

9-4-16

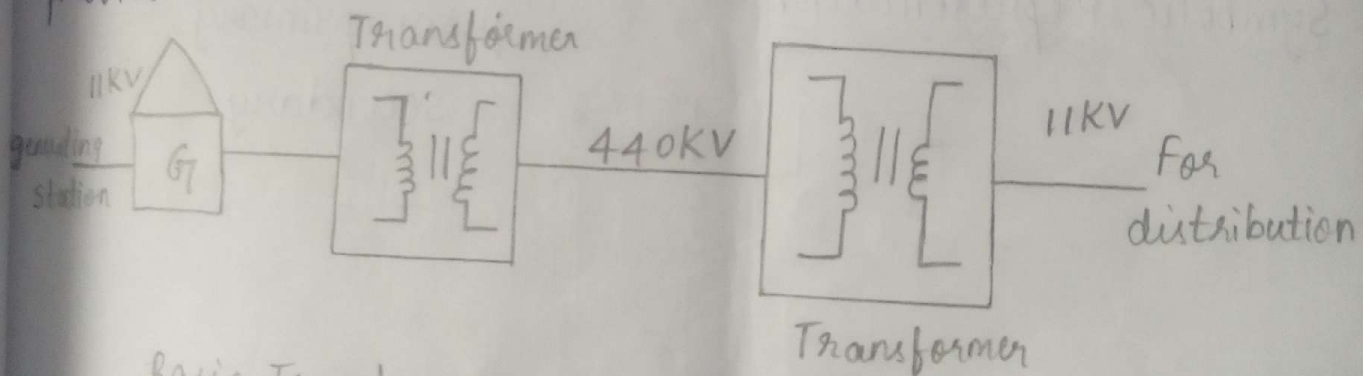
## Unit:5 Transformers

→ A transformer is a static device which transfers electrical energy from one ckt to the other with the desired change in voltage or current & without change in frequency.

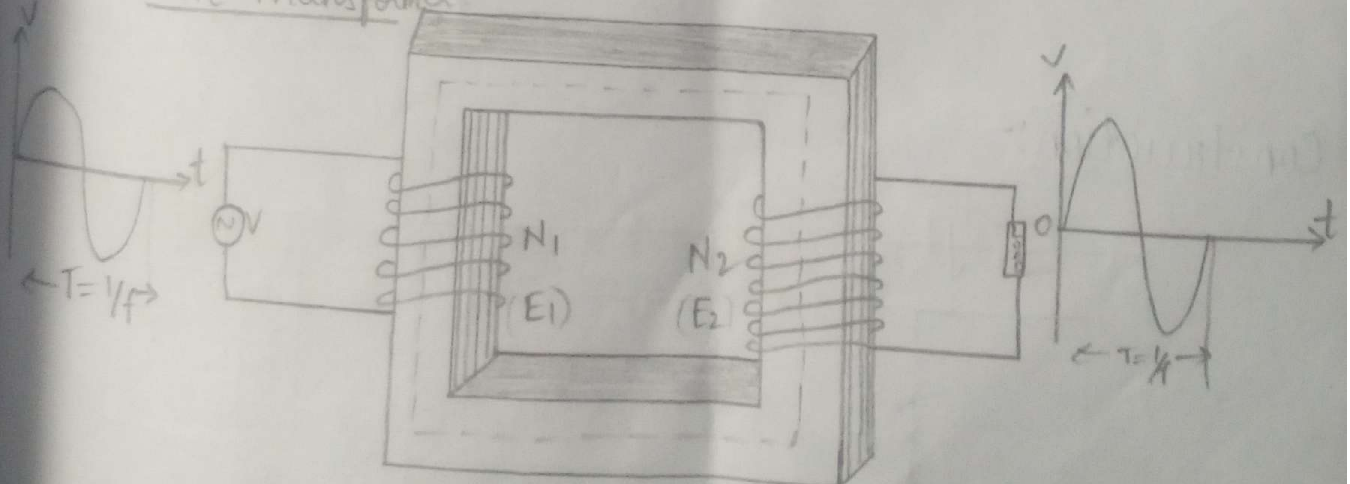
→ Transformer works on the principle of mutual induction.

### Mutual induction:-

When 2 coils are inductively coupled & if current in one coil is changed uniformly, then an emf gets induced in the other coil. This emf can drive a current when a closed path is provided.



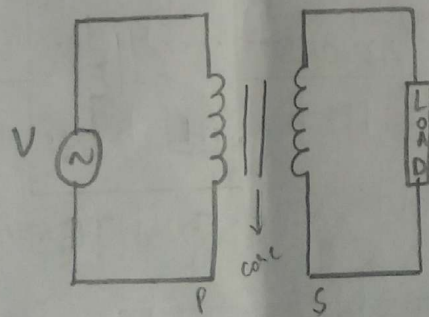
### Basic Transformer:-



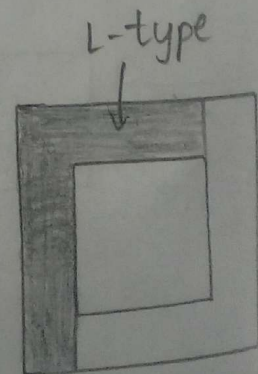
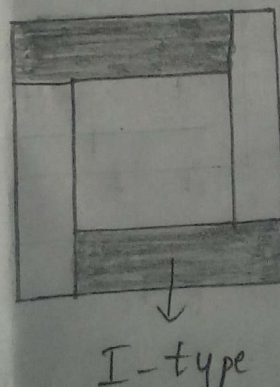
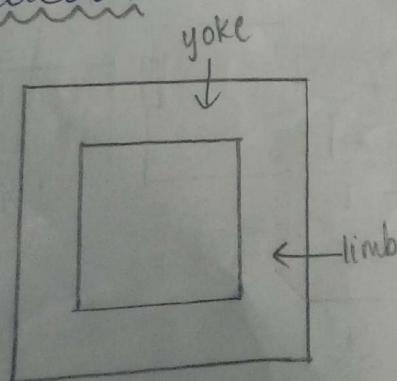
### Operation:-

When primary winding is excited by alternating voltage, it circulates an alternating current. This current produces an alternating flux ( $\phi$ ) which completes its path through common magnetic core represented by a dotted line in fig. Thus an alternating flux links with the secondary winding. As the flux is alternating, acc. to Faraday's laws of electromagnetic induction, mutually induced emf gets developed in Secondary winding. If load is connected to secondary winding, this emf drives a current through it. Thus, though there is no electrical contact b/w 2 windings, an electrical energy gets transferred from primary to secondary.

### Symbolic representation:-



### Construction:-



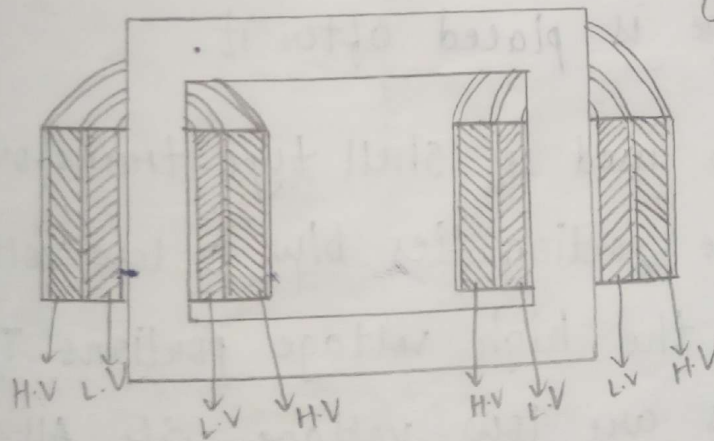


There are 2 basic parts of a trans-former - (1) magnetic core (2) winding or coils.

Magnetic core:- It is square or rectangular in shape. It is divided into 2 parts - the vertical portion on which coils are wound is called limb. The top & bottom horizontal portion is called yoke. Core is made up of laminations to reduce eddy current losses. These laminations are insulated from each other by insulation like varnish. Generally L or I shaped laminations are used to avoid high reluctance at the joints, alternate layers are staggered called staggering.

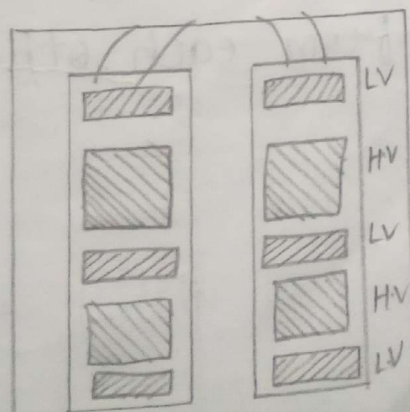
## 2) Types of windings:-

(a)



(b)

Sandwich windings:-



(a) The coils used are wound on the limbs & are insulated from each other. If 2 windings are wound on 2 different limbs, there is a leakage flux, due to this, the transformer performance is badly effected & also the mutual inductance should be very high. To achieve this, 2 windings are split into no. of coils & are wound adjacent to each other on the same limb.

*Cylindrical*

a) Cylindrical coils are used in core type transformers. These coils are mechanically strong & are wound in helical layers. The different layers are insulated from each other by paper, cloth or mica. The low voltage winding is placed near the core & high voltage is placed after it.

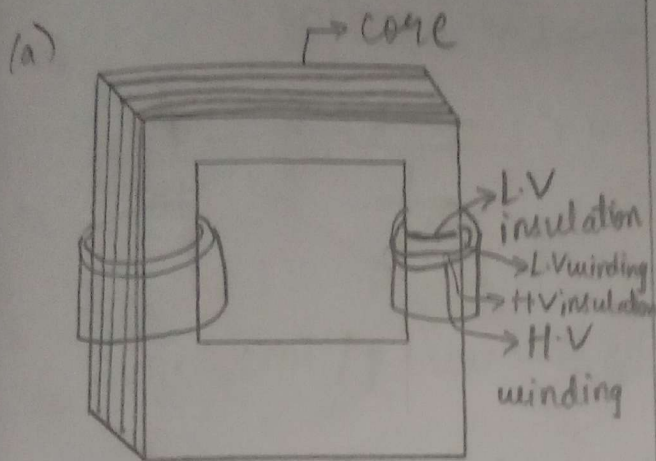
*Sandwich*

b) These are used in shell type transformers. Each high voltage portion lies b/w 2 low voltage portion sandwiching the high voltage portions. The top & bottom coils are low voltage coils. All the portions are insulated from each other by paper.

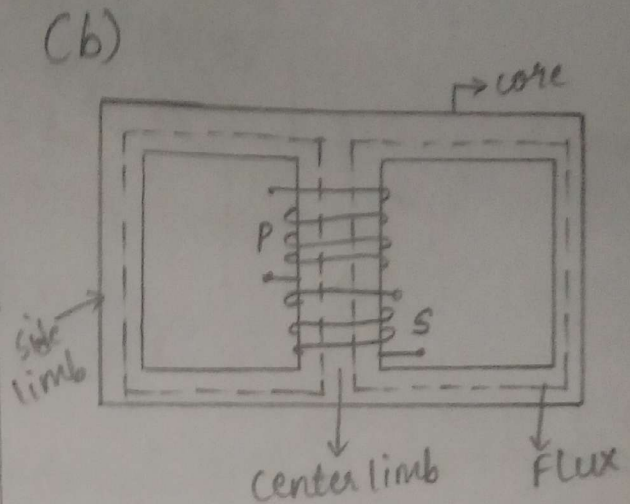


## Types of transformers:-

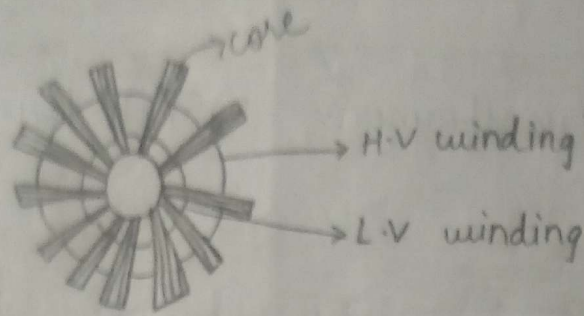
### 1) Core-type transformer:-



### 2) Shell-type transformer:-



### (c) Bellis-Type transformer:-



### EMF equ. of a transformer:-

The various quantities which effect the magnitude of induced emf are  $\phi$  = flux,

$\phi_m$  = max. value of flux.

$N_1$  = <sup>no. of</sup> primary winding turns.

$N_2$  = no. of secondary winding turns.

$f$  = freq. of supplied voltage.

$E_1$  = rms value of primary induced emf.

$E_2$  = rms value of secondary induced emf.

From Faraday's laws of electromagnetic induction, avg. value of emf induced in each turn is proportional to rate of change of flux.

$$\therefore \text{Avg. emf / turn} = \text{avg. rate of change of flux} = \frac{d\phi}{dt}$$

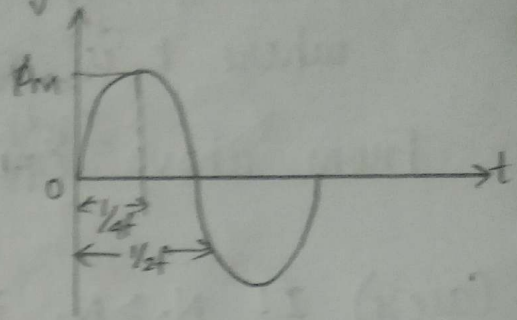


$$\frac{d\phi}{dt} = \frac{\text{change in flux}}{\text{time req. for change in flux.}}$$

Consider  $\frac{1}{4}$ th cycle of the flux as shown in

fig

$$\frac{d\phi}{dt} = \frac{\phi_m}{\frac{1}{4f}} = 4f\phi_m$$



$$\text{avg. emf / turn} = 4f\phi_m \text{ volts}$$

As  $\phi$  is sinusoidal, induced emf is also sinusoidal.

For sinusoidal quantity, form factor is

$$\text{Form factor} = \frac{\text{rms value}}{\text{avg. value}} = 1.11$$

$$\text{rms value} = \text{avg. value} \times 1.11$$

$$\text{rms value of } E = 4f\phi_m \times 1.11$$

$$E = 4.44f\phi_m$$

There are  $N_1$  no. of primary turns. Hence

rms value of induced emf is  $E_1$ ,

$$E_1 = 4.44f\phi_m N_1 \quad \text{--- (1)}$$

There are  $N_2$  no. of secondary turns. Hence

rms value of induced emf is  $E_2$ ,

$$E_2 = 4.44f\phi_m N_2 \quad \text{--- (2)}$$

Voltage ratio:-

Divide (2) by (1), we get

$$\boxed{\frac{E_2}{E_1} = \frac{N_2}{N_1} = k} = \frac{V_2}{V_1}$$

where  $k$  is called as transformation ratio.

From above equ,  $E_2 = kE_1$  ( $k = \frac{N_2}{N_1}$ )

Case:1) If  $N_2 > N_1$ , i.e.  $k > 1$ , then  $E_2 > E_1$ , it is step-up transformer.

Case:2) If  $N_2 < N_1$ , i.e.  $k < 1$ , then  $E_2 < E_1$ , it is step-down transformer.

Case:3) If  $N_2 = N_1$ , i.e.  $k = 1$ , then  $E_2 = E_1$ , it is isolation transformer or 1:1 transformer.

Current ratio:-

For an ideal trans., there are no losses.

Hence product of primary voltage & primary current is same as product of secondary voltage & secondary current.

$$V_1 I_1 = V_2 I_2$$

$$\frac{V_2}{V_1} = \frac{I_1}{I_2} = k$$

Rating of transformer is given in terms of KVA (kilovolt-ampere).

KVA rating of trans. is given by  $\frac{V_1 I_1}{1000} = \frac{V_2 I_2}{1000}$



Pl) A single phase 50Hz transformer has 80 turns on primary winding & 400 turns on secondary winding. The net cross-sectional area of the core is  $200\text{ cm}^2$  if primary winding is connected to 240V, 50Hz supply, determine

- (i) emf induced in secondary winding
- (ii) max. value of flux density in the core.

Sol

$$N_1 = 80, N_2 = 400, f = 50\text{ Hz}, E_1 = 240$$

$$E_2 = 4.44 f \phi_m N_2$$

$$(i) \quad \frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{400}{80} = k$$

$$\therefore k = 5$$

$$E_2 = k E_1 = 5(240) = 1200\text{ V}$$

$$(ii) \text{ flux density } B = \frac{\phi_m}{A}$$

$$\phi_m E_2 = 4.44 f \phi_m$$

$$\phi_m = \frac{1200}{4.44 \times 50} = 0.013\text{ Wb}$$

$$B = \frac{0.013}{200 \times (10^{-2})^2} = 0.65\text{ Wb/m}^2$$

2) The max. flux density <sup>in</sup> of core is of  $\frac{250}{3000} \text{ V, } 50\text{Hz}$   
 single phase transformer is  $1.2 \text{ wb/sq.m}$  if the  
 emf per turn is  $8\text{V}$ . Determine primary &  
 secondary turns & area of the core.

Sol

$$\phi_m = 1.2 \text{ wb/sq.m}, N_1 = ?, N_2 = ?, A = ?$$

$$f = 50\text{Hz}, E_1 = \frac{250}{3000} \text{ V}, E_2 = 8\text{V } 3000\text{V}$$

$$E_1 = 4.44 f \phi_m N_1 \quad \frac{E}{N} = 8\text{V}$$

$$N_1 = \quad \text{i.e } E = 8\text{V}, N = 1 \text{ turn.}$$

$$\frac{E_2}{E_1} = k = 12$$

$$k = \frac{3000}{250} = 12$$

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = k$$

$$E_1 N_2 = E_2 N_1 = k$$

$$\frac{E_2}{N_2} = 8 \Rightarrow N_2 = \frac{3000}{8}$$

$$N_2 = 375$$

$$\frac{E_1}{N_1} = 8 \Rightarrow N_1 = \frac{250}{8}$$

$$N_1 = 31.25 \approx 31$$

$$B_m = \frac{\phi_m}{A} \Rightarrow \phi_m = 1.2 \times A$$



$$E_1 = 4.44 f \phi_m N_1$$

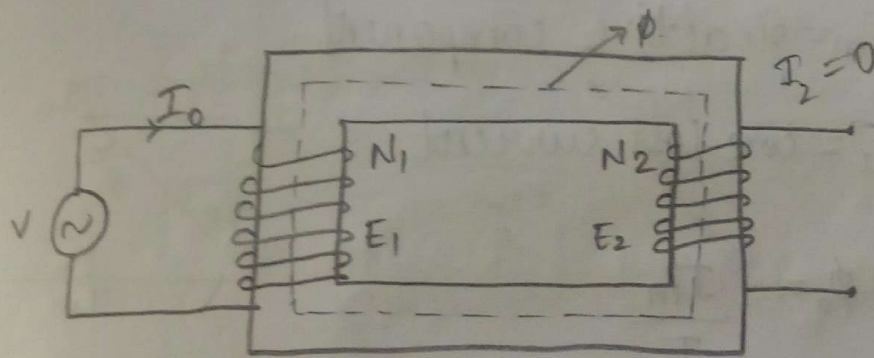
$$250 = 4.44 \times 50 \times \phi_m \times 31$$

$$\phi_m = 0.036$$

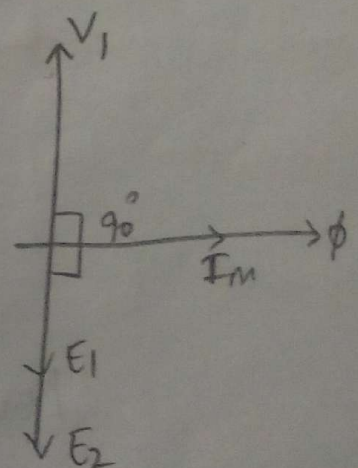
$$\therefore B_m = \frac{\phi_m}{A}$$

$$A = \frac{\phi_m}{B_m} = \frac{0.036}{1.2} = 0.03$$

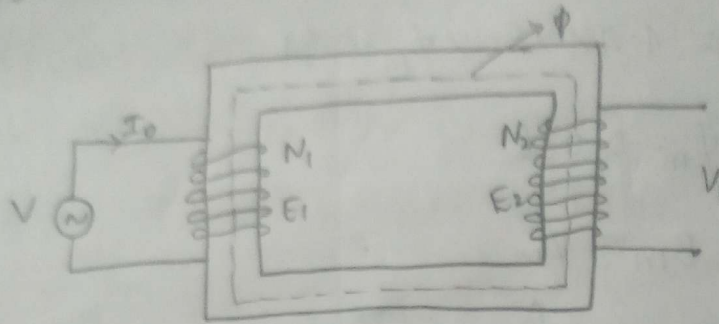
Ideal transformer on no load:-



Phasor diagram



## Practical Transformer without load:-



$$\bar{I}_0 = \bar{I}_m + \bar{I}_c$$

$$|I_0| = \sqrt{I_m^2 + I_c^2}$$

where  $I_m$  = reactive component

$I_c$  = core loss current

$$\sin \phi_0 = \frac{I_m}{I_0}$$

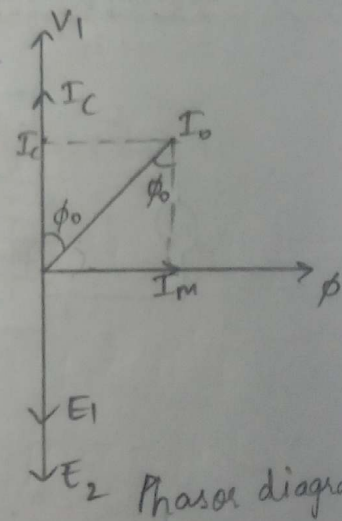
reactive

$$I_m = I_0 \sin \phi_0$$

$$\cos \phi_0 = \frac{I_c}{I_0}$$

active

$$I_c = I_0 \cos \phi_0$$



Phasor diagram

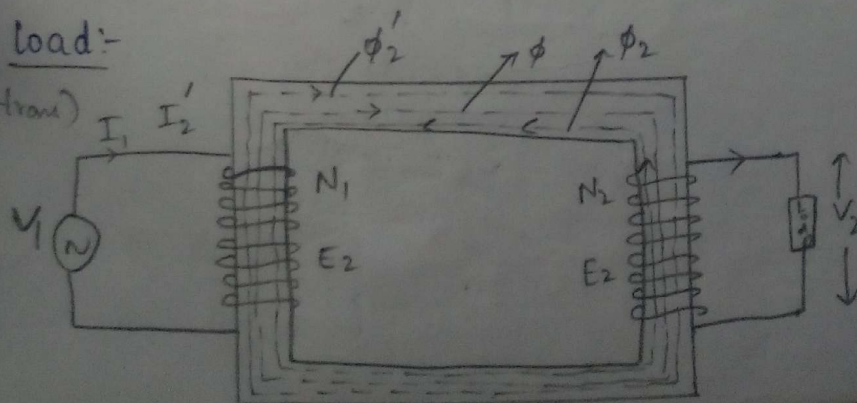
$$\text{Power } W = V_1 I_0 \cos \phi_0$$

$$W_0 = V_1 I_c = P_i = \text{iron loss}$$

In no load condition, Cu losses are very small.

## With load:-

(Practical trans)





This transformer is also known as constant flux generator machine.

$$\begin{aligned} \text{mmf} &= N_2 I_2 \\ &= N_1 I_2' \\ N_2 I_2 &= N_1 I_2' \end{aligned}$$

$$I_2' = \frac{N_2}{N_1} I_2$$

$$I_2' = K I_2$$

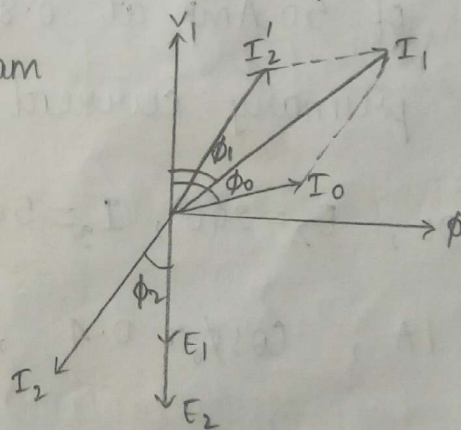
$$\bar{I}_1 = \bar{I}_0 + \bar{I}_2' \quad (\text{where } \bar{I}_0 = \bar{I}_m + \bar{I}_c)$$

$$I_1 = \sqrt{I_0^2 + I_2'^2}$$

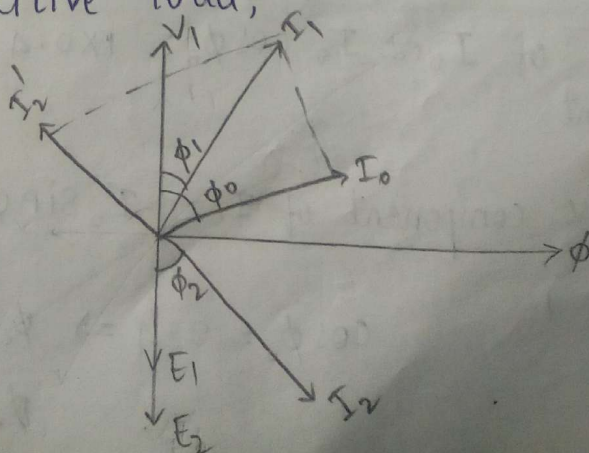
$$I_1 \simeq I_2'$$

a) Consider inductive load, (RL)

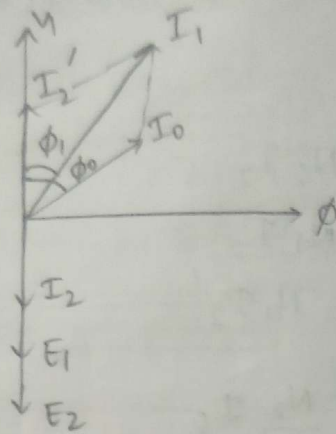
phasor diagram



b) Consider capacitive load,



c) Consider resistive load,



$$\text{mmf} = N_2 I_2 = N_1 I_2' = N_1 I_1$$

$$\Rightarrow \frac{N_2}{N_1} = \frac{I_1}{I_2} = k$$

11) A  $\frac{400}{200}$  V transformer takes 1A at a power factor of 0.4 on no load. If secondary supply is the load current of 50 Amp at 0.8 <sup>lagging</sup> power factor. Calculate the primary current.

Sol  $E_1 = 400$ ,  $E_2 = 200$ ,  $I_2 = 50$  amp,  $I_1 = ?$

$$I_0 = 1 \text{ A}, \cos \phi_0 = 0.4, \cos \phi_2 = 0.8$$

$$I_1 = I_2' + I_0$$

active component of  $I_0 = I_0 \cos \phi_0 = 1 \times 0.4 = 0.4$

reactive component of  $I_0 = I_0 \sin \phi_0 = \sin(66.42^\circ) = 0.916$

$$\cos \phi_0 = 0.4 \Rightarrow \phi_0 = \cos^{-1}(0.4)$$

$$\phi_0 = 66.42^\circ$$



$$\therefore I_0 = \text{active} + j(\text{reactive})$$

$$I_0 = 0.4 + j(0.916)$$

$$\text{WKT } I_2' = k I_2$$

$$k = \frac{E_2}{E_1} = \frac{200}{400} = 0.5$$

$$I_2' = 0.5 \times 50 = 25$$

$$\therefore I_1 = [0.4 + j(0.916)] + 25$$

$$\text{Active component } I_2' \cos \phi_2 = 25 \times 0.8 = 20$$

$$\text{Reactive component } I_2' \sin \phi_2 = 25 \times \sin \phi_2$$

$$\cos \phi_2 = 0.8 \Rightarrow \phi_2 = \cos^{-1}(0.8)$$

$$\phi_2 = 36.86$$

$$I_2' \sin \phi_2 = 25 \times \sin(36.86)$$

$$= 14.99$$

$$\therefore I_2' = 20 + j(14.99)$$

$$\therefore I_1 = [20 + j(14.99)] + [0.4 + j(0.916)]$$

$$= 8 + j18.32 + j5.996 = 13.73$$

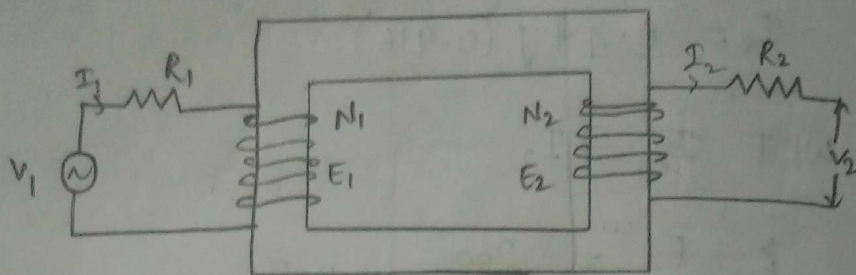
$$I_1 = -5.73 + j(24.316)$$

$$\text{Polar form} = 25.86$$

$$I_1 = 20.4 + j(15.906)$$

$$\text{Polar form} = 25.86 \angle 37.94$$

### Effect of winding resistance:-



$$V_1 = E_1 + I_1 R_1$$

$$E_1 = V_1 - I_1 R_1$$

$$E_2 = V_2 + I_2 R_2$$

Total copper loss :- (when referred to primary)

$$= I_1^2 R_1 + I_2^2 R_2$$

$$= I_1^2 \left[ R_1 + \left( \frac{I_2}{I_1} \right)^2 R_2 \right]$$

$$= I_1^2 \left[ R_1 + \frac{R_2}{k^2} \right] \quad \left( k = \frac{I_1}{I_2} \right)$$

$$= I_1^2 (R_1 + R_2')$$

$$= I_1^2 R_{eq}$$

Effect of leakage resistance:-

$$X_{eq} = X_1 + X_2'$$

$$\text{where } X_2' = \frac{X_2}{k^2}$$

Total copper loss:- (when referred to secondary)

$$= I_1^2 R_1 + I_2^2 R_2$$



$$= I_2^2 \left[ \left( \frac{I_1}{I_2} \right)^2 R_1 + R_2 \right]$$

$$= I_2^2 (k^2 R_1 + R_2)$$

$$= I_2^2 (R_1' + R_2) \quad (k^2 R_1 = R_1')$$

$$= I_2^2 R_{2eq}$$

Effect of leakage resistance :-

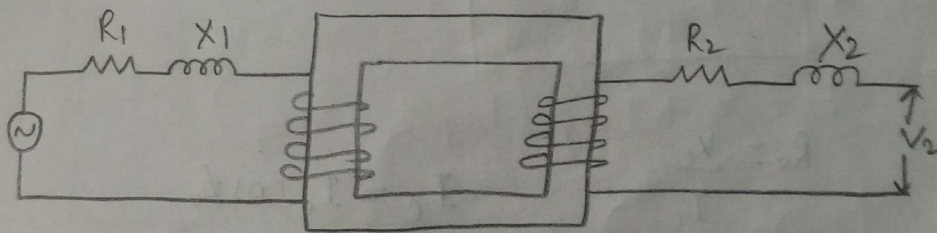
$$X_{2eq} = X_1' + X_2$$

$$X_1' = X_1 k^2$$

Effect of impedance :- (when referred to primary)

$$Z_{1eq} = Z_1 = R_1 + jX_1$$

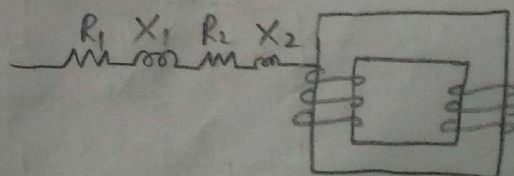
$$Z_2 = R_2 + jX_2$$



Effect of impedance :- (when referred to primary)

$$Z_{1eq} = Z_1 + Z_2'$$

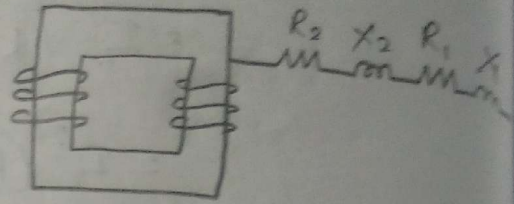
$$Z_2' = \frac{Z_2}{k^2}, \quad Z_{1eq} = \sqrt{R_{1eq}^2 + X_{1eq}^2}$$



Effect of secondary :- (when referred to secondary)

$$Z_{eq} = Z_1' + Z_2$$

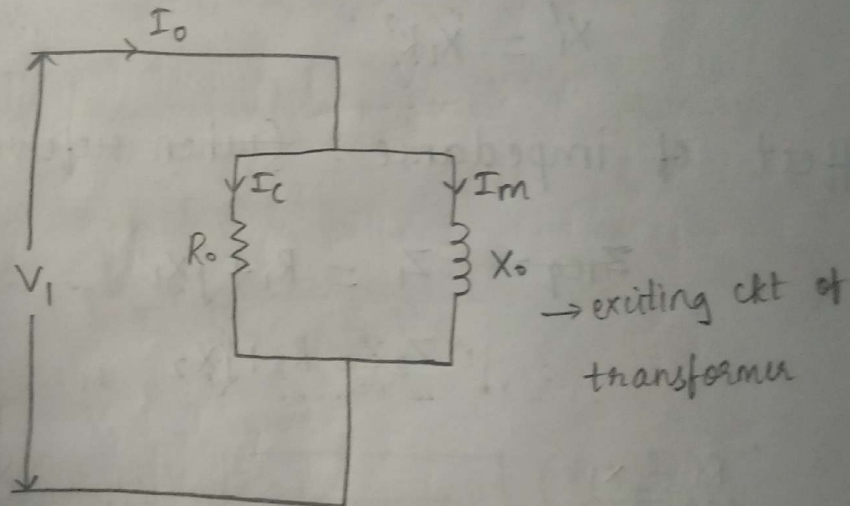
$$Z_1' = \frac{Z_1}{k^2}$$



$$Z_{eq} = R_{2eq} + jX_{2eq}$$

$$|Z_{eq}| = \sqrt{R_{2eq}^2 + X_{2eq}^2}$$

Equivalent ckt of transformer with no load :-



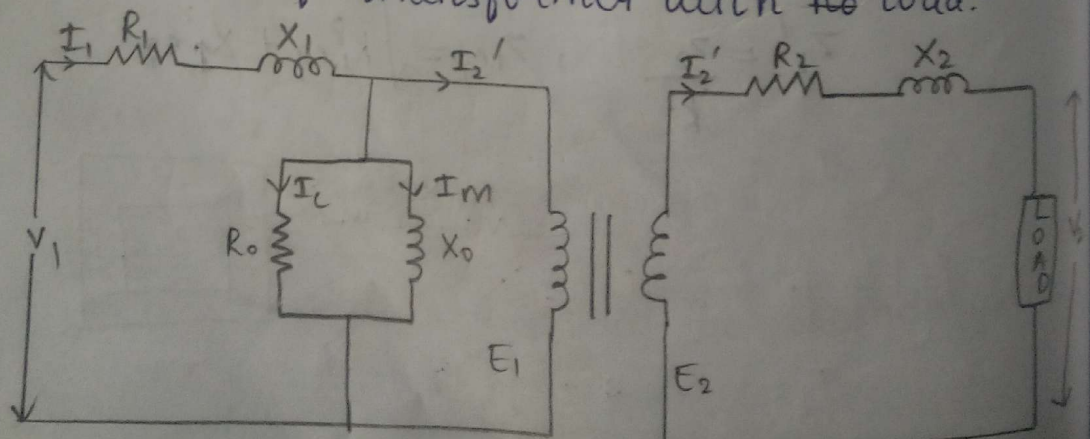
$$R_0 = \frac{V_1}{I_c}$$

$$I_c = I_0 \cos \phi_0$$

$$X_0 = \frac{V_1}{I_m}$$

$$I_m = I_0 \sin \phi_0$$

Equivalent ckt of transformer with no load :-



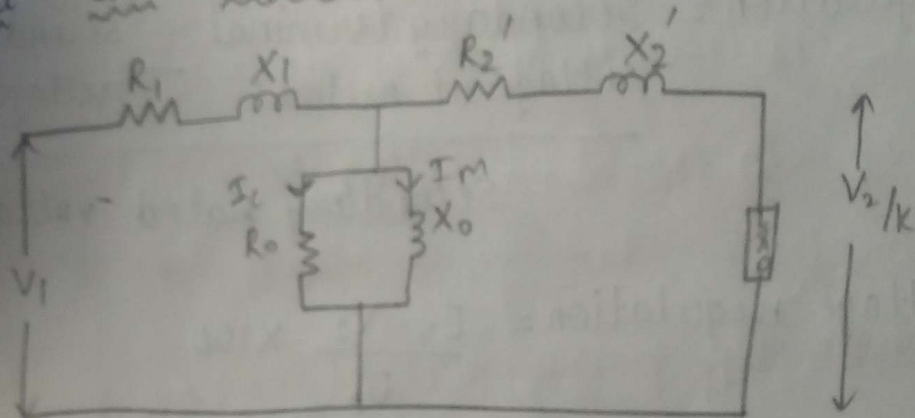


Transferring secondary parameters to primary

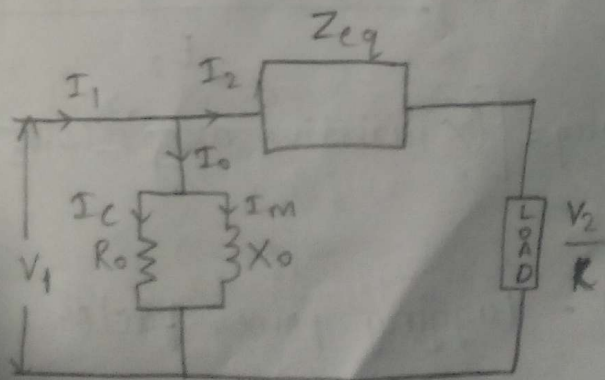
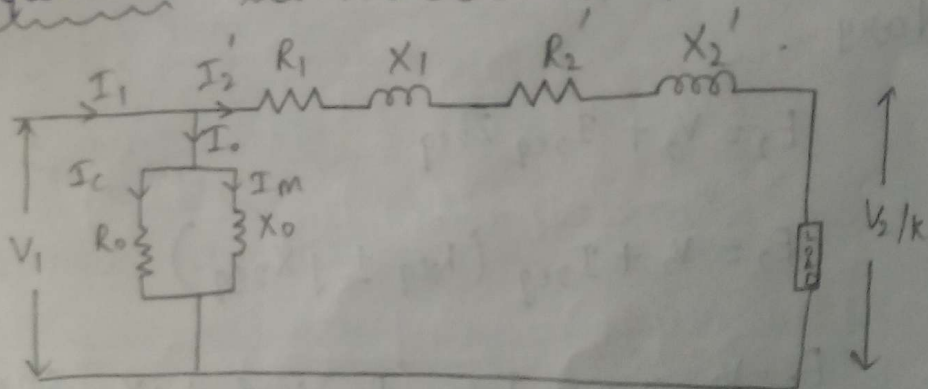
we have  $R_2' = \frac{R_2}{k^2}$  ,  $X_2' = \frac{X_2}{k^2}$  ,  $Z_2' = \frac{Z_2}{k^2}$

$E_2' = \frac{E_2}{k}$  ,  $I_2' = I_2 k$  (where  $k = N_2/N_1$ )

Equivalent ckt referred to primary:-



App. equivalent ckt referred to primary:-



## Voltage regulation of a transformer:-

It is defined as the magnitude of load voltage when load current changes from 0 to full load value. This is expressed as a fraction of secondary rated voltage.

$$\text{Regulation} = \frac{\text{secondary terminal voltage at no load} - \text{secondary terminal voltage at any load}}{\text{secondary rated voltage}}$$

$$\text{voltage regulation} = \frac{E_2 - V_2}{E_2} \times 100$$

By using app. equivalent ckt referred to secondary,

$$E_2 = V_2 + I_{2eq} Z_{2eq}$$

$$E_2 = V_2 + I_{2eq} (R_{2eq} \pm jX_{2eq})$$

→ active      → reactive component

$$\frac{E_2 - V_2}{E_2} \times 100 = \frac{I_{2eq} (R_{2eq} \cos \phi_2 \pm jX_{2eq} \sin \phi_2)}{E_2} \times 100$$

$$\% \text{ voltage reg} = \left[ \underbrace{(\% \text{ resistive drop}) \times \cos \phi_2}_{\substack{(V_R) \\ = I_{2eq} R_{2eq}}} \pm \underbrace{(\% \text{ reactive drop}) \times \sin \phi_2}_{\substack{(V_X) \\ = I_{2eq} X_{2eq}}} \right] \times 100$$

(+) - lagging power factor

(-) - leading power factor



Condition for zero regulation:

Zero voltage regulation is possible only with leading power factors.

$$I_{2eq} [R_{2eq} \cos \phi_2 - X_{2eq} \sin \phi_2] = 0$$

$$R_{2eq} \cos \phi_2 = X_{2eq} \sin \phi_2$$

$$\tan \phi_2 = \frac{R_{2eq}}{X_{2eq}}$$

$$\phi_2 = \tan^{-1} \left( \frac{R_{2eq}}{X_{2eq}} \right)$$

cond:  $\cos \phi_2 = \cos \left[ \tan^{-1} \left( \frac{R_{2eq}}{X_{2eq}} \right) \right]$

Condition for max. voltage regulation:

This is possible with lagging power factor.

$$\text{For max. regulation, } \frac{dR}{d\phi} = 0$$

$$\Rightarrow \frac{d}{d\phi} (V_R \cos \phi + V_X \sin \phi) = 0$$

$$\Rightarrow V_R (-\sin \phi) + V_X (\cos \phi) = 0$$

$$\Rightarrow V_X \cos \phi = V_R \sin \phi$$

$$\Rightarrow \frac{V_X}{V_R} = \tan \phi$$

$$\Rightarrow \phi = \tan^{-1} \left( \frac{V_X}{V_R} \right)$$

$$\Rightarrow \phi = \tan^{-1} \left( \frac{I_{2eq} X_{2eq}}{I_{2eq} R_{2eq}} \right)$$

$$\Rightarrow \phi = \tan^{-1} \left( \frac{X_{2eq}}{R_{2eq}} \right)$$

$$\cos \phi = \cos \left[ \tan^{-1} \left( \frac{X_{2eq}}{R_{2eq}} \right) \right]$$

### Efficiency of a transformer:-

The efficiency of any device is defined as ratio of o/p power to i/p power. Due to the losses in trans., the o/p power of transformer is less than i/p power supply,

$$\therefore \text{o/p power} = \text{i/p power} - \text{losses}$$

(or)

$$\text{i/p} = \text{o/p power} + \text{losses}$$

$$= \text{o/p} + P_i + P_{cu}$$

$$\text{Efficiency } (\eta) = \frac{\text{o/p power}}{\text{i/p power}}$$

$$= \frac{\text{o/p power}}{\text{o/p} + \text{losses}} \times 100$$

$$= \frac{\text{i/p} - \text{losses}}{\text{i/p power}} \times 100$$

$$\% \eta = \frac{\text{o/p power}}{\text{o/p} + \text{total losses}} \times 100$$

$$= \frac{V_2 I_2 \cos \phi}{V_2 I_2 \cos \phi + P_i + P_{cu}} \times 100$$



$$\% \eta = \frac{V_2 I_2 \cos \phi}{V_2 I_2 \cos \phi + P_i + I_{2(FL)}^2 R_{2eq}} \times 100$$

~~n =~~

If trans. is subjected to a fractional load, then using the approximate values of various quantities, the efficiency can be calculated. Let

$$n = \frac{\text{actual load}}{\text{full load}}$$

When load changes, load current changes,

$$\therefore \text{new current } I_2 = n I_{2(FL)} \quad (I_{2(FL)} = \text{full load current})$$

Similarly o/p power  $V_2 I_2 \cos \phi$  also reduces by the same fraction.

new cu. loss due to fractional load is

$$\begin{aligned} P_{cu} &= I_2^2 R_{2eq} \\ &= n^2 I_{2(FL)}^2 R_{2eq} \\ &= n^2 P_{cu(FL)} \end{aligned}$$

$$\% \eta = \frac{n (VA)_{\text{rating}} \times \cos \phi}{n (VA)_{\text{rating}} \times \cos \phi + P_i + n^2 P_{cu(FL)}} \times 100$$

Condition for max. efficiency:

$$\frac{d}{dI_2} (\eta) = 0$$

$$\frac{d}{dI_2} \left[ \frac{V_2 I_2 \cos \phi}{V_2 I_2 \cos \phi + P_i + I_2^2 R_{eq}} \right] = 0$$

$$\Rightarrow \cancel{V_2 \cos \phi (V_2 I_2 \cos \phi + P_i + I_2^2 R_{eq})} - \cancel{(V_2 \cos \phi + 2I_2 R_{eq})} \cdot \cancel{(V_2 I_2 \cos \phi)} = 0$$

$\div NM \& DM \text{ by } I_2$

$$\Rightarrow \cancel{V_2 \cos} \Rightarrow \frac{d}{dI_2} \left[ \frac{V_2 \cos \phi}{V_2 \cos \phi + \frac{P_i}{I_2} + I_2 R_{eq}} \right] = 0$$

$\text{or } \nearrow \text{ w.r.t diff}$

$$\Rightarrow V_2 \cos \phi \left[ \frac{-1}{\left( V_2 \cos \phi + \frac{P_i}{I_2} + I_2 R_{eq} \right)^2} \right] \times \left( \left( -\frac{P_i}{I_2^2} \right) + R_{eq} \right) = 0$$

$\text{or } \searrow \text{ w.r.t diff}$

$$\Rightarrow 0 - (V_2 \cos \phi) \left( 0 - \frac{P_i}{I_2^2} + R_{eq} \right) = 0$$

$$\Rightarrow -(V_2 \cos \phi) \left( -\frac{P_i}{I_2^2} + R_{eq} \right) = 0$$

$$\Rightarrow -\frac{P_i}{I_2^2} + R_{eq} = 0$$

$$\Rightarrow R_{eq} = \frac{P_i}{I_2^2}$$

$$\Rightarrow P_i = I_2^2 R_{eq}$$

$$\Rightarrow P_i = P_{cu}$$

$$\therefore \text{iron loss} = \text{copper loss}$$



Current  $I_{2m}$  at max. efficiency:-  
From  $P_i = I_2^2 R_{eq}$

$$I_2 = \sqrt{\frac{P_i}{R_{eq}}}$$

$$\text{max. current} \Rightarrow I_{2m} = \sqrt{\frac{P_i}{R_{eq}}}$$

$$\frac{I_{2m}}{I_{2(FL)}} = \frac{1}{I_{2(FL)}} \sqrt{\frac{P_i}{R_{eq}}}$$

$$= \sqrt{\frac{P_i}{I_2^2 R_{eq}}}$$

$$\frac{I_{2m}}{I_{2(FL)}} = \sqrt{\frac{P_i}{P_{cu}}}$$

$$I_{2m} = I_{2(FL)} \sqrt{\frac{P_i}{P_{cu(FL)}}}$$

Max KVA supplied to the load at maximum efficiency,

$$\text{KVA rating} = V_2 I_{2m}$$

$$\boxed{\text{KVA rating} = V_2 \sqrt{\frac{P_i}{P_{cu(FL)}}}}$$

P1) An ideal 25 KVA transformer has 500 turns on primary & windings & 40 turns on secondary. The primary is connected 3000V, 50Hz supply. Calculate the

(i) primary & secondary currents on full load.

(ii) secondary emf.

(iii) max. core flux.

Sol

$$\text{KVA rating} = 25 \text{ KVA} = V_1 I_1 = V_2 I_2$$

$$(i) \quad N_1 = 500, \quad N_2 = 40$$

$$E_1 = 3000 \text{ V}, \quad f = 50 \text{ Hz}$$

$$V_1 I_1 = 25 \text{ KVA}$$

$$3000 \times I_1 = 25 \times 10^3$$

$$I_1 = 8.3 \text{ A}$$

$$V_2 I_2 = 25 \times 10^3$$

$$\frac{N_2}{N_1} = \frac{I_1}{I_2}$$

$$I_2 = \frac{I_1 \times N_1}{N_2}$$

$$I_2 = 103.75 \text{ A}$$

(ii) secondary emf,

$$E_2 = 4.44 \times f \phi_m N_2$$



For ideal trans,  $V_2 = E_2$

$$\frac{N_2}{N_1} = \frac{V_2}{V_1}$$

$$V_2 = \frac{N_2 \times V_1}{N_1} = 240 \text{ V}$$

(iii) max. core flux.

$$E_2 = 4.44 \times f \phi_m N_2$$

$$\phi_m = \frac{E_2}{4.44 \times 50 \times 40}$$

$$= 0.027 \text{ wb}$$

$$\phi_m = 27 \text{ mwb}$$

2) A  $\frac{230}{2300}$  V transformer takes a no load current of 6.5 A and absorbs 187 watts. If the resistance of the primary is  $0.06 \Omega$ , find

(i) core loss

(ii) no load power factor

(iii) active component of current

(iv) magnetising current

Sol  $E_1 = 230 \text{ V}$ ,  $E_2 = \frac{2}{1} 2300 \text{ V}$ ,  $I_0 = 6.5 \text{ A}$ ,

$$R_1 = 0.06 \Omega, \text{ Total loss} = 187 \text{ watts}$$

(i) core loss

$$\text{total power loss} = \text{iron loss} + \text{Cu. loss}$$

$$\text{Cu. loss} = I_0^2 R_1$$

$$= 2.53$$

$$\text{iron loss} = \text{total} - \text{Cu. loss}$$

$$= 187 - 2.53$$

$$= 184.47$$

$$(ii) \quad W_o (\text{o/p power}) = V_1 I_o \cos \phi$$

$$187 = 230 (6.5) \cos \phi$$

$$\cos \phi = 0.12 \Rightarrow \phi = 83.10$$

$$(iii) \quad I_c = I_o \cos \phi$$

$$I_c = 0.78 \text{ A}$$

$$(iv) \quad I_m = I_o \sin \phi$$

$$= 6.5 \times \sin(83.10)$$

$$I_m = 6.45 \text{ A}$$

3) A 100 K-VA,  $\frac{2200}{440}$  V transformer has  $R_1 = 0.3 \Omega$ ,

$X_1 = 1.1 \Omega$ ,  $R_2 = 0.01 \Omega$ ,  $X_2 = 0.035 \Omega$ . Calculate

(i) equivalent imp. of trans. referred to primary.

(ii) total Cu. loss.

Sol (i)  $X_{1eq} = X_1 + X_2'$

$$Z_{1eq} = Z_1 + Z_2' \quad \text{or} \quad Z_{1eq} = \sqrt{(R_{1eq})^2 + (X_{1eq})^2}$$



$$Z_1 = R_{1eq} + jX_{1eq}$$

$$Z_2' = \frac{Z_2}{k^2} = \frac{R_{2eq} + jX_{2eq}}{k^2}$$

$$\text{where } R_{1eq} = R_1 + R_2'$$

$$= R_1 + \frac{R_2}{k^2}$$

$$k = \frac{V_2}{V_1} = \frac{440}{2200} = 0.2$$

$$R_{1eq} = 0.3 + \frac{0.01}{(0.2)^2}$$

$$R_{1eq} = 0.55$$

$$X_{1eq} = X_1 + X_2' = X_1 + \frac{X_2}{k^2}$$

$$X_{1eq} = 1.975$$

$$Z_1 = 0.55 + j(1.975)$$

$$\text{Polar form} \Rightarrow Z_1 = 2.05 \angle 74.43^\circ$$

$$Z_2 = R_{2eq} + jX_{2eq}$$

$$Z_{1eq} = \sqrt{(R_{1eq})^2 + (X_{1eq})^2}$$

$$Z_{1eq} = 2.05$$

(ii) total Cu. loss

$$V_1 I_1 = 100 \times 10^3$$

$$I_1 = \frac{100 \times 10^3}{2200} = 45.45$$

$$\text{total cu. loss} = I_1^2 R_{eq} \\ = 1136.13$$

4) A  $\frac{440}{110}$  V trans. has  $R_1 = 0.05 \Omega$ ,  $R_2 = 0.02 \Omega$ , its iron loss at normal i/p is 150 watts. Determine the  
 (i) secondary current at which max. efficiency occurs, & a value of this max. efficiency at a unity power factor.

Sol  $V_1 = 440$ ,  $V_2 = 110$ ,  $R_1 = 0.05 \Omega$ ,  $R_2 = 0.02 \Omega$

$$P_i = 150 \text{ watts}, I_2 = ?$$

$$\text{Cond} \Rightarrow P_i = I_2^2 R_{2eq}$$

$$R_{2eq} = R_2 + R_1'$$

$$R_1' = k^2 R_1 = \left( \frac{N_2}{N_1} \right)^2 \times 0.05$$

$$R_1' = 3.12 \times 10^{-3} = 0.0031$$

$$R_{2eq} = 0.02 + 0.003$$

$$R_{2eq} = 0.023$$

$$P_i = I_2^2 R_{2eq}$$

$$I_2^2 = \frac{P_i}{R_{2eq}} = 6521.73$$

$$I_2 = \sqrt{6521.73} = 80.75$$



$$(ii) \quad \eta = \frac{o/p}{o/p + lcu} = \frac{V_2 I_2 \cos \phi}{V_2 I_2 \cos \phi + 2P_i} \quad (\because P_i = P_{cu})$$

$$\text{given: } \cos \phi = 1 \Rightarrow \frac{110 \times 80.75 \times 1}{(110 \times 80.75 \times 1) + (150 \times 2)}$$

$$= \cancel{0.983} \quad 0.967$$

$$\% \eta \Rightarrow \cancel{98.33\%} \quad 96.7$$

5) The full load Cu & iron loss of a 15 KVA single phase transformer are 320 watts & 200 watts. Calculate the efficiency on

(i) full load

(ii) half load

when PF is 0.8 lagging in each case.

Sol

$$P_{cu} = 320, \quad P_i = 200, \quad V_1 I_1 = 15 \text{ KVA}$$

$$\cos \phi = 0.8$$

$$\% \eta = \frac{o/p}{i/p} \times 100$$

$$\% \eta = \frac{n (\text{VA rating}) \times \cos \phi}{n (\text{VA}) \text{ rating} \times \cos \phi + P_i + n^2 P_{cu} (FL)} \times 100$$

$$n = \frac{\text{actual load}}{\text{full load}} = \frac{\text{full load}}{\text{full load}} = 1$$

$$\% \eta = \frac{1 \times 15 \times 10^3 \times 0.8}{15 \times 10^3 + 320 + 200 \times 1^2} = 95.28 \%$$

(ii) half load

$$n = \frac{\text{actual load}}{\text{full load}} = \frac{1}{2} = 0.5$$

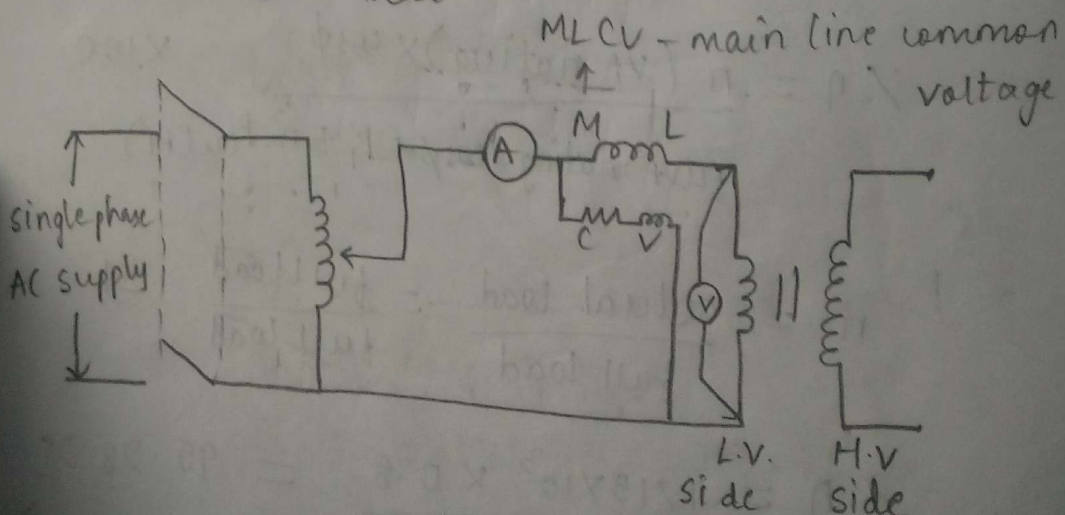
$$\% \eta = \frac{0.5 \times 15 \times 10^3 \times 0.8}{0.5 \times 15 \times 10^3 \times 0.8 + 220 + (0.5)^2 \times 320}$$
$$= 95.2\%$$

Open circuit test:-

Indirect loading test on transformer:-

The efficiency & regulation of a transformer on any load condition & at any power factor condition can be pre-determined by indirect loading method. The equivalent ckt parameters of trans. are determined by conducting oc test and sc test on transformer.

Open circuit (oc) test:-





$V_o (V)$	$I_o (\text{amp})$	$W_o (\text{watt})$
rated		

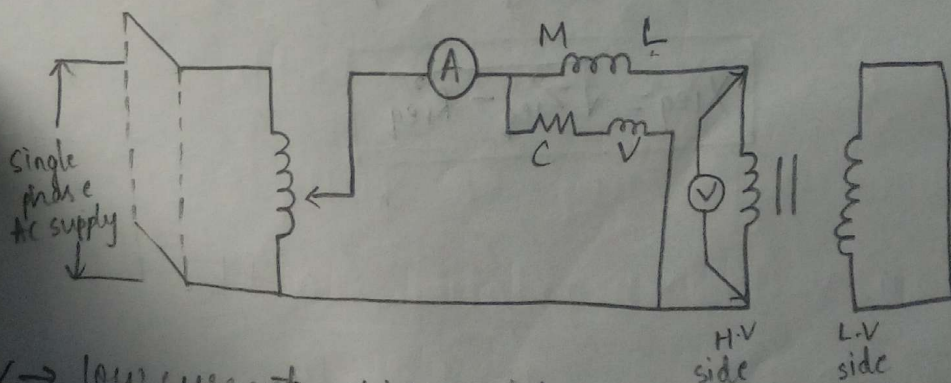
As trans. secondary is open, it is on no load, so current drawn by the primary is no load current  $I_o$ . The 2 components of this no load current are  $I_c$  &  $I_m$ .  $I_c = I_o \cos \phi_o$ ;  $I_m = I_o \sin \phi_o$  where  $\cos \phi_o = \text{no load power factor}$ . Power i/p can be written as  $W_o = V_o I_o \cos \phi_o$ .

$$W_o = P_i = \text{iron loss}$$

$$\cos \phi_o = \frac{W_o}{V_o I_o}$$

The exciting ckt parameters can be determined by  $R_o = \frac{V_o}{I_c}$ ;  $X_o = \frac{V_o}{I_m}$

SC (Short circuit) test:-



HV  $\rightarrow$  low current  $\rightarrow$  high resistance

LV  $\rightarrow$  high "  $\rightarrow$  low "

$V_{sc}$ (volts)	$I_{sc}$ (amp)	$W_{sc}$ (watts)
	rated	

The wattmeter reading is the power loss which is equal to full load Cu. loss as iron losses are very low.

$$W_{sc} = V_{sc} I_{sc} \cos \phi_{sc} = P_{cu}$$

$$\cos \phi_{sc} = \frac{W_{sc}}{I_{sc} \cdot V_{sc}}$$

where  $\cos \phi_{sc}$  = short ckt power factor

$$W_{sc} = I_{sc}^2 R_{eq} = \text{Cu. loss}$$

$$R_{eq} = \frac{W_{sc}}{I_{sc}^2}$$

$$Z_{eq} = \frac{V_{sc}}{I_{sc}}$$

$$Z_{eq} = \sqrt{R_{eq}^2 + X_{eq}^2}$$

$$X_{eq} = \sqrt{Z_{eq}^2 - R_{eq}^2}$$



11) A single phase transformer working at unity power factor has  $\eta = 90\%$  at half of full load and full load of 500kW. Determine its iron & full load Cu loss.

Sol

$$\cos \phi_0 = 1, \quad \eta = 90\%, \quad n = \frac{\text{actual load}}{\text{full load}}$$

$$P_i = ?, \quad P_{cu(FL)} = ?$$

$$P = VI = 500 \text{ kW}$$

$$\eta = \frac{n(VI \cos \phi)}{n(VI \cos \phi) + P_i + \frac{I^2}{I_{FL}^2} P_{cu}}$$

$$\eta = \frac{500 \times 10^3 \times 1 \times 1}{1(500 \times 10^3 \times 1) + P_i + (1)^2 P_{cu}} \quad (n=1) \text{ FL}$$

$$\frac{90}{100} = \frac{500 \times 10^3}{(500 \times 10^3) + P_i + P_{cu}}$$

$$1.8 \times 10^{-6} = \frac{1}{(500 \times 10^3) + P_i + P_{cu}}$$

$$(500 \times 10^3) + P_i + P_{cu} = \frac{1}{1.8 \times 10^{-6}}$$

$$P_i + P_{cu} = 55.55 \quad \text{--- (1)}$$

$$n = \frac{1}{2} = 0.5 \text{ (NL)}$$

$$\eta = \frac{0.5 \times 500 \times 10^3}{(0.5 \times 500 \times 10^3) + P_i + \frac{1}{4} P_{cu}}$$

$$\frac{90}{100} = \frac{0.5 \times 500 \times 10^3}{0.5 \times 500 \times 10^3 \times \frac{P_i}{P_i + P_{cu}}}$$

$$P_i + P_{cu} = 27.77 \text{ --- (1)}$$

$$P_i + (0.5)^2 P_{cu} = 27.77 \text{ --- (2)}$$

Solve equ. (1) & (2)

$$P_i + P_{cu} = 55.55$$

$$P_i + (0.5)^2 P_{cu} = 27.77$$

$$P_{cu} (0.75) = 27.78$$

$$P_{cu} = 37.04 \text{ kW}$$

$$P_i + 37.04 = 55.55$$

$$P_i = 18.51 \text{ kW}$$

2) A 5 KVA,  $\frac{500}{250}$  V, 50 Hz single phase transformer

gave foll. readings.

OC test :-  $\overset{V_o}{500\text{V}}$ ,  $\overset{I_o}{1\text{Amp}}$ , 50 watts

SC test :- 25V, 10 amp, 60 watts

- Determine (i) efficiency on full load, 0.8 lagging power factor. (ii) voltage regulation on FL, 0.8 leading PF (iii)  $\eta$  on 60% FL, 0.8 leading PF (iv) Draw equivalent ckt referred to primary & insert all the values in it.



Sol

(i)

$$V_1 I_1 = 5 \times 10^3, f = 50 \text{ Hz}$$

$$P_i = 50 \text{ W}$$

$$V_1 = 500, V_2 = 250$$

$$P_{cu} = 60 \text{ W}$$

$$\eta = \frac{V I \cos \phi}{V I \cos \phi + P_i + P_{cu}}$$

$$I_1 = \frac{5 \times 10^3}{500}$$

$$\cos \phi = 0.8$$

$$I_1 = 10 \text{ A}$$

$$\eta = \frac{5 \times 10^3 \times 0.8}{(5 \times 10^3 \times 0.8) + 50 + 60}$$

$$\eta = 0.9732$$

$$\eta\% = 97.32\%$$

(ii)

$$\cos \phi = 0.8 \Rightarrow \phi = 36.86$$

$$R_{eq} = \frac{W_{sc}}{I_{sc}^2} = \frac{60}{(10)^2} = 0.6$$

$$\% \text{ voltage reg} = \frac{(I_1 R_{eq} \cos \phi \pm I_1 X_{eq} \sin \phi)}{V_1}$$

$$X_{eq} = \sqrt{Z_{eq}^2 - (R_{eq})^2}$$

$$= \sqrt{\left(\frac{25}{10}\right)^2 - (0.6)^2}$$

$$X_{eq} = 2.42$$

$$\% \text{ voltage reg} = \frac{10(0.6) \times 0.8 - 10(2.42) \sin(36.86)}{V_1}$$

$$= 0.942$$

$$\% \text{ voltage} \Rightarrow 1.94\%$$

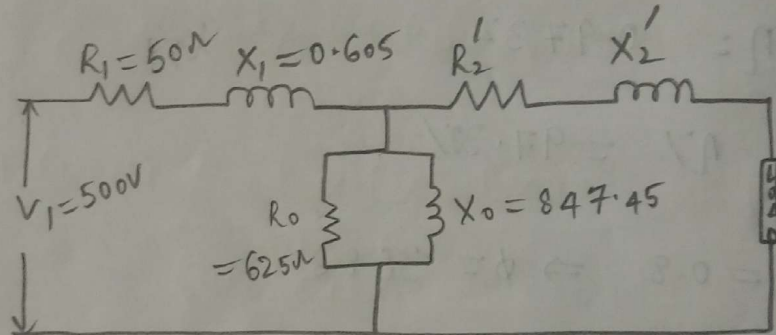
(iii)  $\cos \phi = 0.8$  ,  $n = \frac{60}{100} = 0.6$

$$\eta = \frac{0.6 (5 \times 10^3 \times 0.8)}{0.6 (5 \times 10^3 \times 0.8) + 50 + (0.6)^2 60}$$

$$\eta = 0.9710$$

$$\% \eta = 97.10 \%$$

(iv)



$$I_c = I_0 \cos \phi = 0.8$$

$$R_0 = \frac{V_0}{I_c} = 625$$

$$X_{1eq} = \frac{X_1}{k^2} \quad k = \frac{E_2}{E_1} = 0.05 \quad 0.5$$

$$X_1 = 0.605$$

$$I_m = I_0 \sin \phi$$

$$X_0 = \frac{V}{I_m} = \frac{500}{0.59}$$

$$= 1 \times 0.59$$

$$= 0.59$$

$$= 847.45$$

$$R_{eq} = 0.6 \quad X_{eq} = 2.42$$

