

1. Find the resultant of two forces equal to 50 N and 30 N acting at an angle of 60°. [**Ans.** 70 N ; 21.8°]
2. Two forces of 80 N and 70 N act simultaneously at a point. Find the resultant force, if the angle between them is 150°. [**Ans.** 106.3 N ; 61°]
3. Find the resultant of two forces 130 N and 110 N respectively, acting at an angle whose tangent is 12/5. [**Ans.** 185.7 N ; 30.5°]
4. A push of 180 N and pull of 350 N act simultaneously at a point. Find the resultant of the forces, if the angle between them be 135°. [**Ans.** 494 N ; 30°]
5. Find the angle between two equal forces P, when their resultant is equal to (*i*) P and (*ii*) $P/2$. [**Ans.** 120° N ; 151°]

Hint. When resultant is equal to P, then

$$P = \sqrt{P^2 + P^2 + 2P.P\cos\theta} = P\sqrt{2 + 2\cos\theta}$$

$\therefore \quad 2\cos\theta = -1 \quad$ or $\quad \cos\theta = -0.5 \quad$ or $\quad \theta = 120°$ **Ans.**

When resultant is equal to $P/2$, then

$$0.5P = \sqrt{P^2 + P^2 + 2P.P\cos\theta} = P\sqrt{2 + 2\cos\theta}$$

$\therefore \quad 2\cos\theta = -1.75 \quad$ or $\quad \cos\theta = -0.875 \quad$ or $\quad \theta = 151°$ **Ans.**

6. The resultant of two forces P and Q is R. If Q is doubled, the new resultant is perpendicular to P. Prove that $Q = R$.

Hint. In first case, $\quad R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

In second case, $\quad \tan 90° = \dfrac{(2Q)\sin\theta}{P + (2Q)\cos\theta}$

Since $\tan 90° = \infty$, therefore $P + 2Q\cos\theta = 0$

2.15. LAWS FOR THE RESULTANT FORCE

The resultant force, of a given system of forces, may also be found out by the following laws :
1. Triangle law of forces. **2.** Polygon law of forces.

2.16. TRIANGLE LAW OF FORCES

It states, "*If two forces acting simultaneously on a particle, be represented in magnitude anddirection by the two sides of a triangle, taken in order ; their resultant may be represented in magnitude and direction by the third side of the triangle, taken in opposite order.*"

2.17. POLYGON LAW OF FORCES

It is an extension of Triangle Law of Forces for more than two forces, which states, "*If anumber of forces acting simultaneously on a particle, be represented in magnitude and direction, by the sides of a polygon taken in order ; then the resultant of all these forces may be represented, in magnitude and direction, by the closing side of the polygon, taken in opposite order.*"

EXERCISE 2.2

1. Find the magnitude and direction of the resultant of the concurrent forces of 8 N, 12 N, 15 N and 20 N making angles of 30°, 70°, 120°.25 and 155° respectively with a fixed line. [**Ans.** 39.5 N ; 111.7°]
2. Find magnitude of the resultant force, if 30, 40, 50 and 60 N forces are acting along thelines joining the centre of a square to its vertices. [**Ans.** 28.3 N]
3. Four forces of 25 N, 20 N, 15 N and 10 N are acting simultaneously along straight

lines OA, OB, OC and OD such that

$$\angle AOB = 45°; \angle BOC = 100° \text{ and } \angle COD = 125°.$$

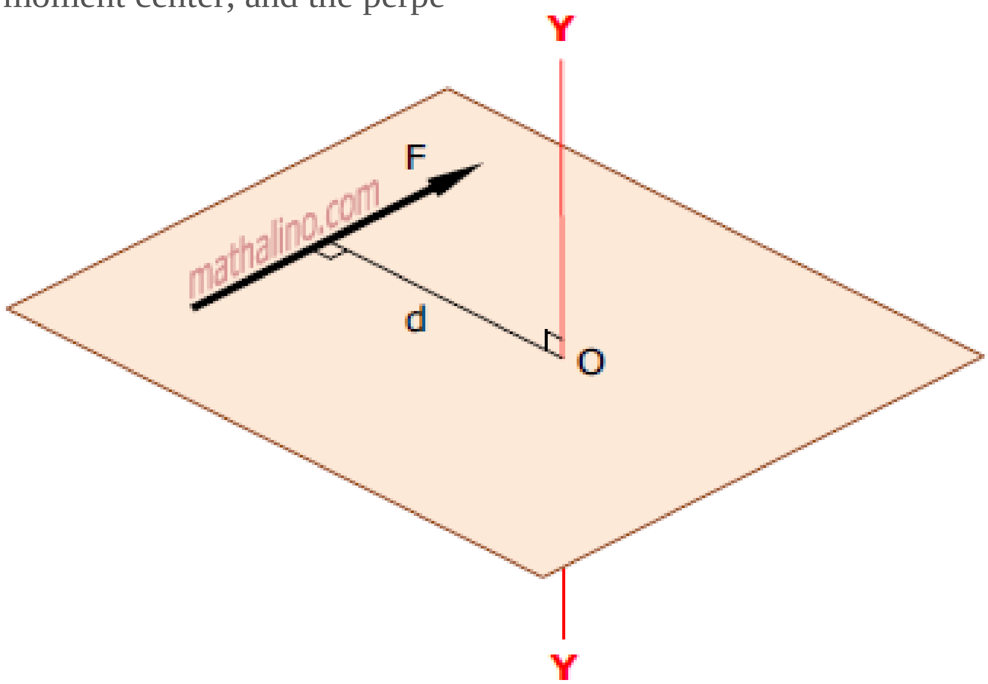
Find graphically magnitude and direction of the resultant force. Also check the answer analytically. [**Ans.** 29.5 N ; 25.4° with OA]

QUESTIONS

1. Define the term 'force', and state clearly the effects of force.
2. What are the various characteristics of a force?
3. Distinguish clearly between resolution of forces and composition of forces.
4. What are the methods for finding out the resultant force for a given system of forces?
5. State and prove parallelogram law of forces.
6. State triangle law of forces and polygon law of forces.
7. Show that the algebraic sum of the resolved part of a number of forces in a given direction, is equal to the resolved part of their resultant in the same direction.
8. Explain clearly the procedure for finding out the resultant force analytically as well as graphically.

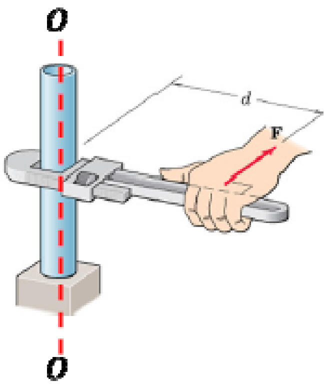
MOMENT OF A FORCE

Moment is the measure of the capacity or ability of the force to produce twisting or turning effect about an axis. This axis is perpendicular to the plane containing the line of action of the force. The magnitude of moment is equal to the product of the force and the perpendicular distance from the axis to the line of action of the force. The intersection of the plane and the axis is commonly called the moment center, and the perpe



ndicular distance from the moment center to the line of action of the force is called moment arm.

Moment of a Force

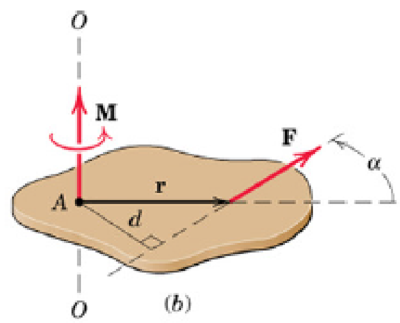


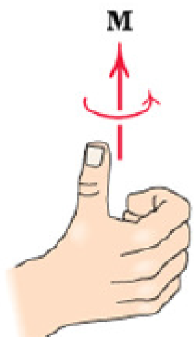
Moment about axis O-O is $M_o = Fd$

Magnitude of M_o measures tendency of **F** to cause rotation of the body about an axis along M_o.

Moment about axis O-O is $M_o = Frsin\alpha$

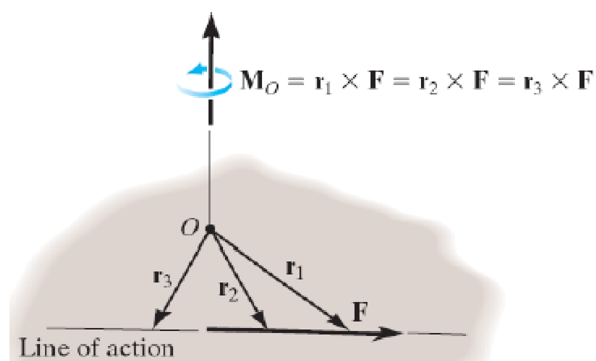
$M_o = r \times F$





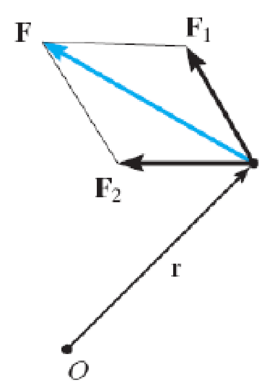
Sense of the moment may be determined by the right-hand rule

Moment of a Force



Principle of Transmissibility
Any force that has the same magnitude and direction as **F**, is *equivalent* if it also has the same line of action and therefore, produces the same moment.

Varignon's Theorem (Principle of Moments)



Moment of a Force about a point is equal to the sum of the moments of the force's components about the point.

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \mathbf{r} \times (\mathbf{F}_1 + \mathbf{F}_2) = \mathbf{r} \times \mathbf{F}_1 + \mathbf{r} \times \mathbf{F}_2$$

4.8. COUPLE

A pair of two equal and unlike parallel forces (*i.e*. forces equal in magnitude, with lines of action parallel to each other and acting in opposite directions) is known as a couple.

As a matter of fact, a couple is unable to produce any translatory motion (*i.e*., motion in a straight line). But it produces a motion of rotation in the body, on which it acts. The simplest example of a couple is the forces applied to the key of a lock, while locking or unlocking it.

A couple is a pair of forces applied to the key of a lock.

4.9. ARM OF A COUPLE

The perpendicular distance (*a*), between the lines of action of the two equal and opposite parallel forces, is known *as arm of the couple* as shown in Fig. 4.11.

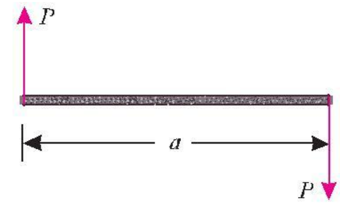


Fig. 4.11.

4.10. MOMENT OF A COUPLE

The moment of a couple is the product of the force (*i.e.*, one of the forces of the two equal and opposite parallel forces) and the arm of the couple. Mathematically:

Moment of a couple = $P \times a$

where P = Magnitude of the force, and

a = Arm of the couple.

4.11. CLASSIFICATION OF COUPLES

The couples may be, broadly, classified into the following two categories, depending upon their direction, in which the couple tends to rotate the body, on which it acts :

1. Clockwise couple, and 2.Anticlockwise couple.

4.12. CLOCKWISE COUPLE

A couple, whose tendency is to rotate the body, on which it acts, in a clockwise direction, is known as a clockwise couple as shown in Fig. 4.12 (*a*). Such a couple is also called positive couple.

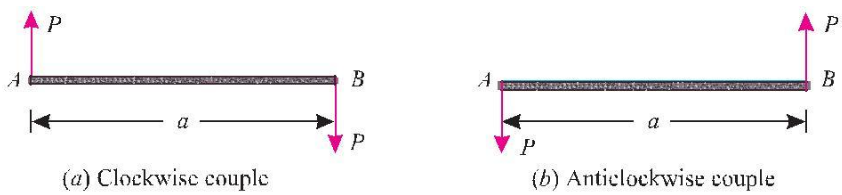


Fig. 4.12.

4.13. ANTICLOCKWISE COUPLE

A couple, whose tendency is to rotate the body, on which it acts, in an anticlockwise direction, is known as an anticlockwise couple as shown in Fig. 4.12 (*b*). Such a couple is also called a negative couple.

4.14. CHARACTERISTICS OF A COUPLE

A couple (whether clockwise or anticlockwise) has the following characteristics :

1. The algebraic sum of the forces, constituting the couple, is zero.
2. The algebraic sum of the moments of the forces, constituting the couple, about any point is the same, and equal to the moment of the couple itself.
3. A couple cannot be balanced by a single force. But it can be balanced only by a couple of opposite sense.
4. Any no. of coplaner couples can be reduced to a single couple, whose magnitude will be equal to the algebraic sum of the moments of all the couples.

Equilibrium of Forces

5.1. INTRODUCTION

In the previous chapter, we have discussed the various methods of finding out resultant force, when a particle is acted upon by a number of forces. This resultant force will produce the same effect as produced by all the given forces.A little consideration will show, that if the resultant of a number of forces, acting on a particle is zero, the particle will be in equilibrium. Such a set of forces, whose resultant is zero, are called equilibrium forces.The force, which brings the set of forces in equilibrium is called an equilibrant.As a matter of fact, the equilibrant is equal to the resultant force in magnitude, but opposite in direction.

5.2. PRINCIPLES OF EQUILIBRIUM

Though there are many principles of equilibrium, yet the following three are important from the subject point of view :

1. *Two force principle.* As per this principle, if a body in equilibrium is acted upon by twoforces, then they must be equal, opposite and collinear.
2. *Three force principle.* As per this principle, if a body in equilibrium is acted upon by threeforces, then the resultant of any two forces must be equal, opposite and collinear with the third force.
3. *Four force principle.* As per this principle, if a body in equilibrium is acted upon by fourforces, then the resultant of any two forces must be equal, opposite and collinear with the resultant of the other two forces.

5.3. METHODS FOR THE EQUILIBRIUM OF COPLANAR FORCES

Though there are many methods of studying the equilibrium of forces, yet the following are important from the subject point of view :

1. Analytical method. 2. Graphical method.

5.4. ANALYTICAL METHOD FOR THE EQUILIBRIUM OF COPLANAR FORCES

The equilibrium of coplanar forces may be studied, analytically, by Lami's theorem as discussed below :

5.5. LAMI'S THEOREM

It states, "*If three coplanar forces acting at a point be inequilibrium, then each force is proportional to the sine of the angle between the other two.*" Mathematically,

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

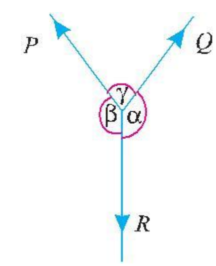
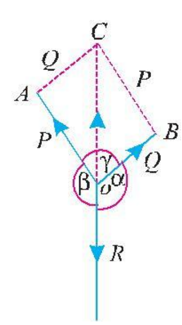


Fig. 5.1. Lami's theorem

where, P, Q, and R are three forces and α, β, γ are the angles as shown in Fig. 5.1.

Proof

Consider three coplanar forces P, Q, and R acting at a point O. Let the opposite angles to three forces be α ,β and γ as shown in Fig. 5.2.



Now let us complete the parallelogram $OACB$ with OA and OB as adjacent sides as shown in the figure. We know that the resultant of two forces P and Q will be given by the diagonal OC both in magnitude and direction of the parallelogram $OACB$.

Since these forces are in equilibrium, therefore the resultant of the forces P and Q must be in line with OD and equal to R, but in opposite direction.

F rom the geometry of the figure, we find

$\angle CAO = 180° - (\angle AOC + \angle ACO)$

$= 180° - [(180° - \beta) + (180° - \alpha)]$

$= 180° - 180° + \beta - 180° + \alpha$

$= \alpha + \beta - 180°$

$\alpha + \beta + \gamma = 360°$

Subtracting 180° from both sides of the above equation, $(\alpha + \beta - 180°) + \gamma = 360° - 180° = 180°$

$\angle CAO = 180° - \gamma$

We know that in triangle *AOC*,

$$\frac{OA}{\sin \angle ACO} = \frac{AC}{\sin \angle AOC} = \frac{OC}{\sin \angle CAO}$$

$$\frac{OA}{\sin(180° - \alpha)} = \frac{AC}{\sin(180° - \beta)} = \frac{OC}{\sin(180° - \gamma)}$$

$$\frac{P}{\sin \alpha} = \frac{Q}{\sin \beta} = \frac{R}{\sin \gamma} \qquad ...[\because \sin(180° - \theta) = \sin \theta]$$

Example 5.1. *An electric light fixture weighting 15 N hangs from a point C, by two strings AC and BC. The string AC is inclined at 60° to the horizontal and BC at 45° to the horizontal as shown in Fig. 5.3*

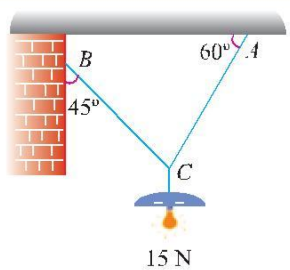


Fig. 5.3.

Using Lami's theorem, or otherwise, determine the forces in the strings AC and BC.

Solution. Given : Weight at C = 15 N

Let T_{AC} = Force in the string *AC*, and T_{BC} = Force in the string *BC*.

The system of forces is shown in Fig. 5.4. From the geometry of the figure, we find that angle between T_{AC} and 15 N is 150° and angle between T_{BC} and 15 N is 135°.

$\therefore \angle ACB = 180° - (45° + 60°) = 75°$

Applying Lami's equation at *C*,

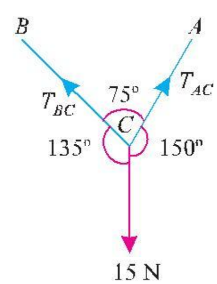


Fig. 5.4.

$$\frac{15}{\sin 75°} = \frac{T_{AC}}{\sin 135°} = \frac{T_{BC}}{\sin 150°}$$

or

$$\frac{15}{\sin 75°} = \frac{T_{AC}}{\sin 45°} = \frac{T_{BC}}{\sin 30°}$$

$$\therefore \quad T_{AC} = \frac{15 \sin 45°}{\sin 75°} = \frac{15 \cdot 0.707}{0.9659} = 10.98 \text{ N} \quad \textbf{Ans.}$$

and $T_{BC} = \frac{15 \sin 30°}{\sin 75°} = \frac{15 \cdot 0.5}{0.9659} = 7.76$ N **Ans.**

Example 5.2. *A string ABCD, attached to fixed points A and D has two equal weihts of 1000 N attached to it at B and C. The weights rest with the portions AB and CD inclined at angles as shown in Fig. 5.5.*

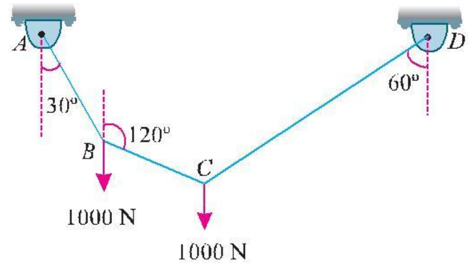
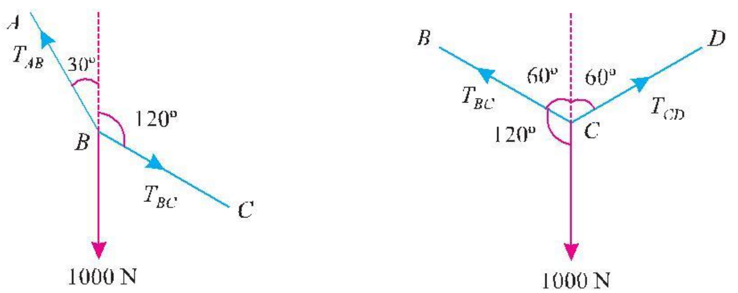


Fig. 5.5.

Find the tensions in the portions AB, BC and CD of the string, if the inclination of the portion BC with the vertical is 120°.

Solution. Given : Load at B = Load at C = 1000 N

For the sake of convenience, let us split up the string *ABCD* into two parts. The system of forces at joints *B* and is shown in Fig. 5.6 (*a*) and (*b*).



(*a*) Joint *B* (*b*) Joint *C*

Fig. 5.6.

Let T_{AB} = Tension in the portion *AB* of the string,

T_{BC} = Tension in the portion *BC* of the string, and

T_{CD} = Tension in the portion *CD* of the string.

Applying Lami's equation at joint *B*,

$$\frac{T_{AB}}{\sin 60°} = \frac{T_{BC}}{\sin 150°} = \frac{1000}{\sin 150°}$$

$$\frac{T_{AB}}{\sin 60°} = \frac{T_{BC}}{\sin 30°} = \frac{1000}{\sin 30°} \qquad \text{...}[\because \sin(180° - \theta) = \sin\theta]$$

$\therefore$ $T_{AB} = \frac{1000 \sin 60°}{\sin 30°} = \frac{1000 \cdot 0.866}{0.5} = 1732$ N **Ans.**

and $T_{BC} = \frac{1000 \sin 30°}{\sin 30°} = 1000$ N **Ans.**

Again applying Lami's equation at joint *C*,

$$\frac{T_{BC}}{\sin 120°} = \frac{T_{CD}}{\sin 120°} = \frac{1000}{\sin 120°}$$

$$\therefore \quad T_{CD} = \frac{1000 \sin 120°}{\sin 120°} = 1000 \text{ N} \quad \textbf{Ans.}$$

Example 5.3. *A light string ABCDE whose extremity A is fixed, has weights W_1 and W_2 attached to it at B and C. It passes round a small smooth peg at D carrying a weight of 300 N at the free end E as shown in Fig. 5.7.*

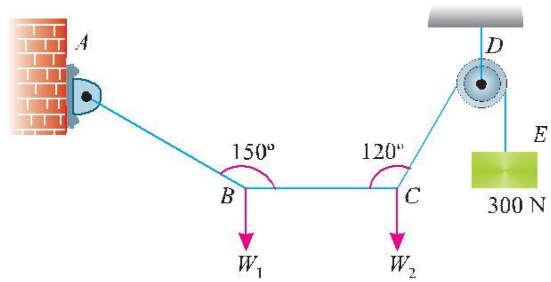
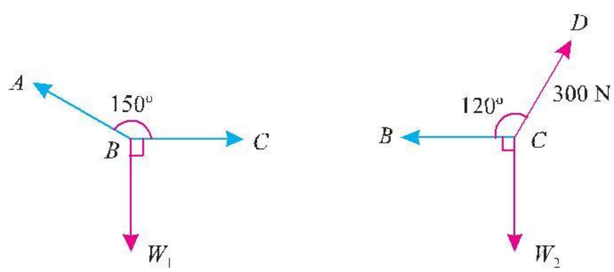


Fig. 5.7.

If in the equilibrium position, BC is horizontal and AB and CD make 150° and 120° with BC, find (i) Tensions in the portion AB, BC and CD of the string and (ii) Magnitudes of W_1 and W_2.

Solution. Given : Weight at E = 300 N

For the sake of convenience, let us split up the string *ABCD* into two parts. The system of forces at joints *B* and *C* is shown in Fig. 5.8. (*a*) and (*b*).



(*a*) Joint *B* (*b*) Joint *C*

Fig. 5.8.

(*i*) *Tensions is the portion AB, BC and CD of the string*

Let T_{AB} = Tension in the portion *AB*, and

T_{BC} = Tension in the portion *BC*,

We know that tension in the portion *CD* of the string. $T_{CD} = T_{DE}$ = 300 N **Ans.**

Applying Lami's equation at C,

$$\frac{T_{BC}}{\sin 150°} = \frac{W_2}{\sin 120°} = \frac{300}{\sin 90°}$$

$$\frac{T_{BC}}{\sin 30°} = \frac{W_2}{\sin 60°} = \frac{300}{1}$$ ...[∵ sin (180° – θ) = sin θ]

∴ T_{BC} = 300 sin 30° = 300 × 0.5 = 150 N **Ans.**

and W_2 = 300 sin 60° = 300 × 0.866 = 259.8 N

Again applying Lami's equation at *B*,

$$\frac{T_{AB}}{\sin 90°} = \frac{W_1}{\sin 150°} = \frac{T_{BC}}{\sin 120°}$$

$$\frac{T_{AB}}{1} = \frac{W_1}{\sin 30°} = \frac{150}{\sin 60°}$$ ...[∵ sin (180° – θ) = sin θ]

∴ $T_{AB} = \frac{150}{\sin 60°} = \frac{150}{0.866}$ = 173.2 N **Ans.**

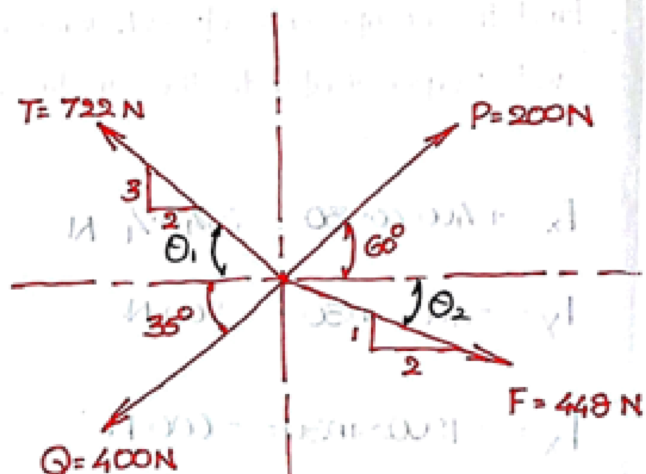
and $W_1 = \frac{150 \sin 30°}{\sin 60°} = \frac{150 \cdot 0.5}{0.866} = 86.6$ N

SINGER EM TEXTBOOK PROBLEMS AND SOLUTIONS

①

1. COMPONENTS OF A FORCE

1. Find x and y components of each of four forces shown in Fig.



Force P

$P_x = 200 \cos 60 = +100$ N

$P_y = 200 \sin 60 = +173.2$ N.

Force T

$T_x = -722 \cos 56.31 = -400.5$ N

$T_y = 722 \sin 56.31 = -600.7$ N.

$\theta_1 = Tan^{-1}\left(\frac{3}{2}\right) = 56.31$

$\theta_2 = Tan^{-1}\left(\frac{1}{2}\right) = 26.56$

Force Q

$Q_x = -400 \cos 35 = -327.6$ N

$Q_y = -400 \sin 35 = -229.4$ N.

Force F

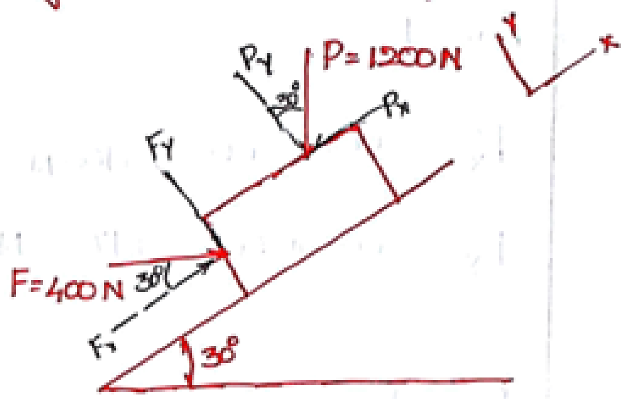
$F_x = 448 \cos 26.56 = 400.72$ N.

$F_y = -448 \sin 26.56 = -200.32$ N.

②

2. The body on the incline is subjected to the vertical and horizontal forces shown.

Find the components of each force along x-y axes oriented parallel and perpendicular to the incline.



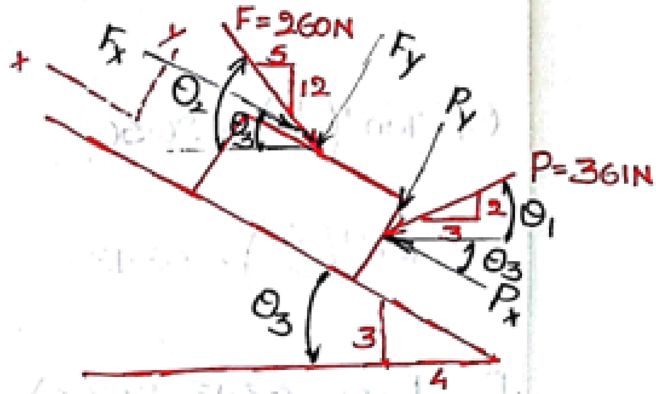
$F_x = +400\cos 30 = 346.4$ N

$F_y = -400\sin 30 = -200$ N

$P_x = -1200\sin 30 = -600$ N

$P_y = -1200\cos 30 = -1039.2$ N.

3. Determine the components of the forces P and Q along the axes Which are parallel and perpendicular to the incline.



$\theta_1 = Tan^{-1}\left(\frac{2}{3}\right) = 33.69$

$\theta_2 = Tan^{-1}\left(\frac{12}{5}\right) = 67.38$

$\theta_3 = Tan^{-1}\left(\frac{3}{4}\right) = 36.87$

$F_x = -260\cos[67.38-36.87] = -224\ N.$

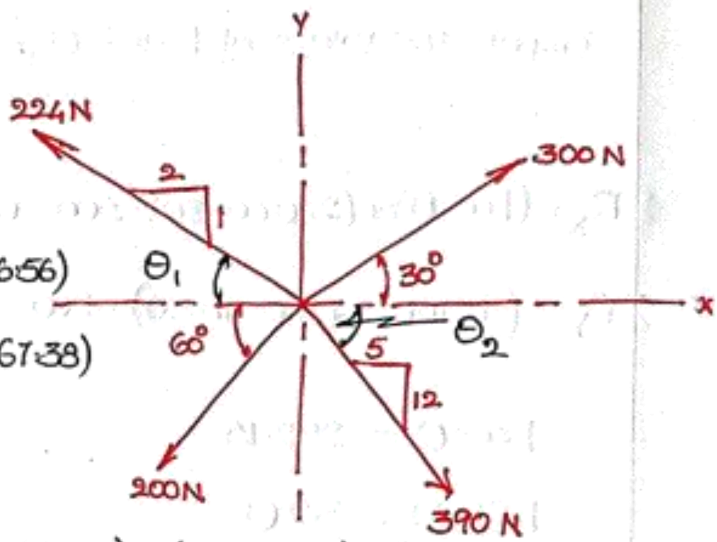
$F_y = -260\sin[67.38-36.87] = -132\ N.$

$P_x = 361\cos[33.69+36.87] = 120.15\ N.$

$P_y = -361\sin[33.69+36.87] = -340.42\ N.$

2. RESULTANT OF A FORCE SYSTEM.

1. Determine the resultant of the four forces acting on the body shown in the figure.



$\theta_1 = Tan^{-1}\left(\frac{1}{2}\right) = 26.56°$

$\theta_2 = Tan^{-1}\left(\frac{12}{5}\right) = 67.38°$

ΣR_x; $(300\cos30) - (224\cos26.56) - (200\cos60) + (390\cos67.38)$

$= 109.45$ N.

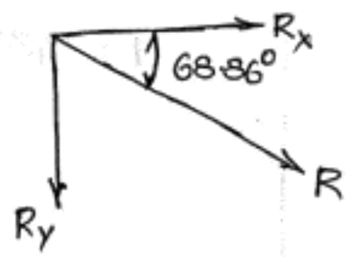
ΣR_y; $(300\sin30) + (224\sin26.56) - (200\sin60) - (390\sin67.38)$

$= -283.05$ N.

Resultan $R = \sqrt{\Sigma R_x^2 + \Sigma R_y^2}$

$R = 303.47$ N. [down to right] [IV Quadrant]

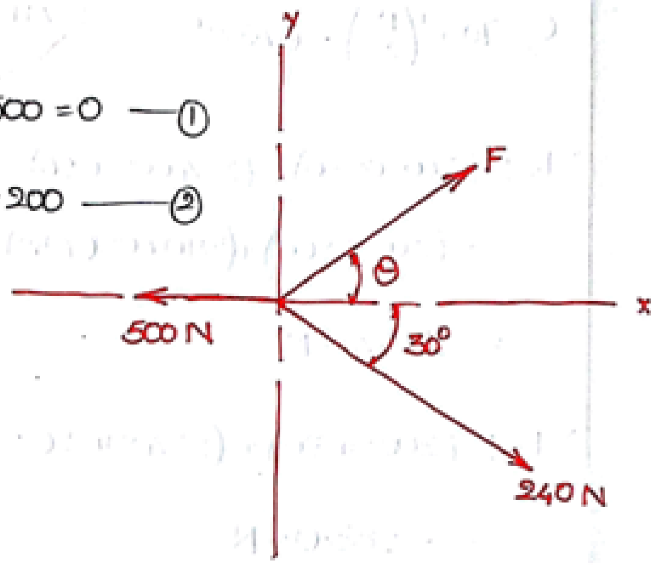
$\theta = Tan^{-1}\left(\frac{F_y}{F_x}\right) = -68.86°$



⑦

2. The force system shown in fig. has a resultant of 200N pointing up along the Y-axis.

Compute the values of F and Θ required to give this resultant.



$\Sigma R_x = (F\cos\theta) + (240\cos 30) - 500 = 0$ —— ①

$\Sigma R_y = (F\sin\theta) - (240\sin 30) = 200$ —— ②

$F\cos\theta = 292.15$

$F\sin\theta = 320$

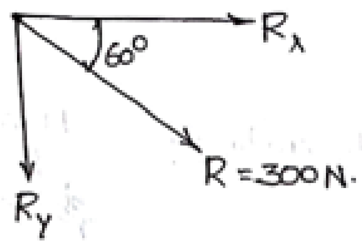
$\tan\theta = 1.095$

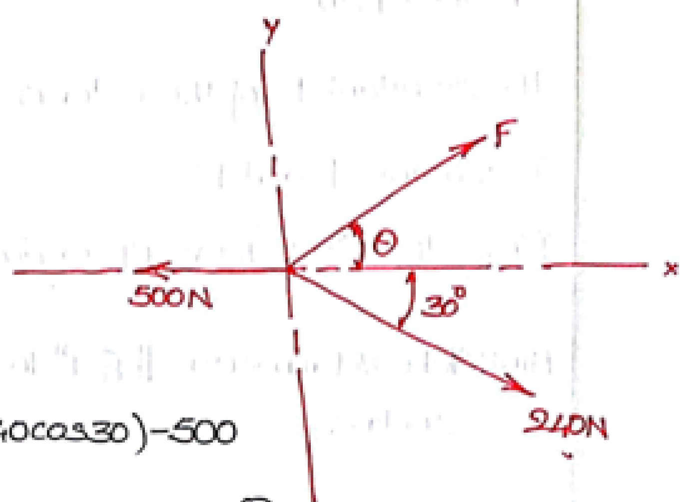
$\theta = 47.6^\circ$

$F = 433.3\,N.$

⑧

3) If the resultant is 300 N down to right at 60° with the x-axis, compute the values of F and θ.





$R_x = 300\cos 60 = (F\cos\theta) + (240\cos 30) - 500$

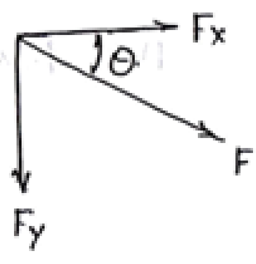
$F\cos\theta = 442.15\ N.$ —— ①

$R_y = -300\sin 60 = (F\sin\theta) - (240\sin 30).$

$F\sin\theta = -139.80\ N$ —— ②.

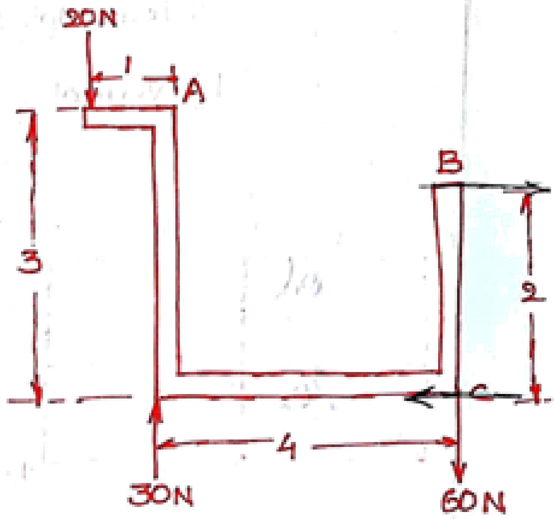
From ① & ②; $\theta = 17.546^\circ$

$F = 463.72\ N.$



3. COUPLES

1\. Replace the system of forces acting on the frame in Fig., by a resultant force R through A and a couple acting horizontally through B and C.



Resultant R through A

$\Sigma R_x = 0$

$\Sigma R_y = -20+30-60 = -50N.$

Acting downwards.

Moment of all forces about A

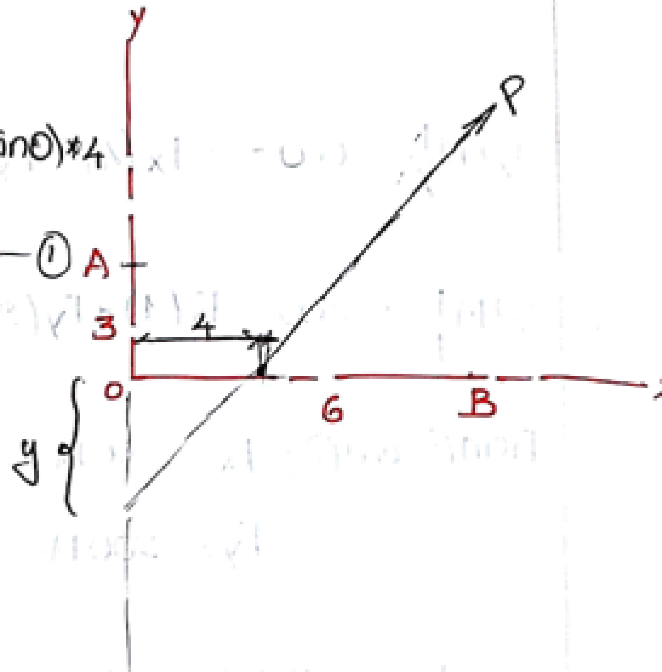
$+\circlearrowright M_A^{All}$; $-(20*1)+(60*4) = 220 N\text{-}m.$

We need couple acting horizontally through B and C.

Force * 2 = 220

Force = 110N Towards right at B and towards left at C.

5. In Fig. a force P intersects the X-axis at 4m to the right of O. If its moment about A is 170 N-m counterclockwise and its moment about B is 40 N-m clockwise, determine its Y-intercept.



$$+\circlearrowleft M_A^P = -170 = -(P\cos\theta)*3 - (P\sin\theta)*4$$

$$170 = 3P\cos\theta + 4P\sin\theta \quad \text{— (1)}$$

$$+\circlearrowleft M_B^P = 40 = (P\sin\theta)(2)$$

$$\therefore \; P\sin\theta = 20$$

From (1); $\quad P\cos\theta = 30.$

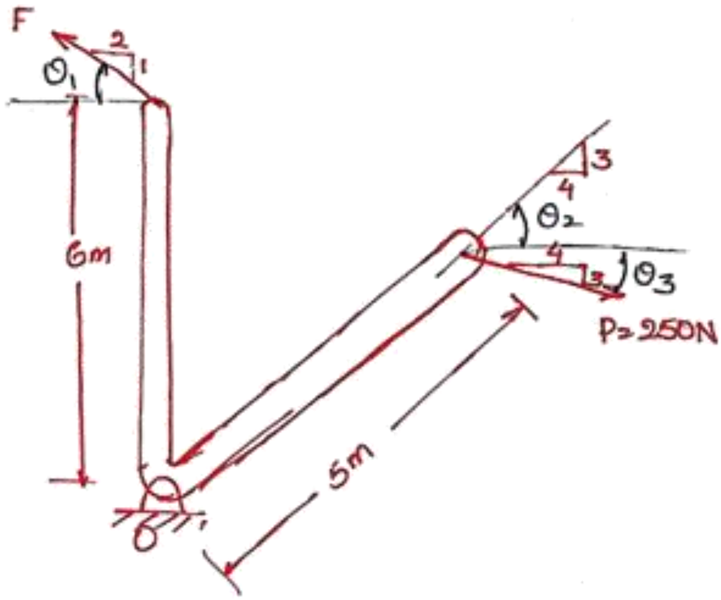
$$\theta = 33.69^0.$$

$$P = 36.05N.$$

$$+\circlearrowleft M_O^P = -[36.05\sin 33.69]*4 = -[36.05\cos 33.69]\, y.$$

$$y = 2.67m.$$

7. In the rocker arm shown in fig., the moment of F about O balances that of P about O. Find F.



$$\theta_1 = \tan^{-1}\left(\frac{1}{2}\right) = 26.56^0$$

$$\theta_2 = \tan^{-1}\left(\frac{3}{4}\right) = 36.87^0$$

$$\theta_3 = \tan^{-1}\left(\frac{3}{4}\right) = 36.87^0$$

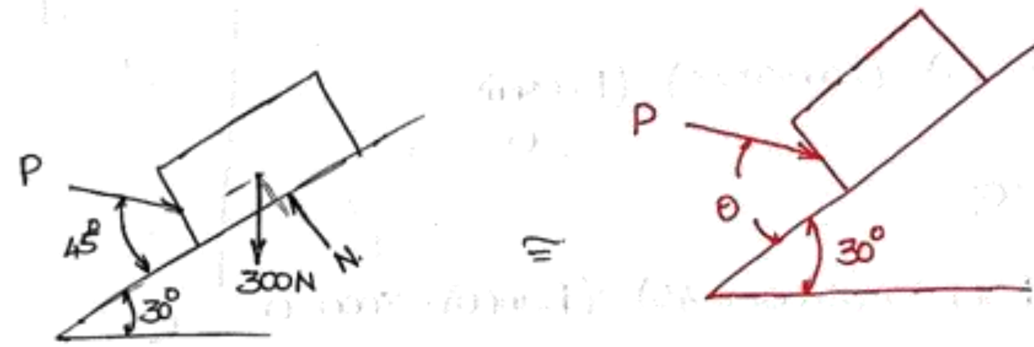
$$+\circlearrowleft M_O^F = +\circlearrowleft M_O^P$$

$$-\left[F\cos(26.56)\right]*6 = 250\sin\left[36.87+36.87\right]*5$$

$$F = \underline{223.59\,N.}$$

EQUILIBRIUM - 1

1. A 300 N box is held at rest on a smooth incline by a force P making an angle θ with the incline as shown in Fig. If θ = 45°, determine the value of P.

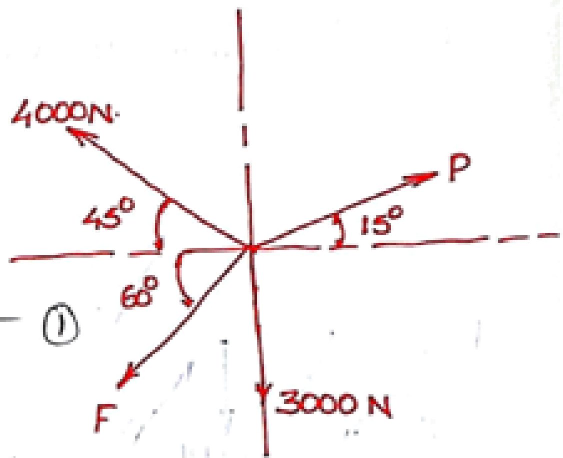


$P\cos 45 = 300\sin 30$

$P = 212.13N.$

2. The forces on the gusset plate of a joint in a bridge truss act as shown in Fig.

Determine the values of P and F to maintain equilibrium of the joint.



$\Sigma F_x = 0;$

$$(P\cos 15) - (4000\cos 45) - (F\cos 60) = 0 \quad \text{—} \ (1)$$

$\Sigma F_y = 0;$

$$(P\sin 15) + (4000\sin 45) - (F\sin 60) - 3000 = 0 \quad \text{—} \ (2)$$

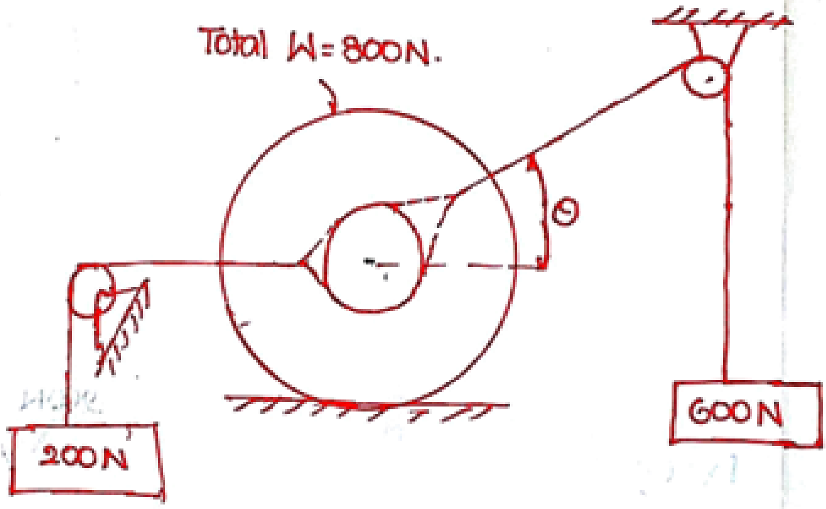
From (1) & (2);

$P = 3342.78\ N$

$F = 800.9\ N.$

4. Cords are looped around a small spacer separating two cylinders each weighing 400 N and pass as shown in fig, over frictionless pulleys to weights of 200 N and 600 N.

Determine the angle θ and the normal reaction N between the cylinders and the smooth horizontal surface.



$\Sigma F_x = 0;$

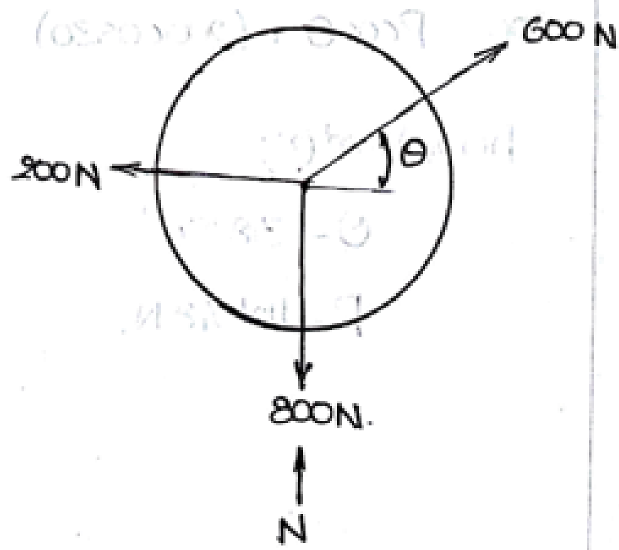
$$600 \cos\theta = 200$$

$$\theta = 70.53^{\circ}$$

$\Sigma F_y = 0;$

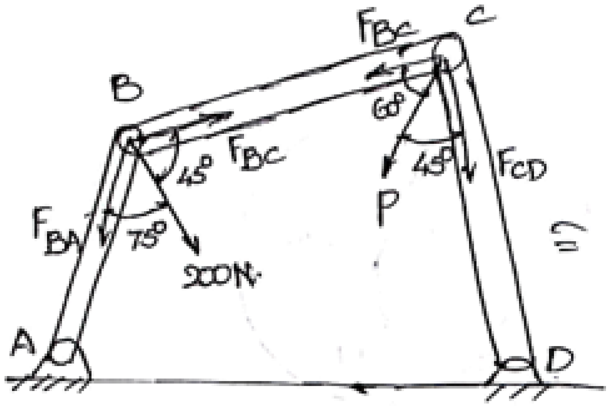
$$N + 600 \sin 70.53 = 800$$

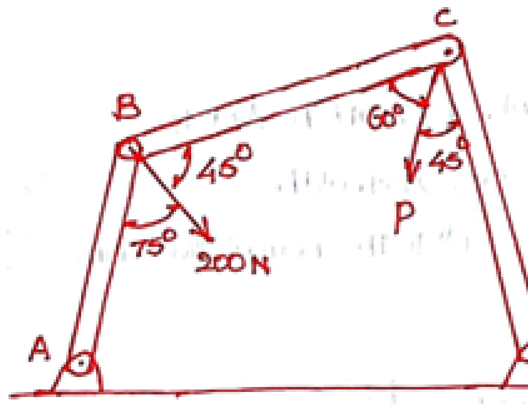
$$N = 234.32 \text{ N.}$$



5. Three bars, pinned together at B and C and supported by hinges at A and D as shown in Fig., form a four-link mechanism. Determine the value of P that will prevent motion.

(28)





At Joint B;

$$\frac{200}{\sin 240} = \frac{F_{BC}}{\sin 75} = \frac{F_{BA}}{\sin 45}$$

$F_{BC} = -223.07\,N$

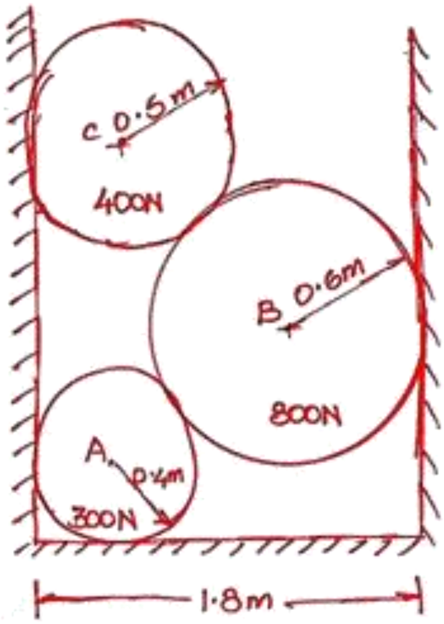
$F_{BA} = -163.29\,N.$

At Joint C

$$\frac{P}{\sin 255} = \frac{-223.07}{\sin 45}$$

$P = 304.72\,N.$

7. Three cylinders are piled in a rectangular ditch as shown in Fig. Neglecting the friction, determine the reaction between cylinder A and the Vertical Wall.



$\sin\theta_1 = \frac{0.7}{1.1}$

$\theta_1 = 39.52$

$\theta_2 = 50.47$

Block C:

$R_1 = R_2 \sin 39.52$

$400 = R_2 \cos 39.52$

$R_2 = 518.54\ N.$

$R_1 = 329.96\ N$

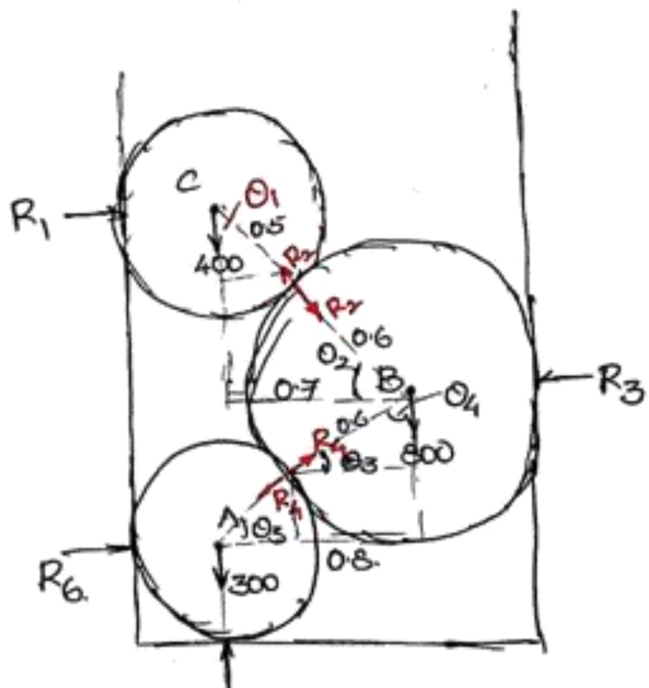
$\cos\theta_3 = \frac{0.8}{1}$

$\theta_3 = 36.87$

$\theta_4 = 53.13$

Block-B

$R_3 - 518.54 \cos 50.47 - R_4 \cos 36.87 = 0$



$800 + R_2 \sin 50.47 = R_4 \sin 36.87.$

$R_3 = 1929.96\ N.$

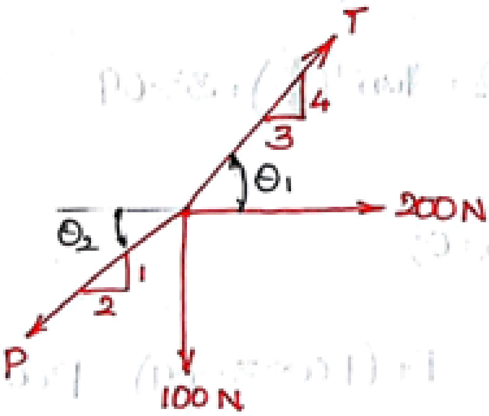
$R_4 = 2{,}000\ N.$

Block A:

$R_6 = R_4 \cos 36.87 = \underline{1600\ N.}$

EQUILIBRIUM - II

1. Determine the value of T, if the force system shown in Fig. is in equilibrium.



$\theta_1 = \tan^{-1}\left(\frac{4}{3}\right) = 53.13$

$\theta_2 = \tan^{-1}\left(\frac{1}{2}\right) = 26.56$

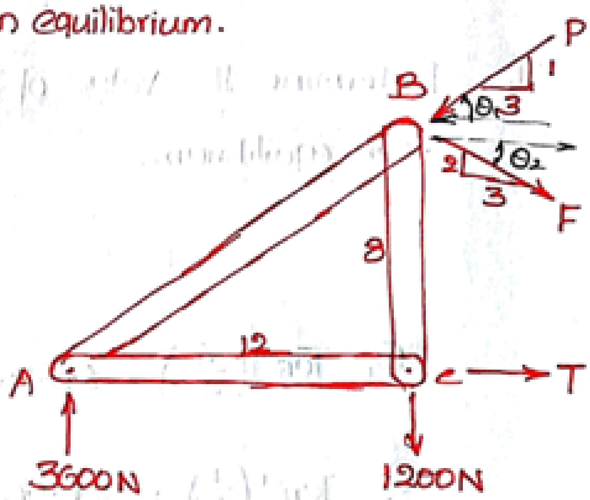
$\Sigma F_x = 0$; $200 + (T\cos 53.13) - (P\cos 26.56) = 0$ —— (1)

$\Sigma F_y = 0$; $(T\sin 53.13) - (P\sin 26.56) - 100 = 0$ —— (2)

From (1) & (2); $T = 399.9$ N.

$P = 491.85$ N.

2. Determine the forces P, F, and T required to keep the triangular frame ABC shown in Fig in equilibrium.



$\theta_1 = \tan^{-1}\left(\frac{1}{3}\right) = 18.43$

$\theta_2 = \tan^{-1}\left(\frac{2}{3}\right) = 33.69$

$\Sigma F_x = 0;$

$T + (F\cos 33.69) - P\cos(18.43) = 0$ ——— (1)

$\Sigma F_y = 0;$ $3600 - 1200 - F\sin 33.69 - P\sin 18.43 = 0$ ——— (2)

$+\circlearrowright M_B^{All} = 0;$ $3600(12) = T(8)$

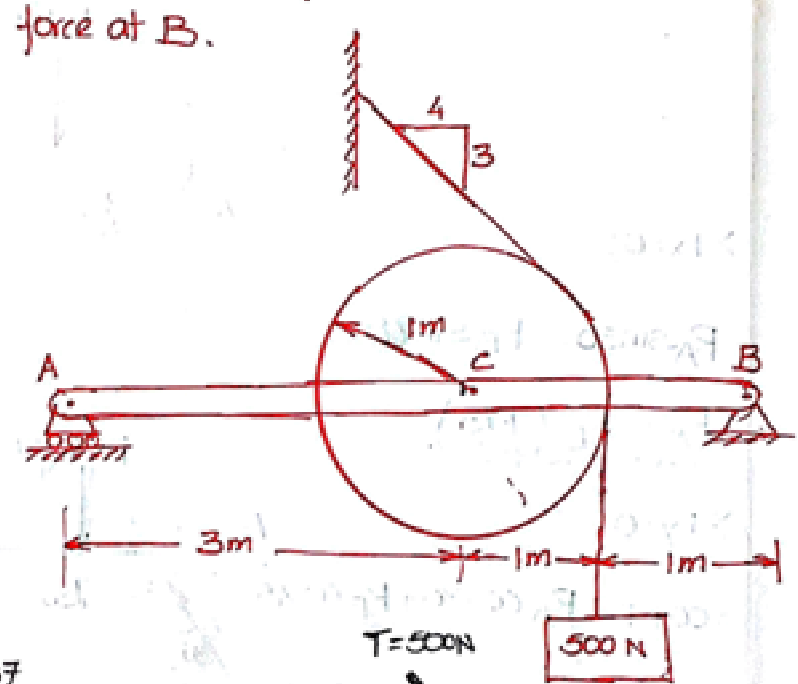
$T = 5400$ N. ——— (3)

$\therefore$ $F = 721.8$ N.

$P = 6324.98$ N.

EQUILIBRIUM-3

1. A pulley of 1-m radius, supporting a load of 500N, is mounted at B on a horizontal beam as shown in Fig.
If the beam weighs 200N and the pulley weighs 50N, find the hinge force at B.



Pulley

$\theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.87$

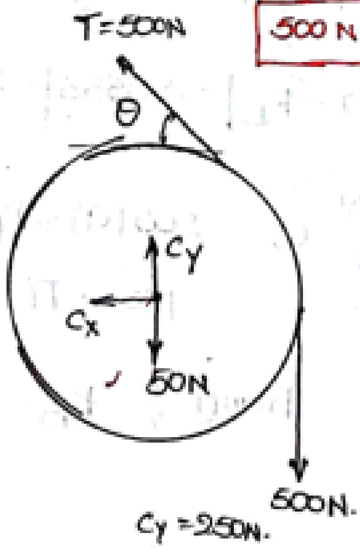
$\Sigma F_x = 0;$

$C_x = -500 \cos 36.87$

$C_x = -400N.$

$50 + 500 = C_y + 500 \sin 36.87$

$C_y = 250N.$



Beam.

$\Sigma F_x = 0;$

$R_{Bx} = -400N.$

$\Sigma F_y = 0;$

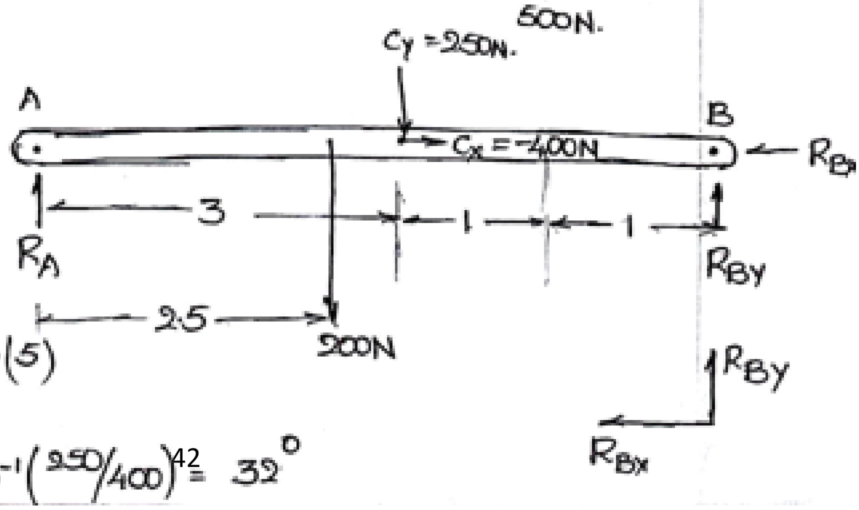
$R_A + R_{By} = 450N.$

$+\circlearrowleft M_A^{All} = 0;$

$200(2.5) + 250(3) = R_{By}(5)$

$R_{By} = 250N.$

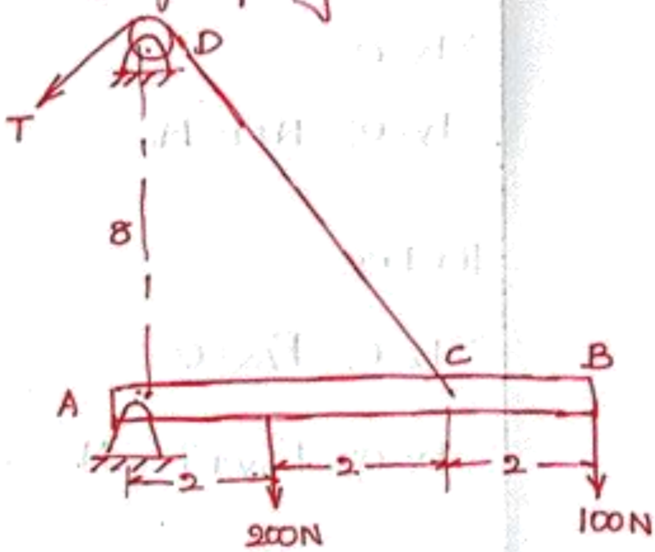
$R = 471.7\ N; \quad \theta = \tan^{-1}(250/400) = 32^{\circ}$



7. A boom AB is supported in a horizontal position by a hinge A and a cable which runs from C over a small pulley at D as shown in Fig.

Compute the tension T in the cable, and the horizontal and vertical components of the reaction at A.

Neglect the weight of the boom and the size of the pulley at D.



$\tan\theta = \frac{8}{4}$

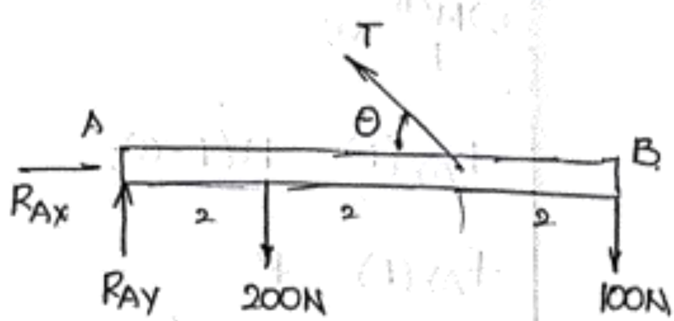
$\theta = 63.43$

$\Sigma F_x = 0;$

$R_{Ax} = T\cos\theta$

$\Sigma F_y = 0;$

$R_{Ay} + T\sin\theta = 200 + 100$



$+\circlearrowleft \Sigma M_A^{All} = 0;$

$T\sin\theta(4) = 200(2) + 100(6)$

$T = 279.52\ N.$

$\therefore\ R_{Ax} = 125.02\ N$
$R_{Ay} = 50\ N.$ } Ans

$R = 134.647\ N.$

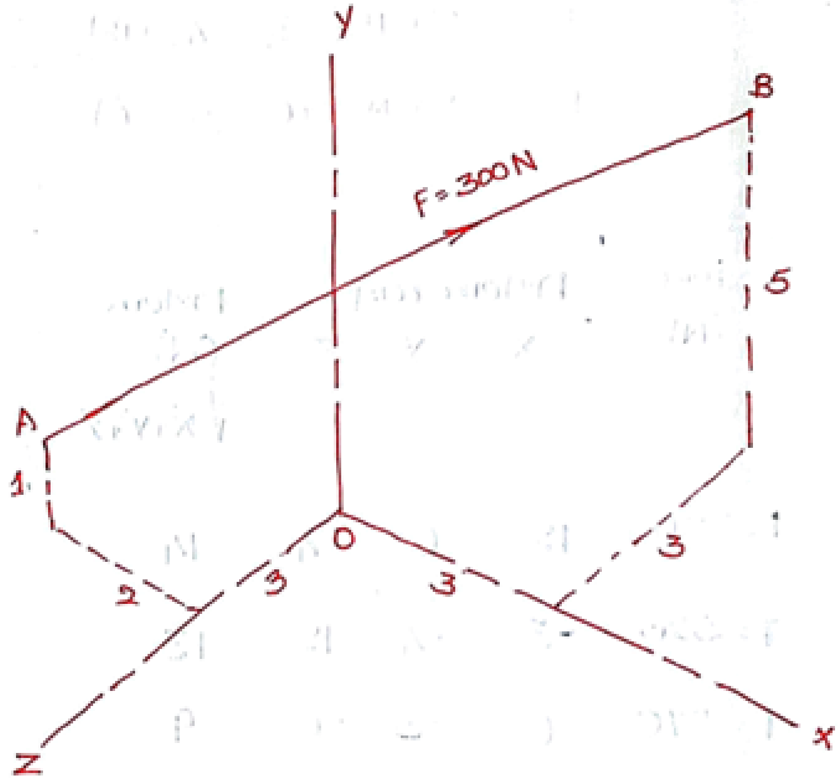
SPATIAL FORCES

1. A force F = 300 N is passing from a point A to B as shown in figure. Find the components of the force.

O(0,0,0)

A(-2,1,3)

B(3,5,-3)



$$\frac{F_x}{x} = \frac{F_y}{y} = \frac{F_z}{z} = \left(\frac{F}{d}\right) \leftarrow F_m$$

$$\frac{F_x}{3+2} = \frac{F_y}{5-1} = \frac{F_z}{-3-3} = \frac{300}{\sqrt{5^2+4^2+6^2}}$$

$F_x = 170.94$ N.

$F_y = 136.75$ N

$F_z = -205.129$ N.

3. Find the resultant of the force system shown in Fig., in which $P = 280N$, $T = 260N$, and $F = 210N$.

Sol:- Co-ordinate points are;

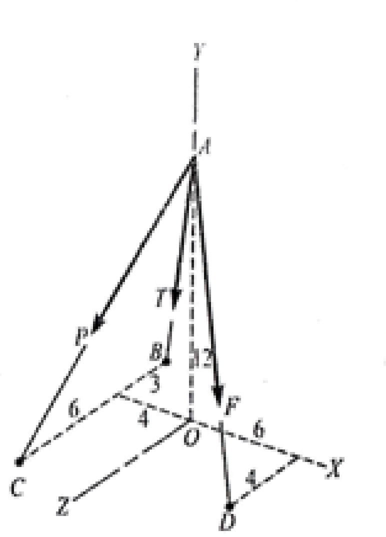
O(0,0,0)

A(0,12,0)

B(-4,0,-3)

C(-4,0,6)

D(6,0,4).



Force : P - A to C

T - A to B

F - A to D.

Force (N)	Distance comp X	Y	Z	Distance (d) $\sqrt{x^2+y^2+z^2}$	Force Multiplier $F_m = F/d$	Force components (N) X-comp	Y-comp	Z-comp
P= 280	-4	-12	6	14	20	-80	-240	120
T= 260	-4	-12	-3	13	20	-80	-240	-60
F= 210	6	-12	4	14	15	90	-180	60
						-70	-660	120

$R_x = -70N$ — Backward

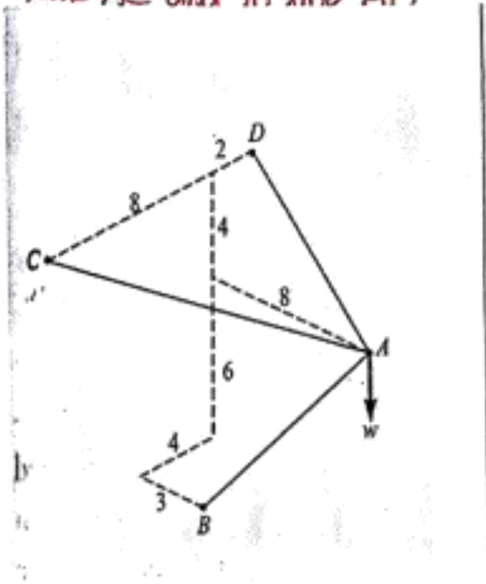
$R_y = -660N$ — Down

$R_z = 120N$. — Right

$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = 674.46N$

CROSS PRODUCT

1. Refering to the cantilever framework shown in Fig., find the shortest distance from point B to line AC and to line AD



Co-ordinate Points;

A (8,0,0)

B (3,-6,4)

C (0,4,8)

D (0,4,-2)

a) Shortest distance from point B to line AC

$$d = \frac{|r_{BA} \times AC|}{|AC|}$$

$$= \frac{|(5i+6j-4k)\times(-8i+4j+8k)|}{\sqrt{64+16+64}}$$

$$\begin{vmatrix} i & j & k \\ 5 & 6 & -4 \\ -8 & 4 & 8 \end{vmatrix} = i(48+16) - j(40-32) + k(20+48)$$

$$64\bar{i} - 8\bar{j} + 68\bar{k}$$

$$d = \frac{\sqrt{64^2+8^2+68^2}}{\sqrt{64+16+64}} = 7.81\,m.$$

b) Shortest distance from point B to line AD.

$$d = \frac{|r_{BA} \times AD|}{|AD|}$$

$$\begin{vmatrix} i & j & k \\ 5 & 6 & -4 \\ -8 & 4 & -2 \end{vmatrix}$$

$$= i(-12+16) - j(-10-32) + k(20+48)$$

$$d = \frac{\sqrt{4^2+42^2+68^2}}{\sqrt{8^2+4^2+2^2}}$$

$$d = 8.73\,m.$$

58

2. In the, A boom AC is supported by a ball-and-socket joint at C and by the cables BE and AD.

The force multiplier of a force F acting from B to E is $F_m = 10\,N/m$,

Find the component of F that is perpendicular to the plane DAC.

Co-ordinate points are;

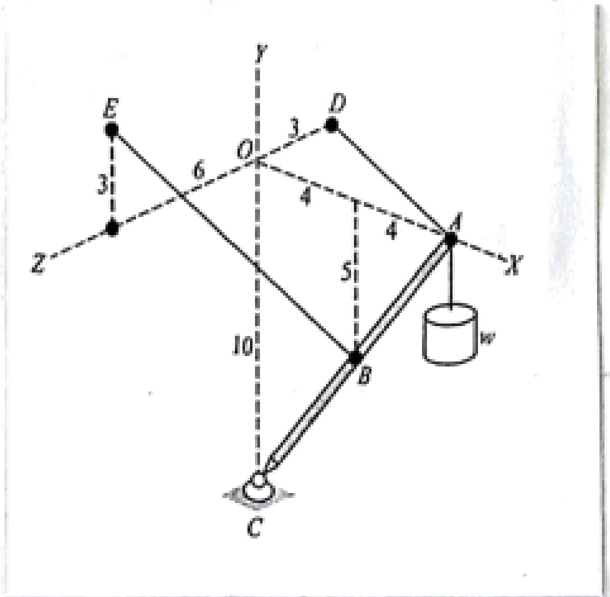
A (8,0,0)

B (4,-5,0)

C (0,-10,0)

D (0,0,-3)

E (0,3,6)



Force; $F = 10(-4i+8j+6k)$

The comp. of F, that is $\perp^{r}$ to the plane DAC

$$F_{\perp^{r}\text{ to the plane DAC}} = \bar{F}\cdot\hat{n}_{DAC}$$

Let us consider a vector $\bar{N}$, $\perp^{r}$ to plane DAC

$$\bar{N} = \bar{AD}\times\bar{AC} = \begin{vmatrix} i & j & k \\ -8 & 0 & -3 \\ -8 & -10 & 0 \end{vmatrix} = i(-30) - j(-24) + k(80)$$

$$= -30i + 24j + 80k.$$

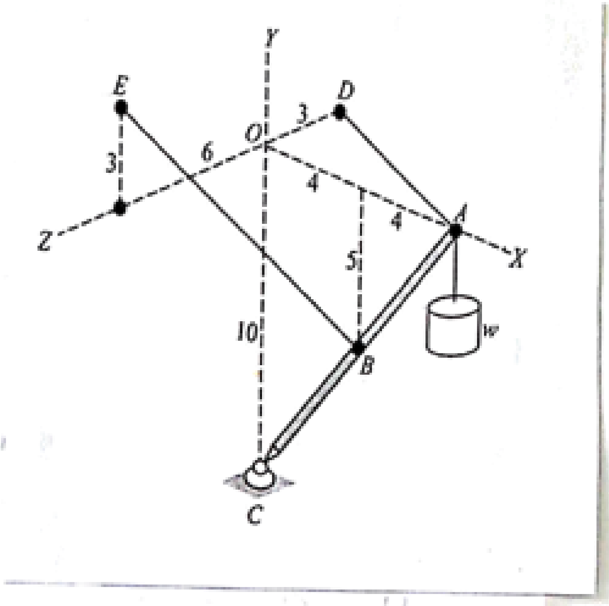
The comp. of F; $10(-4i+8j+6k)\dfrac{(-30i+24j+80k)}{\sqrt{30^2+24^2+80^2}}$

$= \underline{89.24\,N.}$

3. (59)

In the Fig., if the force multiplier of a force P acting from A to D is $P_m = 20\,N/m$, determine the component of P that is perpendicular to the plane defined by points E, A and C.

Co. ordinate points;



A(8,0,0)

B(4,-5,0)

C(0,-10,0)

D(0,0,-3)

E(0,3,6).

$P = 20(-8i - 3k)$

The comp. of P, that is $\perp^{r}$ to the plane EAC.

$$= \bar{P} \cdot \hat{n}_{EAC}$$

Let us consider a vector $\bar{N}$, $\perp^{r}$ to plane EAC

$$\bar{N} = \bar{AE} \times \bar{AC} = \begin{vmatrix} i & j & k \\ -8 & 3 & 6 \\ -8 & -10 & 0 \end{vmatrix}$$

$$= i(60) - j(48) + k(80+24).$$

$$= 60i - 48j + 104k.$$

The comp. of $P = 20(-8i-3k)\dfrac{(60i-48j+104k)}{\sqrt{60^2+48^2+104^2}}$

$$= -122.5\,N.$$