UNIT-I

Introduction to Mechanics: Basic Concepts, system of Forces Coplanar Concurrent Forces -Components in Space Resultant -Moment of Forces and its Application - Couples and Resultant of Force Systems. Equilibrium of system of Forces: Free body diagrams, Equations of Equilibrium of Coplanar Systems.

UNIT 1
INTRODUCTION

1.1. SCIENCE

In this modern age, the word ‘science’ has got different meanings for different people. An ordinary man takes it as ‘something’ beyond his understanding, whereas others may take it as ‘mysteries of research’ which are understood only by a few persons working amidst complicated apparatus in a laboratory. A non-scientist feels that it is a ‘subject’ whose endeavour is aimed to improve the man’s life on the earth. A business executive has the idea that it is ‘something’ which solves our day to day manufacturing and quality control problems, so that the nation’s economic prosperity keeps on improving.

In fact, ‘science’ may be defined as the growth of ideas through observation and experimentation. In

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this sense, the subject of science does not, necessarily, has to contribute something to the welfare of the human life, although the man has received many benefits from the scientific investigations.

1.2. APPLIED SCIENCE

Strictly speaking, the world of science is so vast that the present day scientists and technologists have to group the various spheres of scientific activities according to some common characteristics to facilitate their training and research programmes. All these branches of science, still have the common principle of employing observation and experimentation. The branch of science, which co-ordinates the research work, for practical utility and services of the mankind, is known as Applied Science.

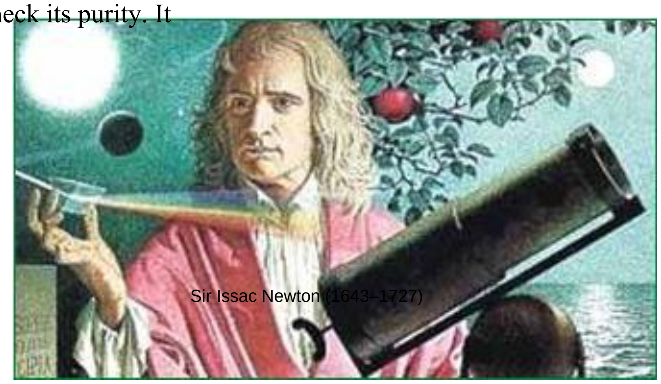
1.3. ENGINEERING MECHANICS

The subject of Engineering Mechanics is that branch of Applied Science, which deals with the laws and principles of Mechanics, alongwith their applications to engineering problems. As a matter of fact, knowledge of Engineering Mechanics is very essential for an engineer in planning, designing and construction of his various types of structures and machines. In order to take up his job more skilfully, an engineer must persue the study of Engineering Mechanics in a most systematic and scientific manner.

1.4. BEGINNING AND DEVELOPMENT OF ENGINEERING MECHANICS

It will be interesting to know, as to how the early man had been curious to know about the different processes going on the earth. In fact, he used to content himself, by holding gods responsible for all the processes. For a long time, the man had been trying to improve his ways of working. The first step, in this direction, was the discovery of a circular wheel, which led to the use of animal driven carts. The study of ancient civilization of Babylonians, Egyptians, Greeks and Roman reveal the use of water wheels and wind mills even during the pre-historic days.

It is believed that the word 'Mechanics' was coined by a Greek philosopher **Aristotle** (384–322 BC). He used this word for the problems of lever and the concept of centre of gravity. At that time, it included a few ideas, which were odd, unsystematic and based mostly on observations containing incomplete information. The first mathematical concept of this subject was developed by Archimedes (287–212 BC). The story, for the discovery of First Law of Hydrostatics, is very popular even today in the history of the development of Engineering Mechanics. In the normal course, Hieron king of Syracuse got a golden crown made for his use. He suspected that the crown has been made with an adultrated gold. The king liked the design of the crown so much that he did not want it to be melted,in order to check its purity. It is said that the king announced a huge reward for a person, who can check the purity of the crown gold without melting it. The legend goes that Archimedes, a pure mathematician, one day sitting in his bath room tub realised that if a body is immersed in water, its apparent weight is reduced. He thought that the apparent loss of weight of the immersed body is equal



Sir Issac Newton (1643–1727)

to the weight of the liquid displaced. It is believed that without further thought, **Archimedes** jumeped out of the bath tub and ran naked down the street shouting ‘Eureka, eureka !’ *i.e*. I have found it, I have found it !’

The subject did not receive any concrete contribution for nearly 1600 years. In 1325, Jean Buridan of Paris University proposed an idea that a body in motion possessed a certain impetus *i.e*. motion. In the period 1325–1350, a group of scientists led by the Thomas Bradwardene ofOxford University did lot of work on plane motion of bodies. Leonarodo Da Vinci (1452–1519), a great engineer and painter, gave many ideas in the study of mechanism, friction and motion of bodies on inclined planes. Galileo (1564–1642) established the theory of projectiles and gave a rudimentary idea of inertia. Huyghens (1629–1695) developed the analysis of motion of a pendulum.

As a matter of fact, scientific history of Engineering Mechanics starts with **Sir Issac Newton** (1643–1727). He introduced the concept of force and mass, and gave Laws of Motion in 1686. James Watt introduced the term horse power for comparing performance of his engines. John Bernoulli (1667–1748) enunciated the priciple of virtual work. In eighteenth century, the subject of Mechanics was termed as Newtonian Mechanics. A further development of the subject led to a controversy between those scientists who felt that the proper measure of force should be change in kinetic energy produced by it and those who preferred the change in momentum. In the nineteenth century, many scientists worked tirelessly and gave a no. of priciples, which enriched the scientific history of the subject.

In the early twentieth century, a new technique of research was pumped in all activities of science. It was based on the fact that progress in one branch of science, enriched most of the bordering branches of the same science or other sciences. Similarly with the passage of time, the concept of Engineering Mechanics aided by Mathematics and other physical sciences, started contributing and development of this subject gained new momentum in the second half of this century. Today, knowledge of Engineering Mechanics, coupled with the knowledge of other specialised subjects *e.g*. Calculus, Vector Algebra, Strength of Materials, Theory of Machines etc. has touched its present height. The knowledge of this subject is very essential for an engineer to enable him in designing his all types of structures and machines.

1.5. DIVISIONS OF ENGINEERING MECHANICS

The subject of Engineering Mechanics may be divided into the following two main groups:

1. Statics, and 2. Dynamics.

1.6. STATICS

It is that branch of Engineering Mechanics, which deals with the forces and their effects, while acting upon the bodies at rest.

1.7. DYNAMICS

It is that branch of Engineering Mechanics, which deals with the forces and their effects, while acting upon the bodies in motion. The subject of Dynamics may be further sub-divided into the following two branches :

1. Kinetics, and 2. Kinematics.

1.8. KINETICS

It is the branch of Dynamics, which deals with the bodies in motion due to the application of forces.

1.9. KINEMATICS

It is that branch of Dynamics, which deals with the bodies in motion, without any reference to the forces which are responsible for the motion.

1.10. FUNDAMENTAL UNITS

The measurement of physical quantities is one of the most important operations in engineering. Every quantity is measured in terms of some arbitrary, but internationally accepted units, called *fundamental units.*

All the physical quantities, met with in Engineering Mechanics, are expressed in terms of three fundamental quantities, *i.e.*

1. length, 2. mass and 3. time.

1.11. DERIVED UNITS

Sometimes, the units are also expressed in other units (which are derived from fundamental units) known as *derived units e.g.* units of area, velocity, acceleration, pressure etc.

1.12. SYSTEMS OF UNITS

There are only four systems of units, which are commonly used and universally recognised. These are known as :

1. C.G.S. units, **2.** F.P.S. units, **3.** M.K.S. units and **4.** S.I. units.

In this book, we shall use only the S.I. system of units, as the future courses of studies are conduced in this system of units only.

1.13. S.I. UNITS (INTERNATIONAL SYSTEM OF UNITS)

The eleventh General Conference* of Weights and Measures has recommended a unified and systematically constituted system of fundamental and derived units for international use. This system of units is now being used in many countries.

In India, the Standards of Weights and Measures Act of 1956 (vide which we switched over to M.K.S. units) has been revised to recognise all the S.I. units in industry and commerce.

In this system of units, the †fundamental units are metre (m), kilogram (kg) and second (s) respectively. But there is a slight variation in their derived units. The following derived units will be used in this book :

Density (Mass density)	kg / m^3
Force	N (Newton)
Pressure	N/mm^2 or N/m^2
Work done (in joules)	J = N-m
Power in watts	W = J/s

International metre, kilogram and second are discussed here.

* It is knwon as General Conference of Weights and Measures (G.C.W.M.). It is an international organisation of which most of the advanced and developing countries (including India) are members. This confer-ence has been ensured the task of prescribing definitions of various units of weights and measures, which are the very basis of science and technology today.

† The other fundamental units are electric current, ampere (A), thermodynamic temperature, kelvin (K) and luminous intensity, candela (cd). These three units will not be used in this book.

1.14. METRE

The international metre may be defined as the shortest distance (at 0°C) between two parallel lines engraved upon the polished surface of the Platinum-Iridium bar, kept at the International Bureau of Weights and Measures at Sevres near Paris.

A bar of platinum - iridium metre kept at a temperature of 0º C.

The standard platinum - kilogram is kept at the International Bureau of Weights and Measures at Serves in France.

1.15. KILOGRAM

The international kilogram may be defined as the mass of the Platinum-Iridium cylinder, which is also kept at the International Bureau of Weights and Measures at Sevres near Paris.

1.16. SECOND

The fundamental unit of time for all the four systems is second, which is $1/(24 \times 60 \times 60)$ = 1/86 400th of the mean solar day. A solar day may be defined as the interval of time between the instants at which the sun crosses the meridian on two consecutive days. This value varies throughout the year. The average of all the solar days, of one year, is called the mean solar day.

1.17. PRESENTATION OF UNITS AND THEIR VALUES

The frequent changes in the present day life are facililtated by an international body known as International Standard Organisation (ISO). The main function of this body is to make recommendations regarding international procedures. The implementation of ISO recommendations in a country is assisted by an organisation appointed for the purpose. In India, Bureau of Indian Standard formerly known as Indian Standards Institution (ISI) has been created for this purpose.

We have already discussed in the previous articles the units of length, mass and time. It is always necessary to express all lengths in metres, all masses in kilograms and all time in seconds. According to convenience, we also use larger multiples or smaller fractions of these units. As a typical example, although metre is the unit of length; yet a smaller length equal to one-thousandth of a metre proves to be more convenient unit especially in the dimensioning of drawings. Such convenient units are formed by using a prefix in front of the basic units to indicate the multiplier.

The full list of these prefixes is given in Table 1.1.

Table 1.1

Factor by which the unit is multiplied	*Standard form*	*Prefix*	*Abbreviation*
1000 000 000 000	10^{12}	Tera	T
1 000 000 000	10^{9}	giga	G
1 000 000	10^{6}	mega	M
1 000	10^{3}	kilo	k
100	10^{2}	hecto*	h
10	10^{1}	deca*	da
0.1	10^{-1}	deci*	d
0.01	10^{-2}	centi*	c
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	µ
0.000 000 001	10^{-9}	nano	n
0.000 000 000 001	10^{-12}	pico	p

Note : These prefixes are generally becoming obsolete probably due to possible confusion.Moreover, it is becoming a conventional practice to use only those powers of ten, which conform to 0^{3n} (where *n* is a positive or negative whole number).

1.18. RULES FOR S.I. UNITS

The Eleventh General Conference of Weights and Measures recommended only the fundamental and derived units of S.I. system. But it did not elaborate the rules for the usage of these units. Later on, many scientists and engineers held a no. of meetings for the style and usage of S.I. units. Some of the decisions of these meetings are :

1. A dash is to be used to separate units, which are multiplied together. For example, a newton-meter is written as N-m. It should no be confused with mN, which stands for millinewton.

2. For numbers having 5 or more digits, the digits should be placed in groups of three separated by spaces (instead of *commas) counting both to the left and right of the decimal point.

3. In a †four digit number, the space is not required unless the four digit number is used in a column of numbers with 5 or more digits.

At the time of revising this book, the author sought the advice of various international authorities regarding the use of units and their values, keeping in view the global reputation of the author as well as his books. It was then decided to ††present the units and their values as per the recommendations of ISO and ISI. It was decided to use :

4500	not	4 500	Or	4,500
7 589 000	not	7589000	Or	7,589,000
0.012 55	not	0.01255	Or	.01255
30×10^{6}	not	3,00,00,000	Or	3×10^{7}

* In certain countries, comma is still used as the decimal marker.

† In certain countries, space is used even in a four digit number.

†† In some question papers, standard values are not used. The author has tried to avoid such questions in the text of the book, in order to avoid possible confusion. But at certain places, such questions have been included keeping in view the importance of question from the reader's angle.

The above mentioned figures are meant for numerical values only. Now we shall discuss about the units. We know that the fundamental units in S.I. system for length, mass and time are metre, kilogram and second respectively. While expressing these quantities, we find it time-consuming to write these units such as metres, kilograms and seconds, in full, every time we use them. As a result of this, we find it quite convenient to use the following standard abberviations, which are internationally recognised. We shall use :

m	for metre or metres
km	for kilometre or kilometres
kg	for kilogram or kilograms
t	for tonne or tonnes
s	for second or seconds
min	for minute or minutes
N	for newton or newtons
N-m	for newton × metres (*i.e.*, work done)
kN-m	for kilonewton × metres
rad	for radian or radians
rev	for revolution or revolutions

1.19. USEFUL DATA

The following data summarises the previous memory and formulae, the knowledge of which is very essential at this stage.

1.20. ALGEBRA

1. $a^0 = 1$; $x^0 = 1$

(*i.e.*, *Anything raised to the power zero is one.*)

2. $x^m \times x^n = x^{m+n}$

(*i.e.*, *If the bases are same*, *in multiplication*, *the powers are added.*)

3. $\dfrac{x^m}{x^n} = x^{m-n}$

(*i.e.*, *If the bases are same in division, the powers are subtracted.*)

4. If $ax^2 + bx + c = 0$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

where a is the coefficient of x^2, b is the coefficient of x and c is the constant term.

1.21. TRIGONOMETRY

In a right-angled triangle *ABC* as shown in Fig. 1.1

1. $\dfrac{b}{c} = \sin\theta$

2. $\dfrac{a}{c} = \cos\theta$

3. $\dfrac{b}{a} = \dfrac{\sin\theta}{\cos\theta} = \tan\theta$

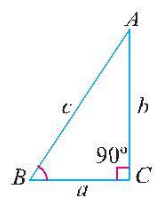


Fig. 1.1.

4. $\frac{c}{b} = \frac{1}{\sin\theta} = \text{cosec}\,\theta$

5. $\frac{c}{a} = \frac{1}{\cos\theta} = \sec\theta$

6. $\frac{a}{b} = \frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta} = \cot\theta$

7. The following table shows values of trigonometrical functions for some typical angles:

angle	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	1
cos	1	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{3}$	1	3	∞

or in other words, for sin write

0°	30°	45°	60°	90°
$\frac{0}{2}$	$\frac{1}{2}$	$\frac{2}{2}$	$\frac{3}{2}$	$\frac{4}{2}$
0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	1

for cos write the values in reverse order ; for tan divide the value of sin by cos for the respective angle.

8. In the first quadrant (*i.e.*, 0° to 90°) all the trigonometrical ratios are positive.

9. In the second quadrant (*i.e.*, 90° to 180°) only sin θ and cosec θ are positive.

10. In the third quadrant (*i.e.*, 180° to 270°) only tan θ and cot θ are positive.

11. In the fourth quadrant (*i.e.*, 270° to 360°) only cos θ and sec θ are positive.

12. In any triangle *ABC*,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

where a, b and c are the lengths of the three sides of a triangle. A, B and C are opposite angles of the sides a, b and c respectively.

13. $\sin(A + B) = \sin A \cos B + \cos A \sin B$

14. $\sin(A - B) = \sin A \cos B - \cos A \sin B$

15. $\cos(A + B) = \cos A \cos B - \sin A \sin B$

16. $\cos(A - B) = \cos A \cos B + \sin A \sin B$

17. $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A . \tan B}$

18. $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A . \tan B}$

19. $\sin 2A = 2 \sin A \cos A$

20. $\sin^2\theta + \cos^2\theta = 1$

21. $1 + \tan^2\theta = \sec^2\theta$

22. $1 + \cot^2\theta = \text{cosec}^2\theta$

23. $\sin^2 A = \underline{1 - \cos 2A}$

2

24. $\cos^2 A = \frac{1 + \cos 2A}{2}$

25. $2 \cos A \sin B = \sin (A + B) - \sin (A - B)$

26. Rules for the change of trigonometrical ratios:

(*A*)	$\sin(-\theta)$	$= -\sin\theta$
	$\cos(-\theta)$	$= \cos\theta$
	$\tan(-\theta)$	$= -\tan\theta$
	$\cot(-\theta)$	$= -\cot\theta$
	$\sec(-\theta)$	$= \sec\theta$
	$\operatorname{cosec}(-\theta)$	$= -\operatorname{cosec}\theta$
(*B*)	$\sin(90^\circ - \theta)$	$= \cos\theta$
	$\cos(90^\circ - \theta)$	$= \sin\theta$
	$\tan(90^\circ - \theta)$	$= \cot\theta$
	$\cot(90^\circ - \theta)$	$= \tan\theta$
	$\sec(90^\circ - \theta)$	$= \operatorname{cosec}\theta$
	$\operatorname{cosec}(90^\circ - \theta)$	$= \sec\theta$
(*C*)	$\sin(90^\circ + \theta)$	$= \cos\theta$
	$\cos(90^\circ + \theta)$	$= -\sin\theta$
	$\tan(90^\circ + \theta)$	$= -\cot\theta$
	$\cot(90^\circ + \theta)$	$= -\tan\theta$
	$\sec(90^\circ + \theta)$	$= -\operatorname{cosec}\theta$
	$\operatorname{cosec}(90^\circ + \theta)$	$= \sec\theta$
(*D*)	$\sin(180^\circ - \theta)$	$= \sin\theta$
	$\cos(180^\circ - \theta)$	$= -\cos\theta$
	$\tan(180^\circ - \theta)$	$= -\tan\theta$
	$\cot(180^\circ - \theta)$	$= -\cot\theta$
	$\sec(180^\circ - \theta)$	$= -\sec\theta$
	$\operatorname{cosec}(180^\circ - \theta)$	$= \operatorname{cosec}\theta$
(*E*)	$\sin(180^\circ + \theta)$	$= -\sin\theta$
	$\cos(180^\circ + \theta)$	$= -\cos\theta$
	$\tan(180^\circ + \theta)$	$= \tan\theta$
	$\cot(180^\circ + \theta)$	$= \cot\theta$
	$\sec(180^\circ + \theta)$	$= -\sec\theta$
	$\operatorname{cosec}(180^\circ + \theta)$	$= -\operatorname{cosec}\theta$

Following are the rules to remember the above 30 formulae :

Rule 1. Trigonometrical ratio changes only when the angle is (90° –θ)or (90° +θ). In allother cases, trigonometrical ratio remains the same. Following is the law of change :

sin changes into cos and cos changes into sin,

tan changes into cot and cot changes into tan,

sec changes into cosec and cosec changes into sec.

Rule 2. Consider the angleθto be a small angle and write the proper sign as per formulae8 to 11 above.

1.22. DIFFERENTIAL CALCULUS

1. $\frac{d}{dx}$ is the sign of differentiation.

2. $\frac{d}{dx}(x)^n = nx^{n-1}$; $\frac{d}{dx}(x)^8 = 8x^7$, $\frac{d}{dx}(x) = 1$

(*i.e.*, *to differentiate any power of x*, *write the power before x and subtract on from thepower*).

3. $\frac{d}{dx}(C) = 0$; $\frac{d}{dx}(7) = 0$

(*i.e.*, *differential coefficient of a constant is zero*).

4. $\frac{d}{dx}(u.v) = u.\frac{dv}{dx} + v.\frac{du}{dx}$

i.e. , *Differential coefficient of product of any two functions* = (1*st function*·*Differential coefficient of* sec *ond function*) + (2*nd function* · *Differential coefficient of first function*)

5. $\frac{d}{dx}\frac{u}{v} = \frac{v\cdot\frac{du}{dx} - u\cdot\frac{dv}{dx}}{v^2}$

i .e., Differential coefficient of two functions when one is divided by the other = $\frac{(\textit{Denominator} \times \textit{Differential coefficient of numerator}) - (\textit{Numerator} \times \textit{Differential coefficient of denominator})}{\textit{Square of denominator}}$

6. Differential coefficient of trigonometrical functions

$$\frac{d}{dx}(\sin x) = \cos x \; ; \; \frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x \; ; \; \frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \,.\, \tan x \; ; \; \frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \,.\, \cot x$$

(*i.e., The differential coefficient, whose trigonometrical function begins with co, is negative*).

7. If the differential coefficient of a function is zero, the function is either maximum or mini-mum. *Conversely*, if the maximum or minimum value of a function is required, then differ-entiate the function and equate it to zero.

1.23. INTEGRAL CALCULUS

1. $\int dx$ is the sign of integration.

2. $\int x^n \, dx = \dfrac{x^{n+1}}{n+1}$; $\int x^6 \, dx = \dfrac{x^7}{7}$

(*i.e.*, *to integration any power of* x, *add one to the power and divide by the new power*).

3. $\int 7 dx = 7x; \int C \, dx = Cx$

(*i.e.*, *to integrate any constant*, *multiply the constant by* x).

4. $\int (ax+b)^n \, dx = \dfrac{(ax+b)^{n+1}}{(n+1)\cdot a}$

(*i.e.*, *to integrate any bracket with power*, *add one to the power and divide by the new powerand also divide by the coefficient of x within the bracket*).

1.24. SCALAR QUANTITIES

The scalar quantities (or sometimes known as scalars) are those quantities which have magnitude only such as length, mass, time, distance, volume, density, temperature, speed etc.

1.25. VECTOR QUANTITIES

The vector quantities (or sometimes known as vectors) are those quantities which have both magnitude and direction such as force, displacement, velocity, acceleration, momentum etc. Following are the important features of vector quantities :

The velocity of this cyclist is an example of a vector quantity.

1. ***Representation of a vector.*** A vector isrepresented by a directed line as shown in Fig. 1.2. It may be noted that the length OA represents the magnitude of the vector $\overrightarrow{OA}$. The direction of the vector is $\overrightarrow{OA}$ is from O (*i.e.*, starting point) to A (*i.e.*, end point). It is also known as vector P.

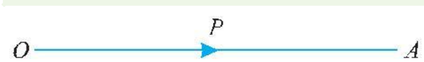
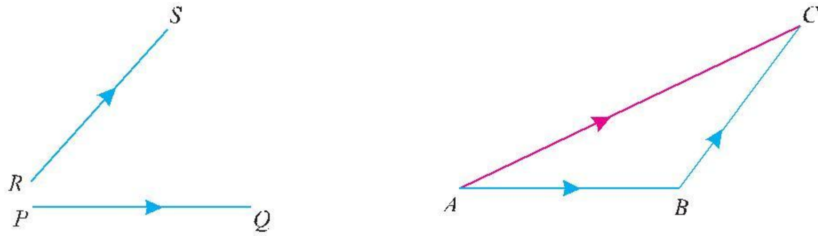


Fig. 1.2. Vector$\overrightarrow{OA}$

2. ***Unit vector.*** A vector, whose magnitude is unity,is known as unit vector.
3. ***Equal vectors.*** The vectors, which are parallel to each other and have same direction (*i.e.*,same sense) and equal magnitude are known as equal vectors.
4. ***Like vectors.*** The vectors, which are parallel to each other and have same sense but unequalmagnitude, are known as like vectors.
5. ***Addition of vectors.*** Consider two vectorsPQandRS, which are required to be added asshown in Fig. 1.3. (*a*).



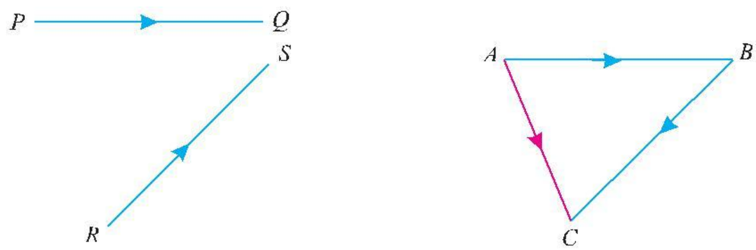
(*a*) Vectors *PQ* and *RS* (*b*) Addition of vectors

Fig. 1.3.

Take a point *A*, and draw line *AB* parallel and equal in magnitude to the vector *PQ* to some convenient scale. Through *B*, draw *BC* parallel and equal to vector *RS* to the same scale. Join *AC* which will give the required sum of vectors *PQ* and *RS* as shown in Fig. 1.3. (*b*).

This method of adding the two vectors is called the Triangle Law of Addition of Vectors. Similarly, if more than two vectors are to be added, the same may be done first by adding the two vectors, and then by adding the third vector to the resultant of the first two and so on. This method of adding more than two vectors is called Polygon Law of Addition of Vectors.

6. *Subtraction of vectors.* Consider two vectors *PQ* and *RS* in which the vector *RS* is requiredto be subtracted as shown in Fig. 1.4 (*a*)



(*a*) Vectors *PQ* and *RS* (*b*) Subtraction of vectors

Fig. 1.4.

Take a point *A*, and draw line *AB* parallel and equal in magnitude to the vector *PQ* to some convenient scale. Through *B*, draw *BC* parallel and equal to the vector *RS*, but in *opposite direction*, to that of the vector *RS* to the same scale. Join *AC*, which will give the resultant when the vector *PQ* is subtracted from vector *RS* as shown in Fig. 1.4 (*b*).

1.1COMPOSITION OF RESOLUTION OF FORCES

2.1. INTRODUCTION

The force is an important factor in the field of Mechanics, which may be broadly *defined as an agent which produces or tends to produce, destroys or tends to destroy motion. *e.g*., a horse applies force to pull a cart and to set it in motion. Force is also required to work on a bicycle pump. In this case, the force is supplied by the muscular power of our arms and shoulders.

Sometimes, the applied force may not be sufficient to move a body, *e.g*., if we try to lift a stone weighing 2 or 3 quintals, we fail to do so. In this case we exert a force, no doubt, but no motion is produced. This shows that a force may not necessarily produce a motion in a body ; but it may, simply, tend to do so. In a tug-of-war the two parties, when balanced, neutralize each other's force. But the moment one party gets weaker, the other party pulls off, in spite of first party's best effort to destroy motion.

2.2. EFFECTS OF A FORCE

A force may produce the following effects in a body, on which it acts :

1. It may change the motion of a body. *i.e*. if a body is at rest, the force may set it in motion. And if the body is already in motion, the force may accelerate it.
2. It may retard the motion of a body.
3. It may retard the forces, already acting on a body, thus bringing it to rest or in equilibrium. We shall study this effect in chapter 5 of this book.
4. It may give rise to the internal stresses in the body, on which it acts. We shall study this effect in the chapters 'Analysis of Perfect Frames' of this book.

2.3. CHARACTERISTICS OF A FORCE

In order to determine the effects of a force, acting on a body, we must know the following characteristics of a force :

1. Magnitude of the force (*i.e*., 100 N, 50 N, 20 kN, 5 kN, etc.)
2. The direction of the line, along which the force acts (*i.e.*, along *OX*, *OY*, at 30° North of East etc.). It is also known as line of action of the force.
3. Nature of the force (*i.e.*, whether the force is push or pull). This is denoted by placing an arrow head on the line of action of the force.
4. The point at which (or through which) the force acts on the body.

2.4. PRINCIPLE OF PHYSICAL INDEPENDENCE OF FORCES

It states, "*If a number of forces are simultaneously acting on a*particle*, *then the resultant ofthese forces will have the same effect as produced by all the forces.* "

2.5. PRINCIPLE OF TRANSMISSIBILITY OF FORCES

It states, "*If a force acts at any point on a †rigid body*, *it may also be considered to act at anyother point on its line of action*, *provided this point is rigidly connected with the body.*"

2.6. SYSTEM OF FORCES

When two or more forces act on a body, they are called to form a *system of forces*. Following systems of forces are important from the subject point of view :

1. ***Coplanar forces.*** The forces, whose lines of action lie on the same plane, are known ascoplanar forces.

2. ***Collinear forces.*** The forces, whose lines of action lie on the same line, are known ascollinear forces.

* A particle may be defined as a body of infinitely small volume and is considered to be concentrated point.

† A rigid body may be defined as a body which can retain its shape and size, even if subjected to some external forces. In actual practice, no body is perfectly rigid. But for the sake of simplicity, we take all the bodies as rigid bodies.

3. ***Concurrent forces.*** The forces, which meet at one point, are known as concurrent forces.The concurrent forces may or may not be collinear.
4. ***Coplanar concurrent forces.*** The forces, which meet at one point and their lines of actionalso lie on the same plane, are known as coplanar concurrent forces.
5. ***Coplanar non-concurrent forces.*** The forces, which do not meet at one point, but theirlines of action lie on the same plane, are known as coplanar non-concurrent forces.
6. ***Non-coplanar concurrent forces.*** The forces, which meet at one point, but their lines ofaction do not lie on the same plane, are known as non-coplanar concurrent forces.
7. ***Non-coplanar non-concurrent forces.*** The forces, which do not meet at one point and theirlines of action do not lie on the same plane, are called non-coplanar non-concurrent forces.

2.7. RESULTANT FORCE

If a number of forces, P, Q, R ... etc. are acting simultaneously on a particle, then it is possible to find out a single force which could replace them *i.e.*, which would produce the same effect as produced by all the given forces. This single force is called *resultant force* and the given forces R ...
etc. are called component forces.

2.8. COMPOSITION OF FORCES

The process of finding out the resultant force, of a number of given forces, is called *compositionof forces* or compounding of forces.

2.9. METHODS FOR THE RESULTANT FORCE

Though there are many methods for finding out the resultant force of a number of given forces, yet the following are important from the subject point of view :

1. Analytical method. 2. Method of resolution.

2.10. ANALYTICAL METHOD FOR RESULTANT FORCE

The resultant force, of a given system of forces, may be found out analytically by the following methods :

1. Parallelogram law of forces. 2. Method of resolution.

2.11. PARALLELOGRAM LAW OF FORCES

It states, "*If two forces, acting simultaneously on a particle, be represented in magnitude anddirection by the two adjacent sides of a parallelogram* ; *their resultant may be represented in magnitude and direction by the diagonal of the parallelogram, which passes through their point of intersection.*"

Mathematically, resultant force,

$$R = \sqrt{F_1^2 + F_2^2 + 2 F_1 F_2 \cos\theta}$$

and

$$\tan\alpha = \frac{F_2 \sin\theta}{F_1 + F_2 \cos\theta}$$

where F_1 and F_2 = Forces whose resultant is required to be found out,

θ = Angle between the forces F_1 and F_2, and

α = Angle which the resultant force makes with one of the forces (say F_1).

Note. It the angle (α) which the resultant force makes with the other force F_2,

then $$\tan \alpha = \frac{F_1 \sin\theta}{F_2 + F_1 \cos\theta}$$

Cor.

1. If θ = 0 *i.e.*, when the forces act along the same line, then

$$R = F_1 + F_2 \qquad \text{...(Since } \cos 0° = 1)$$

2. If θ = 90° *i.e.*, when the forces act at right angle, then

$$\theta = R = \sqrt{F_1^2 + F_2^2} \qquad \text{...(Since } \cos 90° = 0)$$

3. If θ = 180° *i.e.*, when the forces act along the same straight line but in opposite directions, then $R = F_1 - F_2$ ...(Since cos 180° = – 1)

In this case, the resultant force will act in the direction of the greater force.

4. If the two forces are equal *i.e.*, when $F_1 = F_2 = F$ then

$$R = \sqrt{F^2 + F^2 + 2F^2 \cos\theta} = \sqrt{2F^2(1 + \cos\theta)}$$

$$= \sqrt{2F^2 \cdot 2\cos^2 \frac{\theta}{2}} \qquad \text{...} \because 1 + \cos\theta = 2\cos^2\frac{\theta}{2}$$

$$= \sqrt{4F^2 \cos^2 \frac{\theta}{2}} = 2F \cos \frac{\theta}{2}$$

Example 2.1. *Two forces of 100 N and 150 N are acting simultaneously at a point. What isthe resultant of these two forces, if the angle between them is 45°?*

Solution. Given : First force (F_1) = 100 N; Second force (F_2) = 150 N and angle between F_1 and F_2 (θ) = 45°.

We know that the resultant force,

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$= \sqrt{(100)^2 + (150)^2 + 2 \cdot 100 \cdot 150 \cos 45°} \text{ N}$$

$$= \sqrt{10\,000 + 22\,500 + (30\,000 \cdot 0.707)}$$

N = 232 N **Ans.**

Example 2.2. *Two forces act at an angle of 120°. The bigger force is of 40 N and theresultant is perpendicular to the smaller one. Find the smaller force.*

Solution. Given : Angle between the forces $\angle AOC = 120°$, Bigger force (F_1) = 40 N andangle between the resultant and F_2 ($\angle BOC$) = 90° ;

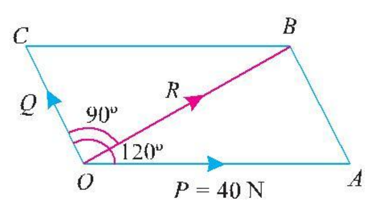


Fig. 2.1.

Let F_2 = Smaller force in N

From the geometry of the figure, we find that $\angle AOB$,

$$\alpha = 120° - 90° = 30°$$

We know that

$$\tan\alpha = \frac{F_2 \sin\theta}{F_1 + F_2\cos\theta}$$

$$\tan 30° = \frac{F_2 \sin 120°}{40 + F_2 \cos 120°} = \frac{F_2 \sin 60°}{40 + F_2(-\cos 60°)}$$

$$\therefore \quad 0.577 = \frac{F_2 \cdot 0.866}{40 - F_2 \cdot 0.5} = \frac{0.866\, F_2}{40 - 0.5\, F_2}$$

$$40 - 0.5\, F_2 = \frac{0.866\, F_2}{0.577} = 1.5\, F_2$$

$$\therefore \quad 2F_2 = 40 \quad \text{or} \quad F_2 = 20 \quad \textbf{Ans.}$$

Example 2.3. *Find the magnitude of the two forces, such that if they act at right angles, their resultant is* $\sqrt{10}$ *N. But if they Act at 60°, their resultant is* $\sqrt{13}$ *N.*

Solution. Given : Two forces $= F_1$ and F_2.

First of all, consider the two forces acting at right angles. We know that when the angle between the two given forces is 90°, then the resultant force (R)

$$\sqrt{10} = \sqrt{F_1^2 + F_2^2}$$

Or $$10 = F_1^2 + F_2^2 \qquad \text{...(Squaring both sides)}$$

Similarly, when the angle between the two forces is 60°, then the resultant force (R)

$$\sqrt{13} = \sqrt{F_1^2 + F_2^2 + 2F_1 F_2 \cos 60°}$$

$$\therefore \quad 13 = F_1^2 + F_2^2 + 2F_1 F_2 \cdot 0.5 \qquad \text{...(Squaring both sides)}$$

Or $$F_1 F_2 = 13 - 10 = 3 \qquad \text{...(Substituting } F_1^2 + F_2^2 = 10)$$

We know that $(F_1 + F_2)^2 = F_1^2 + F_2^2 + 2F_1 F_2 = 10 + 6 = 16$

$$\therefore \quad F_1 + F_2 = \sqrt{16} = 4 \qquad ...(i)$$

Similarly $(F_1 - F_2)^2 = F_1^2 + F_2^2 - 2\, F_1 F_2 = 10 - 6 = 4$

$$\therefore \quad F_1 - F_2 = \sqrt{4} = 2 \qquad ...(ii)$$

Solving equations (*i*) and (*ii*),

$$F_1 = 3 \text{ N} \quad \text{and} \quad F_2 = 1 \text{ N} \quad \textbf{Ans.}$$

2.12. RESOLUTION OF A FORCE

The process of splitting up the given force into a number of components, without changing its effect on the body is called resolution of a force. A force is, generally, resolved along two mutually perpendicular directions. In fact, the resolution of a force is the reverse action of the addition of the component vectors.

2.13. PRINCIPLE OF RESOLUTION

It states, "*The algebraic sum of the resolved parts of a no. of forces, in a given direction, isequal to the resolved part of their resultant in the same direction.*"

Note : In general, the forces are resolved in the vertical and horizontal directions.

Example 2.4. *A machine component 1.5 m long and weight 1000 N is supported by tworopes AB and CD as shown in Fig. 2.2 given below.*

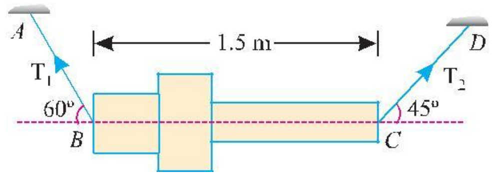


Fig. 2.2.

Calculate the tensions T_1 and T_2 in the ropes AB and CD.

Solution. Given : Weight of the component = 1000 N

Resolving the forces horizontally (*i.e.*, along *BC*) and equating the same,

$$T_1 \cos 60° = T_2 \cos 45°$$

$$\therefore \quad T_1 = \frac{\cos 45°}{\cos 60°} \cdot T_2 = \frac{0.707}{0.5} \cdot T_2 = 1.414\, T_2 \qquad \text{...}(i)$$

and now resolving the forces vertically,

$$T_1 \sin 60° + T_2 \sin 45° = 1000$$

$$(1.414\, T_2)\, 0.866 + T_2 \times 0.707 = 1000$$

$$1.93\, T_2 = 1000$$

$$\therefore \quad T_2 = \frac{1000}{1.93} = 518.1 \text{ N} \qquad \textbf{Ans.}$$

And $$T_1 = 1.414 \times 518.1 = 732.6 \text{ N} \qquad \textbf{Ans.}$$

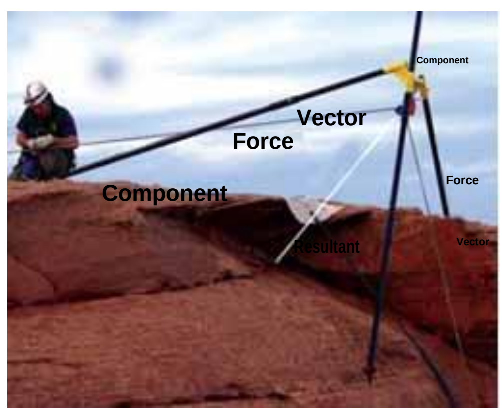
2.14. METHOD OF RESOLUTION FOR THE RESULTANT FORCE

1. Resolve all the forces horizontally and find the algebraic sum of all the horizontal components (*i.e.*, $\sum H$).
2. Resolve all the forces vertically and find the algebraic sum of all the vertical components (*i.e.*, $\sum V$).
3. The resultant *R* of the given forces will be given by the equation :

$$R = \sqrt{(\sum H)^2 + (\sum V)^2}$$

4. The resultant force will be inclined at an angle θ, with the horizontal, such that

$$\tan \theta = \frac{\sum V}{\sum H}$$



Notes : The value of the angle θ will vary depending upon the values of $\sum V$ and $\sum H$ as discussed below :

1. When $\sum V$ is +ve, the resultant makes an angle between 0° and 180°. But when $\sum V$ is –ve, the resultant makes an angle between 180° and 360°.

2. When $\sum H$ is +ve, the resultant makes an angle between 0° to 90° or 270° to 360°. But when $\sum H$ is –ve, the resultant makes an angle between 90° to 270°.

Example 2.5. *A triangle ABC has its side AB=40 mm along positive x-axis and sideBC = 30 mm along positive y-axis. Three forces of 40 N, 50 N and 30 N act along the sides AB, BC and CA respectively. Determine magnitude of the resultant of such a system of forces.*

Solution. The system of given forces is shown in Fig. 2.3.From the geometry of the figure, we find that the triangle *ABC* is a right angled triangle, in which the *side *AC* = 50 mm. Therefore

$$\sin\theta = \frac{30}{50} = 0.6$$

And $$\cos\theta = \frac{40}{50} = 0.8$$

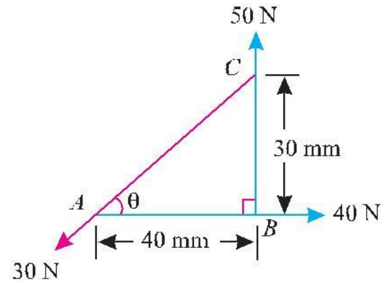


Fig. 2.3.

Resolving all the forces horizontally (*i.e.*, along *AB*), $\sum H = 40 - 30\cos\theta$

$= 40 - (30 \times 0.8) = 16$ N and

now resolving all the forces vertically (*i.e.*, along *BC*)

$$\sum V = 50 - 30\sin\theta$$

$$= 50 - (30 \times 0.6) = 32 \text{ N}$$

We know that magnitude of the resultant force,

$$R = \sqrt{(\sum H)^2 + (\sum V)^2} = \sqrt{(16)^2 + (32)^2} = 35.8 \text{ N} \textbf{ Ans.}$$

Example 2.6. *A system of forces are acting at the corners of a rectangular block as shownin Fig.* 2.4.

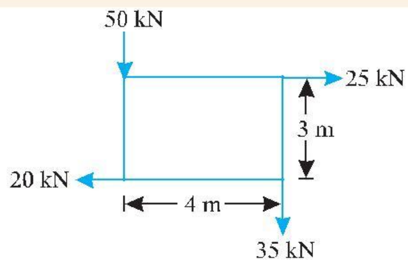


Fig. 2.4.

Determine the magnitude and direction of the resultant force.

Solution. Given : System of forces

Magnitude of the resultant force

Resolving forces horizontally,

$$\sum H = 25 - 20 = 5$$

kNand now resolving the forces vertically

$$\sum V = (-50) + (-35) = -85 \text{ kN}$$

∴ Magnitude of the resultant force

$$R = \sqrt{(\sum H)^2 + (\sum V)^2} = \sqrt{(5)^2 + (-85)^2} = 85.15 \text{ kN} \textbf{ Ans.}$$

---

* Since the side *AB* is along *x*-axis, and the side *BC* is along *y*-axis, there fore it is a right-angled triangle. Now in triangle *ABC*,

$$AC = \sqrt{AB^2 + BC^2} = \sqrt{(40)^2 + (30)^2} = 50 \text{ mm}$$

Direction of the resultant force

Let θ = Angle which the resultant force makes with the horizontal.

We know that

$$\tan\theta = \frac{\sum V}{\sum H} = \frac{-85}{5} = -17 \quad \text{or} \quad \theta = 86.6°$$

Since $\sum H$ is positive and $\sum V$ is negative, therefore resultant lies between 270° and 360°. Thus actual angle of the resultant force

$= 360° - 86.6° = 273.4°$ **Ans.**

Example 2.7. *The forces 20 N,30 N,40 N,50 N and 60 N are acting at one of the angularpoints of a regular hexagon, towards the other five angular points, taken in order. Find the magnitude and direction of the resultant force.*

Solution. The system of given forces is shown in Fig. 2.5

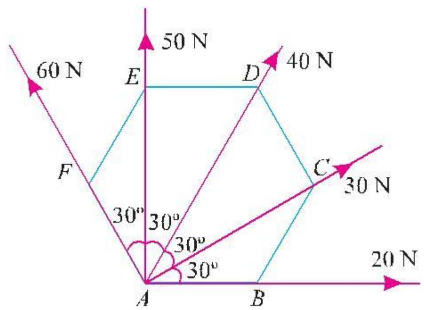


Fig. 2.5.

Magnitude of the resultant force

Resolving all the forces horizontally (*i.e.*, along *AB*),

$$\sum H = 20\cos 0° + 30\cos 30° + 40\cos 60° + 50\cos 90° + 60\cos 120° \text{ N}$$
$$= (20 \times 1) + (30 \times 0.866) + (40 \times 0.5) + (50 \times 0) + 60(-0.5) \text{ N}$$
$$= 36.0 \text{ N} \quad ...(i)$$

and now resolving the all forces vertically (*i.e.*, at right angles to *AB*),

$$\sum V = 20\sin 0° + 30\sin 30° + 40\sin 60° + 50\sin 90° + 60\sin 120° \text{ N}$$
$$= (20 \times 0) + (30 \times 0.5) + (40 \times 0.866) + (50 \times 1) + (60 \times 0.866) \text{ N}$$
$$= 151.6 \text{ N} \quad ...(ii)$$

We know that magnitude of the resultant force,

$$R = \sqrt{(\sum H)^2 + (\sum V)^2} = \sqrt{(36.0)^2 + (151.6)^2} = 155.8 \text{ N} \quad \textbf{Ans.}$$

Direction of the resultant force

Let θ = Angle, which the resultant force makes with the horizontal (*i.e.*, *AB*).

We know that

$$\tan\theta = \frac{\sum V}{\sum H} = \frac{151.6}{36.0} = 4.211 \quad \text{or} \quad \theta = 76.6° \quad \textbf{Ans.}$$

Note. Since both the values of $\sum H$ and $\sum V$ are positive, therefore actual angle of resultantforce lies between 0° and 90°.

Example 2.8. *The following forces act at a point:*(*i*) *20 N inclined at 30° towards North of East*,
(*ii*) *25 N towards North*,
(*iii*) *30 N towards North West*, *and*
(*iv*) *35 N inclined at 40° towards South of West.*

Find the magnitude and direction of the resultant force.

Solution. The system of given forces is shown in Fig. 2.6.

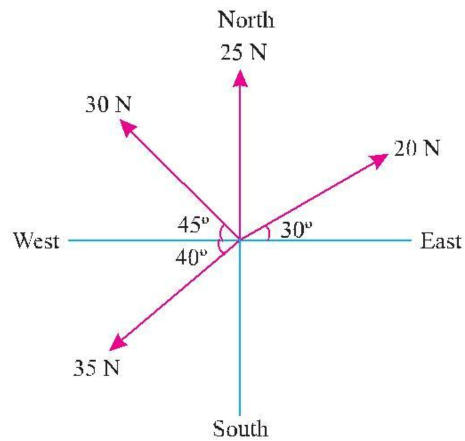


Fig. 2.6.

Magnitude of the resultant force

Resolving all the forces horizontally *i.e.*, along East-West line,

$$\sum H = 20 \cos 30° + 25 \cos 90° + 30 \cos 135° + 35 \cos 220° \text{ N} =$$
$$(20 \times 0.866) + (25 \times 0) + 30\,(-0.707) + 35\,(-0.766) \text{ N}$$
$$= -30.7 \text{ N} \quad ...(i)$$

and now resolving all the forces vertically *i.e.*, along North-South line,

$$\sum V = 20 \sin 30° + 25 \sin 90° + 30 \sin 135° + 35 \sin 220° \text{ N}$$
$$= (20 \times 0.5) + (25 \times 1.0) + (30 \times 0.707) + 35\,(-0.6428) \text{ N}$$
$$= 33.7 \text{ N} \quad ...(ii)$$

We know that magnitude of the resultant force,

$$R = \sqrt{(\sum H)^2 + (\sum V)^2} = \sqrt{(-30.7)^2 + (33.7)^2} = 45.6 \text{ N} \quad \textbf{Ans.}$$

Direction of the resultant force

Let θ = Angle, which the resultant force makes with the East.

We know that

$$\tan \theta = \frac{\sum V}{\sum H} = \frac{33.7}{-30.7} = -1.098 \quad \text{or} \quad \theta = 47.7°$$

Since $\sum H$ is negative and $\sum V$ is positive, therefore resultant lies between 90° and 180°. Thus actual angle of the resultant = 180° – 47.7° = 132.3° **Ans.**

Example 2.9. *A horizontal line PQRS is 12 m long,where PQ=QR=RS=4 m.Forces of1000 N, 1500 N, 1000 N and 500 N act at P, Q, R and* S *respectively with downward direction. The lines of action of these forces make angles of 90°, 60°, 45° and 30° respectively with PS. Find the magnitude, direction and position of the resultant force.*

Solution. The system of the given forces is shown in Fig. 2.7

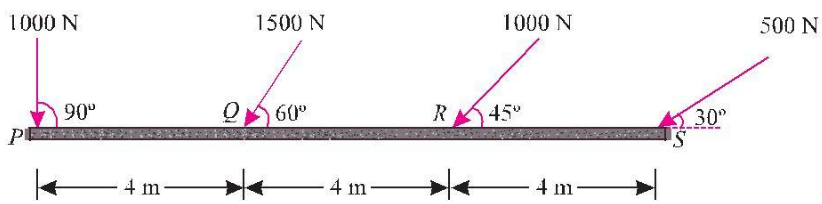


Fig. 2.7.

Magnitude of the resultant force

Resolving all the forces horizontally,

$$\Sigma H = 1000 \cos 90° + 1500 \cos 60° + 1000 \cos 45° + 500 \cos 30° \text{ N}$$
$$= (1000 \times 0) + (1500 \times 0.5) + (1000 \times 0.707) + (500 \times 0.866) \text{ N}$$
$$= 1890 \text{ N} \qquad ...(i)$$

and now resolving all the forces vertically,

$$\Sigma V = 1000 \sin 90° + 1500 \sin 60° + 1000 \sin 45° + 500 \sin 30° \text{ N}$$
$$= (1000 \times 1.0) + (1500 \times 0.866) + (1000 \times 0.707) + (500 \times 0.5) \text{ N}$$
$$= 3256 \text{ N} \qquad ...(ii)$$

We know that magnitude of the resultant force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(1890)^2 + (3256)^2} = 3765 \text{ N } \textbf{Ans.}$$

Direction of the resultant force

Let θ = Angle, which the resultant force makes with *PS.*

$$\therefore \quad \tan \theta = \frac{\Sigma V}{\Sigma H} = \frac{3256}{1890} = 1.722 \quad \text{or} \quad \theta = 59.8° \textbf{ Ans.}$$

Note. Since both the values of ΣH and ΣV are +ve. therefore resultant lies between 0° and 90°.

Position of the resultant force

Let x = Distance between *P* and the line of action of the resultant force.

Now taking moments* of the vertical components of the forces and the resultant force about *P*, and equating the same,

$$3256\, x = (1000 \times 0) + (1500 \times 0.866)\, 4 + (1000 \times 0.707)8 + (500 \times 0.5)12$$
$$= 13\,852$$

$$\therefore \quad x = \frac{13\,852}{3256} = 4.25 \text{ m} \quad \textbf{Ans.}$$

* This point will be discussed in more details in the chapter on 'Moments and Their Applications'.

EXERCISE 2.1

1. Find the resultant of two forces equal to 50 N and 30 N acting at an angle of 60°. [**Ans.** 70 N ; 21.8°]
2. Two forces of 80 N and 70 N act simultaneously at a point. Find the resultant force, if the angle between them is 150°. [**Ans.** 106.3 N ; 61°]
3. Find the resultant of two forces 130 N and 110 N respectively, acting at an angle whose tangent is 12/5. [**Ans.** 185.7 N ; 30.5°]
4. A push of 180 N and pull of 350 N act simultaneously at a point. Find the resultant of the forces, if the angle between them be 135°. [**Ans.** 494 N ; 30°]
5. Find the angle between two equal forces P, when their resultant is equal to (i) P and (ii) $P/2$. [**Ans.** 120° N ; 151°]

Hint. When resultant is equal to P, then

$$P = \sqrt{P^2 + P^2 + 2\,P.\,P\cos\theta} = P\sqrt{2 + 2\cos\theta}$$

$$\therefore \quad 2\cos\theta = -1 \quad \text{or} \quad \cos\theta = -0.5 \quad \text{or} \quad \theta = 120° \quad \textbf{Ans.}$$

When resultant is equal to $P/2$, then

$$0.5\,P = \sqrt{P^2 + P^2 + 2\,P.P\cos\theta} = P\sqrt{2 + 2\cos\theta}$$

$$\therefore \quad 2\cos\theta = -1.75 \quad \text{or} \quad \cos\theta = -0.875 \quad \text{or} \quad \theta = 151° \quad \textbf{Ans.}$$

6. The resultant of two forces P and Q is R. If Q is doubled, the new resultant is perpendicular to P. Prove that $Q = R$.

Hint. In first case, $R = \sqrt{P^2 + Q^2 + 2\,PQ\cos\theta}$

In second case, $\tan 90° = \dfrac{(2Q)\sin\theta}{P + (2Q)\cos\theta}$

Since $\tan 90° = \infty$, therefore $P + 2Q\cos\theta = 0$

2.15. LAWS FOR THE RESULTANT FORCE

The resultant force, of a given system of forces, may also be found out by the following laws :
1. Triangle law of forces. **2.** Polygon law of forces.

2.16. TRIANGLE LAW OF FORCES

It states, "*If two forces acting simultaneously on a particle, be represented in magnitude anddirection by the two sides of a triangle, taken in order ; their resultant may be represented in magnitude and direction by the third side of the triangle, taken in opposite order.*"

2.17. POLYGON LAW OF FORCES

It is an extension of Triangle Law of Forces for more than two forces, which states, "*If anumber of forces acting simultaneously on a particle, be represented in magnitude and direction, by the sides of a polygon taken in order ; then the resultant of all these forces may be represented, in magnitude and direction, by the closing side of the polygon, taken in opposite order.*"

EXERCISE 2.2

1. Find the magnitude and direction of the resultant of the concurrent forces of 8 N, 12 N, 15 N and 20 N making angles of 30°, 70°, 120°.25 and 155° respectively with a fixed line. [**Ans.** 39.5 N ; 111.7°]
2. Find magnitude of the resultant force, if 30, 40, 50 and 60 N forces are acting along thelines joining the centre of a square to its vertices. [**Ans.** 28.3 N]
3. Four forces of 25 N, 20 N, 15 N and 10 N are acting simultaneously along straight lines

OA, *OB*, *OC* and *OD* such that

$$\angle AOB = 45°; \angle BOC = 100° \text{ and} \angle COD = 125°.$$

Find graphically magnitude and direction of the resultant force. Also check the answer analytically. [**Ans.** 29.5 N ; 25.4° with *OA*]

QUESTIONS

1. Define the term 'force', and state clearly the effects of force.
2. What are the various characteristics of a force?
3. Distinguish clearly between resolution of forces and composition of forces.
4. What are the methods for finding out the resultant force for a given system of forces?
5. State and prove parallelogram law of forces.
6. State triangle law of forces and polygon law of forces.
7. Show that the algebraic sum of the resolved part of a number of forces in a given direction, is equal to the resolved part of their resultant in the same direction.
8. Explain clearly the procedure for finding out the resultant force analytically as well as graphically.

OBJECTIVE TYPE QUESTIONS

1. Which of the following statement is correct?

(*a*) A force is an agent which produces or tends to produce motion.
(*b*) A force is an agent which stops or tends to stop motion.
(*c*) A force may balance a given number of forces acting on a body.
(*d*) Both (*a*) and (*b*).

2. In order to determine the effects of a force acting on a body, we must know

(*a*) Its magnitude and direction of the line along which it acts.
(*b*) Its nature (whether push or pull).
(*c*) Point through which it acts on the body.
(*d*) All of the above.

3. If a number of forces are acting simultaneously on a particle, then the resultant of these forces will have the same effect as produced by the all the forces. This is known as

(*a*) Principle of physical independence of forces.
(*b*) Principle of transmissibility of forces.
(*c*) Principle of resolution of forces.
(*d*) None of the above.

4. The vector method, for the resultant force, is also called polygon law of forces

(*a*) Correct (*b*) Incorrect

5. The resultant of two forces P and Q acting at an angle θ is equal to

(*a*) $\sqrt{P^2 + Q^2 + 2PQ\sin\theta}$ (*b*) $\sqrt{P^2 + Q^2 + 2PQ\cos\theta}$
(*c*) $\sqrt{P^2 + Q^2 - 2PQ\sin\theta}$ (*d*) $\sqrt{P^2 + Q^2 - 2PQ\cos\theta}$

6. If the resultant of two forces P and Q acting at an angle (α) with P, then

(*a*) $\tan\alpha = \dfrac{P\sin\theta}{P + Q\cos\theta}$ (*b*) $\tan\alpha = \dfrac{P\cos\theta}{P + Q\cos\theta}$

(*c*) $\tan\alpha = \dfrac{Q\sin\theta}{P + Q\cos\theta}$ (*d*) $\tan\alpha = \dfrac{Q\cos\theta}{P + Q\cos\theta}$

ANSWERS

1. (*d*) **2.** (*d*) **3.** (*a*) **4.** (*b*) **5.** (*b*) **6.** (*c*)

Top

Parallel Forces and Couples

4.1. INTRODUCTION

In the previous chapters, we have been studying forces acting at one point. But, sometimes, the given forces have their lines of action parallel to each other. A little consideration will show, that such forces do not meet at any point, though they do have some effect on the body on which they act. The forces, whose lines of action are parallel to each other, are known as parallel forces.

4.2. CLASSIFICATION OF PARALLEL FORCES

The parallel forces may be, broadly, classified into the following two categories, depending upon their directions :

1. Like parallel forces.
2. Unlike parallel forces.

4.3. LIKE PARALLEL FORCES

The forces, whose lines of action are parallel to each other and all of them act in the same direction as shown in Fig. 4.1 (*a*) are known as like parallel forces.

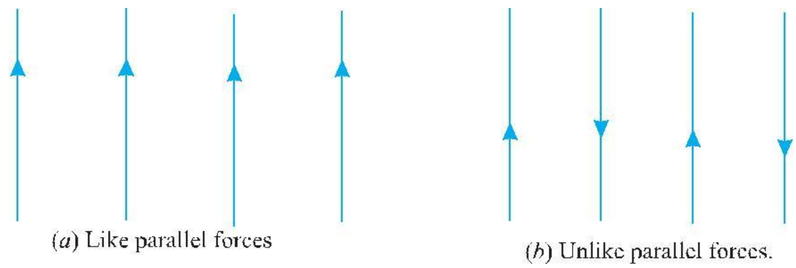


Fig. 4.1.

4.4. UNLIKE PARALLEL FORCES

The forces, whose lines of action are parallel to each other and all of them do not act in the same direction as shown in Fig. 4.1 (*b*) are known as unlike parallel forces.

4.5. METHODS FOR MAGNITUDE AND POSITION OF THE RESULTANT OF PARALLEL FORCES

The magnitude and position of the resultant force, of a given system of parallel forces (like or unlike) may be found out analytically or graphically. Here we shall discuss both the methods one by one.

4.6. ANALYTICAL METHOD FOR THE RESULTANT OF PARALLEL FORCES

In this method, the sum of clockwise moments is equated with the sum of anticlockwise moments about a point.

Example 4.1. *Two like parallel forces of 50 N and 100 N act at the ends of a rod 360 mmlong. Find the magnitude of the resultant force and the point where it acts.*

Solution. Given : The system of given forces is shown in Fig. 4.2

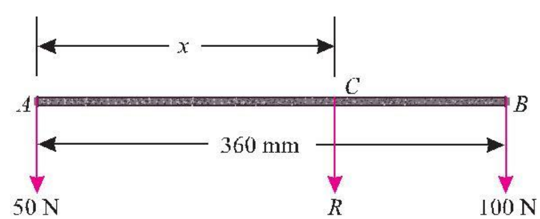


Fig. 4.2.

Magnitude of the resultant force

Since the given forces are like and parallel, therefore magnitude of the resultant

force, $R = 50 + 100 = 150$ N**Ans.**

Point where the resultant force acts

Let x = Distance between the line of action of the resultant force (R) and A (*i.e.* AC) in mm.

Now taking clockwise and anticlockwise moments of the forces about C and equating the same,

$$50 \times x = 100\,(360 - x) = 36\,000 - 100\,x$$

Or $$150\,x = 36\,000$$

$$\therefore \quad x = \frac{36\,000}{150} = 240 \text{ mm} \quad \textbf{Ans.}$$

Example 4.2. *A beam 3 m long weighing 400 N is suspended in a horizontal position by twovertical strings, each of which can withstand a maximum tension of 350 N only. How far a body of 200 N weight be placed on the beam, so that one of the strings may just break ?*

Solution. The system of given forces is shown in Fig. 4.3.

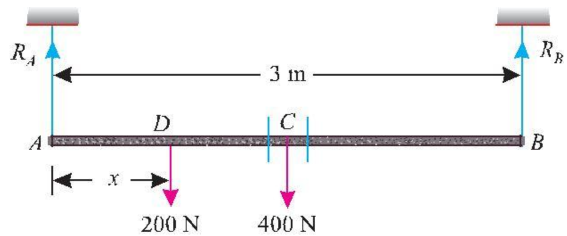


Fig. 4.3.

Let x = Distance between the body of weight 200 N and support A.

We know that one of the string (say A) will just break, when the tension will be 350 N. (*i.e.*, *R_A = 350 N). Now taking clockwise and anticlockwise moments about B and equating the same,

$$350 \times 3 = 200\,(3 - x) + 400 \times 1.5$$

or $$1\,050 = 600 - 200\,x + 600 = 1200 - 200\,x$$

$$\therefore \quad 200\,x = 1\,200 - 1\,050 = 150$$

Or $$x = \frac{150}{200} = 0.75 \text{m} \quad \textbf{Ans.}$$

Example 4.3. *Two unlike parallel forces of magnitude 400 N and 100 N are acting in such away that their lines of action are 150 mm apart. Determine the magnitude of the resultant force and the point at which it acts.*

Solution. Given : The system of given force is shown in Fig. 4.4

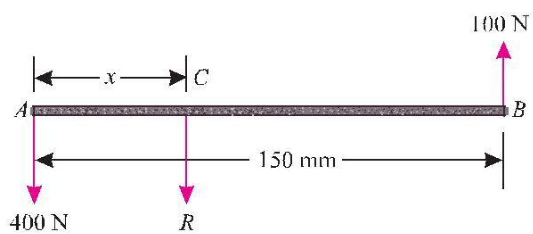


Fig. 4.4.

Magnitude of the resultant force

Since the given forces are unlike and parallel, therefore magnitude of the resultant force, $R = 400 - 100 = 300$ N**Ans.**

* The procedure for finding the reaction at either end will be discussed in the

Point where the resultant force acts

Let x = Distance between the lines of action of the resultant force and A in mm. Now taking clockwise and anticlockwise moments about A and equating the same,

$$300 \times x = 100 \times 150 = 15\ 000$$

$$\therefore \quad x = \frac{15\ 000}{300} = 50 \text{ mm} \quad \textbf{Ans.}$$

Example 4.4. *A uniform beam AB of weight 100 N and 6 m long had two bodies of weights60 N and 80 N suspended from its two ends as shown in Fig. 4.5.*

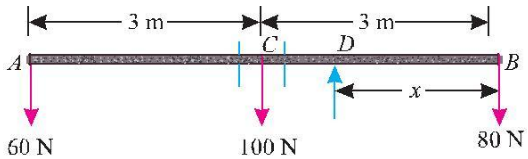


Fig. 4.5.

Find analytically at what point the beam should be supported, so that it may rest horizontally.

Solution. Given : Weight of rodAB= 100 N ; Length of rodAB= 6 mm and weight of the bodies supported at A and$B$ = 60 N and 80 N.

Let x = Distance between B and the point where the beam should be supported.

We know that for the beam to rest horizontally, the moments of the weights should be equal. Now taking moments of the weights about D and equating the same,

$$80x = 60\,(6 - x) + 100\,(3 - x)$$

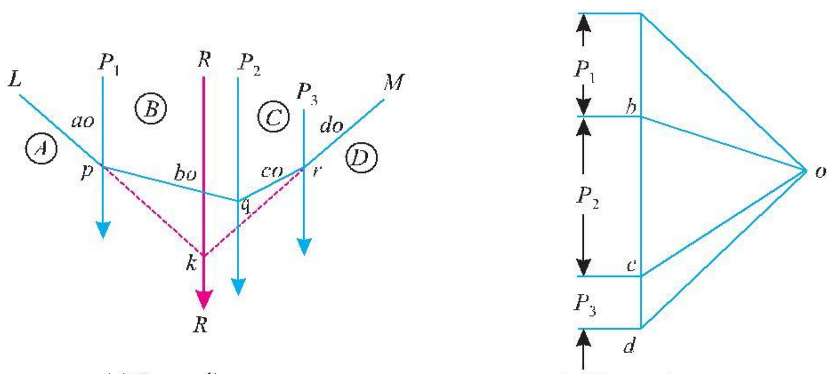
$$= 360 - 60x + 300 - 100x = 660 - 160x$$

$$240x = 660$$

Or $x = \dfrac{660}{240} = 2.75$ m **Ans.**

4.7. GRAPHICAL METHOD FOR THE RESULTANT OF PARALLEL FORCES

Consider a number of parallel forces (say three like parallel forces) P_1, P_2 and P_3 whose resultant is required to be found out as shown in Fig. 4.6 (*a*).



(*a*) Space diagram (*b*) Vactor diagram

Fig. 4.6. Resultant of parallel forces

First of all, draw the space diagram of the given system of forces and name them according to Bow's notations as shown in Fig. 4.6 (*a*). Now draw the vector diagram for the given forces as shown in Fig. 4.6 (*b*) and as discussed below :

1. Select some suitable point a, and draw ab equal to the force AB (P_1) and parallel to it to some suitable scale.

2. Similarly draw *bc* and *cd* equal to and parallel to the forces *BC* (P_2) and *CD* (P_3) respec-tively.
3. Now take some convenient point *o* and joint *oa*, *ob*, *oc* and *od*.
4. Select some point *p*, on the line of action of the force *AB* of the space diagram and through it draw a line *Lp* parallel to *ao*. Now through *p* draw *pq* parallel to *bo* meeting the line of action of the force *BC* at *q*.
5. Similarly draw *qr* and *rM* parallel to *co* and *do* respectively.
6. Now extend *Lp* and *Mr* to meet at *k*. Through *k*, draw a line parallel to *ad*, which gives the required position of the resultant force.
7. The magnitude of the resultant force is given by *ad* to the scale.

Note. This method for the position of the resultant force may also be used for any system offorces *i.e*. parallel, like, unlike or even inclined.

Example 4.5. *Find graphically the resultant of the forces shown in Fig. 4.7. The distancesbetween the forces are in mm.*

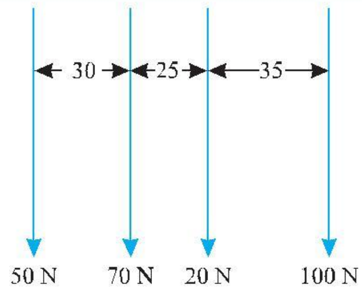


Fig. 4.7.

Also find the point, where the resultant acts.

Solution. Given : forces : 50 N, 70 N, 20 N and 100 N.

First of all, draw the space diagram for the given system of forces and name them according to Bow's notations as shown in Fig. 4.8 (*a*)

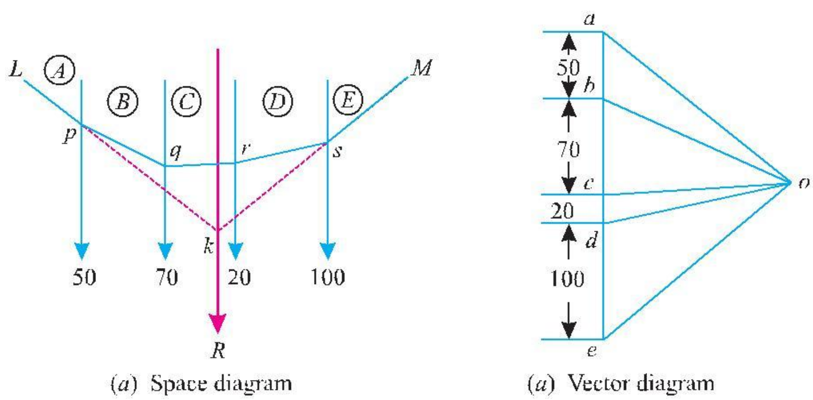


Fig. 4.8.

Now draw the vector diagram for the given forces as shown in Fig. 4.8 (*b*) and as discussed below :

1. Take some suitable point *a* and draw *ab* equal and parallel to force *AB* (*i.e*. 50 N) to some scale. Similarly draw *bc* equal to the force *BC* (*i.e*. 70 N), *cd* equal to the force *CD* (*i.e*. 20 N) and *de* equal to the force *DE* (*i.e*. 100 N) respectively.

2. Now select some suitable point *o*, and join *oa*, *ob*, *oc*, *od* and *oe*.
3. Now take some suitable point *p* on the line of action of the force *AB* of the space diagram. Through *p* draw a line *Lp*, parallel to *ao* of the vector diagram.
4. Now, through *p*, draw *pq* parallel to *bo*, meeting, the line of action of the force *BC* at *q*. Similarly, through *q* draw *qr* parallel to *co*, through *r* draw *rs* parallel to *do* and through *s* draw *sM* parallel to *eo*.
5. Now extend the lines *Lp* and *Ms* meeting each other at *k*. Through *k* draw a line parallel to *ae* which gives the required position of the resultant force.
6. By measurement, we find that resultant force,

$R = ae = 240$ N**Ans.**and line

of action of *k* from force *AB* = 51 mm **Ans.**

Example 4.6. *Find graphically the resultant of the forces shown in Fig. 4.9*

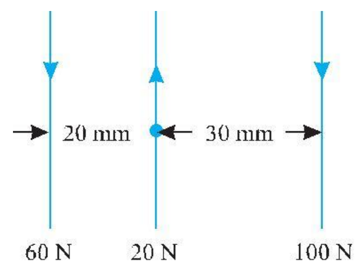


Fig. 4.9.

Also find the point where the resultant force acts.

Solution. Given forces : 60 N; 20 N; and 100 N.

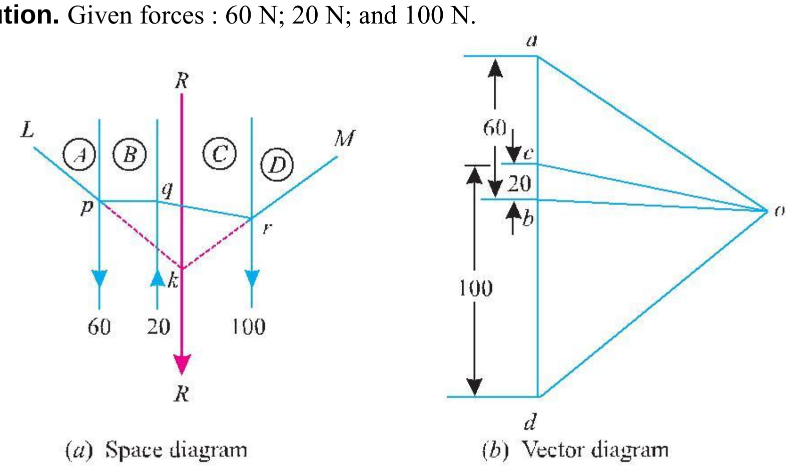


Fig. 4.10.

First of all, draw the space diagram for the given system of forces and name them according to Bow's notations as shown in Fig. 4.10 (*a*).

It may be noted that the force *AB* (equal to 60 N) is acting downwards, force *BC* (equal to 20 N) is acting upwards and the force *CD* (equal to 100 N) is acting downwards as shown in the figure. Now draw the vector diagram for the given forces as shown in Fig. 4.10 (*b*) and as discussed below :

1. Take some suitable point *a* and draw *ab* equal and parallel to force *AB* (*i.e.*, 60 N) to some scale. Similarly, draw *bc* (upwards) equal to force *BC* (*i.e*. 20 N) and *cd* equal to the force *CD* (*i.e*, 100 N) respectively.

2. Now select some suitable point o and join oa, ob, oc and od.
3. Now take some suitable point p on the line of action of the force AB of the space diagram. Through p draw a line Lp parallel to ao of the vector diagram.
4. Now through p, draw pq parallel to bo meeting the line of action of the force BC at q. Similarly through q draw qr parallel to co. Through r draw rM Parallel to do.
5. Now extend the lines Lp and Mr meeting each other at k. Through k draw a line parallel to ad, which gives the required resultant force.
6. By measurement, we find that resultant force,

$$R = ad = 140 \text{ N}$$ **Ans.**

and line of action of k from force AB = 33 mm **Ans.**

Note. In some cases, the lines Lp and rM are parallel and do not meet each other. This happens, when magnitude of the sum of upward forces is equal to sum of the downward forces.

EXERCISE 4.1

1. Two like parallel forces of 10 N and 30 N act at the ends of a rod 200 mm long. Find magnitude of the resultant force and the point where it acts. [**Ans.** 40 N ; 150 mm]

2. Find the magnitude of two like parallel forces acting at a distance of 240 mm, whose resultant is 200 N and its line of action is at a distance of 60 mm from one of the forces. [**Ans.** 50 N ; 150 N]

Hint.
$$P + Q = 200$$
$$Q \times 240 = 200 \times 60 = 12\,000$$
$$\therefore \quad Q = 50 \text{ N and } P = 200 - 50 = 150 \text{ N}$$

3. Two unlike parallel forces are acting at a distance of 450 mm from each other. The forces are equivalent to a single force of 90 N, which acts at a distance of 200 mm from the greater of the two forces. Find the magnitude of the forces. [**Ans.** 40 N ; 130 N]

4. Find graphically the resultant force of the following like parallel forces :

P_1 = 20 N ; P_2 = 50 N ; P_3 = 60 N and P_4 = 70 N

Take distances between P_1 and P_2 as 40 mm, between P_2 and P_3 as 30 mm and between P_3 and P_4 as 20 mm. [**Ans.** 200 N ; 62.5 mm]

4.8. COUPLE

A pair of two equal and unlike parallel forces (*i.e.* forces equal in magnitude, with lines of action parallel to each other and acting in opposite directions) is known as a couple.

As a matter of fact, a couple is unable to produce any translatory motion (*i.e.*, motion in a straight line). But it produces a motion of rotation in the body, on which it acts. The simplest example of a couple is the forces applied to the key of a lock, while locking or unlocking it.

A couple is a pair of forces applied to the key of a lock.

4.9. ARM OF A COUPLE

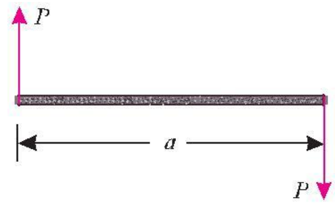


Fig. 4.11.

The perpendicular distance (a), between the lines of action of the two equal and opposite parallel forces, is known *as arm of the couple* as shown in Fig. 4.11.

4.10. MOMENT OF A COUPLE

The moment of a couple is the product of the force (*i.e.*, one of the forces of the two equal and opposite parallel forces) and the arm of the couple. Mathematically:

Moment of a couple = $P \times a$

where P = Magnitude of the force, and

a = Arm of the couple.

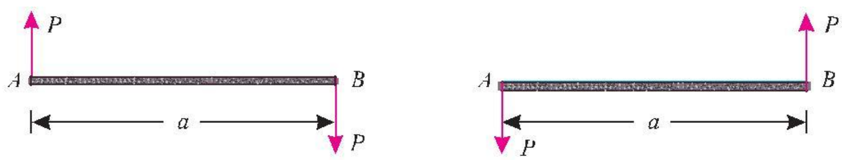
4.11. CLASSIFICATION OF COUPLES

The couples may be, broadly, classified into the following two categories, depending upon their direction, in which the couple tends to rotate the body, on which it acts :

1. Clockwise couple, and 2. Anticlockwise couple.

4.12. CLOCKWISE COUPLE

A couple, whose tendency is to rotate the body, on which it acts, in a clockwise direction, is known as a clockwise couple as shown in Fig. 4.12 (a). Such a couple is also called positive couple.



(a) Clockwise couple (b) Anticlockwise couple

Fig. 4.12.

4.13. ANTICLOCKWISE COUPLE

A couple, whose tendency is to rotate the body, on which it acts, in an anticlockwise direction, is known as an anticlockwise couple as shown in Fig. 4.12 (b). Such a couple is also called a negative couple.

4.14. CHARACTERISTICS OF A COUPLE

A couple (whether clockwise or anticlockwise) has the following characteristics :

1. The algebraic sum of the forces, constituting the couple, is zero.
2. The algebraic sum of the moments of the forces, constituting the couple, about any point is the same, and equal to the moment of the couple itself.
3. A couple cannot be balanced by a single force. But it can be balanced only by a couple of opposite sense.
4. Any no. of coplaner couples can be reduced to a single couple, whose magnitude will be equal to the algebraic sum of the moments of all the couples.

Example 4.7. *A square ABCD has forces acting along its sides as shown in Fig. 4.13. Findthe values of P and Q, if the system reduces to a couple. Also find magnitude of the couple, if the side of the square is 1 m.*

Solution. Given : Length of square = 1 m

Values of P and Q

We know that if the system reduces to a couple, the resultant force in horizontal and vertical directions must be zero. Resolving the forces horizontally,

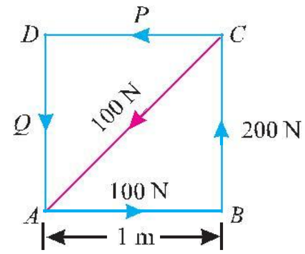


Fig. 4.13.

$$100 - 100\cos 45° - P = 0$$

$$\therefore \quad P = 100 - 100\cos 45° \text{ N}$$

$$= 100 - (100 \times 0.707) = 29.3 \text{ N} \quad \textbf{Ans.}$$

Now resolving the forces vertically,

$$200 - 100\sin 45° - Q = 0$$

$$\therefore \quad Q = 200 - (100 \times 0.707) = 129.3 \text{ N} \quad \textbf{Ans.}$$

Magnitude of the couple

We know that moment of the couple is equal to the algebraic sum of the moments about any point. Therefore moment of the couple (taking moments about A)

$$= (-200 \times 1) + (-P \times 1) = -200 - (29.3 \times 1) \text{ N-m}$$

$$= -229.3 \text{ N-m} \quad \textbf{Ans.}$$

Since the value of moment is negative, therefore the couple is anticlockwise.

Example 4.8. *ABCD is a rectangle,such that AB=CD=a and BC=DA=b. Forces equalto P act along AD and CB and forces equal to Q act along AB and CD respectively. Prove that the perpendicular distance between the resultants of P and Q at A and that of P and Q at C*

$$= \frac{(P\cdot a) - (Q\cdot b)}{\sqrt{(P^2+Q^2)}}$$

Solution. Given : The system of forces is shown in Fig. 4.14.

Let $\quad x$ = Perpendicular distance between the two resultants.

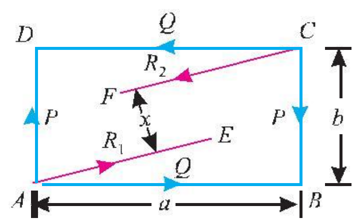


Fig. 4.14.

We know that the resultant of the forces P and Q at A,

$$R_1 = \sqrt{P^2 + Q^2} \quad \text{...}(i)$$

and resultant of the forces P and Q at C,

$$R_2 = \sqrt{P^2 + Q^2} \quad \text{...}(ii)$$

$\therefore$ Resultant $R = R_1 = R_2$ ...[from equations (i) and (ii)]

We know that moment of the force (P) about A,

$$M_1 = P \times a \quad \text{...(+ Due to clockwise)}$$

and moment of the force (Q) about A,

$$M_2 = -Q \times b \quad \text{...(– Due to anticlockwise)}$$

$\therefore$ Net moment of the two couples

$$= (P \times a) - (Q \times b) \quad \text{...}(iii)$$

and moment of the couple formed by the resultants

$$= R \times x = \sqrt{P^2 + Q^2} \cdot x \quad \text{...}(iv)$$

Equating the moments (iii) and (iv),

$$\sqrt{P^2 + Q^2} \cdot x = (P \cdot a) - (Q \cdot b)$$

$$\therefore \quad x = \underline{(P\cdot a) - (Q\cdot b)} \quad \textbf{Ans.}$$

$\sqrt{P^2 + Q^2}$

Example 4.9. *Three forces,acting on a rigid body,are represented in magnitude,directionand line of action by the three sides of a triangle taken in order as shown in Fig.4.15*

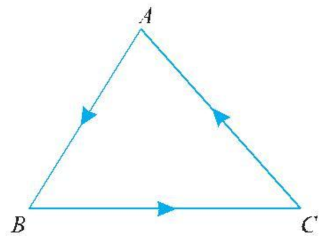
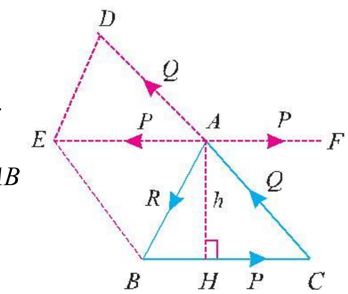


Fig. 4.15.

Prove that the forces are equivalent to a couple whose moment is equal to twice the area of the triangle.

Solution. The system of forces on the triangle*ABC*is shown in Fig. 4.16. Now complete the figure as discussed below :

1. Through *A* draw a line *EF* parallel to *BC*.
2. Extend *CA* to *D*, such that *AD* is equal to *Q* (*i.e. CA*).
3. Now apply two equal and opposite forces (*P*) at *A* represented by *AE* and *AF*.
4. Complete the parallelogram *ABED* with adjacent sides *AB* and *AD*.



We know that the diagonal *AE* represents in magnitude and direction of the resultant of the two forces *R* and *Q*.

Thus the force *AF* (equal to *P*) will set the forces *Q* and *R* in equilibrium. Thus we are left with only two forces *BC* (equal to *P*) and *AE* (equal to *P*) which will constitute a couple. Now from *A*, draw *AH* perpendicular to *BC*. Let this perpendicular be equal to *h*.

We know that moment of the couple,

$$M = P \times a = P \times h \qquad ...(i)$$

and area of triangle $= \frac{1}{2} \cdot \text{Base} \cdot \text{Height} = \frac{1}{2} \cdot P \cdot h \qquad ...(ii)$

From equations (*i*) and (*ii*), we find that moment of the couple =

Twice the area of triangle. **Ans.**

Example 4.10. *A machine component of length 2.5 metres and height 1 metre is carriedupstairs by two men, who hold it by the front and back edges of its lower face.*

If the machine component is inclined at 30° to the horizontal and weighs 100 N, find how much of the weight each man supports ?

Solution. Given : Length of machine component = 2.5 m; Height of the component = 1 m ;Inclination = 30° and weight of component = 100 N

Let P = Weight supported by the man at A.

Q = Weight supported by the man at B.

C = Point where the vertical line throughthe centre of gravity cuts the lower face.

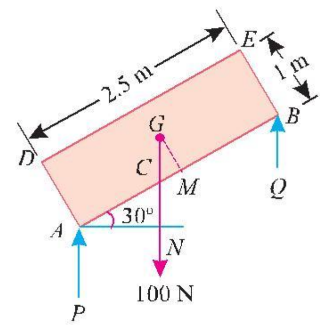


Fig. 4.17.

Now join G (*i.e.*, centre of gravity) with M (*i.e.*, mid-point of AB) as shown in Fig. 4.17.

From the geometry of the figure, we find that

$$GM = \frac{1}{2} = 0.5\text{m}$$

And $$AM = \frac{2.5}{2} = 1.25\text{m}$$

∴ $$AC = AM - CM = 1.25 - GM \tan 30° \qquad \text{... Q } \frac{CM}{GM} = \tan CGM = \tan 30°$$

$$= 1.25 - (0.5 \times 0.577) = 0.96 \text{ m}$$

And $$CB = AB - AC = 2.5 - 0.96 = 1.54 \text{ m}$$

We know that $P \times CA = Q \times CB$

$$P \times 0.96 = (100 - P)\ 1.54 = 154 - 1.54\ P \qquad \text{...(Q}P + Q = 100)$$

∴ $2.5\ P = 154$ or $P = 61.6$ N **Ans.**

and $Q = 100 - 61.6 = 38.4$ N**Ans.**

EXERCISE 4.2.

1. *ABCD* is rectangle, in which $AB = CD = 100$ mm and $BC = DA = 80$ mm. Forces of 100 Neach act along *AB* and *CD* and forces of 50 N each at along *BC* and *DA*. Find the resultant moment of the two couples. [**Ans.** – 13 000 N-mm]
2. A square *ABCD* has sides equal to 200 mm. Forces of 150 N each act along *AB* and *CD* and 250 N act along *CB* and *AD*. Find the moment of the couple, which will keep the system in equilibrium. [**Ans.** – 20 000 N-mm]

QUESTIONS

1. What do you understand by the term 'parallel forces' ? Discuss their classifications.
2. Distinguish clearly between like forces and unlike forces.
3. What is a couple ? What is the arm of a couple and its moment ?
4. Discuss the classification of couples and explain clearly the difference between a positive couple and a negative couple.
5. State the characteristics of a couple.

OBJECTIVE TYPE QUESTIONS

1. The like parallel forces are those parallel forces, which are liked by the scientist and engineers.

   (*a*) Yes (*b*) No

2. A couple consists of

(*a*) two like parallel forces of same magnitude.

(*b*) two like parallel forces of different magnitudes.

(*c*) two unlike parallel forces of same magnitude.

(*d*) two unlike parallel forces of different magnitudes.

3. If the arm of a couple is doubled, its moment will

(*a*) be halved (*b*) remain the same (*c*) be doubled **4.** A couple can be balanced by a force equal to its magnitude.

(*a*) Agree (*b*) Disagree

5. One of the characteristics of a couple is that it can cause a body to move in the direction of the greater force.

(*a*) True (*b*) False

6. In a couple, the lines of action of the forces are

(*a*) parallel (*b*) inclined (*c*) none of the two

ANSWERS

1. (*b*) **2.** (*c*) **3.** (*c*) **4.** (*b*) **5.** (*b*) **6.** (*a*)

Top

CHAPTER

1

Equilibrium of Forces

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5.1. INTRODUCTION

In the previous chapter, we have discussed the various methods of finding out resultant force, when a particle is acted upon by a number of forces. This resultant force will produce the same effect as produced by all the given forces.

A little consideration will show, that if the resultant of a number of forces, acting on a particle is zero, the particle will be in equilibrium. Such a set of forces, whose resultant is zero, are called equilibrium forces.

The force, which brings the set of forces in equilibrium is called an equilibrant.

As a matter of fact, the equilibrant is equal to the resultant force in magnitude, but opposite in direction.

5.2. PRINCIPLES OF EQUILIBRIUM

Though there are many principles of equilibrium, yet the following three are important from the subject point of view :

1. *Two force principle.* As per this principle, if a body in equilibrium is acted upon by twoforces, then they must be equal, opposite and collinear.
2. *Three force principle.* As per this principle, if a body in equilibrium is acted upon by threeforces, then the resultant of any two forces must be equal, opposite and collinear with the third force.
3. *Four force principle.* As per this principle, if a body in equilibrium is acted upon by fourforces, then the resultant of any two forces must be equal, opposite and collinear with the resultant of the other two forces.

5.3. METHODS FOR THE EQUILIBRIUM OF COPLANAR FORCES

Though there are many methods of studying the equilibrium of forces, yet the following are important from the subject point of view :

1. Analytical method. 2. Graphical method.

5.4. ANALYTICAL METHOD FOR THE EQUILIBRIUM OF COPLANAR FORCES

The equilibrium of coplanar forces may be studied, analytically, by Lami's theorem as discussed below :

5.5. LAMI'S THEOREM

It states, "*If three coplanar forces acting at a point be inequilibrium, then each force is proportional to the sine of the angle between the other two.*" Mathematically,

$$\frac{P}{\sin \alpha} = \frac{Q}{\sin \beta} = \frac{R}{\sin \gamma}$$

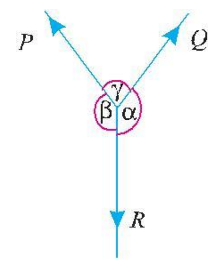


Fig. 5.1. Lami's theorem

where, P, Q, and R are three forces and α, β, γ are the angles as shown in Fig. 5.1.

Proof

Consider three coplanar forces P, Q, and R acting at a point O. Let the opposite angles to three forces be α , β and γ as shown in Fig. 5.2.

Now let us complete the parallelogram $OACB$ with OA and OB as adjacent sides as shown in the figure. We know that the resultant of two forces P and Q will be given by the diagonal OC both in magnitude and direction of the parallelogram $OACB$.

Since these forces are in equilibrium, therefore the resultant of the forces P and Q must be in line with OD and equal to R, but in opposite direction.

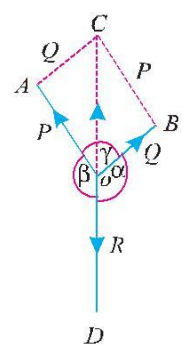


Fig. 5.2. Proof of Lami's theorem

From the geometry of the figure, we find

$$BC = P \text{ and } AC = Q$$

$$\angle AOC = (180° - \beta)$$

and $$\angle ACO = \angle BOC = (180° - \alpha)$$

$$\angle CAO = 180° - (\angle AOC + \angle ACO)$$

$$= 180° - [(180° - \beta) + (180° - \alpha)]$$

$$= 180° - 180° + \beta - 180° + \alpha$$

$$= \alpha + \beta - 180°$$

$$\alpha + \beta + \gamma = 360°$$

But Subtracting 180° from both sides of the above equation, $(\alpha + \beta - 180°) + \gamma = 360° - 180° = 180°$

$$\angle CAO = 180° - \gamma$$

or We know that in triangle AOC,

$$\frac{OA}{\sin \angle ACO} = \frac{AC}{\sin \angle AOC} = \frac{OC}{\sin \angle CAO}$$

$$\frac{OA}{\sin (180° - \alpha)} = \frac{AC}{\sin (180° - \beta)} = \frac{OC}{\sin (180° - \gamma)}$$

or

$$\frac{P}{\sin \alpha} = \frac{Q}{\sin \beta} = \frac{R}{\sin \gamma} \qquad \ldots[\because \sin (180° - \theta) = \sin \theta]$$

Example 5.1. *An electric light fixture weighting 15 N hangs from a point C, by two strings AC and BC. The string AC is inclined at 60° to the horizontal and BC at 45° to the horizontal as shown in Fig. 5.3*

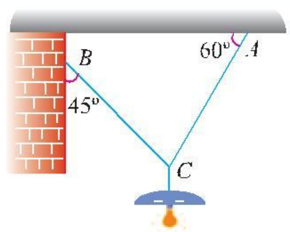


Fig. 5.3.

Using Lami's theorem, or otherwise, determine the forces in the strings AC and BC.

Solution. Given : Weight at C = 15 N

Let T_{AC} = Force in the string AC, and T_{BC} = Force in the string BC.

The system of forces is shown in Fig. 5.4. From the geometry of the figure, we find that angle between T_{AC} and 15 N is 150° and angle between T_{BC} and 15 N is 135°.

$\therefore \angle ACB = 180° - (45° + 60°) = 75°$

Applying Lami's equation at C,

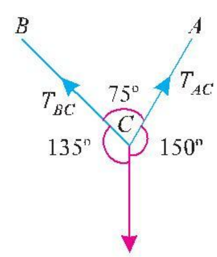


Fig. 5.4.

$$\frac{15}{\sin 75°} = \frac{T_{AC}}{\sin 135°} = \frac{T_{BC}}{\sin 150°}$$

or

$$\frac{15}{\sin 75°} = \frac{T_{AC}}{\sin 45°} = \frac{T_{BC}}{\sin 30°}$$

$$\therefore \quad T_{AC} = \frac{15 \sin 45°}{\sin 75°} = \frac{15 \cdot 0.707}{0.9659} = 10.98 \text{ N} \quad \textbf{Ans.}$$

and $$T_{BC} = \frac{15 \sin 30°}{\sin 75°} = \frac{15 \cdot 0.5}{0.9659} = 7.76 \text{ N} \quad \textbf{Ans.}$$

Example 5.2. *A string ABCD,attached to fixed points A and D has two equal weihts of1000 N attached to it at B and C. The weights rest with the portions AB and CD inclined at angles as shown in Fig. 5.5.*

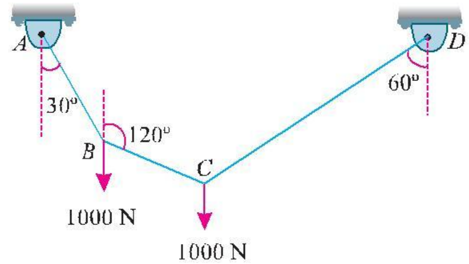


Fig. 5.5.

Find the tensions in the portions AB, BC and CD of the string, if the inclination of the portion BC with the vertical is 120°.

Solution. Given : Load at B = Load at C = 1000 N

For the sake of convenience, let us split up the string *ABCD* into two parts. The system of forces at joints *B* and is shown in Fig. 5.6 (*a*) and (*b*).

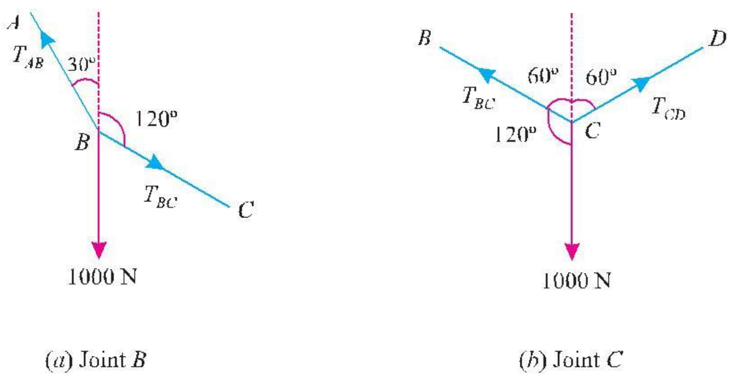


Fig. 5.6.

Let T_{AB} = Tension in the portion *AB* of the string,

T_{BC} = Tension in the portion *BC* of the string, and

T_{CD} = Tension in the portion *CD* of the string.

Applying Lami's equation at joint *B*,

$$\frac{T_{AB}}{\sin 60°} = \frac{T_{BC}}{\sin 150°} = \frac{1000}{\sin 150°}$$

$$\frac{T_{AB}}{\sin 60°} = \frac{T_{BC}}{\sin 30°} = \frac{1000}{\sin 30°} \qquad \text{...[Q } \sin (180° - \theta) = \sin \theta]$$

$$\therefore \quad T_{AB} = \frac{1000 \sin 60°}{\sin 30°} = \frac{1000 \cdot 0.866}{0.5} = 1732 \text{ N} \quad \textbf{Ans.}$$

and $$T_{BC} = \frac{1000 \sin 30°}{\sin 30°} = 1000 \text{ N} \quad \textbf{Ans.}$$

Again applying Lami's equation at joint *C*,

$$\frac{T_{BC}}{\sin 120°} = \frac{T_{CD}}{\sin 120°} = \frac{1000}{\sin 120°}$$

$$\therefore \quad T_{CD} = \frac{1000 \sin 120°}{\sin 120°} = 1000 \text{ N} \quad \textbf{Ans.}$$

Example 5.3. *A light string ABCDE whose extremity A is fixed,has weights W_1 and W_2 attached to it at B and C. It passes round a small smooth peg at D carrying a weight of 300 N at the free end E as shown in Fig. 5.7.*

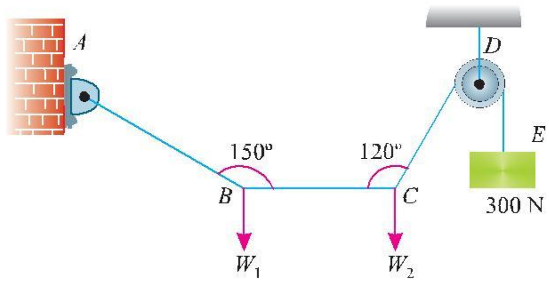
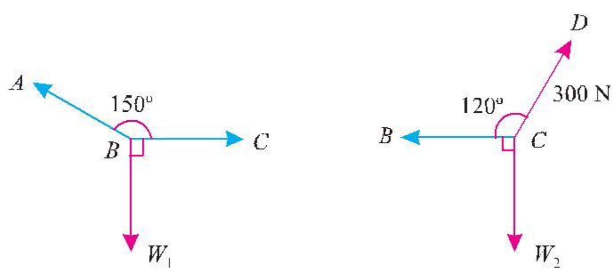


Fig. 5.7.

If in the equilibrium position, BC is horizontal and AB and CD make 150° and 120° with BC, find (i) Tensions in the portion AB, BC and CD of the string and (ii) Magnitudes of W_1 and W_2.

Solution. Given : Weight at *E* = 300 N

For the sake of convenience, let us split up the string *ABCD* into two parts. The system of forces at joints *B* and *C* is shown in Fig. 5.8. (*a*) and (*b*).



(*a*) Joint *B* (*b*) Joint *C*

Fig. 5.8.

(*i*) *Tensions is the portion AB, BC and CD of the string*

Let T_{AB} = Tension in the portion *AB*, and

T_{BC} = Tension in the portion *BC*,

We know that tension in the portion *CD* of the string. $T_{CD} = T_{DE}$ = 300 N **Ans.**

Applying Lami's equation at C,

$$\frac{T_{BC}}{\sin 150°} = \frac{W_2}{\sin 120°} = \frac{300}{\sin 90°}$$

$$\frac{T_{BC}}{\sin 30°} = \frac{W_2}{\sin 60°} = \frac{300}{1}$$ ...[∵ sin (180° – θ) = sin θ]

∴ $T_{BC} = 300 \sin 30° = 300 \times 0.5 = 150$ N **Ans.**

and $W_2 = 300 \sin 60° = 300 \times 0.866 = 259.8$ N

Again applying Lami's equation at B,

$$\frac{T_{AB}}{\sin 90°} = \frac{W_1}{\sin 150°} = \frac{T_{BC}}{\sin 120°}$$

$$\frac{T_{AB}}{1} = \frac{W_1}{\sin 30°} = \frac{150}{\sin 60°}$$ ...[∵ sin (180° – θ) = sin θ]

∴ $T_{AB} = \frac{150}{\sin 60°} = \frac{150}{0.866} = 173.2$ N **Ans.**

and $W_1 = \frac{150 \sin 30°}{\sin 60°} = \frac{150 \cdot 0.5}{0.866} = 86.6$ N

(*ii*) *Magnitudes of W_1 and W_2*

From the above calculations, we find that the magnitudes of W_1 and W_2 are 86.6 N and 259.8 N respectively. **Ans.**

EXERCISE 5.1

1. Two men carry a weight of 2 kN by means of two ropes fixed to the weight. One rope is inclined at 45° and the other at 30° with their vertices. Find the tension in each rope.
   [**Ans.** 1.04 kN ; 1.46 kN]
2. Three forces acting on a particle are in equilibrium. The angles between the first and second is 90° and that between the second and third is 120°. Find the ratio of the forces.
   [**Ans.** 1.73 : 1 : 2]
3. A smooth sphere of weight W is supported by a string fastened to a point A on the smooth vertical wall, the other end is in contact with point B on the wall as shown in Fig. 5.9

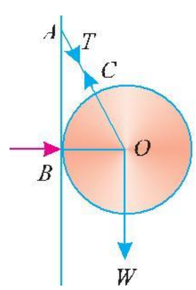


Fig. 5.9.

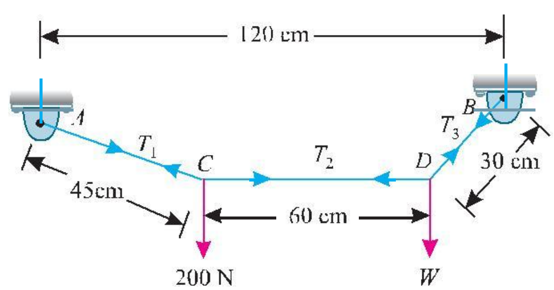


Fig. 5.10.

If length of the string AC is equal to radius of the sphere, find tension (T) in the string and reaction of the wall. [**Ans.** 1.155 W ; 0.577 W]

Hint. Since $AO = 2OB$, therefore $\angle AOB = 60°$

4. A rope is connected between two points A and B 120 cm apart at the same level. A load of 200 N is suspended from a point C on the rope 45 cm from A as shown in Fig. 5.10. Find the load, that should be suspended from the rope D 30 cm from B, which will keep the rope CD horizontal. [**Ans.** 400 N]

5. A uniform sphere of weight W rests between a smooth vertical plane and a smooth plane inclined at an angle θ with the vertical plane. Find the reaction at the contact surfaces. [**Ans.** W cot θ ; W cosec θ]

Example 5.4. *Two equal heavy spheres of 50 mm radius are in equilibrium within a smoothcup of 150 mm radius. Show that the reaction between the cup of one sphere is double than that between the two spheres.*

Solution. Given : Radius of spheres = 50 mm and radius of the cup = 150 mm.

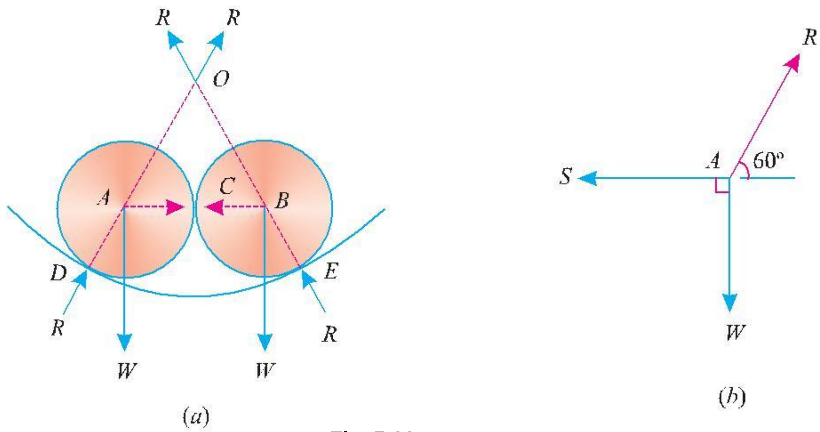


Fig. 5.11.

The two spheres with centres A and B, lying in equilibrium, in the cup with O as centre are shown in Fig. 5.11 (a). Let the two spheres touch each other at C and touch the cup at D and E respectively.

Let R = Reactions between the spheres and cup, and S = Reaction between the two spheres at C.

From the geometry of the figure, we find that OD = 150 mm and AD = 50 mm. Therefore OA = 100 mm. Similarly OB = 100 mm. We also find that AB = 100 mm. Therefore OAB is an equilateral triangle. The system of forces at A is shown in Fig. 5.11 (b).

Applying Lami's equation at A,

$$\frac{R}{\sin 90°} = \frac{W}{\sin 120°} = \frac{S}{\sin 150°}$$

$$\frac{R}{1} = \frac{W}{\sin 60°} = \frac{S}{\sin 30°}$$

$$\therefore \quad R = \frac{S}{\sin 30°} = \frac{S}{0.5} = 2S$$

Hence the reaction between the cup and the sphere is double than that between the two spheres. **Ans.**

Example 5.5. *A smooth circular cylinder of radius 1.5 meter is lying in a triangular groove,one side of which makes 15° angle and the other 40° angle with the horizontal. Find the reactions at the surfaces of contact, if there is no friction and the cylinder weights 100 N.*

Solution. Given : Weight of cylinder = 100 N

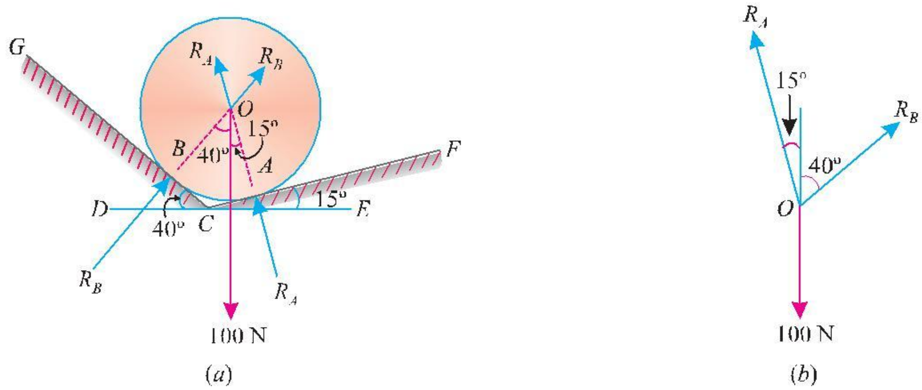


Fig. 5.12.

Let R_A = Reaction at A, and

R_B = Reaction at B.

The smooth cylinder lying in the groove is shown in Fig. 5.12 (*a*). In order to keep the system in equilibrium, three forces *i.e.* R_A, R_B and weight of cylinder (100 N) must pass through the centre of the cylinder. Moreover, as there is no *friction, the reactions R_A and R_B must be normal to the surfaces as shown in Fig. 5.12 (*a*). The system of forces is shown in Fig. 5.12 (*b*).

Applying Lami's equation, at O,

$$\frac{R_A}{\sin(180° - 40°)} = \frac{R_B}{\sin(180° - 15°)} = \frac{100}{\sin(15° + 40°)}$$

or
$$\frac{R_A}{\sin 40°} = \frac{R_B}{\sin 15°} = \frac{100}{\sin 55°}$$

$\therefore$
$$R_A = \frac{100 \cdot \sin 40°}{\sin 55°} = \frac{100 \cdot 0.6428}{0.8192} = 78.5 \text{ N} \quad \textbf{Ans.}$$

and
$$R_B = \frac{100 \cdot \sin 15°}{\sin 55°} = \frac{100 \cdot 0.2588}{0.8192} = 31.6 \text{ N} \quad \textbf{Ans.}$$

Example 5.6. *Two cylinders P and Q rest in a channel as shown in Fig. 5.13.*

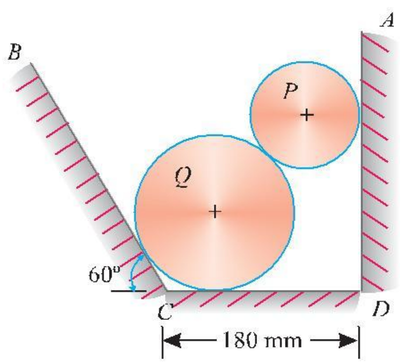


Fig. 5.13. *The cylinder P has diameter of 100 mm and weighs 200 N, whereas the cylinder Q has diameter of 180 mm and weighs 500 N.*

* This point will be discussed in more details in the chapter of Principles of Friction.

If the bottom width of the box is 180 mm, with one side vertical and the other inclined at 60°, determine the pressures at all the four points of contact.

Solution. Given : Diameter of cylinder P = 100 mm ; Weight of cylinder P = 200 N ; Diameter of cylinder Q = 180 mm ; Weight of cylinder Q = 500 N and width of channel = 180 mm.

First of all, consider the equilibrium of the cylinder P. It is in equilibrium under the action of the following three forces which must pass through A *i.e.*, the centre of the cylinder P as shown in Fig. 5.14 (*a*).

1. Weight of the cylinder (200 N) acting downwards.
2. Reaction (R_1) of the cylinder P at the vertical side.
3. Reaction (R_2) of the cylinder P at the point of contact with the cylinder Q.

From the geometry of the figure, we find that

$$ED = \text{Radius of cylinder } P = \frac{100}{2} = 50 \text{ mm}$$

Similarly $$BF = \text{Radius of cylinder } Q = \frac{180}{2} = 90 \text{ mm}$$

and $$\angle BCF = 60°$$

$$\therefore CF = BF \cot 60° = 90 \cdot 0.577 = 52 \text{ mm}$$

$$\therefore FE = BG = 180 - (52 + 50) = 78 \text{ mm}$$

and $$AB = 50 + 90 = 140 \text{ mm}$$

$$\therefore \cos \angle ABG = \frac{BG}{AB} = \frac{78}{140} = 0.5571$$

or $$\angle ABG = 56.1°$$

The system of forces at A is shown in Fig. 5.14 (*b*).

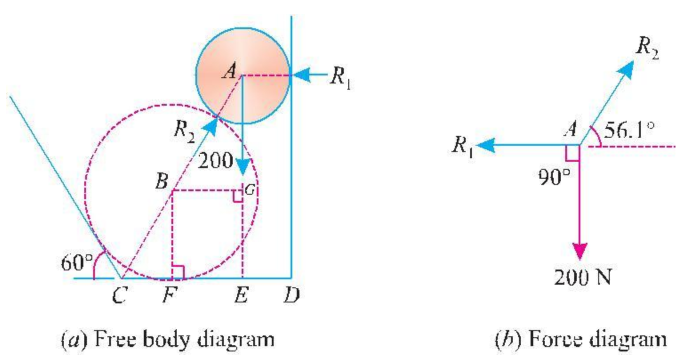


Fig. 5.14.

Applying Lami's equation at A,

$$\frac{R_1}{\sin(90° + 56.1°)} = \frac{R_2}{\sin 90°} = \frac{200}{\sin(180° - 56.1°)}$$

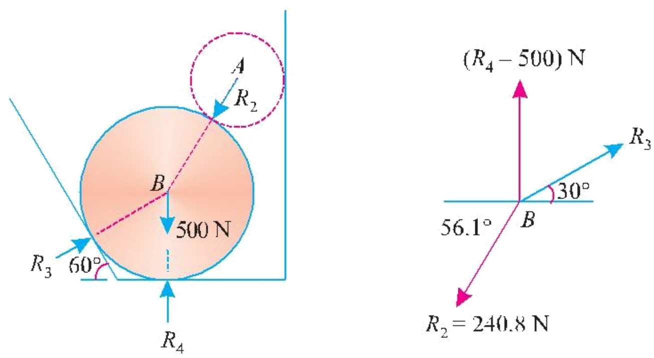
$$\frac{R_1}{\cos 56.1°} = \frac{R_2}{1} = \frac{200}{\sin 56.1°}$$

$\therefore$ $$R_1 = \frac{200 \cos 56.1°}{\sin 56.1°} = \frac{200 \cdot 0.5571}{0.830} = 134.2 \text{ N} \quad \textbf{Ans.}$$

and $$R_2 = \frac{200}{\sin 56.1°} = \frac{200}{0.8300} = 240.8 \text{ N} \quad \textbf{Ans.}$$

Now consider the equilibriXum of the cylinder Q. It is in equilibrium under the action of the following four forces, which must pass through the centre of the cylinder as shown in Fig. 5.15 (*a*).

1. Weight of the cylinder Q (500 N) acting downwards.
2. Reaction R_2 equal to 240.8 N of the cylinder P on cylinder Q.
3. Reaction R_3 of the cylinder Q on the inclined surface.
4. Reaction R_4 of the cylinder Q on the base of the channel.



(*a*) Free body diagram (*b*) Force diagram

Fig. 5.15.

A little consideration will show, that the weight of the cylinder Q is acting downwards and the reaction R_4 is acting upwards. Moreover, their lines of action also coincide with each other.

$\therefore$ Net downward force = $(R_4 - 500)$ N

The system of forces is shown in Fig. 5.15

(*b*). Applying Lami's equation at B,

$$\frac{R_3}{\sin (90° + 56.1°)} = \frac{240.8}{\sin 60°} = \frac{R_4 - 500}{\sin (180° + 30° - 56.1°)}$$

$$\frac{R_3}{\cos 56.1°} = \frac{240.8}{\sin 60°} = \frac{R_4 - 500}{\sin 26.1°}$$

$\therefore$ $$R_3 = \frac{240.8 \cdot \cos 56.1°}{\sin 60°} = \frac{240.8 \cdot 0.5577}{0.866} = 155 \text{ N} \quad \textbf{Ans.}$$

and $$R_4 - 500 = \frac{240.8 \cdot \sin 26.1°}{\sin 60°} = \frac{240.8 \cdot 399}{0.866} = 122.3 \text{ N}$$

$\therefore$ $$R_4 = 122.3 + 500 = 622.3 \text{ N} \quad \textbf{Ans.}$$