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Unit: 4 DC Machines

Machine:-

A machine deals with the energy transfer either from mechanical to electrical form or electrical to mechanical form. This process is called electromechanical energy conversion.

The machines which are related to electrical energy of alternating type are called ac machines, and which are related to energy of direct type are called dc machines.

DC machines are classified into 2 types:-

- 1) Generator (converts mechanical energy to electrical energy)
- 2) Motor (converts electrical to mechanical energy)

The construction of dc machines remains same for generator and motor.

Electromagnetic induction:-

When magnetic flux linking with the conductor changes, an emf is induced in the conductor. If this conductor forms a closed path, then current flows through it. This phenomenon is called electromagnetic induction.

The principle of dc generator is dynamically

induced emf. (Faraday's laws of electromagnetic induction).

Faraday's -I law:-

When the magnetic flux linking a conductor or coil changes, an emf is induced in it.

Faraday's -II law:-

The magnitude of the emf induced is equal to the rate of change of flux linkages.

Induced emf:-

The change in magnetic flux linkages can be brought in 2 ways :-

1) The conductor is moved in a stationary magnetic field in such a way that the magnetic flux linking it changes in magnitude, the emf induced in this way is called dynamically induced emf.

2) The conductor is stationary and magnetic field is changing, the emf induced in this way is called statically induced emf.

Generating action requires following basic components :-

- 1) Coil or conductor.
- 2) Magnetic field / magnetic flux.
- 3) Relative motion b/w conductor & flux.

To have the rotation of the conductors, the conductors are placed on the armature and are rotated with the help of the external device called 'prime mover'. ^{Examples:-} (Steam engines, water turbines)

Magnetism:-

The property by virtue of which a piece of a solid body attracts iron pieces is called magnetism.

The solid body is called natural magnet. The 2 ends are called poles. When 2 magnets are brought near each other, their behaviour governed by laws called laws of magnetism.

Law: 1

It states that like magnetic poles repel each other & unlike magnetic poles attract each other.

Law: 2 (Coulomb's law)

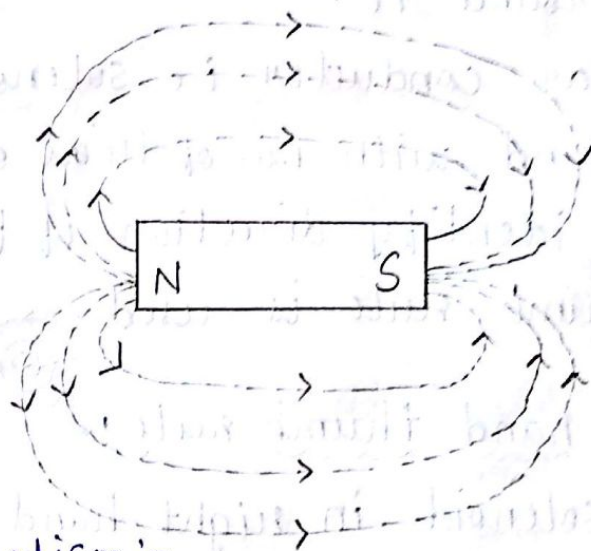
The force exerted by one pole on other pole is directly prop. to the product of pole strengths & inversely prop. to square of distance b/w them. $F \propto \frac{M_1 M_2}{d^2} \Rightarrow F = k \frac{M_1 M_2}{d^2}$

Magnetic field:-

The region around the magnet within which the influence of the magnet can be experienced is called magnetic field. It is represented by imaginary lines around the magnet called the magnetic lines of force.

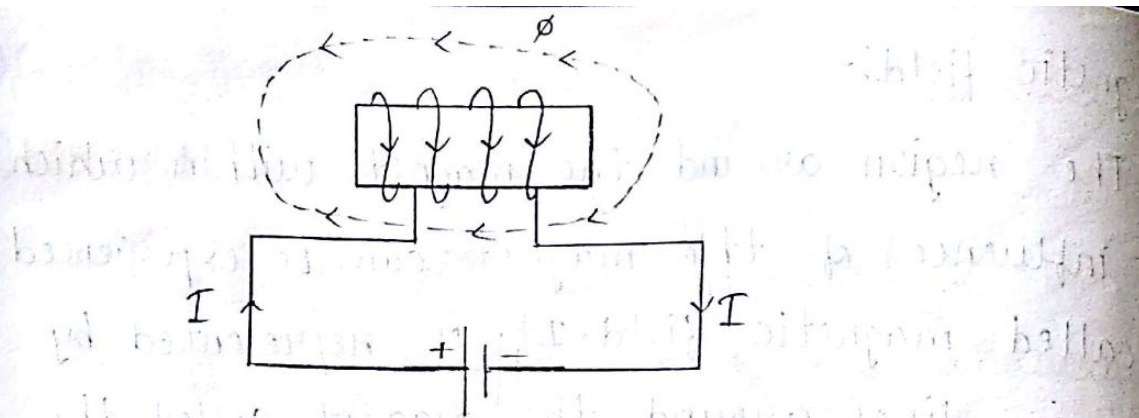
The total no. of lines of force is called magnetic flux (ϕ), measured in unit weber.

$$1 \text{ weber} = 10^8 \text{ lines of force}$$



Electromagnetism:-

When a conductor carries current, it creates a magnetic field around it, the direction of such magnetic field depends on direction of current passing through the conductor. The rule to determine the direction of flux is right hand thumb rule.



Right hand thumb rule:-

It states that hold the current carrying conductor in right hand such that the thumb is pointing in the direction of current and curled fingers point in the direction of magnetic field or flux around it.

For a circular conductor i.e. solenoid (a long conductor is wound with no. of turns on a core to form a coil) to identify direction of flux, modified right hand thumb rule is used.

Modified right hand thumb rule:-

Hold the solenoid in right hand such that curled fingers point in the direction of current then the thumb represents the direction of flux lines inside the core.

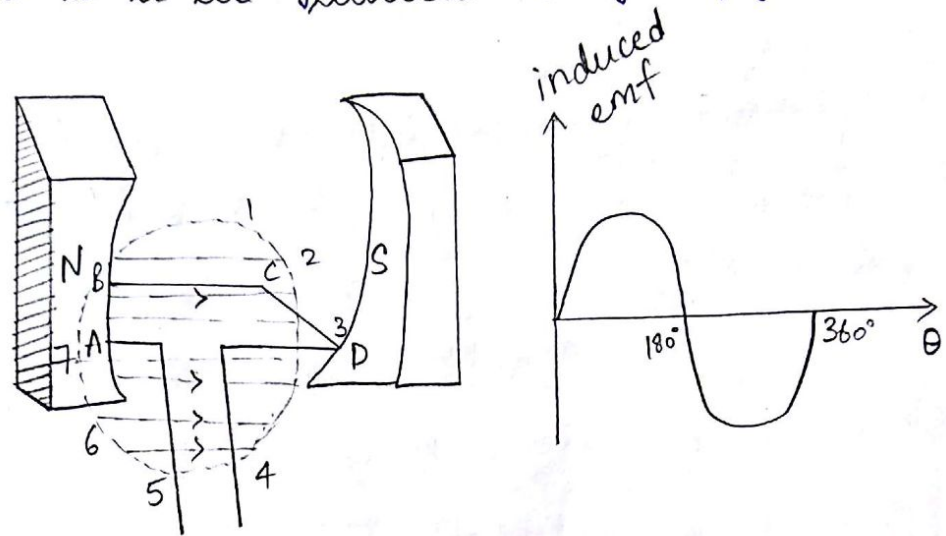
The direction of induced emf can be obtained by Fleming's right hand rule.

- (dot) indicates current direction towards the observer
- x (cross) indicates current direction away from the observer

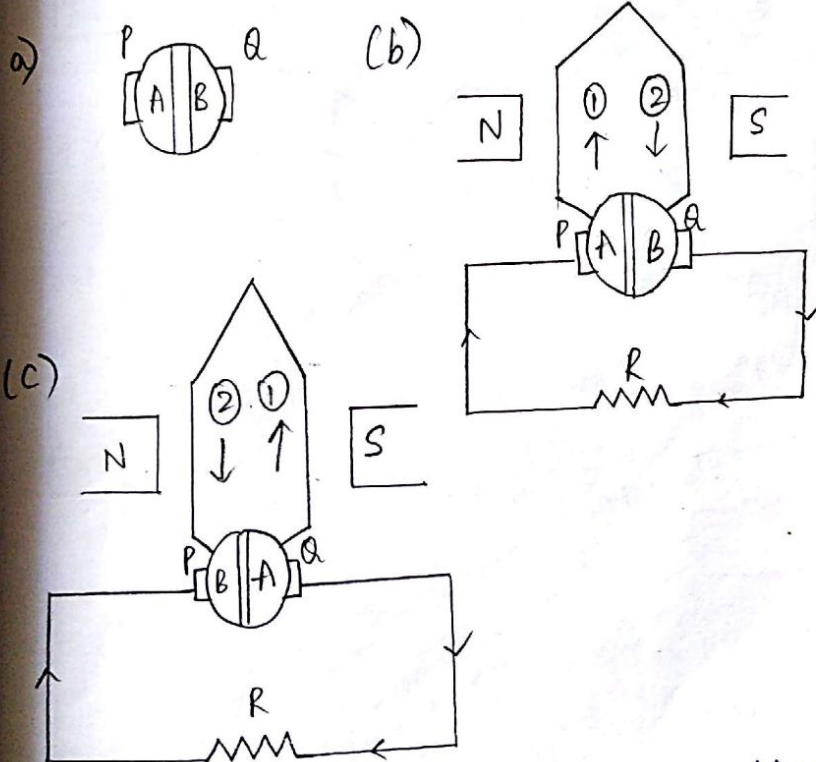
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Operation of a DC generator:- (Single loop generator)



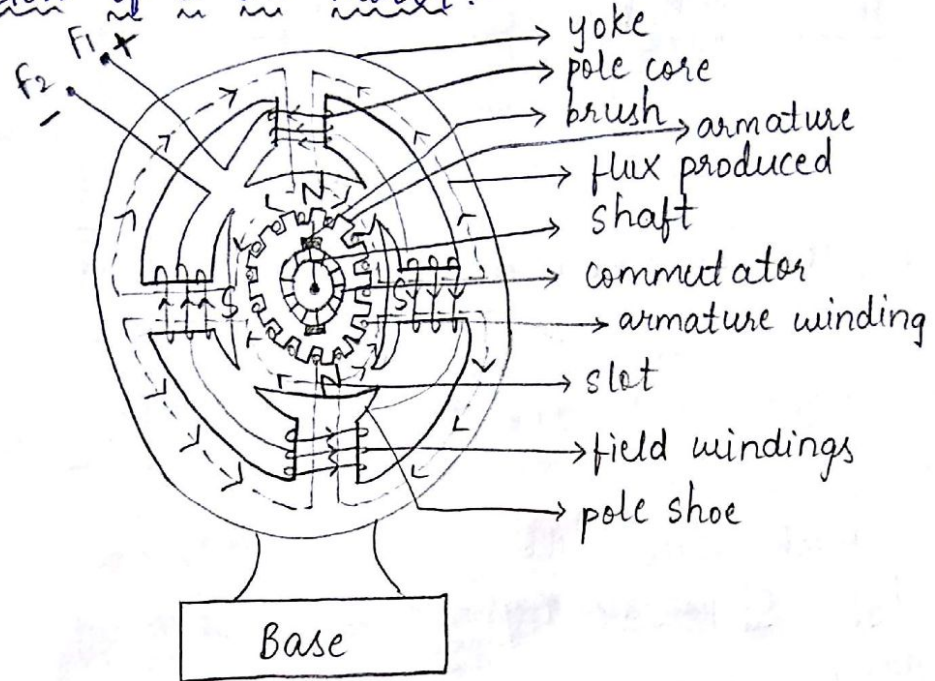
Action of a commutator:-



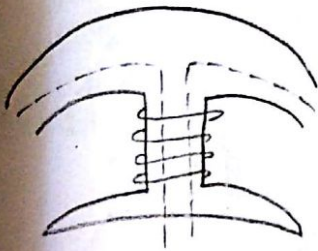
* Flemming's right hand rule:- If 3 fingers of our right hand are outstretched such that each finger is right angled to each other then index finger is made to pt in the direction of lines of flux, thumb in the direction of relative motion of conductor & outstretched middle finger gives direction of emf induced.

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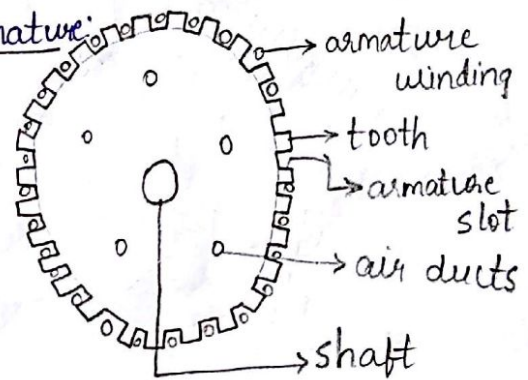
Construction of a DC motor:-



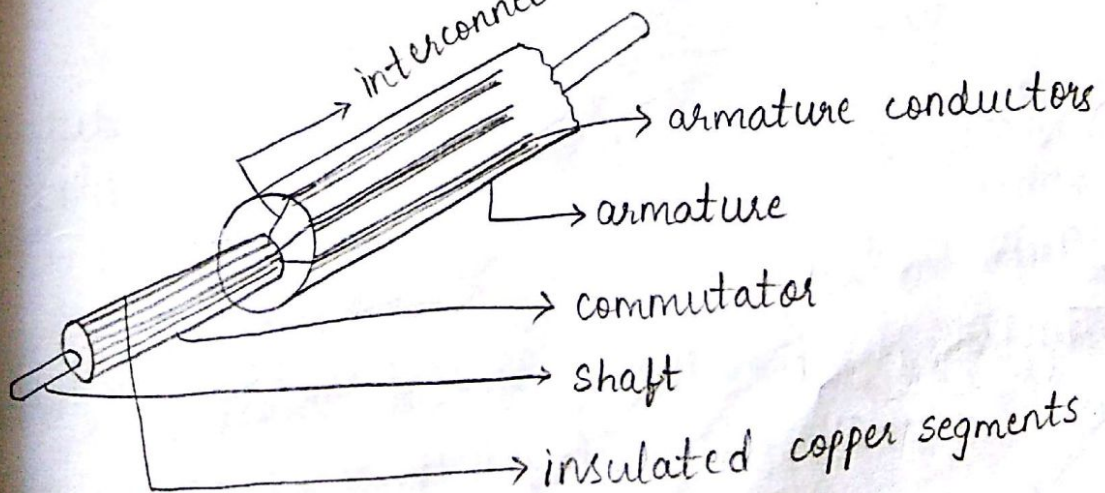
Poles:-



Armature:-



Armature & commutator:-



yoke :- protection
low reluctance
mechanical support

Poles → pole core - windings for producing field
→ pole shoes - spread out the flux

armature → armature core -
→ " windings } material used is
cast steel or cast iron.

commutator - mechanical rectifier (alternator to direct current)

brush - collects the current, they are stationary. Material
from commutator & gives to
external load.

used for making brushes is carbon.

windings - Cu or Al are used as materials.

Parts	Purpose	Material used
Yoke	1) It acts as protective layer for entire machine from the harmful atm. 2) It acts as mechanical support to the poles. 3) It provides the low reluctance path for the flux to avoid wastage of power.	1) Cast iron for smaller machines 2) Silicon steel, cast steel, roller steel for larger machines.
Pole core	It consists of field windings to produce flux.	cast iron (or) cast steel.
Pole shoe	Spreads out the flux and is shaped such that the armature is fit into the system.	Cast iron (or) cast steel
Field windings	To electromagnetise the poles and to produce flux.	Copper
Armature core	Rotating part of a d.c machine	Steel laminations (to avoid eddy current losses).
Armature windings	Placed in the armature slots and are insulated from each other to carry armature current.	Copper (Cu)

Commutator

converts ac voltages induced in the armature conductors into the dc voltages in the external circuit.

Copper segments insulated with mica.

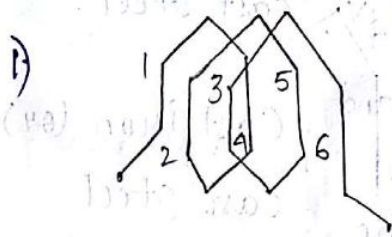
Brush

Collects the current from commutator & delivers it to the load.

Carbon or graphite.

Types of windings:- Depending upon types of winding from armature to commutator, windings are classified into 2 types:-

Lap winding



1) No. of $\parallel^d$ paths (A) is equal to no. of poles (P).

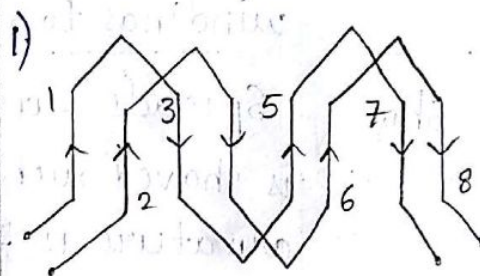
$$A = P.$$

2) No. of brush sets is equal to no. of poles.

3) Preferable for high current, low voltage capacity generators.

4) Normally used for

Wave winding



1) No. of $\parallel^d$ paths (A) is equal to two. (irrespective of no. of poles).

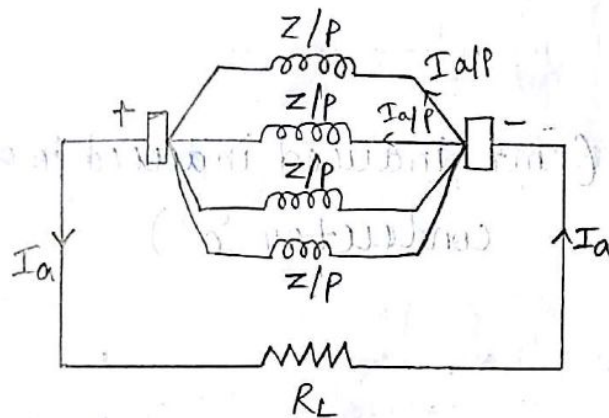
2) No. of $\parallel^d$ paths is equal to 2. (i.e. of no. of brush sets).

3) Preferable for high voltage, low current capacity generators.

4) Preferred for generators

generators of capacity more than 500amp

5) Each $\parallel^l$ path has $\frac{Z}{p}$ conductors in series.
(Z - no. of conductors and p - no. of poles).

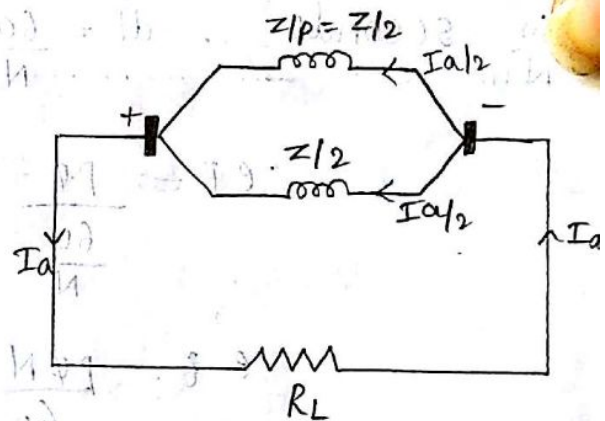


6) Current in each $\parallel^l$ path is I_a/p .

7) EMF in a $\parallel^l$ path is equal to emf per $\parallel^l$ path.

of capacity less than 500 amp.

5) Each $\parallel^l$ path has $\frac{Z}{2}$ conductors in series.



6) Current in each $\parallel^l$ path is $I_a/2$.

7) EMF generated = emf per $\parallel^l$ path.

* EMF equ. of a DC generator:-

Let p = no. of poles of a generator,

ϕ = flux produced by each pole in webers.

N = Speed of armature in rpm.

Z = total no. of armature conductors

A = no. of $\parallel^l$ paths

Avg. value of emf induced in each armature conductor is $E = \frac{d\phi}{dt}$. In one revolution, conductor will cut total flux produced by all the poles. (i.e. $p\phi$) $d\phi = p\phi$

Time required to complete one revolution is $\frac{60}{N}$ seconds. $dt = \frac{60}{N}$

$$\therefore e = \frac{p\phi}{\frac{60}{N}} \quad (\text{emf induced in one conductor 'e'})$$

$$e = \frac{p\phi N}{60}$$

As there are z conductors with A // paths, no. of conductors are in series & emf remains same across all // paths, total emf is

$$E = \frac{p\phi N}{60} \times \frac{z}{A} \quad \text{volts.}$$

where $A = p$ (lap winding)

$A = 2$ (wave winding)

* $\rightarrow$ Single turn or one turn of a coil consists of 2 conductors. Total no. of conductors, $z = 2 \times \text{no. of turns.}$

P1.) A 6-pole lap wound dc generator has 600 conductors on its armature. The flux per pole is 0.02 webers. Calculate

a) the speed at which generator must be run to generate 300V.

b) What would be the speed if the generator were wave wound.

Sol $P = 6$, $Z = 600$, $\phi = 0.02$ webers

$$A = P \Rightarrow A = 6 \text{ (lap)}$$

a) $E = 300V$, $A = 6$

$$E = \frac{P \phi N}{60} \times \frac{Z}{A}$$

$$300 = \frac{6 \times 0.02 \times N \times 600}{60 \times 6}$$

$$N = 1500 \text{ rpm}$$

b) $A = 2$, (wave), $E = 300V$

$$E = \frac{P \phi N}{60} \times \frac{Z}{A}$$

$$300 = \frac{6 \times 600 \times 0.02 \times N}{60 \times 2}$$

$$N = 500 \text{ rpm}$$

2) A 4-pole lap wound dc generator has a useful flux of 0.07 weber/pole. Calculate the generated emf when it is rotated at a speed of 900 rpm with the help of prime mover.

Armature consists of 440 no. of conductors.

Also calculate the generated emf if lap wound armature is replaced by wave wound armature.

Sol $P = 4$, $\phi = 0.07$ webers, $N = 900$ rpm

$$Z = 440, E = ?$$

$$E = \frac{4 \times 0.07 \times 900 \times 440}{60 \times 4} (\because A = P = 4)$$

$$E = 462 \text{ volts}$$

if $A = 2$ (wave wound)

$$E = \frac{4 \times 0.07 \times 900 \times 440}{60 \times 2}$$

$$E = 924 \text{ volts}$$

3) A 4-pole lap wound dc generator has 42 coils with 8 turns/coil. It is driven at 1120 rpm if useful flux per pole is 21 mweber. Calculate the generated emf & Find the speed at which it is to be driven to generate the same emf as

calculate above with wave wound armature

Sol

$$P = 4, \quad Z = 42, \quad N = 1120 \text{ rpm}$$

$$\phi = 21 \times 10^{-3} \text{ Wb}$$

No. of conductors $Z = 2 \times \text{no. of turns}$

$$\text{no. of turns} = \frac{Z}{2} = \frac{42}{2} = 21$$

$$Z = 2 \times 336 \text{ cond.}$$

$$Z = 672$$

$$(i) \quad E = \frac{4 \times 21 \times 10^{-3} \times 1120 \times 672}{60 \times 2}$$

$$E = 263.42 \text{ volts}$$

(ii) $A = 2$ (wave wound)

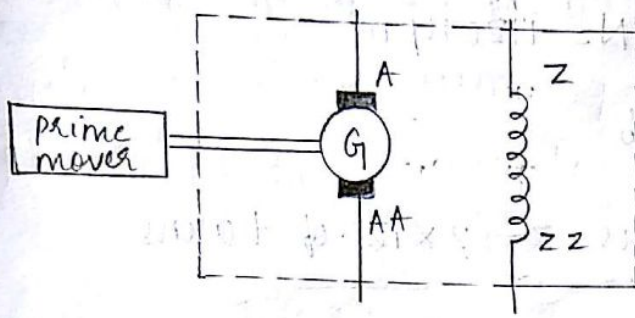
$$E = 263.42, \quad N = ?$$

$$263.42 = \frac{4 \times 21 \times 10^{-3} \times N \times 672}{60 \times 2}$$

$$N = 559.99 \text{ rpm}$$

$$N = 560 \text{ rpm}$$

Symbolic representation of a dc generator:-



Excitation:-

The supplying current to the field winding is called excitation. The way of supplying exciting current is called method of excitation - 2 types.

- (i) Separate excitation (ii) Self excitation

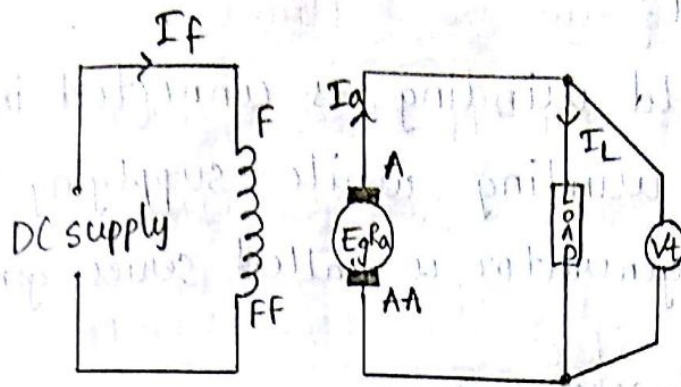
Depending on method of excitation, dc generators are classified as:-

- 1) Separately excited generators
- 2) Self excited generators

1) Separately excited generators:-

When the field winding is supplied from external, separate dc supply i.e. excitation of field winding is separate, then the generator is called separately excited generator.

Schematic representation of separately excited generators is:-



Armature current $I_a = I_L$

Generated emf $E_g = V_t + I_a R_a + V_{brush}$

Terminal voltage $V_t = E_g - I_a R_a$

Power developed in armature $P_a = E_g I_a$

Power delivered to load $P_L = E_g I_a - I_a^2 R_a$

$$P_L = I_a (E_g - I_a R_a)$$

$$P_L = V_t I_a$$

$$P_L = V_t I_L$$

2) Self excited generators:-

When the field winding is supplied from the armature of the generator itself, then it is called self excited generator.

Based on how field winding is connected to the armature, self excited generators are classified into 3 types:-

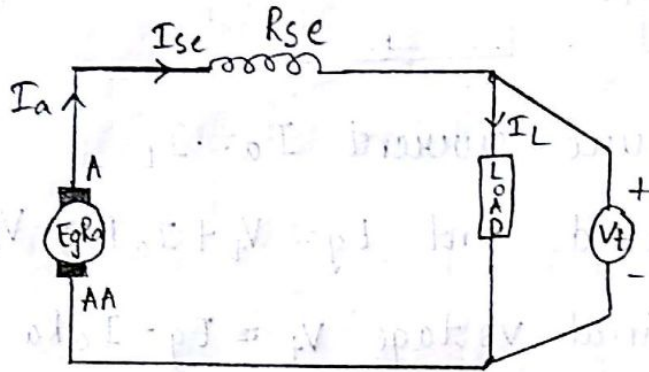
(i) Series generator

(ii) Shunt generator

(iii) Compound generator

(i) Series generator:-

When the field winding is connected in series with armature winding while supplying the load, then the generator is called series generator.



$$I_a = I_{se} = I_L$$

$$E_g = V_t + I_a R_a + I_{se} R_{se} + V_{brush}$$

$$E_g = V_t + I_a (R_a + R_{se})$$

$$V_t = E_g - I_a (R_a + R_{se})$$

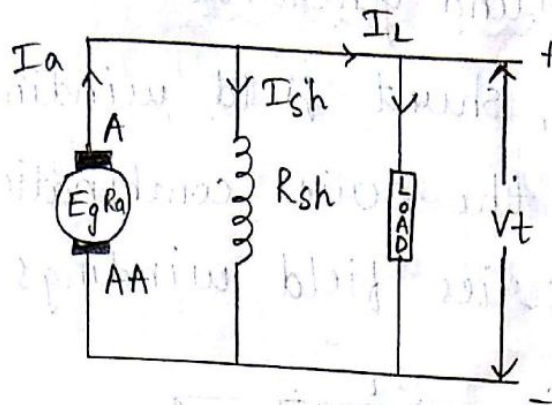
$$P_a = E_g I_a$$

$$P_L = E_g I_a - I_a^2 R_a - I_{se}^2 R_{se}$$

$$P_L = I_a (E_g - I_a (R_a + R_{se})) = I_a V_t$$

(ii) Shunt generator:-

When the field winding is connected in parallel with armature winding ~~while~~ and the combination across the load, then the generator is called shunt generator.



$$I_a = I_{sh} + I_L$$

$$V_t = E_g - I_a R_a$$

$$I_{sh} = \frac{V_t}{R_{sh}}$$

$$P_a = E_g I_a$$

$$P_L = V_t I_L$$

(iii) Compound generators:-

In this type, a part of field winding is connected in // with armature and a part in series with armature. Both series and shunt windings are mounted on the same poles.

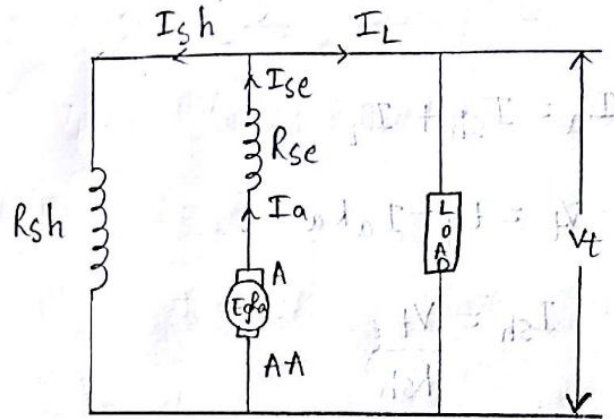
Depending upon the connection of shunt and series winding, compound generator is further classified into:-

a) Long shunt compound generator.

b) Short shunt compound generator.

a) Long shunt compound generator:-

In this type, shunt field winding is connected across the series combination of armature and series field windings.



$$I_a = I_{se} = I_{sh} + I_L$$

$$V_t = E_g - I_a R_a - I_{se} R_{se}$$

$$V_t = E_g - I_a (R_a + R_{se})$$

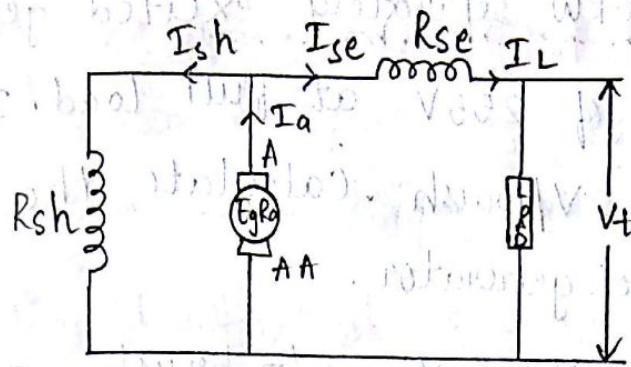
$$I_{sh} = \frac{V_t}{R_{sh}}$$

$$P_a = E_g I_a$$

$$P_L = V_t I_L$$

b) Short shunt compound generator:-

In this type, shunt field winding is connected only across the armature excluding series field windings.



$$I_a = I_{sh} + I_{se}$$

$$I_{se} = I_L, \quad I_{sh} = \frac{V_t + R_{se} I_{se}}{R_{sh}}$$

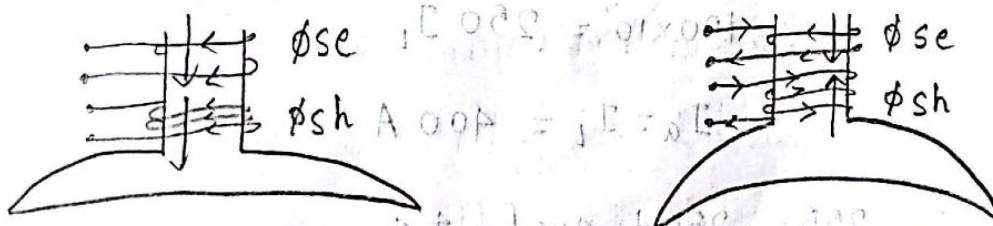
$$V_t + I_{se} R_{se} = E_g - I_a R_a$$

$$V_t = E_g - I_a R_a - I_{se} R_{se}$$

$$P_a = E_g I_a$$

$$P_L = V_t \cdot I_L$$

→ Compound generators are further classified as cumulative compound generators and differential compound generators which are as shown:-

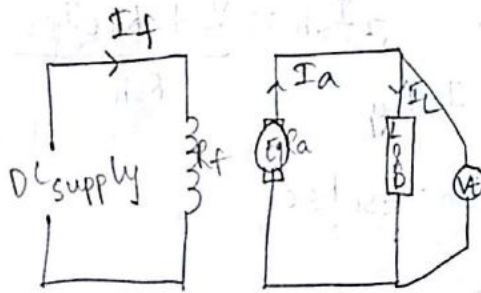


$$\phi_T = \phi_{se} + \phi_{sh}$$

$$\phi_T = \phi_{sh} - \phi_{se}$$

P1) A 250-poles, 100kW separately excited generator has an induced emf of 255V at full load. If the brush drop is 2V/brush. Calculate the armature resistance of the generator.

Sol $E_g = 255V$, $V_{\text{brush}}/\text{brush} = 2 \Rightarrow 2 \times 2 = 4$
 $V_t = 250V$, $P_L = 100kW$



$$I_a = I_L$$

$$E_g = V_t + I_a R_a + V_{\text{brush}}$$

$$255 = 250 + I_a R_a + 4$$

WKT $P = VI_L$

$$100 \times 10^3 = 250 I_L$$

$$I_a = I_L = 400 A$$

$$\therefore 255 = 250 + 400 R_a + 4$$

$$R_a = 2.5 \times 10^{-3} = 0.0025 \Omega$$

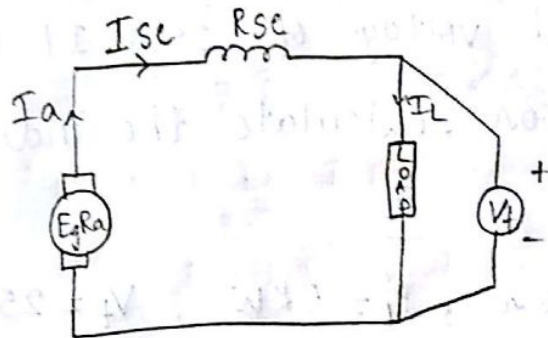
2) A dc series generator has armature resistance of 0.5Ω and series field resistance of 0.03Ω . It drives a load of 50A, if it has 6 turns/coil and total 540 coils on armature and is

driven at 1500 rpm. Calculate terminal voltage at the load. Assume 4 poles lap-type winding flux/pole as 2mwb and total brush drop as 2V.

Sol $R_a = 0.5 \Omega$, $R_{sc} = 0.03 \Omega$, $I_L = 50A$

$V_b = 2V$, $N = 1500 \text{ rpm}$, $p = 4$ (lap) 6 turns/coil

$\therefore 540 \text{ coils} \Rightarrow 3240 \text{ turns}$



$\phi = 2 \text{ mwb}$
 $= 2 \times 10^{-3} \text{ wb}$

Here $I_a = I_{sc} = I_L$

$$E_g = V_t + I_a R_a + I_a R_{sc} + V_{\text{brush}}$$

$E_g = ?$, $V_t = ?$

$$E_g = \frac{\phi Z N}{60} \times \frac{p}{A}$$

$Z = 2 \times \text{no. of turns}$

$$\begin{aligned} \text{no. of turns} &= \frac{6 \text{ turns} \times 540 \text{ coils}}{\text{coils}} \\ &= 3240 \text{ turns} \end{aligned}$$

$$Z = 2 \times 3240 = 6480 \text{ conductors}$$

$$E_g = \frac{2 \times 10^{-3} \times 6480 \times 1500}{60} \times \frac{4}{4}$$

$$E_g = 324$$

$$324 = V_t + 50(0.5) + 50(0.03) + 2^3$$

$$V_t = 295.5$$

3) A 250V dc ^{shunt} generator has shunt field windings resistance of 100Ω, it is supplied to a load of 5kW at a load voltage of 250V. ~~It is supplied~~ If its R_a is ^{0.22Ω} ~~250A~~, calculate the induced emf of generator.

Sol

$$R_{sh} = 100 \Omega, P_L = 5 \text{ kW}, V_t = 250 \text{ V}$$

$$R_a = \frac{0.22 \Omega}{250 \text{ A}}, E_g = ?$$

$$I_a = I_{sh} + I_L \quad \left(I_{sh} = \frac{V_t}{R_{sh}} = 2.5 \text{ A} \right)$$

$$V_t = E_g - I_a R_a \quad P_L = V_t I_L$$

$$250 = E_g - \frac{0.22}{250} (I_a) \quad I_L = 20 \text{ A}$$

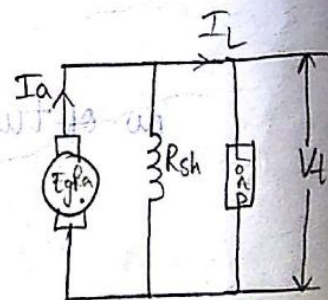
$$I_a = 2.5 + 20$$

$$I_a = 22.5 \text{ A}$$

$$250 = E_g - \frac{0.22}{250} (22.5)$$

$$E_g = 587.5$$

$$E_g = 254.95$$



4) A short shunt compound dc generator supplies a current of 75A at a voltage of 225V. Calculate the generated voltage if $R_a = 0.04 \Omega$, shunt & series

field windings are 90Ω & 0.02Ω resp.

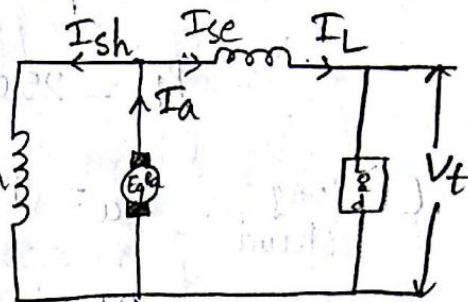
Sol $I_L = 75A$, $V_t = 225$, $E_g = ?$

$$R_a = 0.04\Omega, R_{se} = 0.02\Omega$$

$$R_{sh} = 90\Omega$$

$$V_t = E_g - I_a R_a - I_{se} R_{se} - R_{sh} I_{sh}$$

$$I_{se} = I_L = 75A$$



$$V_t + I_{se} R_{se} = R_{sh} I_{sh} \quad \left| \begin{array}{l} E_g = V_t + I_a R_a + I_{se} R_{se} \\ E_g - I_a R_a = R_{sh} I_{sh} \end{array} \right.$$

$$I_a = I_{sh} + I_{se}$$

$$I_{sh} = \frac{V_t + I_{se} R_{se}}{R_{sh}}$$

$$= 2.51$$

$$I_a = 2.51 + 75$$

$$I_a = 77.51$$

$$\therefore 225 = E_g - 77.51(0.04) - 75(0.02)$$

$$E_g = 229.6$$

5) A compound generator is to supply a load of 250 lamps, each ~~stated~~ rated at $100W, 250V$. The armature, series & shunt windings have res. of $0.06\Omega, 0.04\Omega$ & 50Ω resp. Determine E_g when machine is connected in (i) long shunt (ii) short shunt. Take drop per brush as $1V$.

Sol

$$R_a = 0.06 \Omega, R_{se} = 0.04 \Omega, R_{sh} = 50 \Omega$$

$$E_g = ? \text{ 1 lamp} \rightarrow 100 \text{ W}$$

$$250 \text{ lamps} \rightarrow ? \quad 25000 \text{ W}$$

$$\therefore P_L = 25000 \text{ W}, V_t = 250 \text{ V}$$

(i) long shunt: $I_a = I_{se} = I_{sh} + I_L$

draw the
(diagram)

$$V_t = E_g - I_a R_a - I_{se} R_{se} + V_b$$

$$I_{sh} = \frac{V_t}{R_{sh}} = 5 \text{ A}$$

$$P_L = V_t I_L$$

$$I_L = \frac{25000}{250} = 100 \text{ A}$$

$$I_a = I_{se} = 5 + 100 = 105 \text{ A}$$

$$250 = E_g - 105(0.06) - 105(0.04) + V_{brush} = 2$$

$$E_g = 262.5$$

(ii) Short shunt

$$I_a = I_{sh} + I_{se}$$

$$I_{se} = I_L = 100 \text{ A}, I_{sh} = \frac{V_t + R_{se} I_{se}}{R_{sh}}$$

$$I_{sh} = 5.08 \text{ A}$$

$$I_a = 105.08 \text{ A}$$

$$V_t + I_{se} R_{se} = E_g - I_a R_a + V_{brush}$$

$$250 + 100(0.04) = E_g - (105.08)(0.06) + 2$$

$$E_g = 262.3$$

6) A 100 kW, 240 V shunt generator has a field resistance of 55Ω and armature resistance of 0.067Ω . Find full load rated voltage.

Sol $R_{sh} = 55 \Omega$, $R_a = 0.067 \Omega$, $V_t = 240 V$,

$$P_L = 100 \text{ kW}, E_g = ?$$

$$V_t = E_g - I_a R_a$$

$$I_a = I_{sh} + I_L$$

$$I_{sh} = \frac{V_t}{R_{sh}} = 4.36 \text{ A}$$

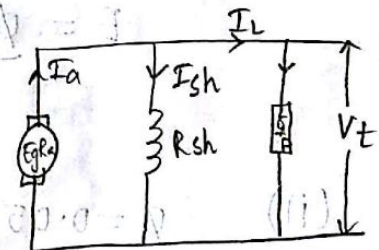
$$P_L = V_t I_L$$

$$I_L = 416.6 \text{ A}$$

$$I_a = 420.96 \text{ A}$$

$$240 = E_g - (420.96)(0.067)$$

$$E_g = 268.2$$



7) A wave wound 6-pole long shunt compound⁷ dc generator has 600 armature conductors, the generator is driven at 300 rpm. Calculate emf generated if flux/pole is 0.06 webers if now, the generator is required to produce emf of 550V at a reduced value of flux/pole of 0.05 webers. Calculate the speed at which the armature of the generator must be driven.

Sol $p = 6$ $\therefore$ wave $\Rightarrow A = 2$

(i) $Z = 600$, $N = 300 \text{ rpm}$, $\phi = 0.06$

$$E = \frac{\phi p N \times Z}{60 \times A} = 540 \text{ V}$$

(ii) $\phi = 0.05$, $E = 550 \text{ V}$, $N = ?$

$$550 = \frac{0.05 \times 6 \times \cancel{300}^N \times 600}{60 \times 2}$$

$$N = 366.66 \text{ rpm}$$

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Characteristics of dc generators:-

The dc generators have following characteristics

- 1) Open circuit characteristics (OCC) / No load char
or magnetisation characteristics :-

It gives the relation b/w generated emf on no-load and field current I_f when machine is driven at its rated speed.

- 2) External char. / Load characteristics :-

It gives the relation b/w terminal voltage V_t and the load current I_L when the machine is driven at rated speed.

- 3) Internal characteristics :-

It gives the relation b/w generated emf with load and load current when the machine is run at rated speed.

The above sets of 3 char. are the same for all types of generators.

Magnetisation characteristics:-

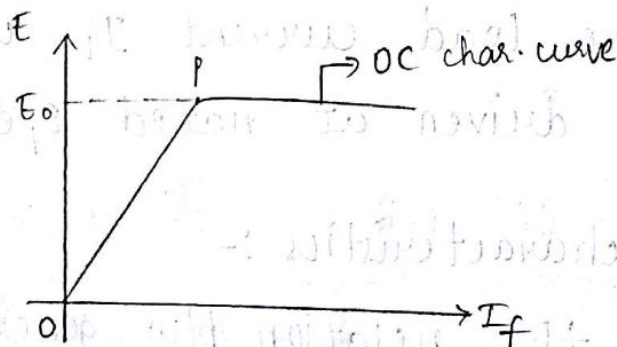
$$E = \frac{\phi Z N}{60} \times \frac{p}{A}$$

$$E_0 \propto \phi$$

$$\phi \propto I_f$$

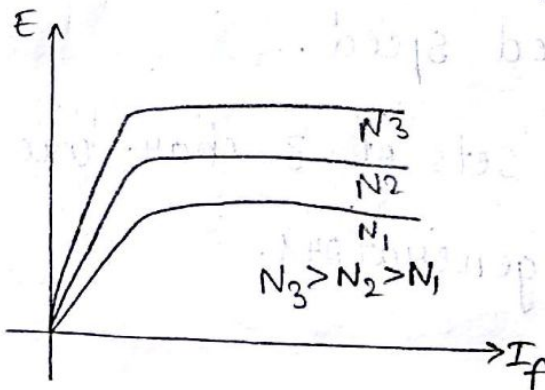
$$\therefore E_0 \propto I_f$$

Thus induced emf increases directly as I_f increases but after certain I_f , core gets saturated and flux also remains constant though I_f increases. The char. are as shown in fig.



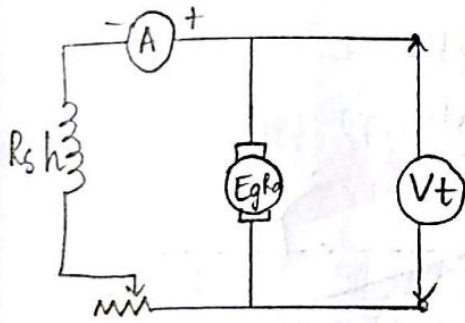
Induced emf also varies with speed. $E \propto N \phi$

i.e. $E \propto N \phi$

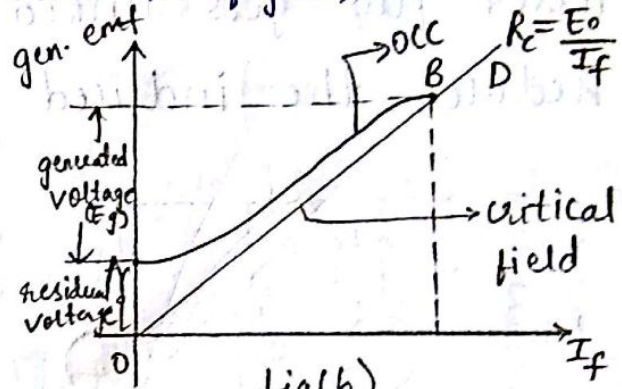


DC char. of dc shunt generator:-

In shunt wound dc generator, field winding is connected in $\parallel^d$ with armature winding as shown in fig (a). The OCC of the generator at constant speed is shown in fig (b).



fig(a)



fig(b)

Due to residual magnetism in the poles, some emf is generated in the armature when $I_f = 0$, so curve starts from 'A' not from origin. The line OD represents critical field resistance. The field current increases until OCC reaches point B. OC is the max. value of emf that can be generated.

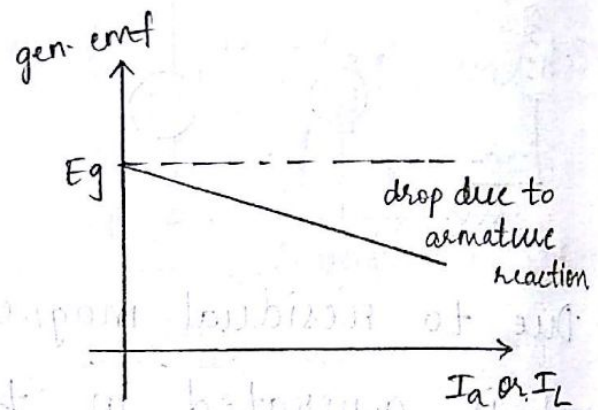
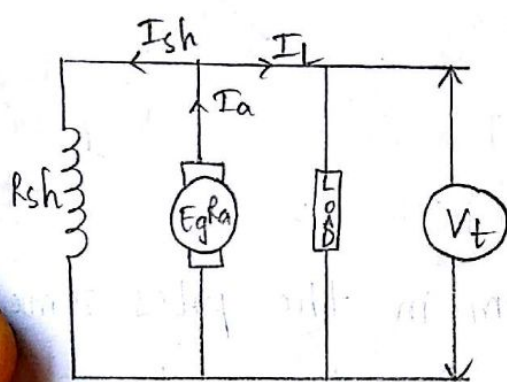
Load char. of dc shunt generator:-

1) Internal characteristics:-

Ideally the induced emf is not dependent on I_L or I_a but as load current increases, armature current I_a increases to supply load demand. As $I_a \uparrow$, armature flux increases.

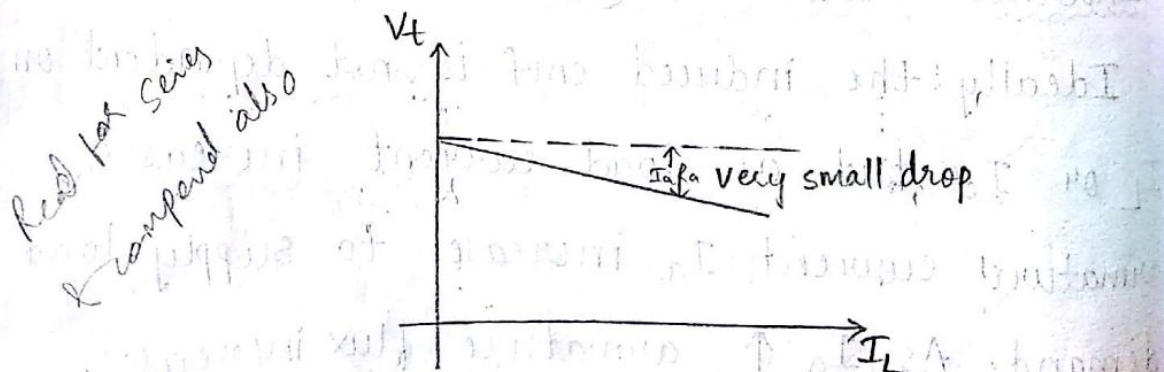
The effect of flux produced by armature on the main flux produced by the field winding is called an armature reaction.

Due to this, main flux gets distorted. Hence lesser flux gets linked with the conductors and reduces the induced emf.



2) External characteristics:-

For a dc generator, $E_g - I_a R_a = V_t$, Neglecting other drops, so as load current I_L increases, I_a increases. Thus the drop $I_a R_a$ $\uparrow$ es and V_t $\downarrow$ es. As the value of R_a is very small, drop in V_t is also very small. Hence dc shunt generator is called constant voltage generator.



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DC Motors

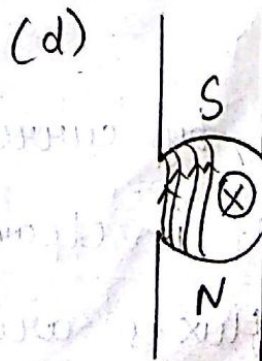
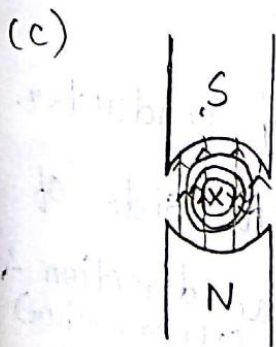
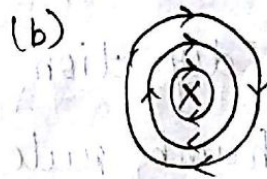
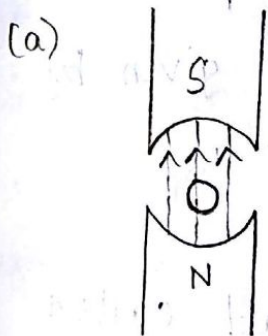
→ A motor is a device which converts electrical energy into mechanical energy. The energy conversion is opposite to that of a generator.

→ In the generator, mechanical energy is supplied by the prime mover while in a dc motor, the electrical energy is supplied by a dc supply.

→ Construction is same as generator.

Principle of a motor:-

When a current carrying conductor is placed in the magnetic field, it experiences mechanical force.



→ The individual force experienced by the conductors acts as twisting or turning force on the armature which is called torque.

→ The torque is the product of force and radius at which the force acts. So overall armature experiences a torque & starts rotating.

→ Consider a single conductor placed in a magnetic field as shown in fig (a). The magnetic field is produced by permanent magnet. (practically it is produced by field winding).

→ Consider a conductor carrying the current away from the observer as in fig (b). Any current carrying conductor produces its own magnetic field around it & hence produces its own flux. The direction of flux is given by right hand thumb rule.

→ There are 2 fluxes:-

(i) flux produced by permanent magnet called main flux.

(ii) flux produced by current carrying conductor.

→ From fig c, it is clear that on one side of conductor both fluxes are in the same direction (add each other) on the other side, fluxes are in opp. direction.

and hence try to cancel each other.

→ There exists high flux density area on the left & low flux density area on right as shown in fig(d).

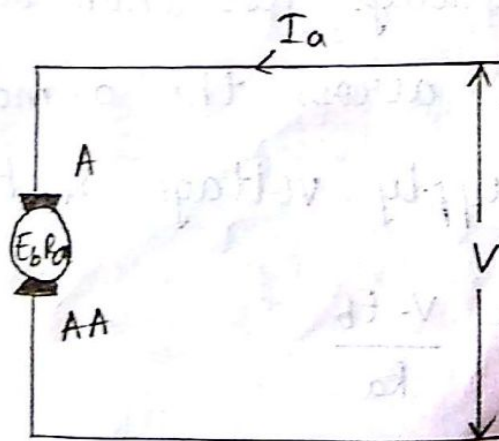
→ The magnitude of force experienced by the conductor in a motor is given by $F = BIL$.

Units: newtons

→ The direction of such force can be determined by Fleming's left hand rule.

Fleming's left hand rule:-

Outstretch 3 fingers of left hand such that each is mutually $\perp$ to each other, point the index finger in the direction of magnetic field, middle finger in direction of current, then thumb gives direction of force experienced by conductor.



After a motoring action, there exists a generating action. There is an induced emf in the rotating armature conductors according to Faraday's law of electromagnetic induction.

The induced emf acts in the opposite direction of supply voltage according to Lenz's law.

Lenz's law:-

The direction of induced emf is always so as to oppose the cause producing it, so this emf is called back emf E_b .

$$E_b = \frac{\phi Z N}{60} \times \frac{P}{A}$$

Voltage equ. of a dc motor:-

$$V = E_b + I_a R_a + V_{\text{brush}}$$

$$V = E_b + I_a R_a$$

Back emf is always less than supply voltage.

The net voltage across the armature is the difference b/w supply voltage & back emf.

$$I_a = \frac{V - E_b}{R_a}$$

Back emf as regulating mechanism:-

Due to the presence of back emf, the dc motor becomes a regulating machine i.e. the motor adjusts itself to draw the armature current just enough to satisfy the load demand.

The basic principle of this is $E_b \propto N$. When load is put on the motor, motor slows down
Speed $\downarrow$, $E_b \downarrow$, net voltage $\uparrow$, $I_a \uparrow$, force $\uparrow$ and torque $\uparrow$.

The increase in the torque is sufficient to satisfy increased load demand. The motor speed stops decreasing when the armature current is enough to produce torque demanded by new load.

When load on the motor is decreased, speed $\uparrow$, $E_b \uparrow$, net voltage $\downarrow$, $I_a \downarrow$, force $\downarrow$ and torque $\downarrow$.

The motor speed stops increasing when the armature current is enough to produce less torque required by new load.

Expression for power equ:-

From voltage equ, we have

$$V = E_b + I_a R_a + V_{\text{brush}}$$

$$V = E_b + I_a R_a$$

$$I_a = \frac{V - E_b}{R_a}$$

$$V = E_b + I_a R_a \text{ --- (1)}$$

$$VI_a = \underbrace{E_b I_a}_{P_m} + I_a^2 R_a$$

$$P_m = VI_a - I_a^2 R_a \text{ --- (2) (Power equ)}$$

where VI_a = electric power supplied to armature

$E_b I_a$ = gross mechanical o/p power developed by armature.

$I_a^2 R_a$ = electric power wasted in the armature (loss).

input = o/p power + losses

o/p = input - losses

Condition for max. power:-

From the above equ, we have

$$P_m = VI_a - I_a^2 R_a$$

For max. power, diff P_m wrt I_a & equate to zero

$$\text{i.e. } \frac{dP_m}{dI_a} = 0$$

$$0 = V - 2I_a R_a$$

$$V = 2I_a R_a$$

$$\boxed{I_a R_a = \frac{V}{2}} \quad \text{--- (3)}$$

Mechanical power developed in armature is max. when back emf is equal to half the applied voltage.

Torque equ. of a dc motor:-

The twisting or the turning force about an axis is called torque.

Consider a wheel of radius 'r' meters acted upon by a circumferential force 'F' newtons. Let the speed of the wheel rotating be 'N' rpm when angular freq of wheel

$$\omega = \frac{2\pi N}{60} \text{ radians per sec.}$$

Work done in one revolution = is

$$W = F \times \text{dist travelled in one revolution}$$

$$W = F \times 2\pi r \text{ Joules}$$

$$P - \text{power developed} = \frac{\text{work done}}{\text{time}} = \frac{F \times 2\pi r}{\frac{60}{N}}$$

$$P = (F \times R) \times \frac{2\pi N}{60}$$

$$P = \gamma \times \omega \quad \text{watts}$$

where γ = torque in 'N' meters

Let γ_a be the cross torque developed by the armature of motor, it is called the armature torque. The cross mechanical power developed in armature is $E_b I_a$.

Power in armature = armature torque $\times \omega$

$$P_m = \gamma_a \times \omega$$

$$\text{WKT } E_b I_a = \gamma_a \times \omega$$

$$\text{WKT } E_b = \frac{\phi Z N}{60} \times \frac{P}{A}$$

$$\therefore E_b I_a = \gamma_a \times \omega$$

$$\frac{\phi Z N}{60} \times \frac{P}{A} \times I_a = \gamma_a \times \frac{2\pi N}{60}$$

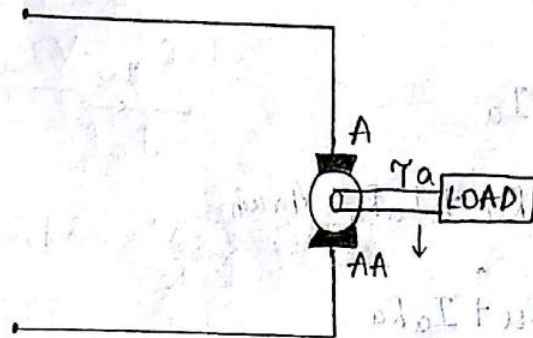
$$\gamma_a = \frac{1}{2\pi} \phi I_a \cdot \frac{P Z}{A}$$

$$\gamma_a = 0.159 \phi I_a \frac{Z P}{A} \text{ Nm}$$

Note: Total armature torque is equal to sum of less torque and load torque.

$$\gamma_a = \gamma_f + \gamma_{sh}$$

Types of torque in motor:-



The mechanical power developed in armature is transmitted to the load through the shaft of a motor. There is a power loss due to friction and iron loss, to overcome these losses the torque required is called lost torque τ_f (stray loss).

The torque which is available at shaft for doing useful work is called load torque or shaft torque (τ_{sh} or τ_L).

$$\tau_a = \tau_f + \tau_{sh}$$

Net o/p of motor $\tau_{out} = \tau_{sh} \times \omega$

Types of motors:-

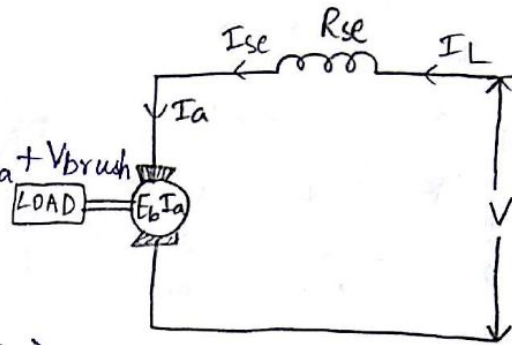
1) Series motor:-

$$I_L = I_{se} = I_a$$

$$V_b = E_b + I_{se} R_{se} + I_a R_a + V_{brush}$$

$$V = E_b + I_{se} R_{se} + I_a R_a$$

$$V = E_b + I_a (R_{se} + R_a)$$



2) Shunt motor:-

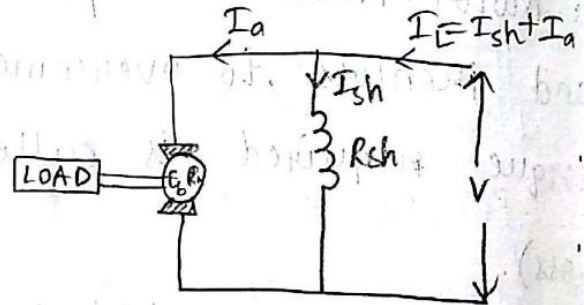
$$I_L = I_{sh} + I_a$$

$$V = E_b + I_a R_a + V_{brush}$$

$$V = E_b + I_a R_a$$

$$V = I_{sh} R_{sh}$$

$$I_{sh} = \frac{V}{R_{sh}}$$



3) Compound motor:-

They are of 2 types:-

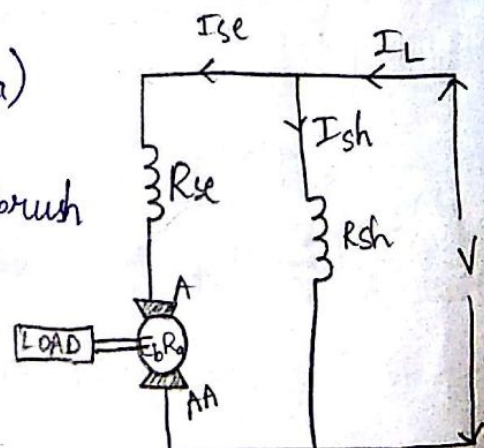
(i) Long shunt:-

$$I_L = I_{se} + I_{sh} \quad (I_{se} = I_a)$$

$$V = E_b + I_a R_a + I_{se} R_{se} + V_{brush}$$

$$V = E_b + I_a (R_a + R_{se})$$

$$P_a = E_b I_a$$



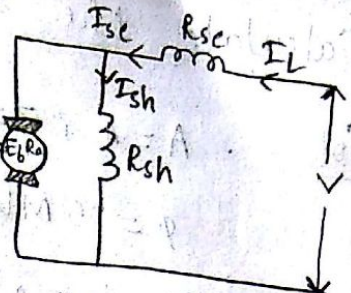
(ii) Short Shunt:-

$$V - I_{se} R_{se} = E_b$$

$$I_L = I_{se} = I_a + I_{sh} + I_{aRa} + V_{\text{brush}}$$

$$V = E_b + I_{se} R_{se} + I_a R_a$$

$$I_{sh} = \frac{V - I_{se} R_{se}}{R_{sh}}$$



Pl) A 4-pole DC motor takes 50A armature current, the armature has lap connected 480 conductors. The flux per pole is 20 Mwb. Calculate the brass torque developed by armature of the motor.

Sol

$$\tau = 0.159 \phi I_a \frac{Zp}{A}$$

$$Z = 480, \quad A = p = 4$$

$$I_a = 50, \quad \phi = 20 \times 10^{-3} \text{ Wb}$$

$$\tau = \frac{0.159 \times 50 \times 20 \times 10^{-3} \times 480}{4}$$

$$\tau = 76.32 \text{ Nm}$$

2) A 4-pole 250V DC series motor has a wave connected armature with 200 conductors the flux per pole is 25 Mwb (when motor is drawing 60A from the supply armature resistance is 0.15Ω

while series field winding resistance is 0.5Ω .
Calculate the speed under this condition.

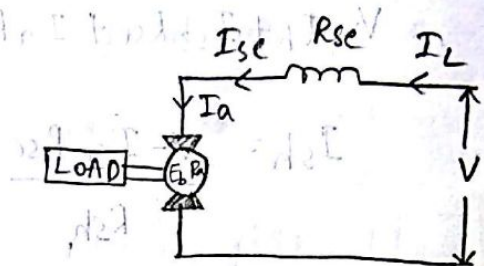
Sol

$$A = 4, Z = 200$$

$$\phi = 25 \text{ mWb}, I_a = 60$$

$$R_a = 0.15 \Omega, R_{se} = 0.2 \Omega$$

$$E_b = \frac{\phi Z n}{60} \times \frac{P}{A}$$



$$E_b = 250 - 60 (0.15 + 0.2)$$

$$= 229 \text{ V}$$

$$N = \frac{229 \times 60 \times 2}{25 \times 10^{-3} \times 200 \times 4}$$

$$N = 1379 \text{ rpm}$$

3.) A 250V DC shunt motor takes a line current of 200 amp, resistance of shunt field winding of 200Ω & $R_a = 0.3 \Omega$. Find armature current & back emf

Sol

$$I_L = I_{sh} + I_a$$

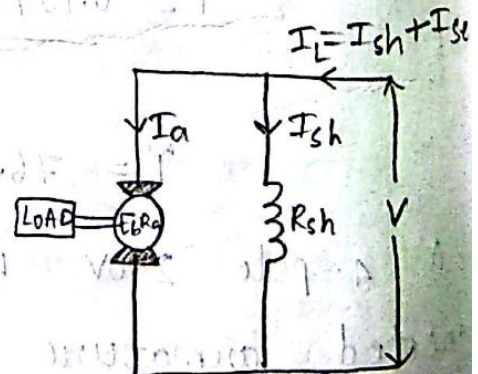
$$R_a = 0.3$$

$$V = E_b + I_a R_a$$

$$E_b = V - I_a R_a$$

$$E_b = 250 - 20 (200 \times 0.3)$$

$$I_{sh} = \frac{250}{200} = 1.25$$



Losses in a DC machine:-

They are classified into 3 types:-

- 1) Copper losses $\left\{ \begin{array}{l} \rightarrow \text{armature Cu-loss} \\ \rightarrow \text{series Cu-loss} \\ \rightarrow \text{shunt field Cu-loss} \end{array} \right.$
- 2) Iron/core losses $\left\{ \begin{array}{l} \rightarrow \text{hysteresis loss} \\ \rightarrow \text{eddy current loss} \end{array} \right.$
- 3) Mechanical losses $\left\{ \begin{array}{l} \rightarrow \text{windage loss} \\ \rightarrow \text{frictional loss} \end{array} \right.$

Constant losses:-

- (i) Iron loss
 - (ii) Mechanical loss
 - (iii) Shunt field Cu-loss
- } stray losses

Copper losses:-

The copper losses are the losses taking place due to current flowing in the winding there are basically 2 windings in a DC machine (armature & field winding).

1) Armature Cu-loss:-

$$\text{Series field Cu-loss} = I_{se}^2 R_{se}$$

$$\text{Shunt field Cu-loss} = I_{sh}^2 R_{sh}$$

Iron or core losses:-

These losses are also called as magnetic losses, these includes hysteresis loss or eddy current loss. These are due to the rotation of armature in the magnetic field of poles.

1) Hysteresis loss:-

When piece AB under north pole the magnetic lines pass from a to b after half revolution the same piece of iron is under south pole the magnetic lines pass from b to a so that magnetism in iron is reused. The continuous reversal of magnetism in the lead to some amount of power loss which is called hysteresis law.



$$P_h = B_{\max} f v \quad K_h - \text{hysteresis loss const.}$$
$$= K_h B_{\max}^{1.6} f v$$

$B_{\max}$ - max magnetic flux density in armature

f - frequency, v - volume of armature

Eddy current laws:-

When armature rotates in magnetic field of poles & emf induced at which calculates eddy current in armature core, the power loss due to these eddy current is called eddy current loss.

$$P_E = K_E B_{\max}^2 f^2 t^2 V$$

where P_E = thickness of lamination

K_E = eddy current constant

Mechanical loss :-

These are due to friction and windage.

Windage loss :-

i.e. Air friction of rotating armature.

Friction loss :-

Brush friction, bearing friction.

$$\text{Iron losses} + \text{mech. losses} = \text{stray losses}$$

Constant losses :- (Stray, shunt field losses)

The losses in the DC generator which remains constant at all loads are known as constant losses.

(i) Iron loss (ii) Mech. loss (iii) Shunt field Cu. loss

Variable loss :-

The losses in a machine which varies the load is called variable loss.

(i) Cu. loss in armature winding

(ii) Cu. loss in series field winding

Total losses = constant loss + variable loss

P1) A counter emf of a shunt motor is 227V, the field resistance is 160 Ω & field resistance current is 1.5A, if line current is 39.5A, find armature current when motor is stationary.

Sol

$$E_b = V = 227V, R_{sh} = 160 \Omega$$

$$I_{sh} = 1.5A, I_L = 39.5A$$

$$R_a = ?, I_a = ?$$

$$E_b = V - I_a R_a$$

$$V = E_b + I_a R_a$$

$$I_L = I_{sh} + I_a$$

$$I_a = 39.5 - 1.5$$

$$= 38A$$

$$V = I_{sh} R_{sh}$$

$$= 1.5 \times 100 = 240V$$

$$V = E_b + I_a R_a$$

$$R_a = 0.392 \Omega$$

$E_b = 0$ motor is kept stationary!

$$V = E_b + I_a R_a$$

$$240 = 0 + I_a (0.392)$$

$$I_a = 701.7 A$$

P2) A 20KHz 250V DC shunt generator by armature & field resistance of 0.1Ω & 125Ω respectively, calculate total armature power developed when running as

(i) generator has delivering 20KW o/p?

(ii) has a motor taking 20KW i/p

Sol — $f = 20\text{Hz}$, $V_t = 250\text{V}$, $R_a = 0.1\Omega$
 $R_{sh} = 0.1\Omega$, $R_{sh} = 125\Omega$, $P_a = ?$

(i) $P_a = E_g I_a$, $P_L = 20\text{KW}$

$$I_a = I_{sh} + I_L$$

$$P_L = V_t I_L$$

$$I_L = \frac{P_L}{V_t} = \frac{20 \times 10^3}{250} = 80\text{A}$$

$$I_L = 80\text{A}$$

$$I_{sh} = \frac{V_t}{R_{sh}} = \frac{250}{125} = 2\text{A}$$

$$I_a = 2 + 80$$

$$= 82\text{A}$$

$$E_g = V_t + I_a R_a$$

$$E_g = 250 + 82 \times 0.1$$

$$= 258.2\text{V}$$

$$\therefore P_a = 21.172\text{KW}$$

$$(ii) P = 20 \text{ kW}$$

$$P_a = E_b I_a$$

$$I_L = I_{sh} + I_a$$

$$P = V I_L$$

$$I_L = \frac{P}{V} = \frac{20 \times 10^3}{250}$$

$$I_L = 80 \text{ A}$$

$$I_{sh} = \frac{V}{R_{sh}}$$

$$= \frac{250}{125}$$

$$I_{sh} = 2 \text{ A}$$

$$I_a = I_L - I_{sh}$$

$$I_a = 80 - 2$$

$$I_a = 78 \text{ A}$$

$$E_b = 250 - 78 \times 0.1 \text{ V}$$

$$= 242.2 \text{ V}$$

$$P_a = E_b I_a$$

$$= 242.2 \times 78$$

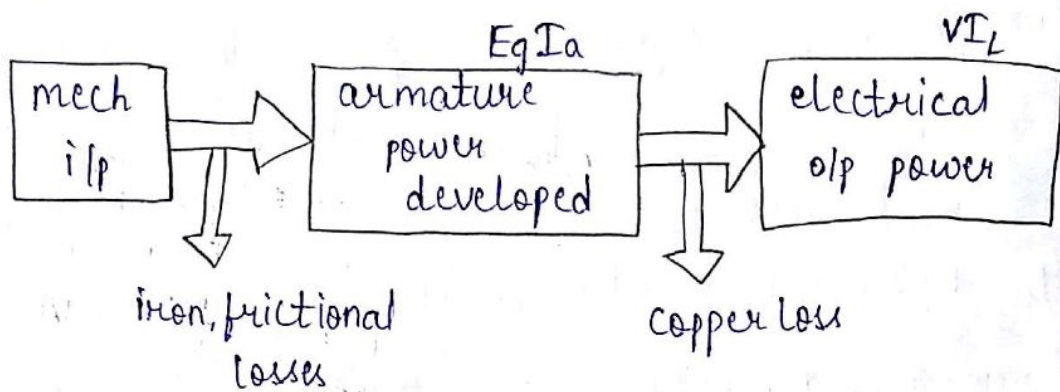
$$P_a = 18.891 \text{ kW}$$

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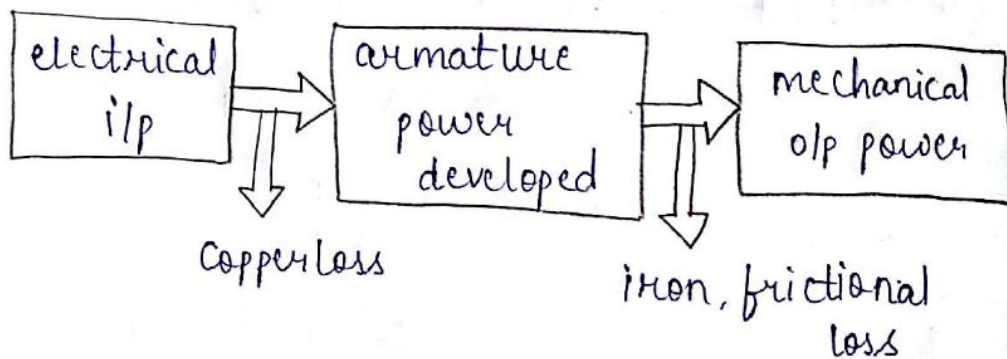
Efficiency of the machine:-

The power flow and energy transformation at various stages which takes place are represented diagrammatically as shown in fig.

(i) Generator :-



(ii) Motor :-



$$\% \eta \text{ (efficiency)} = \frac{\text{total o/p}}{\text{total i/p}} \times 100$$

$$= \frac{\text{o/p power} + \text{losses}}{\text{o/p power} + \text{losses}} \times 100$$

$$\% \eta = \frac{\text{i/p} - \text{losses}}{\text{i/p}} \times 100$$

$$\text{Mech. efficiency, } \eta_m = \frac{E_g I_a}{\text{mech i/p}}$$

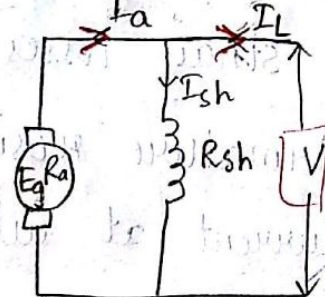
Electrical efficiency $\eta_e = \frac{VI_L}{E_g I_a} \times 100$

Overall efficiency $\eta = \frac{\text{electrical o/p power}}{\text{mech i/p power}}$

$\eta = \frac{VI_L}{\text{mech i/p}}$

Consider a shunt generator

$\eta = \frac{VI_L}{VI_L + W_c + I_a^2 R_a}$



∴ No & DA with $V \cdot I_L$

$I_a = I_L + I_{sh}$

$\eta = \frac{V \cdot I_L}{1 + \frac{W_c}{V \cdot I_L} + \frac{I_a^2 R_a}{V \cdot I_L}}$

$\eta = \frac{1}{1 + \frac{W_c}{V \cdot I_L} + \frac{I_L R_a}{V}}$

The efficiency is max when denominator is minimum.

i.e. $\frac{d\eta}{dI_L} = 0$

$\frac{d}{dI_L} \left[1 + \frac{W_c}{V \cdot I_L} + \frac{I_L R_a}{V} \right] = 0$

$$0 + \frac{w_c}{V I_L^2} = \frac{R_a}{V}$$

$$w_c = I_L^2 R_a$$

i.e. constant loss = variable losses

Pl.) The shunt generator delivers full load current of 200 amp at 240V. Shunt field resistance is 60Ω & full load efficiency is 90%. The stray losses are 800 watts. Find

(i) Armature resistance

(ii) Current at which max. efficiency occurs.

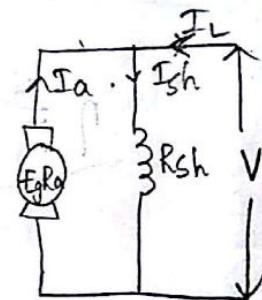
Sol $I_L = 200 \text{ amp}$, $V = 240 \text{ V}$, $R_{sh} = 60\Omega$,

$$\% \eta = 90\% = \frac{o/p}{i/p} \times 100$$

Stray losses = 800 watts.

$$R_a = ?$$

$$V_t = E_g - I_a R_a$$



Total loss = Const. loss + variable loss

$$\begin{aligned} \text{Total loss} &= V \cdot I_L \rightarrow o/p \\ &= 48000 = 48 \text{ kW} \end{aligned}$$

$$90 = \frac{48}{i/p} \times 100 \Rightarrow i/p = 53.3 \text{ kW}$$

$$\text{Total losses} = i/p - o/p$$

$$= 53.3 - 48$$

$$= 5.33 \text{ kW}$$

$$\text{Total losses} = \text{const} + \text{variable losses}$$

$$= \text{stray loss} + \text{shunt field Cu. loss} + \text{variable loss}$$

$$5.33 \times 10^3 = 800 + I_{sh}^2 R_{sh} + I_a^2 R_a$$

$$= 800 + \frac{V^2}{(R_{sh})} + I_a^2 R_a$$

$$\text{WKT } I_a = I_L + I_{sh}$$

$$= 200 + \frac{V}{R_{sh}}$$

$$I_a = 204$$

$$5.33 \times 10^3 = 800 + 16 \times 60 + (204)^2 \times R_a$$

$$\therefore R_a = 0.085$$

(ii)

$$w_c = I_L^2 R_a$$

$$\text{const. loss} = \text{variable loss}$$

$$\text{stray + shunt field Cu. loss} = I_a^2 R_a$$

$$800 + I_{sh}^2 R_{sh} = I_L^2 R_a$$

$$800 + 4^2 \times 60 = I_L^2$$

$$I_L = 0.085$$

$$I_L = 1438 \text{ amp}$$

2.) A 6-pole 500 volts wave connected shunt motor has 1200 armature conductors & useful flux/pole of 20 Mwb. Armature & field resist are 0.5Ω & 250Ω . What will be the speed and torque developed by the motor when it draws 20 amp from supply, neglect armature reaction. If magnetic & mechanical losses are 900 watts, find

(i) useful torque

(ii) efficiency at this load.

Sol $p = 6$, $V = 500V$, $Z = 1200$, $I_L = 20\text{ amp}$

$\phi = 20\text{ Mwb}$, $R_a = 0.5\Omega$, $R_{sh} = 250\Omega$.

$$E_b = \frac{\phi Z N}{60} \times \frac{P}{A}$$

$$V = E_b + I_a R_a$$

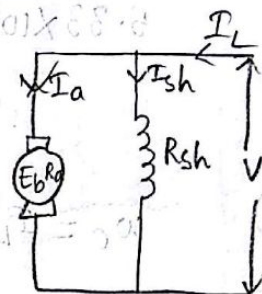
$$E_b = V - I_a R_a$$

$$I_a = I_L - I_{sh}$$

$$I_a = 20 - \frac{V}{R_{sh}}$$

$$I_a = \frac{18}{22} \text{ amp}$$

$$E_b = 500 - 0.5 \times \frac{18}{22} = 489.491$$



$$V = I_a R_a + I_{sh} R_{sh} + E_b$$

$$\frac{491}{489} = \frac{20 \times 10^{-3} \times 1200 \times N \times 6}{60 \times 2}$$

$$N = \frac{409.16}{407.5} \text{ rpm}$$

(i) Torque (T) = $0.159 \phi I_a \times \frac{ZP}{A}$

$$T_a = 206.06 \text{ Nm}$$

Useful torque $T_{sh} = \frac{P_{out}}{\omega}$

o/p power = $P_m - \text{losses}$

$$P_m = E_b I_a - 900$$

$$P_m = 7938$$

$$\omega = \frac{2\pi N}{60} = 42.8$$

$$\therefore T_{sh} = \frac{7938}{42.8} = 185.46 \text{ Nm}$$

(ii) $\% \eta = \frac{o/p}{i/p} \times 100$

$$i/p \text{ power} = V \times I_L$$

$$= 500 \times 20$$

$$= 10000$$

$$\% \eta = \frac{7938}{10000} \times 100$$

$$= 79.38 \%$$

Speed control of DC motors:-

(7)

$$E_b = \frac{\phi Z N}{60} \times \frac{P}{A}$$

(for material)

$$E_b \propto \phi N$$

$$N \propto \frac{E_b}{\phi}$$

$$\text{WKT } E_b = V - I_a R_a$$

$$N \propto \frac{(V - I_a R_a)}{\phi}$$

$\therefore R_a$ is very small, it is neglected.

From the ^{below} above equ. speed can be controlled in 2 methods.

$$N \propto \frac{V}{\phi}$$

(i) Armature or voltage control method

(ii) Flux control method

Armature control method:-

