A Refresher Course

On

"ENGINEERING MECHANICS"

Organized By

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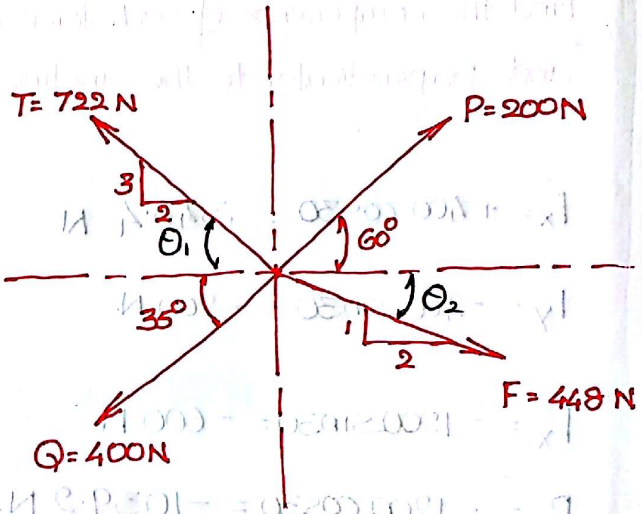
1. Components Of A Force

1. Find x and y components of each of four forces shown in Fig.



$\theta_1 = \tan^{-1}\left(\frac{3}{2}\right) = 56.31$

$\theta_2 = \tan^{-1}\left(\frac{1}{2}\right) = 26.56$

Force P

$P_x = 200 \cos 60 = +100\ N$

$P_y = 200 \sin 60 = +173.2\ N.$

Force T

$T_x = -722 \cos 56.31 = -400.5\ N$

$T_y = 722 \sin 56.31 = -600.7\ N.$

Force Q

$Q_x = -400 \cos 35 = -327.6\ N$

$Q_y = -400 \sin 35 = -229.4\ N.$

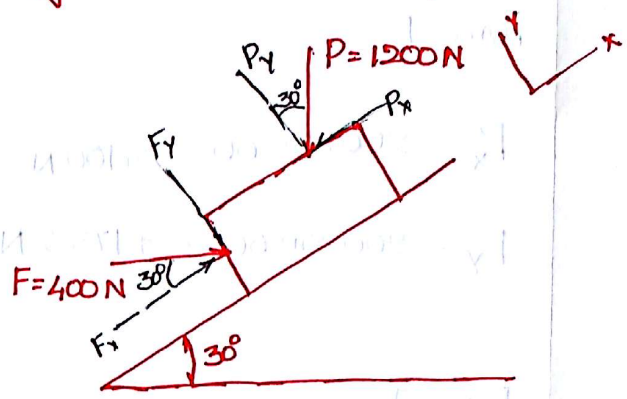
Force F

$F_x = 448 \cos 26.56 = 400.72\ N.$

$F_y = -448 \sin 26.56 = -200.32\ N.$

2. The body on the incline is subjected to the vertical and horizontal forces shown.

Find the components of each force along x-y axes oriented parallel and perpendicular to the incline.



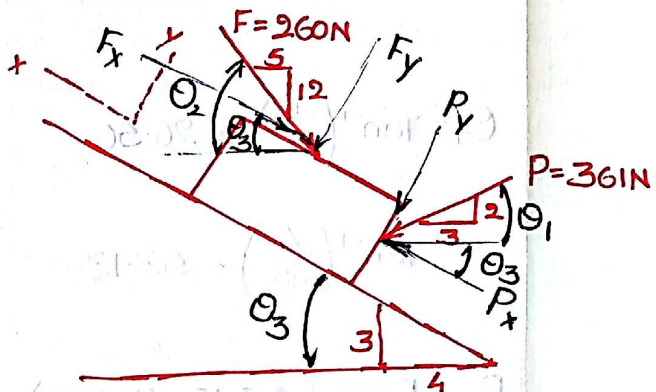
$F_x = +400 \cos 30 = 346.4$ N

$F_y = -400 \sin 30 = -200$ N

$P_x = -1200 \sin 30 = -600$ N

$P_y = -1200 \cos 30 = -1039.2$ N.

3. Determine the components of the forces P and Q along the axes which are parallel and perpendicular to the incline.



$\theta_1 = \tan^{-1}\left(\frac{2}{3}\right) = 33.69$

$\theta_2 = \tan^{-1}\left(\frac{12}{5}\right) = 67.38$

$\theta_3 = \tan^{-1}\left(\frac{3}{4}\right) = 36.87$

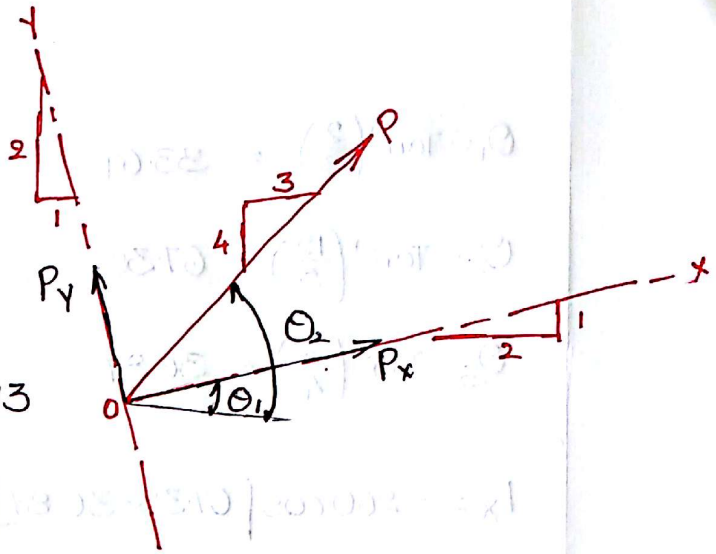
$F_x = -260\cos\left[67.38 - 36.87\right] = -224\ N.$

$F_y = -260\sin\left[67.38 - 36.87\right] = -132\ N.$

$P_x = 361\cos\left[33.69 + 36.87\right] = 120.15\ N.$

$P_y = -361\sin\left[33.69 + 36.87\right] = -340.42\ N.$

4. If the x-component of P is 893 N, Determine P and its y-component.



$\theta_1 = \tan^{-1}\left(\frac{1}{2}\right) = 26.56$

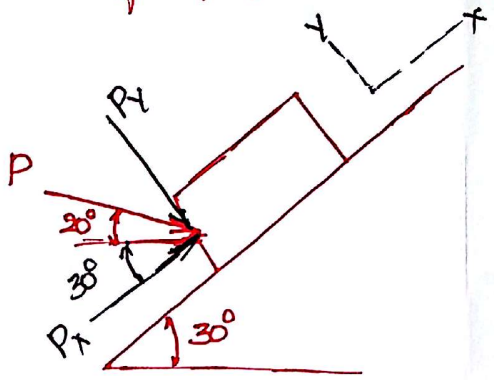
$\theta_2 = \tan^{-1}\left(\frac{4}{3}\right) = 53.13$

$P_x = P\cos(53.13 - 26.56) = 893$

$P = 998.45\,N.$

$P_y = P\sin(53.13 - 26.56) = 446.59\,N.$

5. The body on the 30° incline is acted upon by a force P inclined at 20° with the horizontal. If P is resolved into components parallel and perpendicular to the incline and the value of the parallel component is 300 N, find the value of the perpendicular component and P.



$P_x = P\cos[20+30] = 300$

$P = 466.72\ N.$

$P_Y = -P\sin[20+30]$

$= -466.72 \sin 50 = -357.52\ N.$

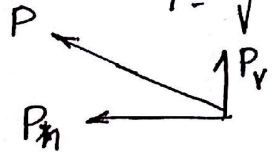
6. The horizontal and vertical components of several forces are given below. Find the each force.

$P_h = -100\ N$; $P_v = 200\ N$

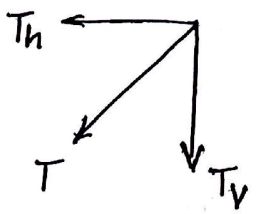
$T_h = -400N$; $T_V = -500\ N.$

Force P: $P = \sqrt{100^2+200^2} = 223.61N.$; $\theta = \tan^{-1}\left(\frac{-200}{100}\right) = -63.43$

$T = \sqrt{400^2+500^2} = 640.31N$; $\theta = \tan^{-1}\left(\frac{500}{400}\right) = 51.34$



II Quadrant



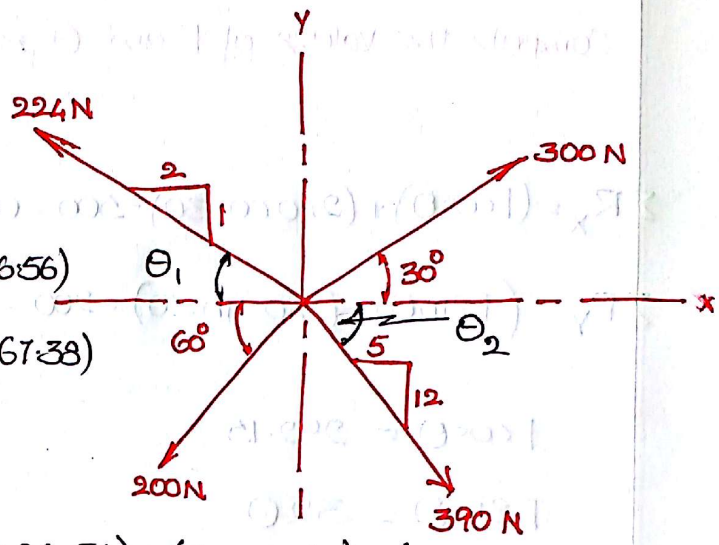
III Quadrant.

2. RESULTANT OF A FORCE SYSTEM.

1. Determine the resultant of the four forces acting on the body shown in the figure.

$\theta_1 = \tan^{-1}\left(\frac{1}{2}\right) = 26.56^\circ$

$\theta_2 = \tan^{-1}\left(\frac{12}{5}\right) = 67.38^\circ$



ΣR_x; $(300\cos 30) - (224\cos 26.56) - (200\cos 60) + (390\cos 67.38)$

$= 109.45$ N.

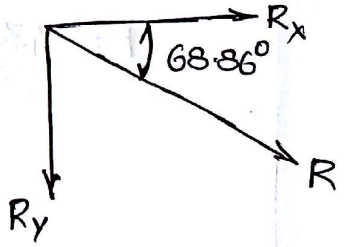
ΣR_y; $(300\sin 30) + (224\sin 26.56) - (200\sin 60) - (390\sin 67.38)$

$= -283.05$ N.

Resultan $R = \sqrt{\Sigma R_x^2 + \Sigma R_y^2}$

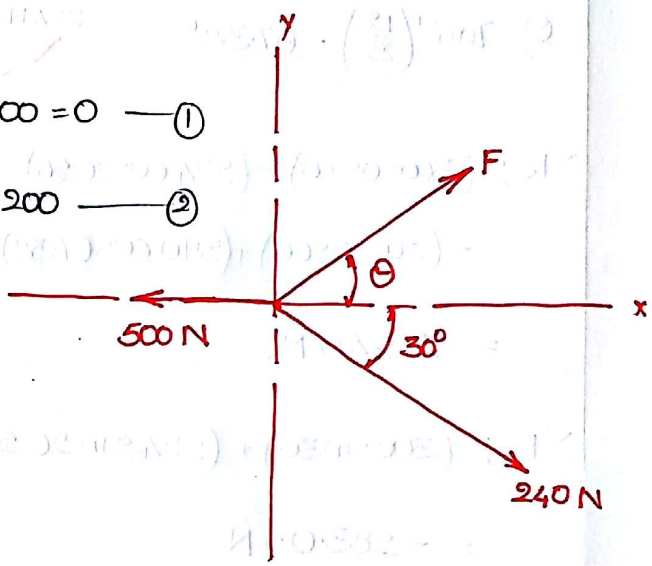
$R = 303.47$ N. [doon-to right]
[IV Quadrant]

$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = -68.86^\circ$



2. The force system shown in fig. has a resultant of 200N pointing up along the Y-axis.

Compute the values of F and θ required to give this resultant.



$$\Sigma R_x = (F\cos\theta) + (240\cos 30) - 500 = 0 \quad \text{—①}$$

$$\Sigma R_y = (F\sin\theta) - (240\sin 30) = 200 \quad \text{—②}$$

$$F\cos\theta = 292.15$$

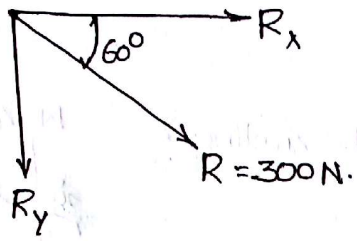
$$F\sin\theta = 320$$

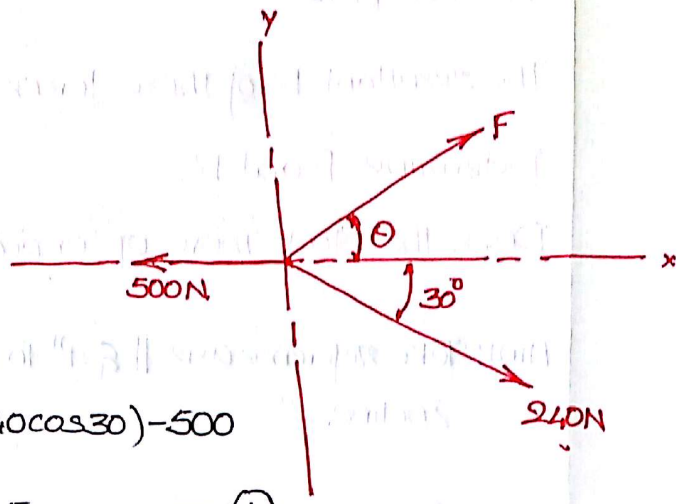
$$\tan\theta = 1.095$$

$$\theta = 47.6^\circ$$

$$F = 433.3\ \text{N}.$$

3) If the resultant is 300 N down to right at 60° with the x-axis, compute the values of F and θ.





$R_x = 300\cos 60 = (F\cos\theta) + (240\cos 30) - 500$

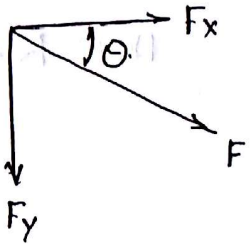
$F\cos\theta = 442.15$ N. —— (1)

$R_y = -300\sin 60 = (F\sin\theta) - (240\sin 30)$.

$F\sin\theta = -139.80$ N —— (2).

From (1) & (2); $\theta = 17.546^0$

$F = 463.72$ N.

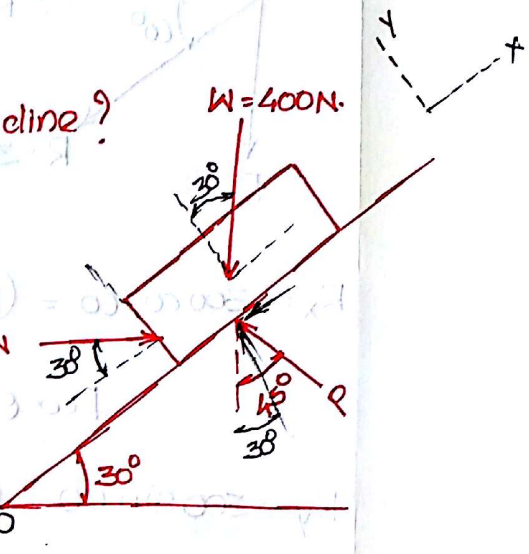


4. The block shown in figure is acted on by its weight W = 400 N, a horizontal force F = 600 N, and the pressure P exerted by the inclined plane.

The resultant R of these forces is parallel to the incline.

Determine P and R.

Does the block move up or down the incline?



Hint: Take reference axis ∥ & ⊥ᴿ to the incline.

$\Sigma R_x = (600\cos 30) - (400\sin 30) - (P\sin 15) = R$

$\Sigma R_y = -(600\sin 30) - (400\cos 30) + (P\cos 15) = 0$

$P = 669.21\,N.$

$R = 146.41\,N.$

Here; R is +ve; means block moves up along the incline.

5. Two locomotives on opposite banks of a canal pull a vessel moving parallel to the banks by means of two horizontal ropes. The tensions in these ropes are 2000N and 2400N while the angle between them is 60°.

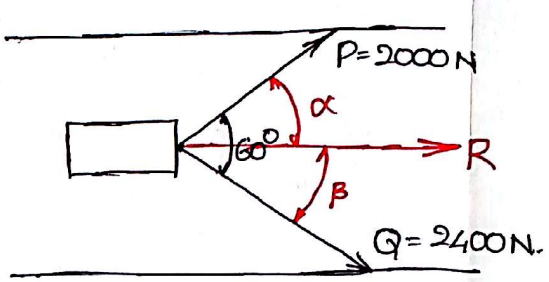
Find the resultant pull on the vessel and the angle between each of the ropes and the sides of the canal.

ΣR_x; $(2000\cos\alpha)+(2400\cos\beta)=R$

ΣR_y; $(2000\sin\alpha)-(2400\sin\beta)=0$

(3) unknowns, (2) equations

Not possible.



Using Law of Parallelogram;

$$R=\sqrt{P^2+Q^2+2PQ\cos\theta}$$

$$R=\sqrt{2000^2+2400^2+2(2000)(2400)\cos 60}$$

$$R=3815.15\ N.$$

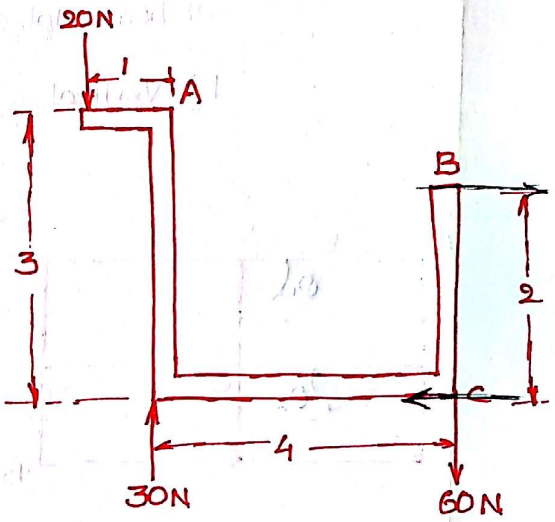
$$\tan\alpha=\frac{Q\sin\theta}{P+Q\cos\theta}=\frac{2400\sin 60}{2000+2400\cos 60}.$$

$$\alpha=33^\circ$$

$$\beta=60-33=27^\circ$$

3. COUPLES

1. Replace the system of forces acting on the frame in Fig., by a resultant force R through A and a couple acting horizontally through B and C.



Resultant R through A

$\Sigma R_x = 0$

$\Sigma R_y = -20 + 30 - 60 = -50N.$

Acting downwards.

Moment of all forces about A

$+\circlearrowleft M_A^{All}$; $-(20*1) + (60*4) = 220N\text{-}m.$

We need couple acting horizontally through B and C.

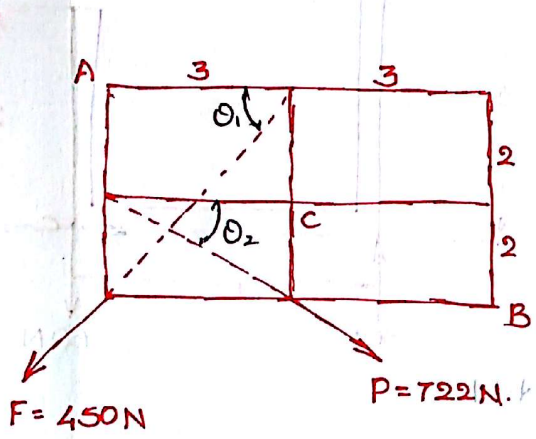
Force * 2 = 220

Force = 110N towards right at B and towards left at C.

2. Figure shows a plate subjected to loads. Replace the loads by an equivalent force through C and a couple acting through A and B. Solve if the forces of the couple are

a) horizontal and

b) Vertical.



$\theta_1 = \tan^{-1}\left(\frac{4}{3}\right) = 53.13^{0}.$

$\theta_2 = \tan^{-1}\left(\frac{2}{3}\right) = 33.69^{0}$

$\Sigma R_x = -(450\cos 53.13) + (722\cos 33.69) = 330.74\ N.$

$\Sigma R_y = -(450\sin 53.13) - (722\sin 33.69) = -760.49\ N.$

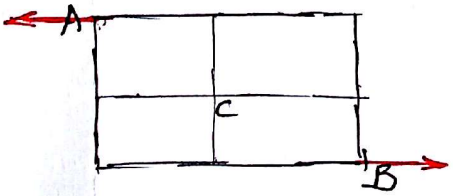
Resultant, $R = \sqrt{(\Sigma R_x)^2 + (\Sigma R_y)^2} = 829.29\ N.$

It acts in IV Quadrant.

$+\circlearrowright \Sigma M_C : -F_x(2) - P_y(3)$

$= -[450\cos 53.13]*2 - [722\sin 33.69]*3 = -1741.47\ N\text{-}m.$

a) Horizontal

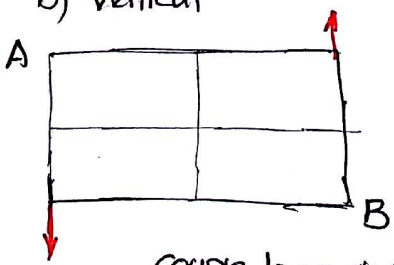


Couple force * 4 = 1741.47

Couple force = 435.37 N.

(Left at A and Right at B).

b) Vertical



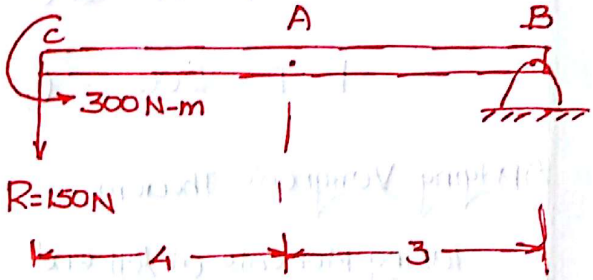
Couple force * 6 = 1741.47

Couple force = 290.25 N.

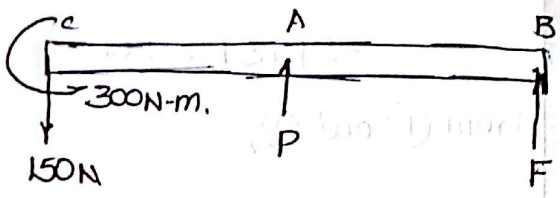
(Down at A and up at B)

3) A vertical force P at A and another vertical force F at B which act on the bar shown in Figure, produce a resultant force of 150 N down at D and produce a CCW couple C = 300 N-m.

Find the magnitude and direction of the forces P and F.



Let us assume the forces P and F are acting upwards.



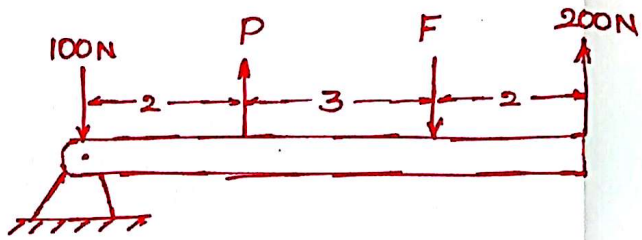
$\therefore \quad P + F = -150.$

$+\circlearrowright M_B^{All} \quad P(3) = -300 - 150(7)$

$P = -450\ N$ – Acting downwards

$F = 300\ N.$ – Acting upwards.

4. Find the Values of P and F so that the four forces shown in figure produce an upward resultant of 300 N acting at 4 m from the left end of the bar.



$$\Sigma R_y = -100 + P - F + 200 = 300$$

$$P - F = 200 \quad -(1)$$

Applying Varignon's Theorem.

Taking Moments at left end

$$-P(2) + F(5) - 200(7) = -300(4)$$

$$-2P + 5F = 200 \quad -(2)$$

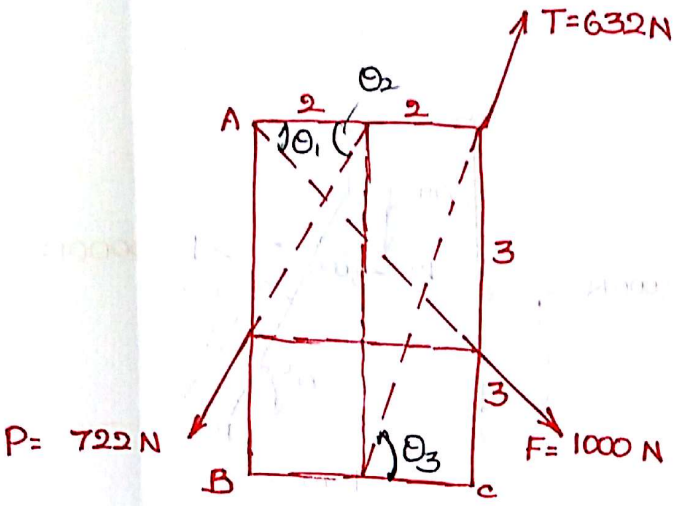
From (1) and (2);

$$P = 400\,N$$

$$F = 200\,N$$

5. Compute the resultant of the three forces acting on the plate shown in fig.

Locate its intersection with AB and Bc.



$\theta_1 = \tan^{-1}\left(\frac{3}{4}\right) = 36.87$

$\theta_2 = \tan^{-1}\left(\frac{3}{2}\right) = 56.31$

$\theta_3 = \tan^{-1}\left(\frac{6}{2}\right) = 71.56$

ΣR_x; $(1000\cos 36.87) - (722\cos 56.31) + (632\cos 71.56)$.

$= 599.41$ N.

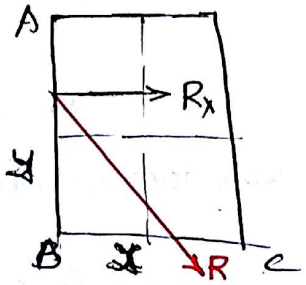
ΣR_y; $-(1000\sin 36.87) - (722\sin 56.31) + (632\sin 71.56) = -601.19$ N.

Resultant, $R = \sqrt{(\Sigma R_x)^2 + (\Sigma R_y)^2} = 848.95$ N. → (IV Quadrant)

$\theta = \tan^{-1}\left(\frac{\Sigma R_y}{\Sigma R_x}\right) = 45.08^0$

Using Varignon's theorem;

$\Sigma M_B^{All} = \Sigma M_B^R$



$(1000\cos 36.87)*6 - (722\cos 56.31)*6 + (722\sin 56.31)*2$

$- (632\sin 71.56)*2 = 599.41(Y) = 601.19(X)$.

$x = 3.99$ m.

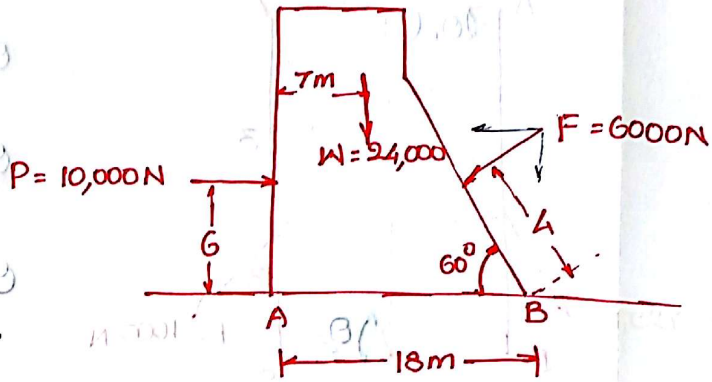
$Y = 4$ m

On AB; Resultant is 4 m from B

i.e; on Bc; Resultant is 3.99 m from B.

6. Determine the resultant of three forces acting on the dam shown in fig. and locate its intersection with the base AB.

For good design, this intersection should occur within the middle third of the base. Does it?



ΣF_x; (10,000) − (6000 sin 60)

= 4803.85 N.

ΣF_y; −24,000 − (6000 cos 60).

= −27,000 N.

Resultant, $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$ = 27,424.02 N. (in 4th Quadrant).

$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$ = 79.91°.

Applying Varignon's theorem;

ΣM_B = Σ Moment of R_B.

(10,000 * 6) − (24,000 * 11) − (6000 * 4) = − 27,000 (x).

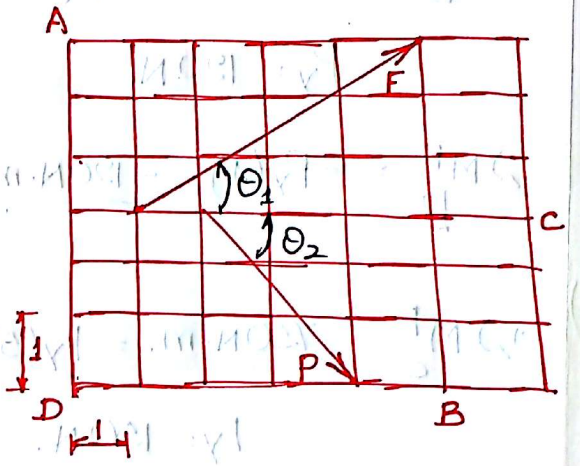
x = 8.44 m.

Resultant cuts the base AB at 8.44 m from B.

[6 < 8.44 < 12
Resultant lies within the middle third of the base.] − Ans.

MOMENT OF A FORCE.

1. In the figure, assuming clockwise moments as positive, compute the moment of force F = 450 N and of force P = 361 N about points A, B, C and D.



$\theta_1 = \tan^{-1}\left(\frac{3}{4}\right) = 36.87^\circ$

$\theta_2 = \tan^{-1}\left(\frac{3}{2}\right) = 56.31^\circ$

$+\circlearrowright M^F_A$

$F_x = (450 \cos 36.87) = 360\ N.$

$F_y = (450 \sin 36.87) = 270\ N.$

$+\circlearrowright M^F_A = -(360*3) - (270*1) = -1350\ N\text{-}m$

$+\circlearrowright M^F_B = (360*3) + (270*4) = 2160\ N\text{-}m$

$+\circlearrowright M^F_C = (270*5) = 1350\ N\text{-}m.$

$+\circlearrowright M^F_D = (360*3) - (270) = 810\ N\text{-}m.$

$P_x = 361 \cos 56.31 = 200.25\ N.$

$P_y = -361 \sin 56.31 = -300.37\ N.$

$+\circlearrowright M^P_A = -(200.25*3) + (300.37*2) = 0$

$+\circlearrowright M^P_B = (200.25*3) - (300.37*3) = -300.36\ N\text{-}m$

$+\circlearrowright M^P_C = -(300.37*4) = -1201.48\ N\text{-}m$

$+\circlearrowright M^P_D = (200.25*3) + (300.37*2) = 1201.49\ N\text{-}m.$

2. In the previous fig., if the moment of P is 480 N-m about D, find the moment of P about B.

Also, if the moment of F about C is 750 N-m, find its moment about D.

i) $+\circlearrowright M_D^P = 480\ \text{N-m} = P_y(4)$

$P_y = 120\ \text{N}.$

$+\circlearrowright M_B^P = -P_y(1) = -120\ \text{N-m}.$

ii) $+\circlearrowright M_C^F = 750\ \text{N-m} = F_y(5)$

$F_y = 150\ \text{N}.$

$+\circlearrowright M_D^F = F_x(3) - F_y(1).$ ——— What is F_x = ?

$+\circlearrowright M_C^F = F_x(3) + F_y(1) = 750.$

$F_x(3) + 150 = 750$

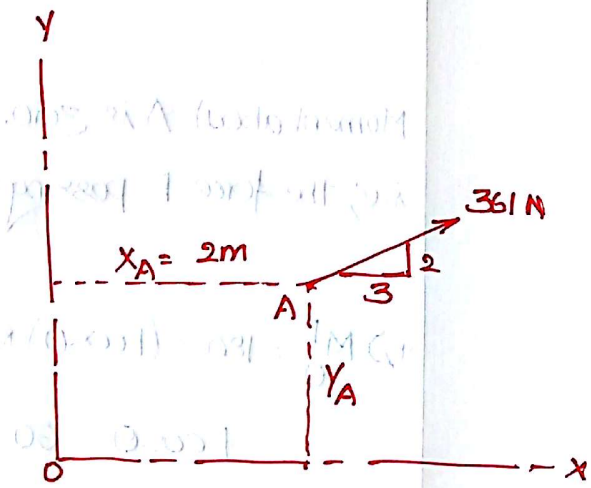
$F_x = 200\ \text{N}.$

$\therefore +\circlearrowright M_D^F = (200*3) - 150 = 450\ \text{N-m}.$

3. Find the Y- co-ordinate of point A so that the 361N force will have a clockwise moment of 400 N-m about O.

Also determine the Y- intercept of the action line of the force.

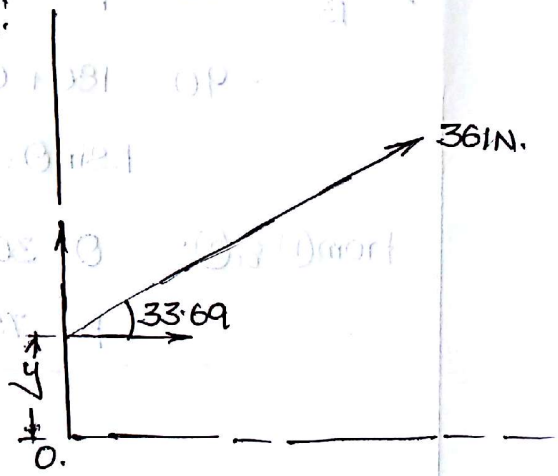
$\theta = \tan^{-1}\left(\frac{2}{3}\right) = 33.69^\circ$



$+\circlearrowleft M_O^F = 400 = (361\cos 33.69) * y_A - (361 \sin 33.69) * 2$

$y_A = 2.67$ m.

Y- intercept of the action line of the force:

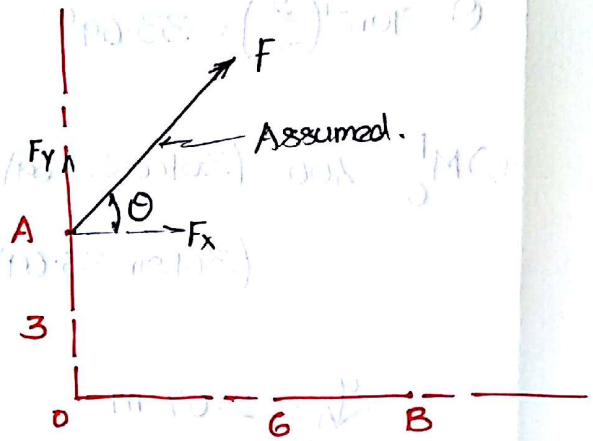


$+\circlearrowleft M_O^F = 400 = [361\cos 33.69] * y$

$y = 1.33$ m. above O.

4. In the figure, the moment of a certain force F is 180 N-m clockwise about O and 90 N-m counterclockwise about B.

If its moment about A is zero. Determine the force F.



Moment about A is zero.

i.e; the force F passing through A.

$+\circlearrowright M_O^F = 180 = (F\cos\theta) * 3$

$F\cos\theta = 60$ ——— (1)

$+\circlearrowright M_B^F = -90 = [F\cos\theta * 3] + (F\sin\theta * 6)$

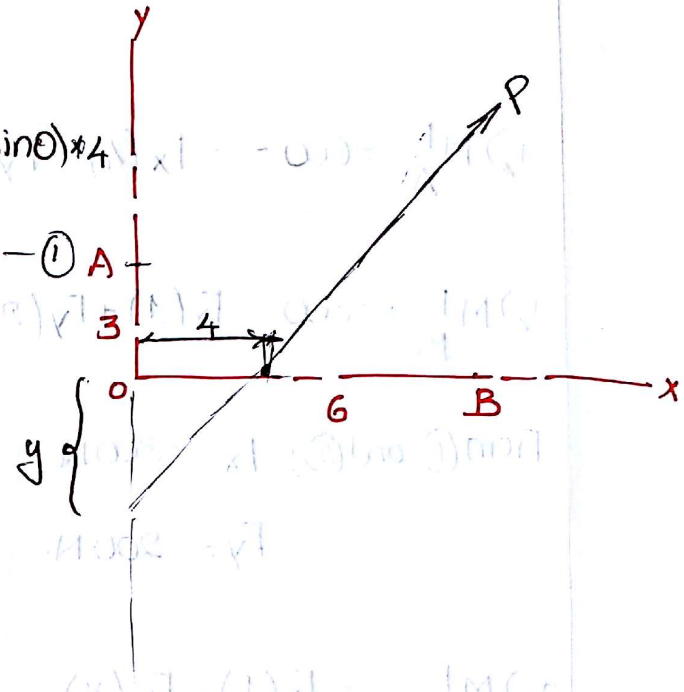
$-90 = 180 + 6F\sin\theta$

$F\sin\theta = -45$ ——— (2)

From (1) & (2); $\theta = 36.87^\circ$.

$F = 75N.$ → Acting down to the right at 36.87°.

5. In Fig. a force P intersects the X-axis at 4m to the right of O. If its moment about A is 170 N-m counterclockwise and its moment about B is 40 N-m clockwise, determine its Y-intercept.



$$+\circlearrowleft M_A^P = -170 = -(P\cos\theta)*3 - (P\sin\theta)*4$$

$$170 = 3P\cos\theta + 4P\sin\theta \quad -(1)$$

$$+\circlearrowleft M_B^P = 40 = (P\sin\theta)(2)$$

$$\therefore P\sin\theta = 20$$

From (1); $P\cos\theta = 30$.

$$\theta = 33.69^0.$$

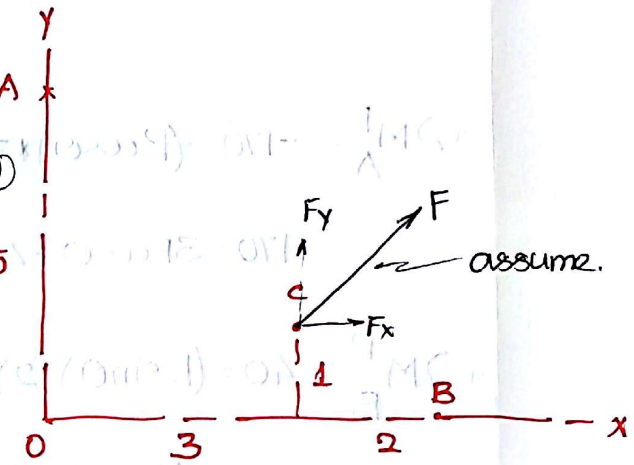
$$P = 36.05N.$$

$$+\circlearrowleft M_O^P = -[36.05\sin 33.69]*4 = -[36.05\cos 33.69]y.$$

$$y = 2.67m.$$

6. In fig. a force F passing through C produces a clockwise moment of 600 N-m about A and a counter clockwise moment of 300 N-m about B.

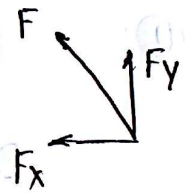
Determine the moment of F about O.



$+\circlearrowleft M^F_A = 600 = -F_x(4) - F_y(3) \quad -(1)$

$+\circlearrowleft M^F_B = -300 = F_x(1) + F_y(2) \quad -(2)$

From (1) and (2); $F_x = -300\,N$, $F_y = 200\,N$. } II Quadrant.



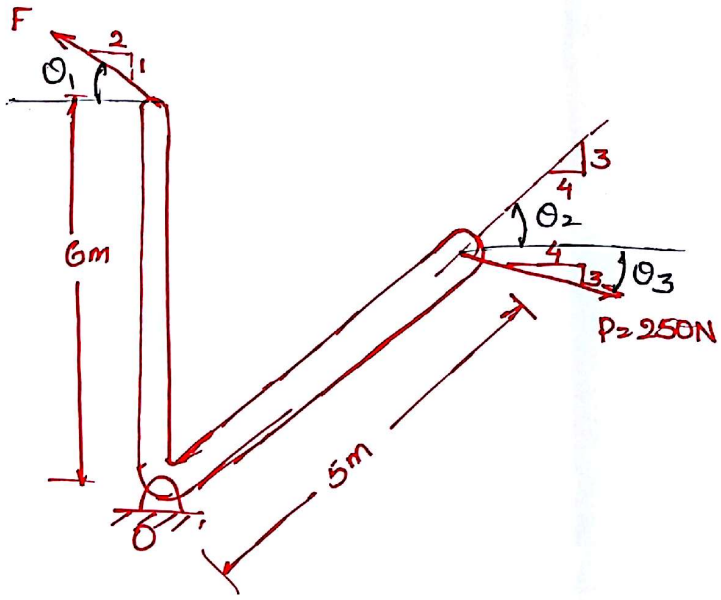
$+\circlearrowleft M^F_O = -F_x(1) - F_y(3)$

$= -300 - 600$

$= -900\,N\text{-}m.$

7. In the rocker arm shown in fig., the moment of F about O balances that of P about O.

Find F.



$\theta_1 = \tan^{-1}\left(\frac{1}{2}\right) = 26.56^0$

$\theta_2 = \tan^{-1}\left(\frac{3}{4}\right) = 36.87^0$

$\theta_3 = \tan^{-1}\left(\frac{3}{4}\right) = 36.87^0$

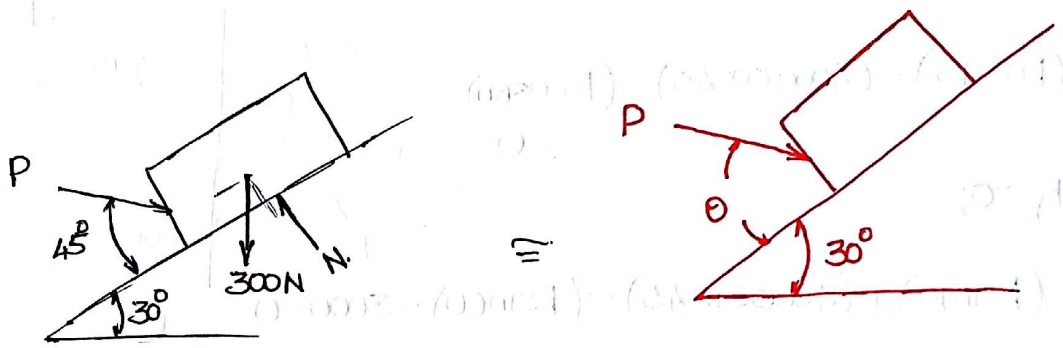
$+\circlearrowleft M_O^F = +\circlearrowleft M_O^P$

$-[F\cos(26.56)] * 6 = 250\sin[36.87 + 36.87] * 5$

$F = 223.59\ N.$

EQUILIBRIUM - 1

1. A 300 N box is held at rest on a smooth incline by a force P making an angle θ with the incline as shown in Fig. If $\theta = 45^\circ$, determine the value of P.

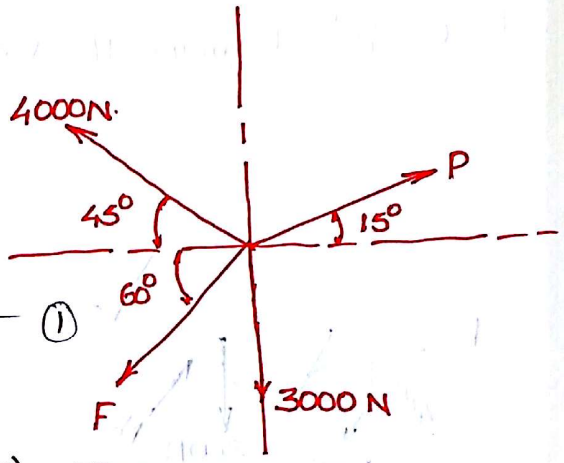


$P\cos 45 = 300 \sin 30$

$P = 212.13\,N.$

2. The forces on the gusset plate of a joint in a bridge truss act as shown in Fig.

Determine the values of P and F to maintain equilibrium of the joint.



$\Sigma F_x = 0$;

$(P\cos 15) - (4000\cos 45) - (F\cos 60) = 0$ — (1)

$\Sigma F_y = 0$;

$(P\sin 15) + (4000\sin 45) - (F\sin 60) - 3000 = 0$ — (2)

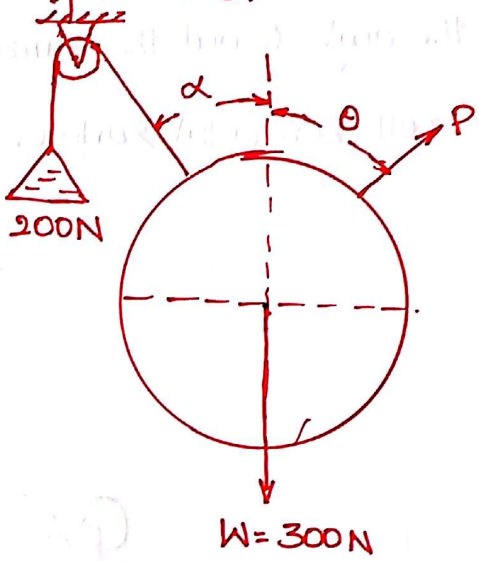
From (1) & (2);

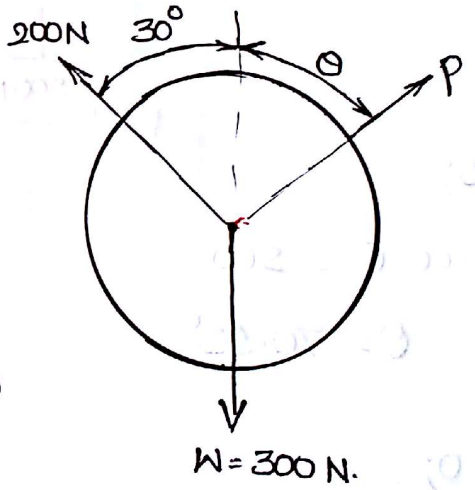
$P = 3342.78$ N

$F = 800.9$ N.

3. The 300N sphere in Fig is supported by the pull P and a 200N weight passing over a frictionless pulley.

If $\alpha = 30^\circ$, compute the values of P and θ.





$\Sigma F_x = 0$;

$P\sin\theta = 200\sin 30$ ——(1)

$\Sigma F_y = 0$;

$300 = P\cos\theta + (200\cos 30)$ —(2)

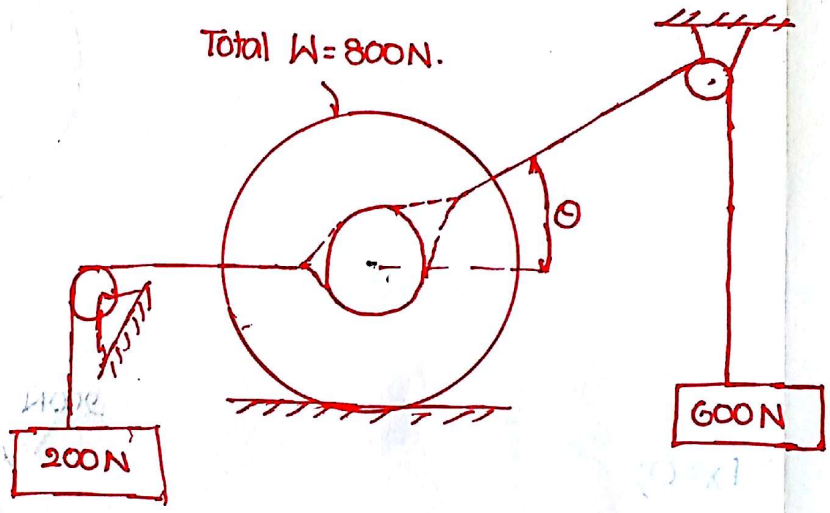
From (1) and (2);

$\theta = 38.26^\circ$

$P = 161.48\,N.$

4. Cords are looped around a small spacer separating two cylinders each weighing 400 N and pass as shown in fig, over frictionless pulleys to weights of 200 N and 600 N.

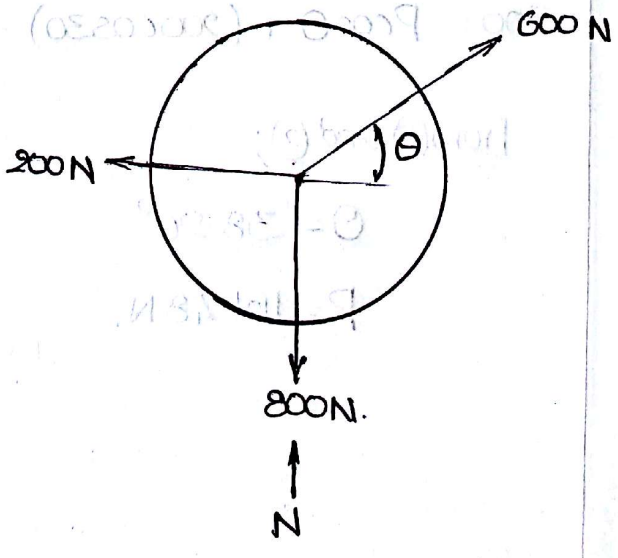
Determine the angle θ and the normal reaction N between the cylinders and the smooth horizontal surface.



$\Sigma F_x = 0$;

$600 \cos\theta = 200$

$\theta = 70.53^\circ$

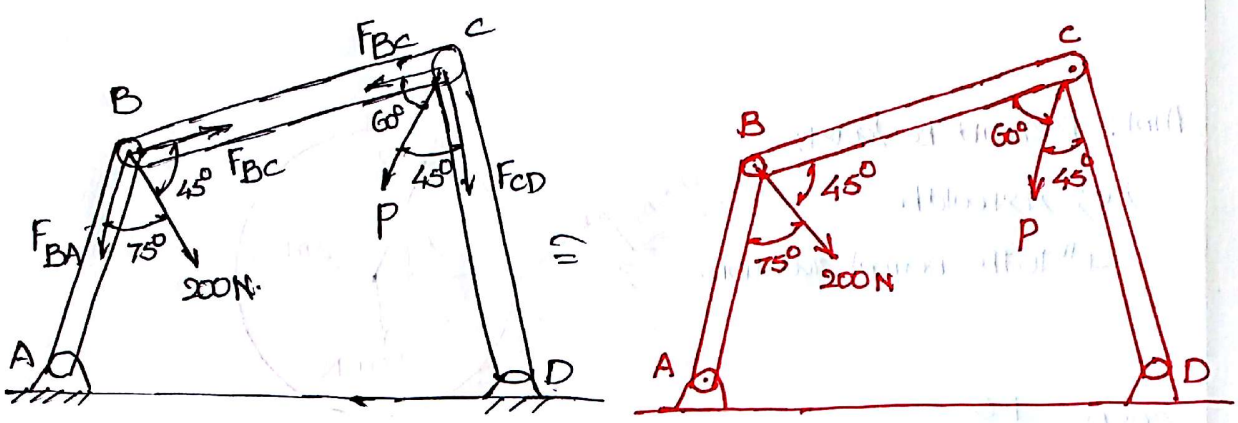


$\Sigma F_y = 0$;

$N + 600 \sin 70.53 = 800$

$N = 234.32\ N.$

5. Three bars, pinned together at B and C and supported by hinges at A and D as shown in Fig., form a four-link mechanism. Determine the value of P that will prevent motion.



At Joint B;

$$\frac{200}{\sin 240} = \frac{F_{BC}}{\sin 75} = \frac{F_{BA}}{\sin 45}$$

$F_{BC} = -223.07\ N$

$F_{BA} = -163.29\ N.$

At Joint C

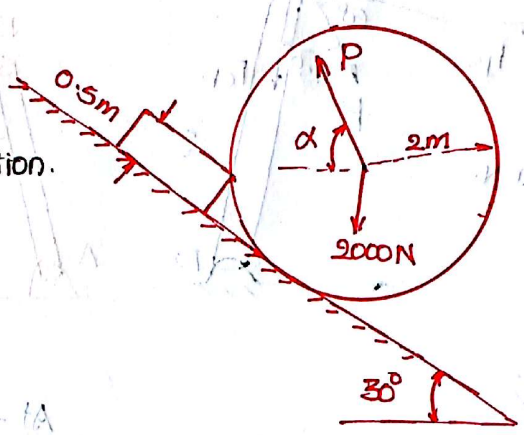
$$\frac{P}{\sin 255} = \frac{-223.07}{\sin 45}$$

$P = 304.72\ N.$

6. Determine the amount and direction of the smallest force P required to start the wheel in Fig. over the block.

What is the reaction at the block?

Hint: P should be least;
i.e; it should be
$\perp^r$ to the normal reaction.

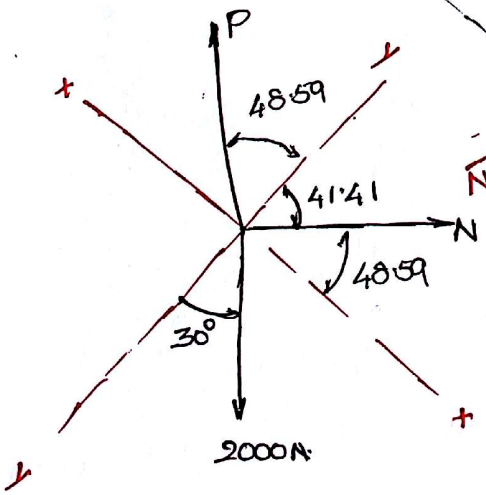


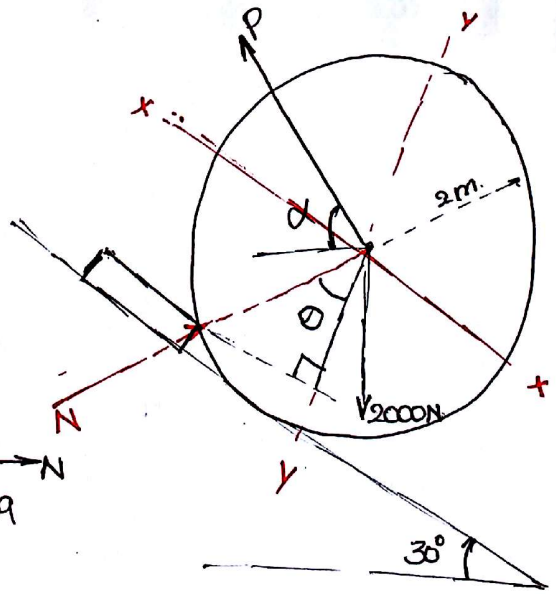
$\cos\theta = \frac{1.5}{2}$

$\theta = 41.41^0$

$\cos\theta = \frac{1.5}{2}$

$\theta = 41.41^0$



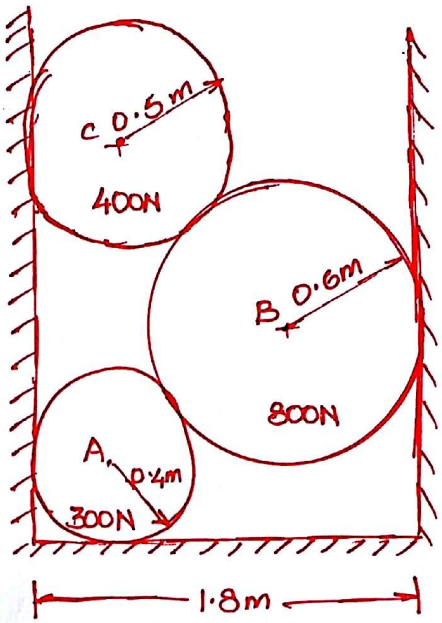


$\frac{2000}{\sin 90} = \frac{P}{\sin 48.59} = \frac{N}{\sin 161.41}$

$P = 1895.64\ N$

$N = 637.58\ N.$

7. Three cylinders are piled in a rectangular ditch as shown in Fig. Neglecting the friction, determine the reaction between cylinder A and the Vertical Wall.



$\sin\theta_1 = \frac{0.7}{1.1}$

$\theta_1 = 39.52$

$\theta_2 = 50.47$

Block C:

$R_1 = R_2 \sin 39.52$

$400 = R_2 \cos 39.52$

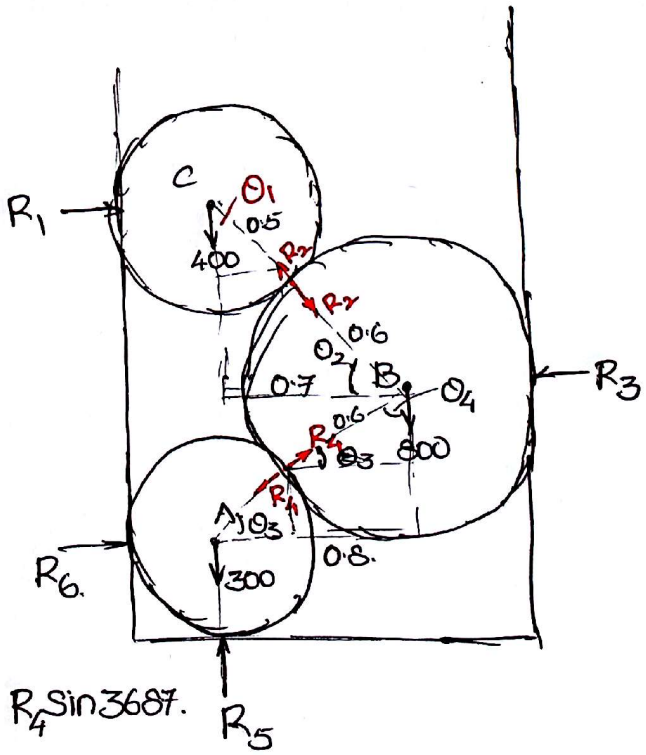
$R_2 = 518.54\ N.$

$R_1 = 329.96\ N$

$\cos\theta_3 = \frac{0.8}{1}$

$\theta_3 = 36.87$

$\theta_4 = 53.13.$



Block-B

$R_3 - 518.54 \cos 50.47$

$-R_4 \cos 36.87 = 0$

$800 + R_2 \sin 50.47 = R_4 \sin 36.87.$

$R_3 = 1929.96\ N.$

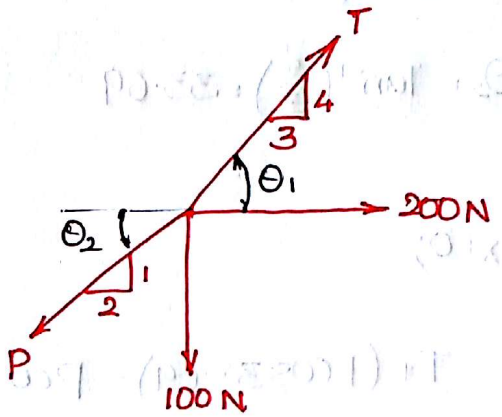
$R_4 = 2,000\ N.$

Block A:

$R_6 = R_4 \cos 36.87 = 1600\ N.$

EQUILIBRIUM - II

1. Determine the value of T, if the force system shown in Fig. is in equilibrium.



$\theta_1 = \tan^{-1}\left(\frac{4}{3}\right) = 53.13$

$\theta_2 = \tan^{-1}\left(\frac{1}{2}\right) = 26.56$

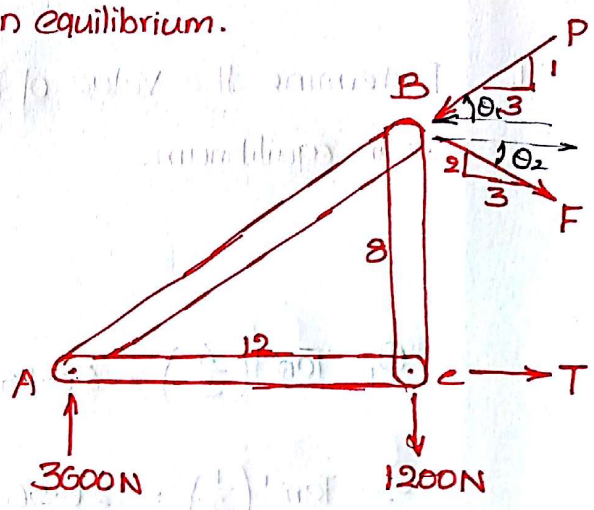
$\Sigma F_x = 0;\quad 200 + (T\cos 53.13) - (P\cos 26.56) = 0$ ——— (1)

$\Sigma F_y = 0;\quad (T\sin 53.13) - (P\sin 26.56) - 100 = 0$ ——— (2)

From (1) & (2); $T = 399.9$ N.

$P = 491.85$ N.

2. Determine the forces P, F, and T required to keep the triangular frame ABC shown in Fig in equilibrium.



$\theta_1 = \tan^{-1}\left(\frac{1}{3}\right) = 18.43$

$\theta_2 = \tan^{-1}\left(\frac{2}{3}\right) = 33.69$

$\Sigma F_x = 0;$

$$T + (F\cos 33.69) - P\cos(18.43) = 0 \quad \text{——— (1)}$$

$\Sigma F_y = 0;$ $\quad 3600 - 1200 - F\sin 33.69 - P\sin 18.43 = 0$ ——— (2)

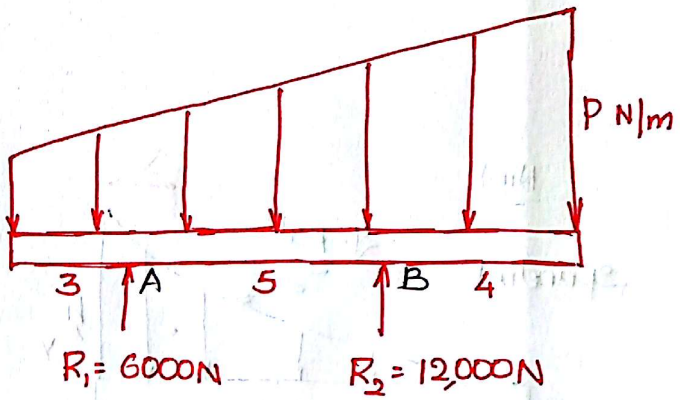
$+\circlearrowleft M_B^{All} = 0;$ $\quad 3600(12) = T(8)$

$$T = 5400\ N. \quad \text{——— (3)}$$

$\therefore$ $\quad F = 721.8\ N.$

$P = 6324.98\ N.$

3. A beam supports a load varying uniformly from an intensity of W N/m at the left end to P N/m at the right end. Find the values of W and P to cause the reactions shown in Fig.



$\Sigma F_y = 0$;

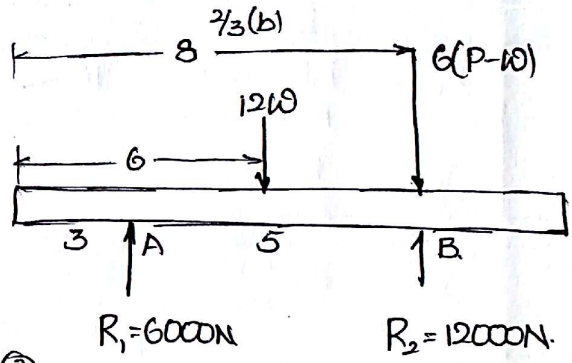
$R_1 + R_2 = W(12) + \frac{1}{2}(12)(P-W)$

$18{,}000 = 12W + 6P - 6W$

$6W + 6P = 18{,}000$ ——— (1)

Moments about A = 0

$+\circlearrowright M_A^{All} = 0$;



$12000(5) = 12W(3) + 6(P-W)5$

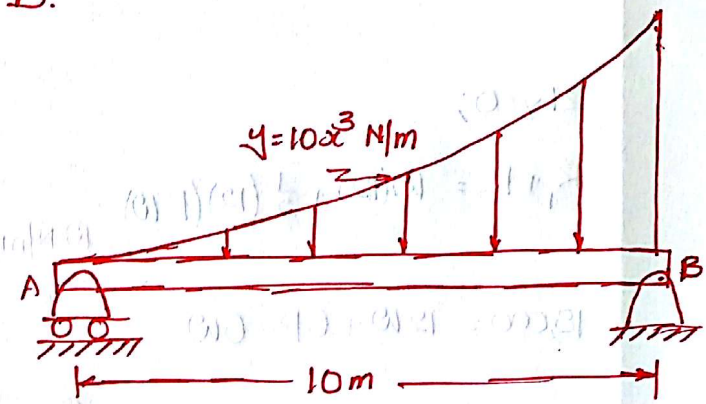
$60{,}000 = 36W + 30P - 30W$ ——— (2)

From (1) and (2); $W = 1250$ N/m

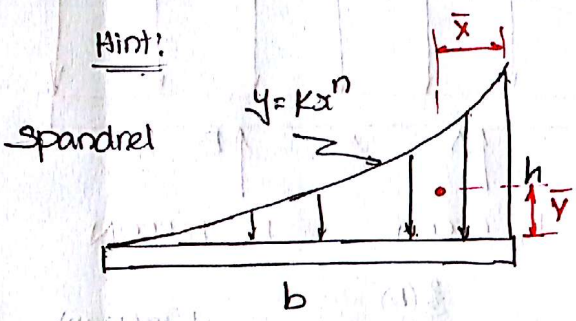
$P = 1750$ N/m.

4. As shown in Fig., the intensity of loading on a simply supported beam 10 m long is given by $y = 10x^3$, where y is in N/m and x is in m measured from A.

Find the reactions at A and B.



Hint:



Area $= \frac{bh}{n+1}$

$\bar{x} = \frac{b}{n+2}$

$\bar{y} = h\left(\frac{n+1}{n+2}\right)$

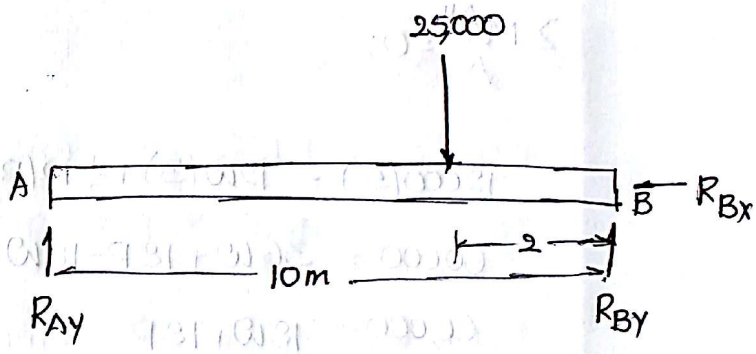
$y = 10x^3$

$h = 10(10^3) = 10{,}000$ N/m

Area $= \frac{(10)(10{,}000)}{4} = 25{,}000$

$\bar{x} = \frac{10}{5} = 2$

$\bar{y} = 10{,}000\left(\frac{4}{5}\right) = 8{,}000$



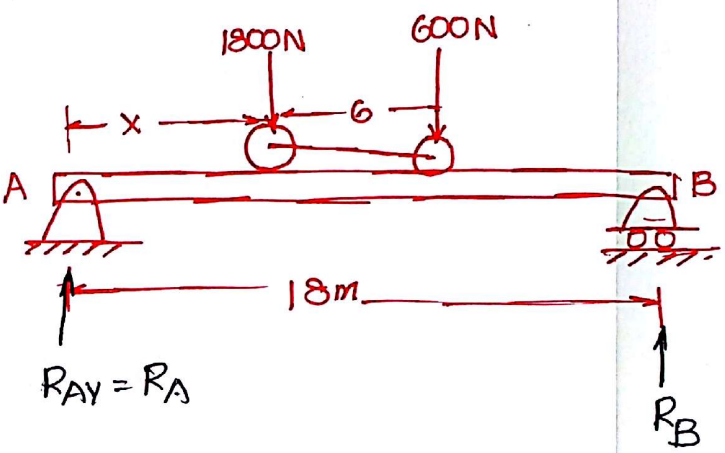
$\Sigma F_x = 0$; $R_{Bx} = 0$

$\Sigma F_y = 0$; $R_{AY} + R_{BY} = 25{,}000$

$+\circlearrowleft M_A^{All} = 0$; $R_{BY}(10) = 25{,}000(8)$ $\Rightarrow$ $R_{BY} = 20{,}000$ N

$R_{AY} = 5{,}000$ N.

5. The wheel loads on a small tractor crossing an 18m span are given in Fig. Determine the distance x at which the support reaction at A is twice that at B.



$R_A = 2R_B$ —— (1)

$\Sigma F_X = 0$; $R_{AX} = 0$

$\Sigma F_Y = 0$; $R_{AY} + R_B = 2400$

$+\circlearrowright \Sigma M$ $R_A + R_B = 2400$

$2R_B + R_B = 2400$

$R_B = 800\ N$

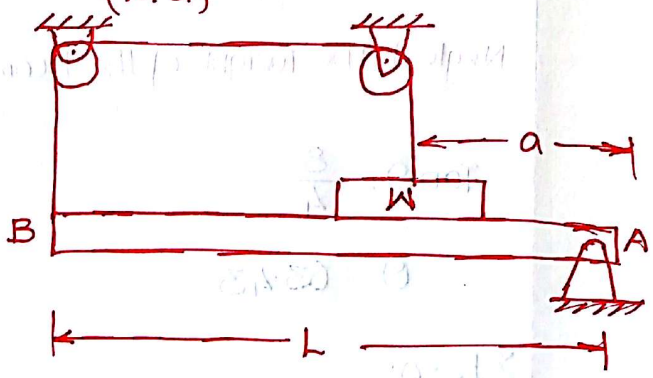
$R_A = 1600\ N.$

$+\circlearrowright M_A^{All} = 0$; $800(18) = 1800(x) + 600(x+6)$

$800(18) = 2400x + 3600$

$x = 4.5\ m.$

6. A Weight W rests on the bar AB as shown in Fig. The cable connecting W and B passes over frictionless pulleys. If bar AB has negligible weight, show that the reaction at A is $\frac{W(L-a)}{(L+a)}$



For Block;

$\Sigma F_x = 0$

$\Sigma F_y = 0$; $N + T = W$.

For Bar;

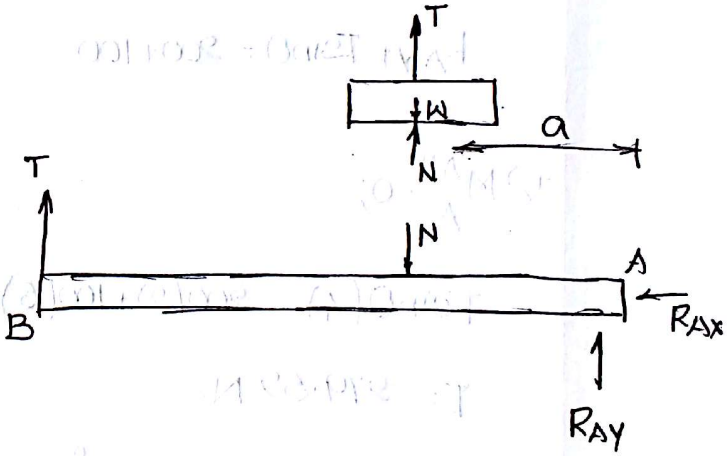
$\Sigma F_x = 0$; $R_{Ax} = 0$

$\Sigma F_y = 0$; $R_{Ay} + T = N$.

$+\circlearrowleft \sum M_B^{All} = 0$;

$R_{Ay}(L) = N(L-a)$.

~~$R_{Ay}(L) = R$~~



$+\circlearrowleft \sum M_A^{All} = 0$; $T(L) = N(a)$

$T(L) = (W-T)a$.

$T(L) = Wa - Ta$.

$T = \frac{Wa}{L+a}$.

$R_{Ay}(L) = (W-T)(L-a)$

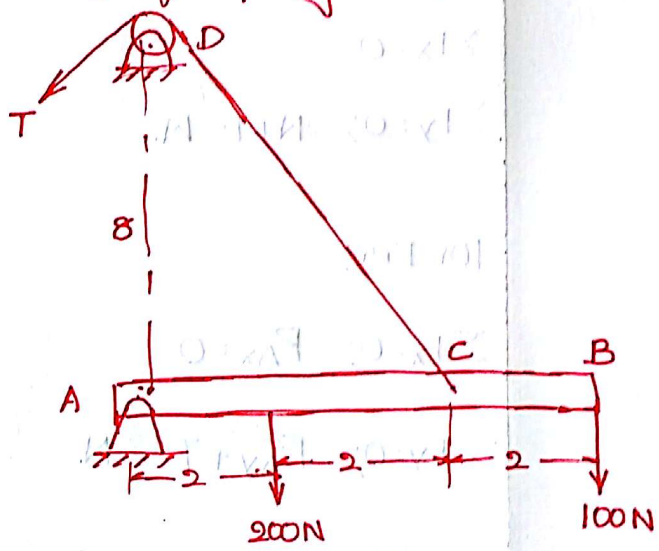
$R_{Ay}(L) = \left(W - \frac{Wa}{L+a}\right)(L-a)$

$$R_{Ay} = \frac{W(L-a)}{(L+a)}, \quad R_A = \frac{W(L-a)}{(L+a)}$$

7. A boom AB is supported in a horizontal position by a hinge A and a cable which runs from C over a small pulley at D as shown in Fig.

Compute the tension T in the cable, and the horizontal and vertical components of the reaction at A.

Neglect the weight of the boom and the size of the pulley at D.



$\tan\theta = \frac{8}{4}$

$\theta = 63.43$

$\Sigma F_x = 0;$

$R_{AX} = T\cos\theta$

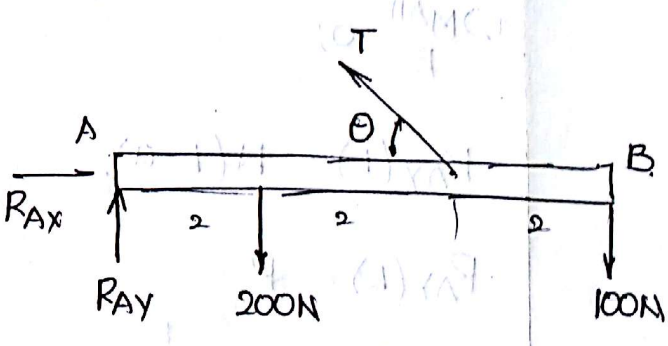
$\Sigma F_y = 0;$

$R_{AY} + T\sin\theta = 200 + 100$

$+\circlearrowleft \Sigma M_A^{All} = 0;$

$T\sin\theta(4) = 200(2) + 100(6)$

$T = 279.52\ N.$



$\therefore R_{AX} = 125.02\ N$

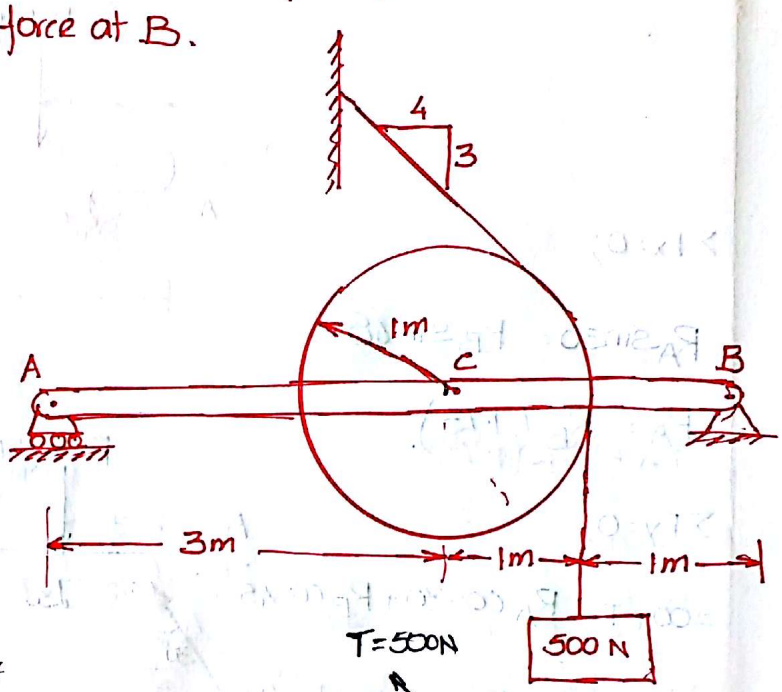
$R_{AY} = 50\ N.$

} Ans

$R = 134.647\ N.$

EQUILIBRIUM-3

1. A pulley of 1-m radius, supporting a load of 500N, is mounted at B on a horizontal beam as shown in Fig.
If the beam weighs 200N and the pulley weights 50N, find the hinge force at B.



Pulley

$\theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.87$

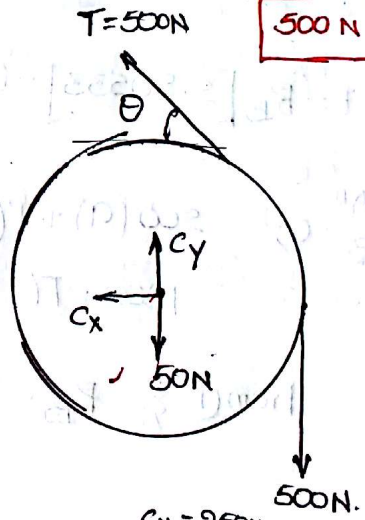
$\Sigma F_x = 0;$

$C_x = -500\cos 36.87$

$C_x = -400\,N.$

$50 + 500 = C_y + 500\sin 36.87$

$C_y = 250\,N.$



Beam.

$\Sigma F_x = 0;$

$R_{Bx} = -400\,N.$

$\Sigma F_y = 0;$

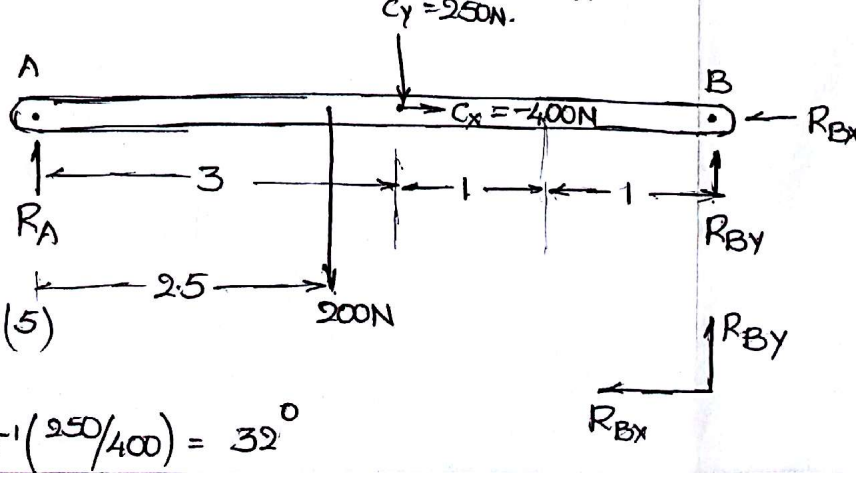
$R_A + R_{By} = 450\,N.$

$+\circlearrowleft \Sigma M_A^{All} = 0;$

$200(2.5) + 250(3) = R_{By}(5)$

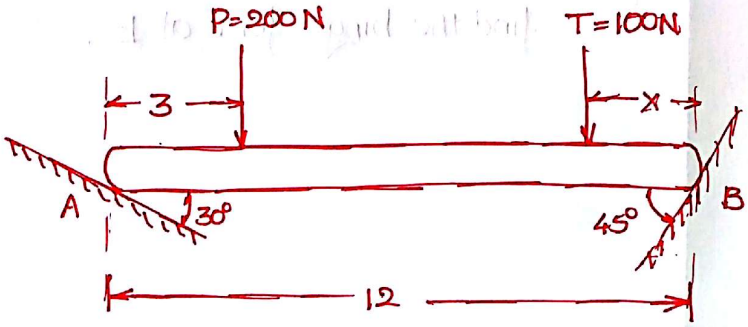
$R_{By} = 250\,N.$

$R = 471.7\,N;\quad \theta = \tan^{-1}\left(\frac{250}{400}\right) = 32^{0}$



2 A 12m bar of negligible weight rests in a horizontal position on the smooth inclines shown in Fig.

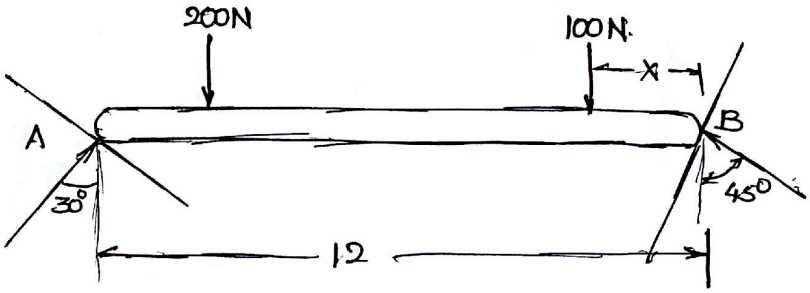
Compute the distance x at which load T = 100 N should be placed from point B to keep the bar horizontal.



$\Sigma F_x = 0$;

$R_A \sin 30 = R_B \sin 45$

$R_A = R_B (1.4142)$



$\Sigma F_y = 0$;

$300 = R_A \cos 30 + R_B \cos 45$

$R_B = 155.29 N.$

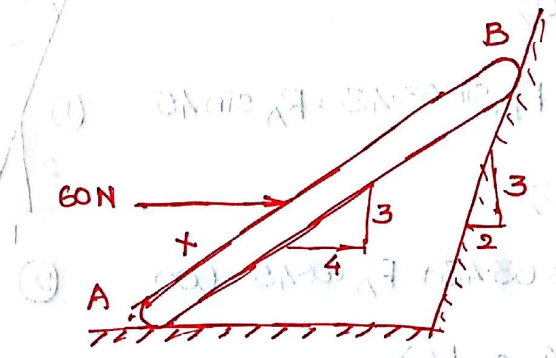
$+\circlearrowleft M_A^{All} = 0$; $(155.29 \cos 45) * 12 = 200(3) + 100(12 - x)$

$x = 4.82 m.$

3. Find the distance X - measured along AB at which a horizontal force of 60N should be applied to hold the uniform bar AB in the position shown in Fig.

Bar AB is 10 m long and weighs 140N.

The incline and the floor are smooth.



$\Sigma F_x = 0;$

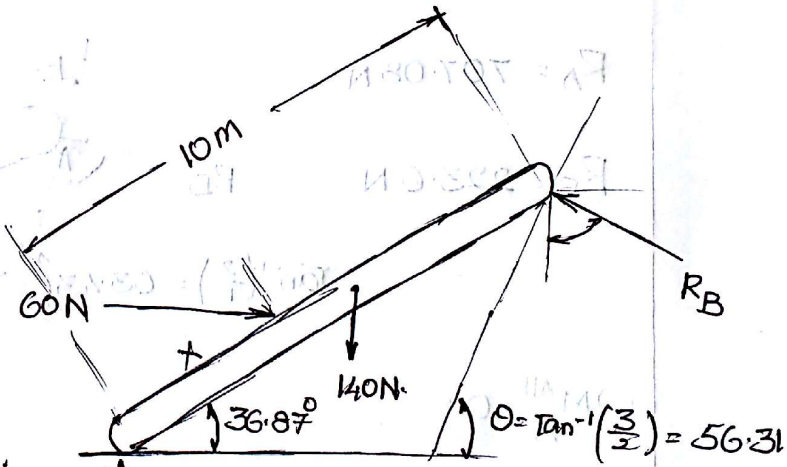
$60 = R_B \sin 56.31$

$R_B = 72.11\ N.$

$\Sigma F_y = 0;$

$R_A + R_B \cos 56.31 = 140$

$R_A = 100\ N.$



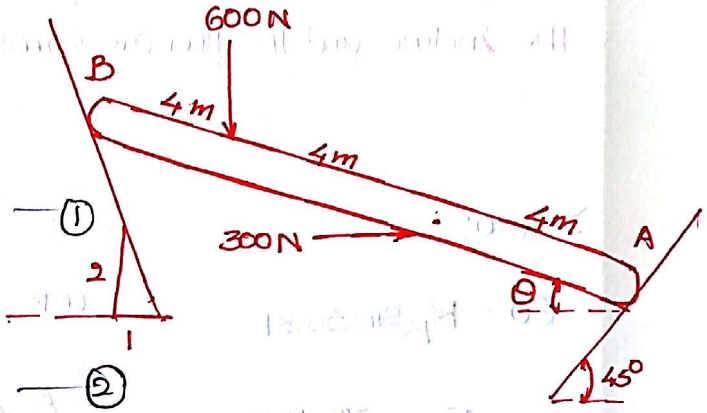
$+\circlearrowleft M_B^{All} = 0;$

$R_A[10\cos 36.87] - 140(5\cos 36.87) = 60\sin 36.87(10-x)$

$X = \underline{3.33\ m.}$

4. Bar AB of negligible weight is subjected to a vertical force of 600 N and a horizontal force of 300 N applied as shown in Fig. Find the angle θ at which equilibrium exists.

Assume smooth incline surfaces.



$\Sigma F_x = 0$;

$300 + R_B \sin 63.43 = R_A \sin 45$ — (1)

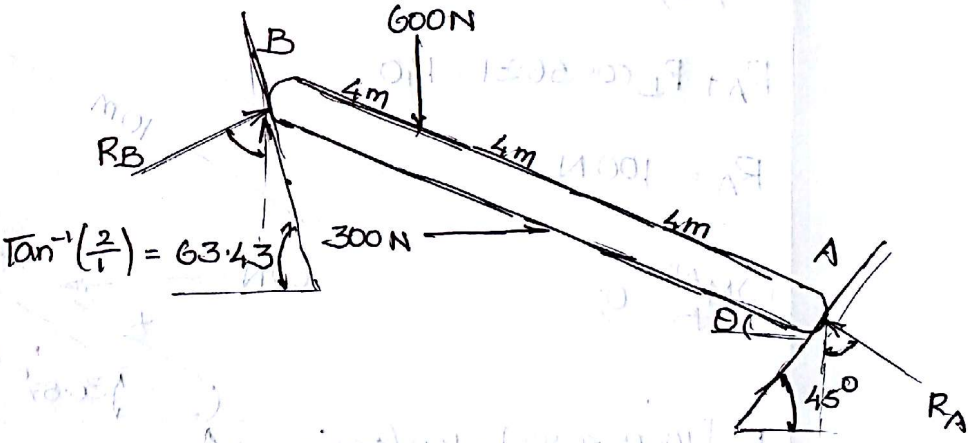
$\Sigma F_y = 0$;

$R_B \cos 63.43 + R_A \cos 45 = 600$ — (2)

From (1) and (2);

$R_A = 707.08\ N$

$R_B = 223.6\ N$



$+\circlearrowleft M_A^{All} = 0$;

$$\left[R_B \cos 63.43\,(12\cos\theta)\right] + \left[(R_B \sin 63.43)(12\sin\theta)\right] - 600\,(8\cos\theta) + 300\,(4\sin\theta) = 0$$

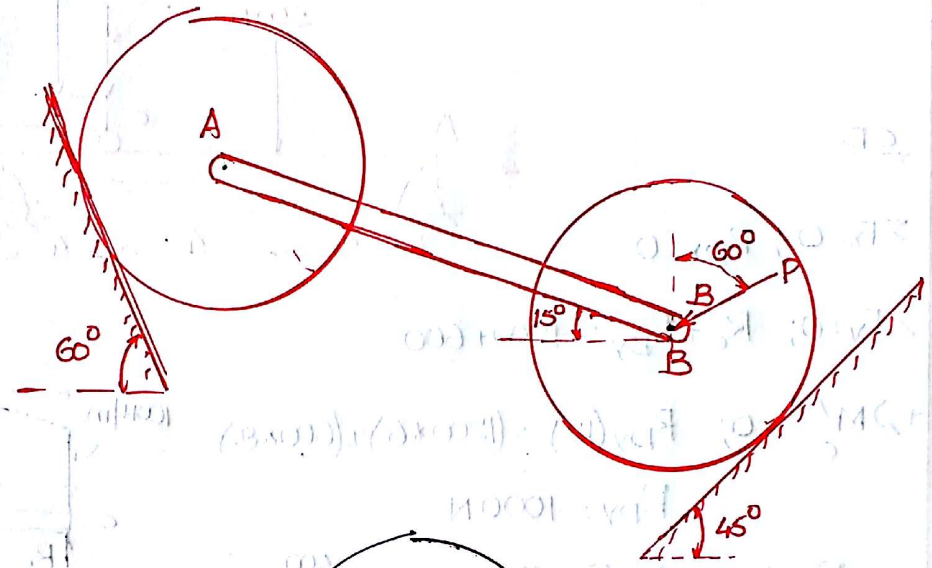
$$1200.17\cos\theta + 2399.82\sin\theta - 4800\cos\theta + 1200\sin\theta = 0.$$

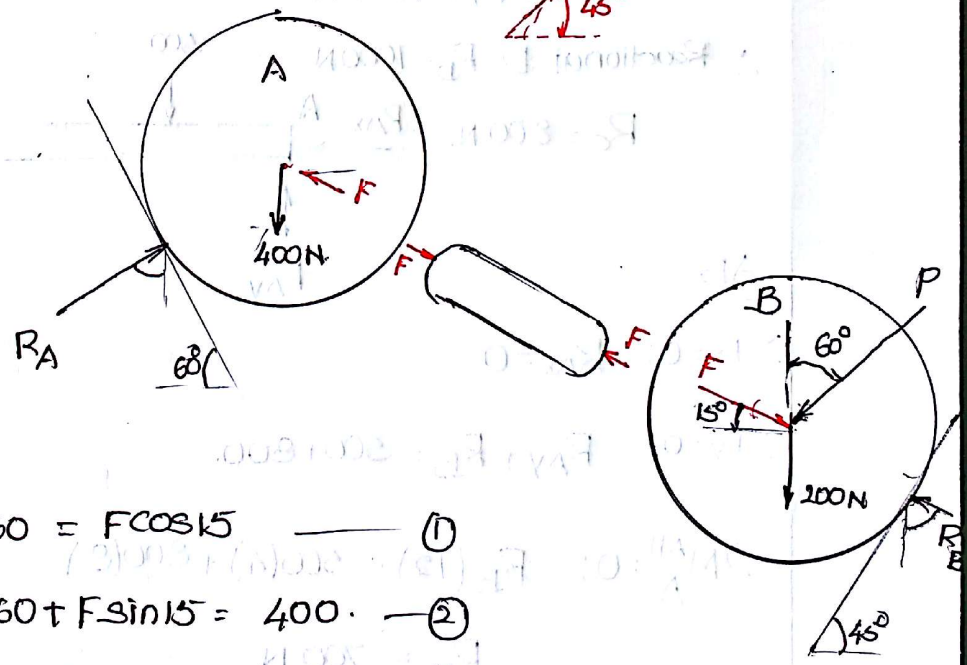
$$3599.82 \sin\theta = 3599.83\cos\theta.$$

$$\theta = 45^\circ$$

5. Two cylinders, A of Weight 400N and B of Weight 200N, rest on smooth inclines. They are connected by a bar of negligible weight hinged to each cylinder at its geometric centre by smooth pins.

Find the force P acting as shown that will hold the system in the given position.





cylinder A

$\Sigma F_x = 0$; $R_A \sin 60 = F \cos 15$ —— (1)

$\Sigma F_y = 0$; $R_A \cos 60 + F \sin 15 = 400$ —— (2)

From (1) and (2) $R_A = 546.41$ N.

$F = 489.89$ N.

cylinder B

$\Sigma F_x = 0$; $489.89 \cos 15 = P \sin 60 + R_B \sin 45$

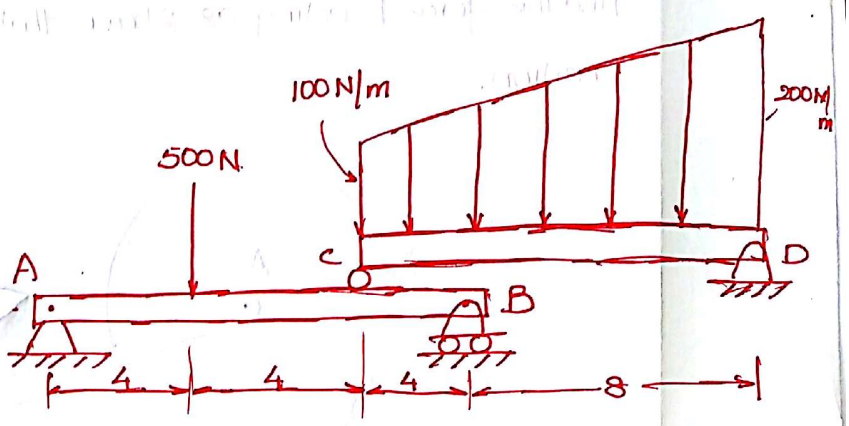
$\Sigma F_y = 0$; $489.89 \sin 15 + P \cos 60 + 200 = R_B \cos 45$

$P = 107.17$N.

$R_B = 538$N.

6. Two beams, supported as shown in Fig and separated by a roller at C, jointly carry the indicated loads.

Determine the reactions at A, B and D.



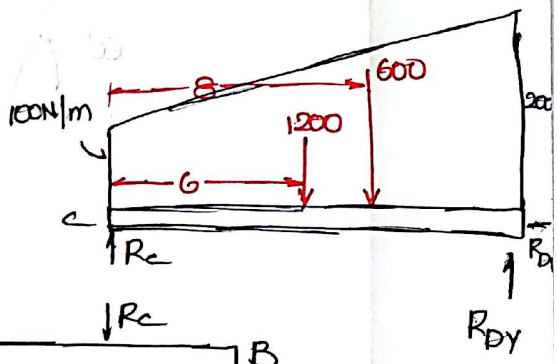
CD

$\Sigma F_x = 0$; $R_{Dx} = 0$

$\Sigma F_y = 0$; $R_C + R_{Dy} = 1200 + 600$

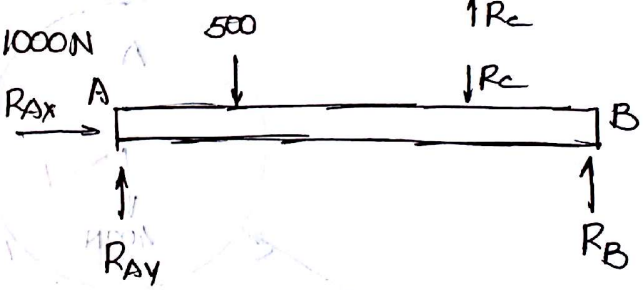
$+\circlearrowleft \Sigma M_C^{All} = 0$; $R_{Dy}(12) = (1200 * 6) + (600 * 8)$

$R_{Dy} = 1000\ N.$



$\therefore$ Reaction at $D = R_D = 1000N$

$R_C = 800\ N.$



AB

$\Sigma F_x = 0$; $R_{Ax} = 0$

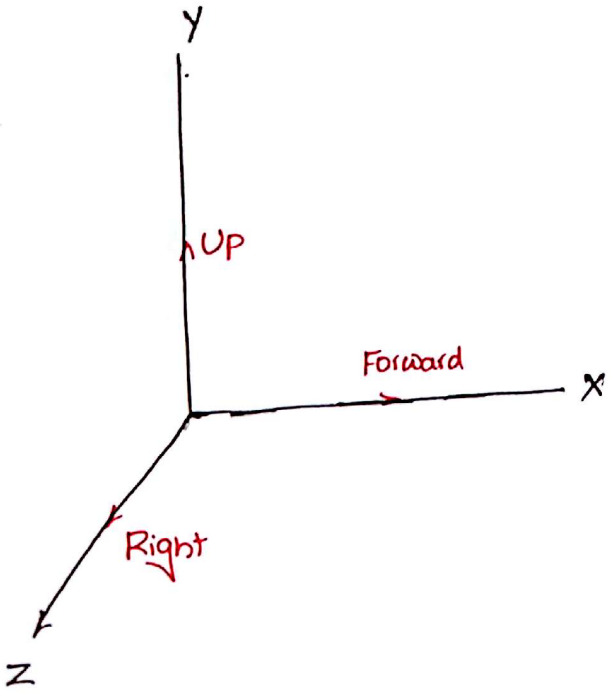
$\Sigma F_y = 0$; $R_{Ay} + R_B = 500 + 800.$

$+\circlearrowleft \Sigma M_A^{All} = 0$; $R_B(12) = 500(4) + 800(8)$

$R_B = 700\ N$

$R_{Ay} = 600N. \Rightarrow R_A = 600N.$

	(+ve)	(−ve)
x:	Forward	Backward
y:	Up	Down
z:	Right	Left.



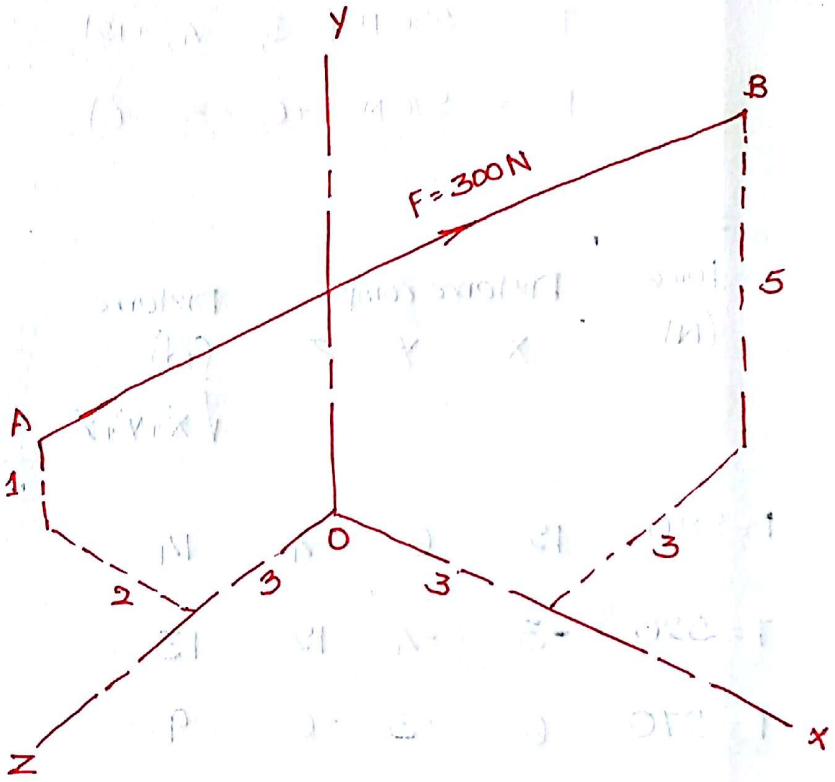
SPATIAL FORCES

1. A force $F = 300$ N is passing from a point A to B as shown in figure. Find the components of the force.

O(0,0,0)

A(-2,1,3)

B(3,5,-3)



$$\frac{F_x}{x} = \frac{F_y}{y} = \frac{F_z}{z} = \left(\frac{F}{d}\right)_{Fm}$$

$$\frac{F_x}{3+2} = \frac{F_y}{5-1} = \frac{F_z}{-3-3} = \frac{300}{\sqrt{5^2+4^2+6^2}}$$

$F_x = 170.94$ N.

$F_y = 136.75$ N

$F_z = -205.129$ N.

2. Determine the resultant of the system of concurrent forces having the following magnitudes and passing through the origin and the indicated points:

P = 280 N (+12, +6, −4)

T = 520 N (−3, −4, +12)

F = 270 N (+6, −3, −6)

Force (N)	Distance comp. X	Y	Z	Distance (d) $\sqrt{X^2+Y^2+Z^2}$	Force Multiplier Fm = F/d	Force components (N) X-comp	Y-comp	Z-comp.
P = 280	12	6	−4	14	20	240	120	−80
T = 520	−3	−4	12	13	40	−120	−160	480
F = 270	6	−3	−6	9	30	180	−90	−180
						300	−130	220

∴ R_X = 300 N — Forward

R_Y = −130 N — Down

R_Z = 220 N. — Right

$R = \sqrt{R_X^2+R_Y^2+R_Z^2}$ = 394.08 N. pointing forward, down and to the right.

3. Find the resultant of the force system shown in Fig., in which P = 280N, T = 260N, and F = 210N.

Sol:- Co-ordinate points are,

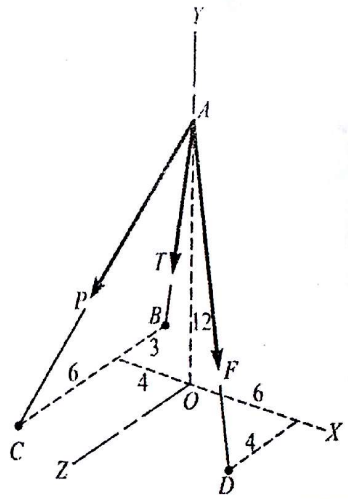
O(0,0,0)

A(0,12,0)

B(-4,0,-3)

C(-4,0,6)

D(6,0,4).



Force : P - A to C

T - A to B

F - A to D.

Force (N)	Distance comp X	Y	Z	Distance (d) $\sqrt{x^2+y^2+z^2}$	Force Multiplier $Fm = F/d$	Force components (N) X-comp	Y-comp	Z-com
P = 280	-4	-12	6	14	20	-80	-240	120
T = 260	-4	-12	-3	13	20	-80	-240	-60
F = 210	6	-12	4	14	15	90	-180	60
						-70	-660	120

$R_x = -70N$ — Backward

$R_y = -660N$ — Down

$R_z = 120N$ — Right

$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \underline{674.46N}$

4. In Fig., a vertical boom AE is supported by guy wires from A to B, C and D.

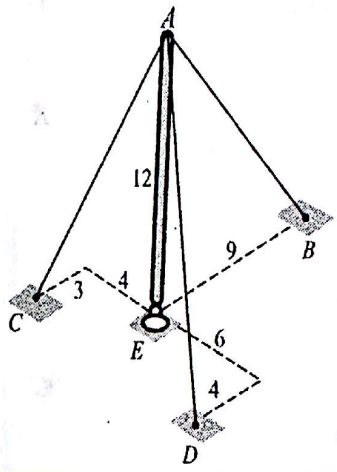
If the tensile load in AD = 252 N, find the forces in AC and AB so that the resultant force on A will be vertical.

A (0, 12, 0)

B (0, 0, −9)

C (−4, 0, 3)

D (6, 0, 4)



The tensile forces are;

Let: A to B = P

A to C = Q

A to D = 252 N.

Force (N)	Distance comp x	y	z	Distance (d) $\sqrt{x^2+y^2+z^2}$	Force Multiplier Fm = F/d	Force components (N) x-comp	y-comp	z-comp.
AB = P	0	−12	−9	15	$\frac{P}{15}$	0	$\frac{-12P}{15}$	$\frac{-9P}{15}$
AC = Q	−4	−12	3	13	$\frac{Q}{13}$	$\frac{-4Q}{13}$	$\frac{-12Q}{13}$	$\frac{3Q}{13}$
AD = 252	6	−12	4.	14.	18	108	−216	72
						$\frac{-4Q}{13}+108$	$\frac{-12P}{15}-\frac{12Q}{13}$ −216	$\frac{-9P}{15}+\frac{3Q}{13}$ +72

Resultant is vertical; $R_y = R$

$R_x = \frac{-4Q}{13} + 108 = 0$; $Q = 351$ N.

$R_z = \frac{-9P}{15} + \frac{3Q}{13} + 72 = 0$; $P = 255$ N.

5. Three concurrent forces P, T and F have a resultant of 10N directed forward and up to the right at $\theta_x = 60^\circ$, $\theta_y = 60^\circ$, $\theta_z = 45^\circ$. P equals 21N and passes from the origin through point (3,2,6). The value of T is 18 N and directed from the origin toward point (-6,6,-3). Determine the magnitude of the third force F and the angles it makes with the reference axes.

Resultant; R = 10N – Forward, Up and Right.

$$\frac{R_x}{\cos 60} = \frac{R_y}{\cos 60} = \frac{R_z}{\cos 45} = 10$$

$R_x = 5\,N$

$R_y = 5N$

$R_z = 7.07\,N.$

Force (N)	Distance comp X	Y	Z	Distance (d) $\sqrt{x^2+y^2+z^2}$	Force Multiplier $F_m = F/d$	Force components (N) X-comp	Y-comp	Z-comp.
P = 21	3	2	6	7	3	9	6	18
T = 18	-6	6	-3	9	2	-12	12	-6
F	–	–	–	–	–	F_x	F_y	F_z

$\therefore$ $9 - 12 + F_x = 5 \Rightarrow F_x = 8\,N.$

$6 + 12 + F_y = 5 \Rightarrow F_y = -13\,N.$

$18 - 6 + F_z = 7.07 \Rightarrow F_z = -4.93\,N.$

$$F = \sqrt{8^2 + 13^2 + 4.93^2}$$

$F = 16.04\,N.$

Forward, down to the left.

$$\frac{8}{\cos\theta_x} = \frac{-13}{\cos\theta_y} = \frac{-4.93}{\cos\theta_z} = 16.04.$$

$\theta_x = 60.08^\circ$; $\theta_y = 35.85^\circ$; $\theta_z = 72.1^\circ$.

DOT PRODUCT

1. Two forces F and P are given as $F = 4i - 3j - 2k$ and $P = 3i + 2j - k$.

Find; i) F·P

ii) Angle between F and P.

i) $F \cdot P = (4i - 3j - 2k) \cdot (3i + 2j - k)$

$= 12 i \cdot i + 8 \cancel{i \cdot j}^{0} - 4 i \cdot k - 9 j \cdot i - 6 j \cdot j + 3 j \cdot k - 6 k \cdot i - 4 k \cdot j + 2 k \cdot k$

$= 12 - 6 + 2$

$= \underline{8}$

ii) Angle between F and P.

$$\cos\theta = \frac{a \cdot b}{ab} = \hat{n}_a \cdot \hat{n}_b$$

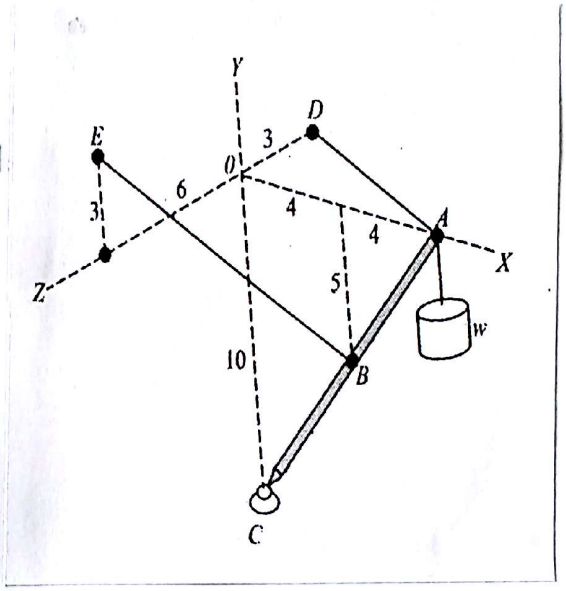
$$\cos\theta = \frac{(4i - 3j - 2k)}{\sqrt{16+9+4}} \cdot \frac{(3i + 2j - k)}{\sqrt{9+4+1}}$$

$$= \frac{12 - 6 + 2}{\sqrt{29}\ \sqrt{14}}$$

$$\theta = \underline{66.607^{0}}$$

2. In Fig. a boom AC is supported by a ball-and-socket joint at C and by the cables BE and AD.

If the force multiplier of a force F acting from B to E is $F_m = 10\ N/m$ and that of a force P acting from A to D is $P_m = 20\ N/m$, find the component of each force along AC.



A(8,0,0)

B(4,-5,0)

C(0,-10,0)

D(0,0,-3)

E(0,3,6)

$F = 10(-4i+8j+6k)$

$P = 20(-8i-3k)$

Vector $AC = -8i-10j$

i) component of force $\bar{F}$ along $\bar{AC}$;

$$F_{AC} = \bar{F}\cdot\hat{n}_{AC}$$

$$= \frac{10(-4i+8j+6k)\,(-8i-10j)}{\sqrt{64+100}} = \frac{10}{\sqrt{164}}(32-80)$$

$$F_{AC} = -37.48\ N.$$

ii) component of force $\bar{P}$ along $\bar{AC}$;

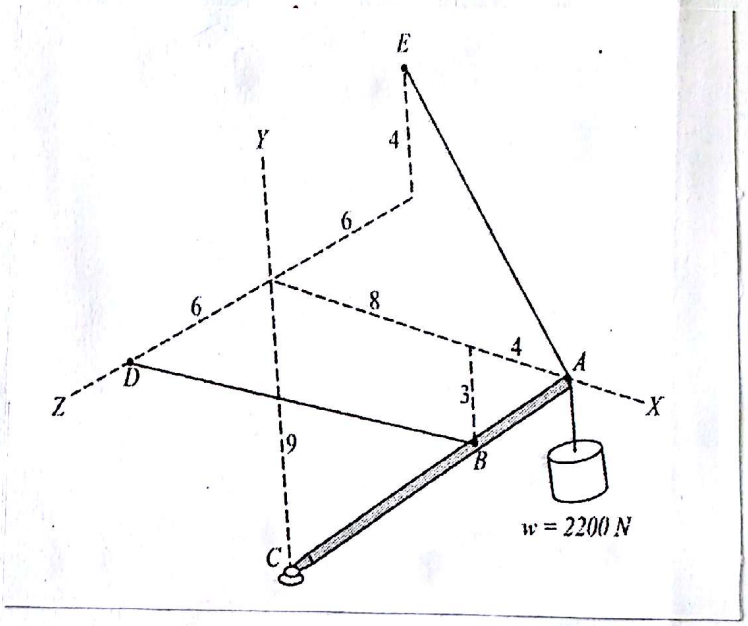
$$P_{AC} = \bar{P}\cdot\hat{n}_{AC};\quad \frac{20(-8i-3k)\cdot(-8i-10j)}{\sqrt{164}}$$

$$P_{AC} = 99.95\ N.$$

3. In the system shown in Fig, it is found that the force multiplier of force F acting from B to D is $F_m = 150$ N/m and that of force P acting from A to E is $P_m = 100$ N/m.

Find the component of force along AC and DE.

What angle does each force make with AC?



O(0,0,0)
A(12,0,0)
B(8,-3,0)
C(0,-9,0)
D(0,0,6)
E(0,4,-6).

$\bar{F} = 150(-8\hat{i}+3\hat{j}+6\hat{k})$

$\bar{P} = 100(-12\hat{i}+4\hat{j}-6\hat{k})$

i) comp. of force F along AC

$F_{AC} = \bar{F}\cdot\hat{n}_{AC}$

$= 150(-8\hat{i}+3\hat{j}+6\hat{k})\dfrac{(-12\hat{i}-9\hat{j})}{\sqrt{144+81}}$

$= 10(96-27) = 690\text{N}.$

ii) comp. of force F along DE

$F_{DE} = \bar{F}\cdot\hat{n}_{DE}$

$= 150(-8\hat{i}+3\hat{j}+6\hat{k})\dfrac{(4\hat{j}-12\hat{k})}{\sqrt{16+144}}$

$= \dfrac{150}{\sqrt{160}}(12-72) = -711.5\text{N}$

i) comp. of force P along AC

$P_{AC} = \bar{P}\cdot\hat{n}_{AC}$

$= 100(-12\hat{i}+4\hat{j}-6\hat{k})\cdot\dfrac{(-12\hat{i}-9\hat{j})}{\sqrt{144+81}}$

$= \dfrac{100}{15}(144-36) = 720\text{N}.$

ii) comp. of force P along DE

$P_{DE} = \bar{P}\cdot\hat{n}_{DE}$

$= 100(-12\hat{i}+4\hat{j}-6\hat{k})\cdot\dfrac{(4\hat{j}-12\hat{k})}{\sqrt{16+144}}$

$= 695.7\text{N}.$

From DOT product definition

$\bar{F} \cdot \overline{AC} = F\,AC\cos\theta$

$\cos\theta = \hat{F} \cdot \hat{n}_{AC}$

$= \dfrac{-8i+3j+6k}{\sqrt{64+9+36}} \cdot \dfrac{(-12i-9j)}{\sqrt{144+81}}$

$\theta = 63.85^{\circ}$

$\bar{P} \cdot \overline{AC} = P.AC\cos\theta$

$\cos\theta = \hat{P} \cdot \hat{n}_{AC}$

$= \dfrac{(-12i+4j-6k)}{\sqrt{(144+16+36)}} \cdot \dfrac{(-12i-9j)}{\sqrt{144+81}}$

$\theta = 59.05^{\circ}$

4. In the unsymmetrical cantilever frame work shown in Fig points C and D are in the same vertical plane while B is 3m in front of this plane.

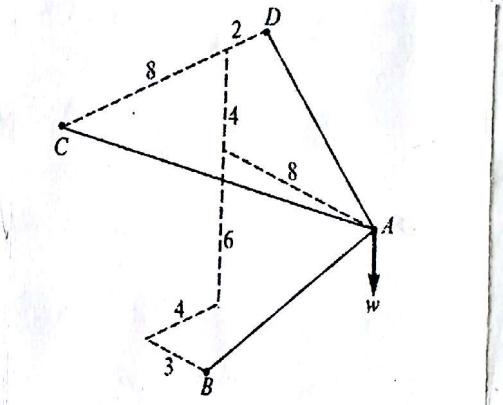
Assume a force F to act from A to C and another force P to act from A to D.

In terms of their force multipliers, find the components of these forces along the direction from A to B.

What is the angle between F and P?

Sol: Co-ordinate points are;

A (8,0,0)
B (3,-6,4)
C (0,4,8)
D (0,4,-2).



$\bar{F} = F_m(-8\hat{i}+4\hat{j}+8\hat{k})$

$\bar{P} = P_m(-8\hat{i}+4\hat{j}-2\hat{k})$

i) Comp. of $\bar{F}$ along the direction AB

$F_{AB} = \bar{F} \cdot \hat{n}_{AB}$

$= F_m(-8\hat{i}+4\hat{j}+8\hat{k}) \dfrac{(-5\hat{i}-6\hat{j}+4\hat{k})}{\sqrt{25+36+16}}$

$F_{AB} = 5.47 F_m$

ii) Comp. of $\bar{P}$ along the direction AB

$P_{AB} = \bar{P} \cdot \hat{n}_{AB}$

$= P_m(-8\hat{i}+4\hat{j}-2\hat{k}) \cdot \dfrac{(-5\hat{i}-6\hat{j}+4\hat{k})}{\sqrt{25+36+16}}$

$P_{AB} = 0.912 P_m$.

iii) Angle between F and P: $\cos\theta = \hat{n}_F \cdot \hat{n}_P = \dfrac{F_m(-8\hat{i}+4\hat{j}+8\hat{k})}{F_m\sqrt{64+16+64}} \cdot \dfrac{P_m(-8\hat{i}+4\hat{j}-2\hat{k})}{P_m(\sqrt{64+16+4}}$

$\theta = 54.4^0$.

5. Refering to the Fig. find the magnitude of the force F acting from D toward A that will have a component of 100N along DB.

co-ordinate points:

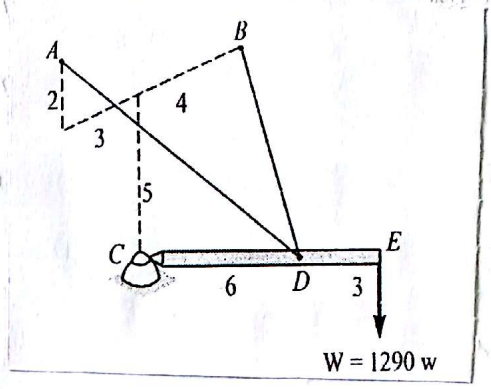
A(0, 2, 3)

B(0, 0, -4)

C(0, -5, 0)

D(6, -5, 0)

E(9, -5, 0).



$F = F_m(-6i + 7j + 3k)$

$F_{DB} = \bar{F} \cdot \hat{n}_{DB} = 100$

$= F_m(-6i + 7j + 3k) \cdot \dfrac{(-6i + 5j - 4k)}{\sqrt{36 + 25 + 16}} = 100$

$= F_m \dfrac{(36 + 35 - 12)}{\sqrt{77}} = 100$

$F_m = 14.87\ N.$

$F_m = \dfrac{F}{d} = \dfrac{F}{\sqrt{36 + 49 + 9}} = 14.87$

$F = 144.2\ N.$

6. The force $F = \bar{\imath} + 2\bar{j} + F_z\bar{k}$ makes an angle of 60° with the line $L = 4\bar{\imath} + 3\bar{j}$.

Find the value of F_z ?

$$\cos\theta = \hat{n}_F \cdot \hat{n}_L$$

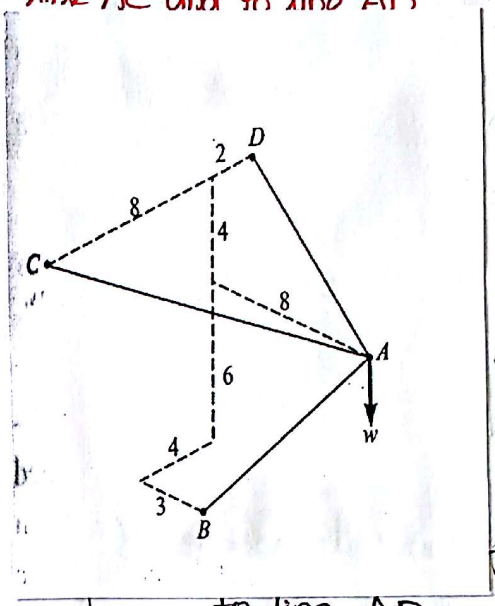
$$\cos 60 = \frac{(\bar{\imath} + 2\bar{j} + F_z\bar{k})}{\sqrt{1+4+F_z^2}} \cdot \frac{(4\bar{\imath} + 3\bar{j})}{\sqrt{16+9}}$$

$$0.5 = \frac{4+6}{\sqrt{5+F_z^2}} \cdot 5$$

$$F_z = \pm 3.32$$

CROSS PRODUCT

1. Refering to the cantilever framework shown in Fig., find the shortest distance from point B to line AC and to line AD



Co-ordinate Points;

A(8,0,0)

B(3,-6,4)

C(0,4,8)

D(0,4,-2)

a) Shortest distance from point B to line AC

$$d = \frac{|\vec{r}_{BA} \times \vec{AC}|}{|AC|}$$

$$\frac{|(5\hat{i}+6\hat{j}-4\hat{k}) \times (-8\hat{i}+4\hat{j}+8\hat{k})|}{\sqrt{64+16+64}}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 6 & -4 \\ -8 & 4 & 8 \end{vmatrix} = \hat{i}(48+16) - \hat{j}(40-32) + \hat{k}(20+48)$$

$$64\hat{i} - 8\hat{j} + 68\hat{k}$$

$$d = \frac{\sqrt{64^2+8^2+68^2}}{\sqrt{64+16+64}} = \underline{7.81 \text{ m.}}$$

from point B to line AD.

$$d = \frac{|\vec{r}_{BA} \times \vec{AD}|}{|AD|}.$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 6 & -4 \\ -8 & 4 & -2 \end{vmatrix}$$

$$= \hat{i}(-12+16) - \hat{j}(-10-32) + \hat{k}(20+48)$$

$$d = \frac{\sqrt{4^2+42^2+68^2}}{\sqrt{8^2+4^2+2^2}}$$

$$d = \underline{8.73 \text{ m.}}$$

2. In the, A boom AC is supported by a ball-and-socket joint at C and by the cables BE and AD.

The force multiplier of a force F acting from B to E is $F_m = 10\ N/m$,

Find the component of F that is perpendicular to the plane DAC.

Co-Ordinate points are;

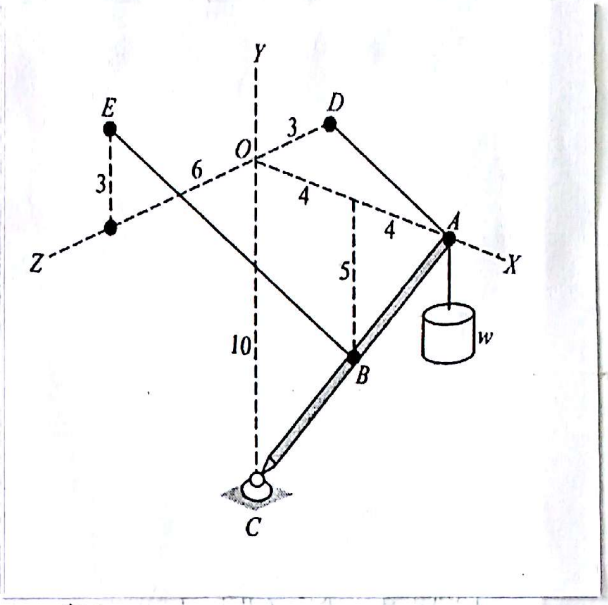
A (8, 0, 0)

B (4, −5, 0)

C (0, −10, 0)

D (0, 0, −3)

E (0, 3, 6)



Force, $F = 10(-4i + 8j + 6k)$

The comp. of F, that is $\perp^{lr}$ to the plane DAC

$$F_{\perp^{lr} \text{ to the plane DAC}} = \bar{F} \cdot \hat{n}_{DAC}$$

Let us consider a vector $\bar{N}$, $\perp^{lr}$ to plane DAC

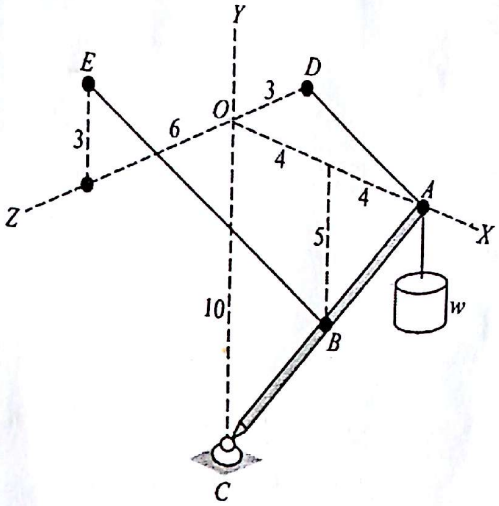
$$\bar{N} = \bar{AD} \times \bar{AC} = \begin{vmatrix} i & j & k \\ -8 & 0 & -3 \\ -8 & -10 & 0 \end{vmatrix} = i(-30) - j(-24) + k(80)$$

$$= -30i + 24j + 80k.$$

The comp. of F; $10(-4i + 8j + 6k) \dfrac{(-30i + 24j + 80k)}{\sqrt{30^2 + 24^2 + 80^2}}$

$$= \underline{89.24\ N.}$$

3. In the Fig., if the force multiplier of a force P acting from A to D is $P_m = 20\ N/m$, determine the component of P that is perpendicular to the plane defined by points E, A and C.



Co-ordinate points;

A (8, 0, 0)

B (4, −5, 0)

C (0, −10, 0)

D (0, 0, −3)

E (0, 3, 6).

$P = 20(-8i - 3k)$

The comp. of P, that is $\perp^{r}$ to the plane EAC.

$$= \bar{P} \cdot \hat{n}_{EAC}$$

Let us consider a vector $\bar{N}$, $\perp^{r}$ to plane EAC

$$\bar{N} = \bar{AE} \times \bar{AC} = \begin{vmatrix} i & j & k \\ -8 & 3 & 6 \\ -8 & -10 & 0 \end{vmatrix}$$

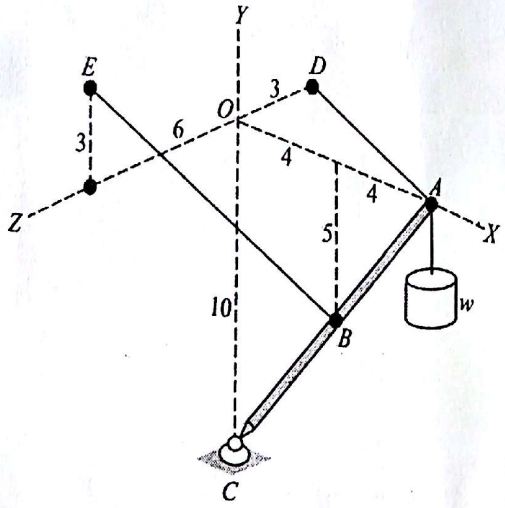
$$= i(60) - j(48) + k(80 + 24).$$

$$= 60i - 48j + 104k.$$

$$\text{The comp. of } P = 20(-8i - 3k) \frac{(60i - 48j + 104k)}{\sqrt{60^2 + 48^2 + 104^2}}$$

$$= -122.5\ N.$$

4) Refer to the system in Fig., and find the length of the common perpendicular between lines BE and AD



co-ordinate points:

A(8,0,0)

B(4,-5,0)

C(0,-10,0)

D(0,0,-3)

E(0,3,6).

Length of the common ⊥ᴿ between lines BE and AD.

$$d = \vec{n} \cdot \frac{\overline{BE} \times \overline{AD}}{|\overline{BE} \times \overline{AD}|}$$

$\vec{n}$ is joining $\overline{BE}$ and $\overline{AD}$.

Let $\overline{AB}$ be that vector.

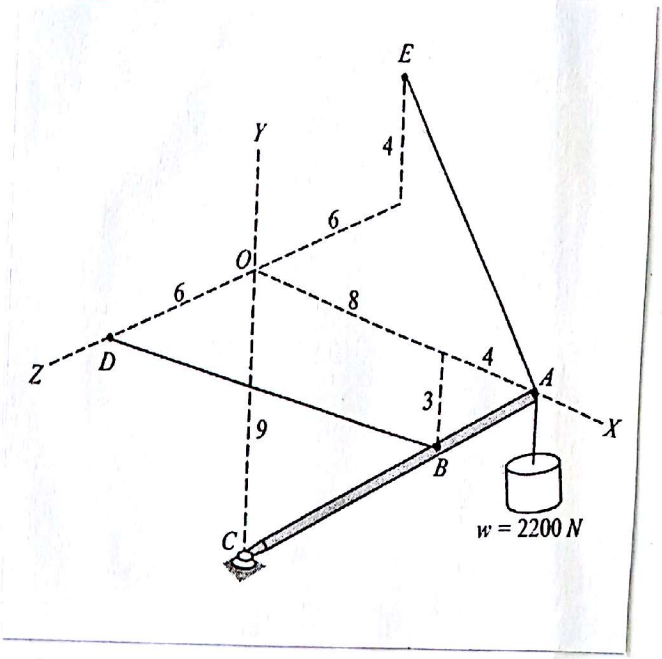
$$\overline{BE} \times \overline{AD} = \begin{vmatrix} i & j & k \\ -4 & 8 & 6 \\ -8 & 0 & -3 \end{vmatrix} = i(-24) - j(12+48) + k(64)$$

$$= -24i - 60j + 64k.$$

$$d = \frac{(-4i - 5j) \cdot (-24i - 60j + 64k)}{\sqrt{24^2 + 60^2 + 64^2}}$$

$$d = \underline{4.35\ m}.$$

5. In the system shown in Fig., determine the length of the common perpendicular between lines AE and BD.



Co-Ordinate Points:

O(0,0,0)

A(12,0,0)

B(8,-3,0)

C(0,-9,0)

D(0,0,6)

E(0,4,-6)

Length of the common $\perp^{r}$ between lines AE and BD.

$$d = \bar{n} \cdot \frac{\overline{AE} \times \overline{BD}}{|\overline{AE} \times \overline{BD}|}$$

Let $\bar{n}$ is joining AE and BD.

Let $\overline{AB}$ be that vector.

$$\overline{AE} \times \overline{BD} = \begin{vmatrix} i & j & K \\ -12 & 4 & -6 \\ -8 & 3 & 6 \end{vmatrix} = i(24+18) - j(-72-48) + K(-36+32)$$

$$= i(42) + j(120) - 4K$$

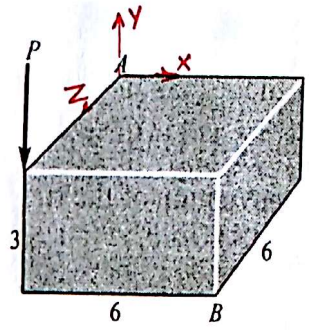
$$d = (-4i - 3j) \cdot \frac{(42i + 120j - 4K)}{\sqrt{(42^2 + 120^2 + 4^2)}}$$

$$d = \underline{-4.15\ m.}$$

SPATIAL MOMENTS

1. Find the moment of force P about the diagonal AB.

Use this result to show that the length of the common perpendicular between P and AB is 4m.



A (0,0,0)
B (6,-3,6)
C (0,0,6)
D (0,-3,6).

M^P_{AB} = comp. of Moment in the direction AB

$$M^P_{AB} = M^P_A \cdot \hat{n}_{AB}$$

$$M^P_A = \overline{AC} \times \overline{P}$$

$$= \begin{vmatrix} i & j & k \\ 0 & 0 & 6 \\ 0 & -P_y & 0 \end{vmatrix}$$

$$= i(6P_y)$$

$$M^P_{AB} = i(6P_y) \cdot \frac{(6i-3j+6k)}{\sqrt{36+9+36}}$$

$$= \frac{36P_y}{9} = 4P_y \cong 4P$$

Also, Moment of P about AB = Force * Moment Arm.

$\therefore$ Moment Arm = 4m.

$\therefore$ Length of the common $\perp^r$ between P and AB = 4m.

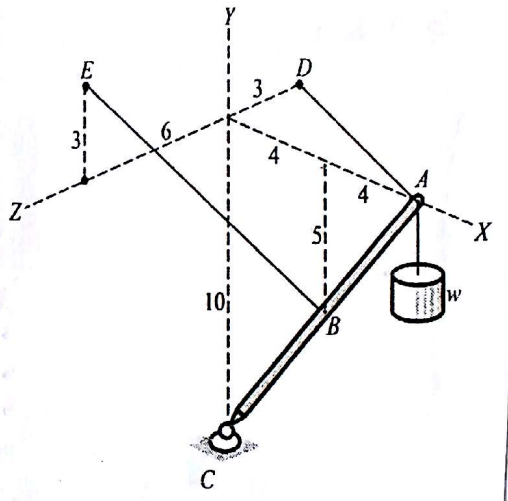
2. In the Fig. a boom AC is supported by a ball-and-socket joint at C and by the cables BE and AD.

If the force multiplier of force F acting from B to E is $F_m = 10\ N/m$, Find;

a) Moment of F about point C

b) Moment of F about point D

c) Moment of F about a line directed from C to D.



Co-ordinate Points:

A (8, 0, 0)

B (4, -5, 0)

C (0, -10, 0)

D (0, 0, -3)

E (0, 3, 6)

$F = 10(-4i + 8j + 6K)$

a) $M_C^F = \bar{r} \times \bar{F}$

$= \overline{CB} \times \bar{F}$

$$= \begin{vmatrix} i & j & K \\ +4 & 5 & 0 \\ -40 & 80 & 60 \end{vmatrix}$$

$$= 10 \begin{vmatrix} i & j & K \\ 4 & 5 & 0 \\ -4 & 8 & 6 \end{vmatrix}$$

$= 10[30i - 24j + 52K]\ N\text{-}m.$

b) $M_D^F = \overline{DB} \times \bar{F}$

$$= 10 \begin{vmatrix} i & j & K \\ 4 & -5 & 3 \\ -4 & 8 & 6 \end{vmatrix}$$

$= 10[-54i - 36j + 12K]\ N\text{-}m.$

c) $M_{CD}^F = M_C^F \hat{n}_{CD}$

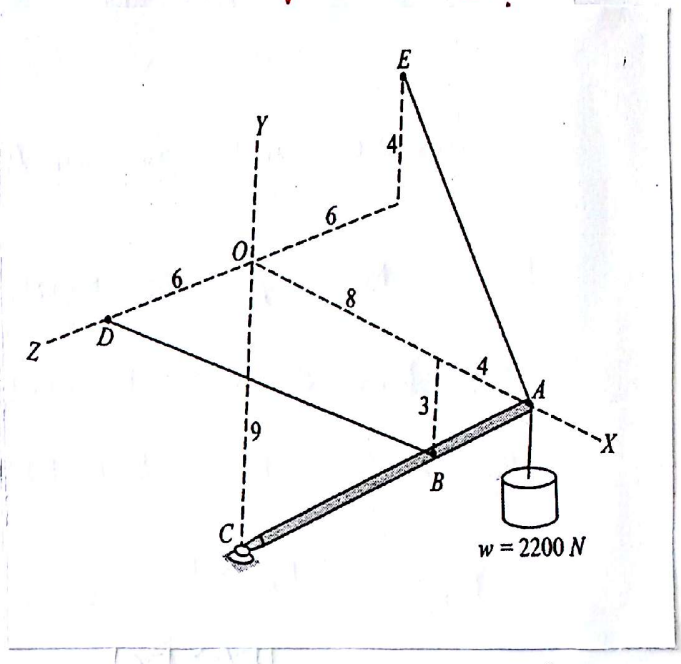
$= 10[-54i - 36j + 12K] \dfrac{(10j - 3K)}{\sqrt{109}}$

$= -379.29\ N\text{-}m.$

3. For the system shown in Fig, the force multiplier of Pacting from A to E is $P_m = 100\ N/m$.

Find the moment of P about C and about D.

What is the moment of P about an axis directed from C to D?



Co-ordinate Points:

A (12, 0, 0)

B (8, −3, 0)

C (0, −9, 0)

D (0, 0, 6)

E (0, 4, −6)

$P = 100\left[-12i + 4j - 6k\right]$

a) $M_C^P = \overline{CA} \times \overline{P}$

$$= 100\begin{vmatrix} i & j & k \\ 12 & 9 & 0 \\ -12 & 4 & -6 \end{vmatrix}$$

$= 100\left[i(-54) - j(-72) + k(156)\right]$

$= 100\left[-54i + 72j + 156k\right]$

b) $M_D^P = \overline{DE} \times \overline{P}$

$$= 100\begin{vmatrix} i & j & k \\ 0 & 4 & -12 \\ -12 & 4 & -6 \end{vmatrix}$$

$= 100\left[24i - 144j + 48k\right]$

c) $M_{CD}^P = M_C^P \cdot \hat{n}_{CD}$

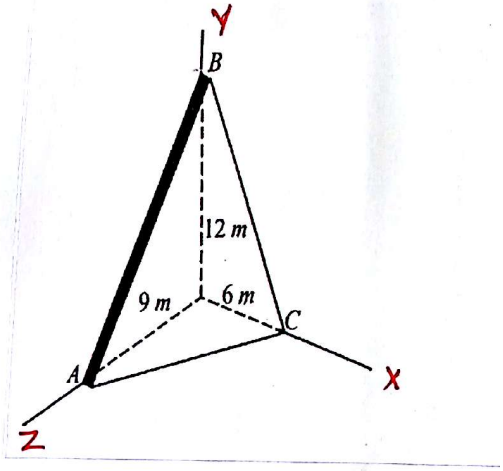
$= 100(-54i + 72j + 156k) \cdot \dfrac{(9j + 6k)}{\sqrt{81 + 36}}$

$= 14644.08\ N\text{-}m.$

4. As shown in Fig. a triangular plate rests against the corner of a room.

A moment of 260 N-m is required to start rotating the plate about the edge AC.

What is the least force applied at B to start this rotation?



Component of F in the direction AC

$$M^F_{AC} = M^F_C \cdot \hat{n}_{AC} = 260 \text{ N-m}.$$

If the force applied at B should be least, then it should be $\perp^{r}$ to the plane ABC

$$\hat{n}_N = \frac{(\overline{BA} \times \overline{Bc})}{|\overline{BA} \times \overline{Bc}|}$$

A(0,0,9)
B(0,12,0)
C(6,0,0).

$$\overline{BA} \times \overline{Bc} = \begin{vmatrix} i & j & k \\ 0 & -12 & 9 \\ 6 & -12 & 0 \end{vmatrix} = i(108) - j(-54) + k(72)$$

$$= 108i + 54j + 72k.$$

$$\hat{n}_N = \frac{108i + 54j + 72k}{\sqrt{19,764}}$$

$$\overline{F} = \frac{Fm}{\sqrt{19764}}(108i + 54j + 72k)$$

$$M^F_A = \overline{AB} \times \overline{F} = \frac{Fm}{\sqrt{19764}} \begin{vmatrix} i & j & k \\ -6 & 12 & 0 \\ 108 & 54 & 72 \end{vmatrix}$$

$$= \frac{Fm}{\sqrt{19764}} \left[i(864) + j(432) + k(-1620) \right]$$

5. Reduce the following set of forces to a single resultant force acting at the origin of the co-ordinate axes and a couple.

E = 140N acting from A(3,7,-1) toward B(5,1,2)

P = 260N acting from A(-6,4,1) toward D(6,7,5)

T = 270N acting from E(-2,4,1) toward G(4,-2,4)

$$\bar{E} = E_m(x\bar{i} + y\bar{j} + z\bar{k})$$

$$E_m = \frac{E}{d} = \frac{E}{\sqrt{x^2+y^2+z^2}}$$

$$\therefore \bar{E} = \frac{140}{\sqrt{2^2+6^2+3^2}}[2\bar{i} - 6\bar{j} + 3\bar{k}]$$

$$\bar{E} = \frac{140}{\sqrt{49}}[2i - 6j + 3k] = 40i - 120j + 60k$$

$$\bar{P} = \frac{260}{\sqrt{169}}[12i + 3j + 4k] = 240i + 60j + 80k$$

$$\bar{T} = \frac{270}{\sqrt{81}}[6i - 6j + 3k] = 180i - 180j + 90k.$$

$$\therefore \text{Resultant } \bar{R} = \bar{E} + \bar{P} + \bar{T} = 460\bar{i} - 240\bar{j} + 230k.$$

Couple = Moment of all the forces about origin.

$$= M_O^E + M_O^P + M_O^T.$$

A C E

$$\begin{vmatrix} i & j & k \\ 3 & 7 & -1 \\ 40 & -120 & 60 \end{vmatrix} + \begin{vmatrix} i & j & k \\ -6 & 4 & 1 \\ 240 & 60 & 80 \end{vmatrix} + \begin{vmatrix} i & j & k \\ -2 & 4 & 1 \\ 180 & -180 & 90 \end{vmatrix}$$

$$= 1100i + 860j - 2320k.$$

$$MF_{AC} = MF_C \cdot \hat{n}_{AC}$$

$$= \frac{Fm}{\sqrt{19764}} \left[864\hat{i} + 432\hat{j} - 1620\hat{k}\right] \cdot \frac{(6\hat{i} - 9\hat{k})}{\sqrt{117}} = 260$$

$$Fm = 20\,N.$$

6. In Fig., a force T acts along BE and a force P acts along AD. Assuming their force multipliers to be $T_m = 20 N/m$ and $P_m = 10 N/m$, find the force F to be applied at C to reduce their resultant to a couple? What is this resultant couple?

Co-ordinate points:

A(0, 0, 10)

B(0, -4, 6)

C(0, -10, 0)

D(-8, -4, 0)

E(7, 0, 0).

$\bar{T} = 20\left[7\bar{i} + 4\bar{j} - 6\bar{k}\right] = 140i + 80j - 120k$

$\bar{P} = 10\left[-8\bar{i} - 4\bar{j} + 10\bar{k}\right] = -80i - 40j - 100k.$

Let the Resultant $\bar{R} = \bar{T} + \bar{P} = 60\bar{i} + 40\bar{j} - 220\bar{k}$

A force $\bar{F}$ is applied at C will be equal and opposite to $\bar{R}$ so that the system reduces to a couple.

$\therefore \bar{F} = -60i - 40j + 220k.$ is to be applied at C.

To find the couple; Take moment of all the forces about origin.

i.e; $M_O^T + M_O^P + M_O^F$.

$$= \begin{vmatrix} i & j & k \\ 0 & -4 & 6 \\ 140 & 80 & -120 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 0 & 0 & 10 \\ -80 & -40 & -100 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 0 & -10 & 0 \\ -60 & -40 & 220 \end{vmatrix}$$

$= -1800\bar{i} + 40j - 40k$; N-m.