

Unit: 3 Filters & Symmetrical attenuators

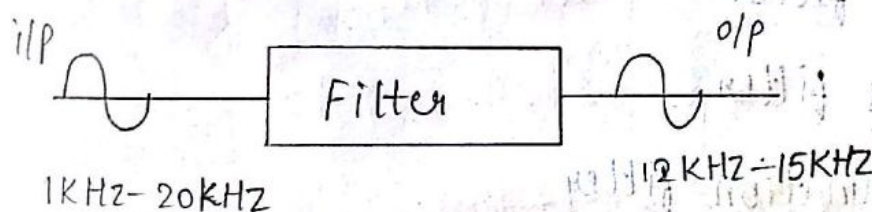
B-1-16

Filters:- * Filter is a pure reactive n/w

A filter is a network which allows ^{certain} range of frequencies to pass through it and eliminates the remaining unwanted frequencies from i/p.

→ The range of frequencies which are passing through the filter is known as pass band or transmission band.

→ The range of frequencies which are eliminated by the filter is known as stop band or attenuation band.



pass band - 12 KHz to 15 KHz

stop band - $1 < f < 12$, $15 < f < 20$

Cut off frequency:-

It is a designated value of frequency which separates the pass band and stop band. It is denoted by f_c , it may be a single or multiple frequency depending on type of filter used.

Characteristics of filters:-

- 1) Filter should possess 0 attenuation in the transmission band i.e. $\alpha = 0$ where α is attenuation.
- 2) The attenuation in stop band should be infinite.

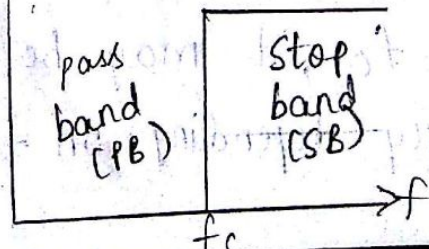
Classification of filters:-

Based on range of frequencies transmitted through a filter, filters are classified as

- 1) Lowpass filter
- 2) Highpass filter
- 3) Bandpass filter
- 4) Band elimination filter

1) Lowpass filter:-

A filter which allows frequencies below the designated frequency & eliminates the above frequencies is called as LPF.

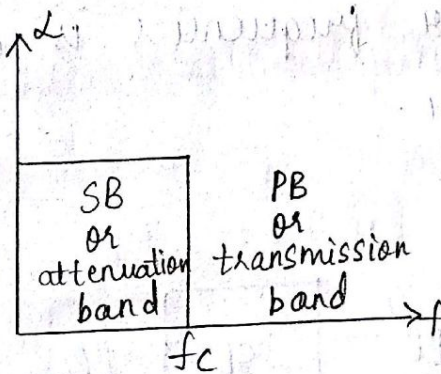


pass band : $0 < f < f_c$

stop band : $f > f_c$

2) Highpass filter:-

A filter which allows the frequencies above the designated frequency and eliminates the below frequencies is called as highpass filter.

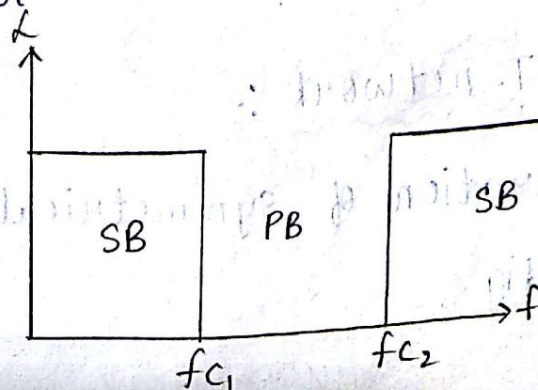


pass band: $f > f_c$

stop band: $0 < f < f_c$

3) Bandpass filter:-

A filter which allows a certain range of frequencies b/w 2 designated ~~values~~ frequencies and eliminates remaining frequencies is called bandpass filter.



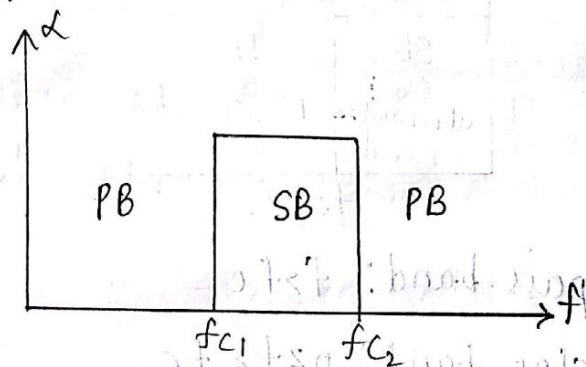
pass band: $f_{c1} < f < f_{c2}$

stop band: $0 < f < f_{c1}$

$f > f_{c2}$

4) Band elimination filter:-

A filter which eliminates a certain range of frequencies in b/w 2 designated frequencies and allows other frequencies is called as band elimination filter.



pass band: $0 < f < f_{c1}$; $f > f_{c2}$

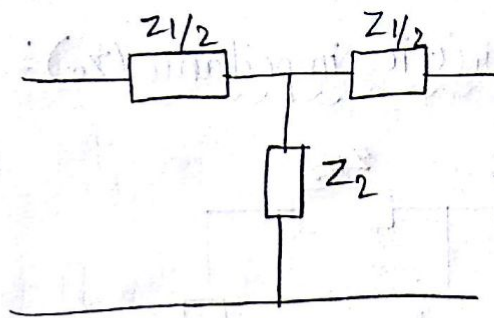
stop band: $f_{c1} < f < f_{c2}$

Filter networks:-

The filter networks are usually of 2 types:-

1) Symmetrical T-network:-

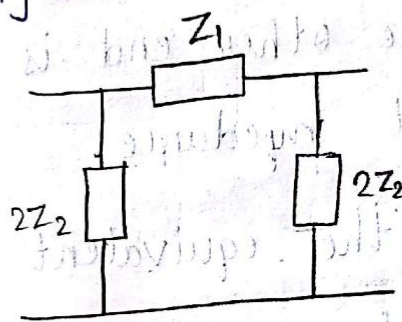
The configuration of symmetrical T-n/w is shown in fig.



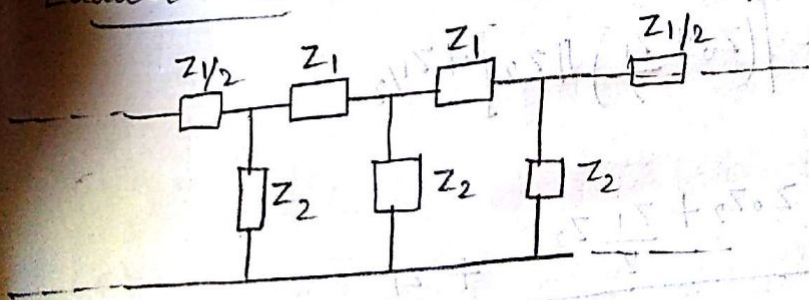
where Z_1 is series arm impedance,
 Z_2 is shunt arm impedance.

2) Symmetrical π -network:-

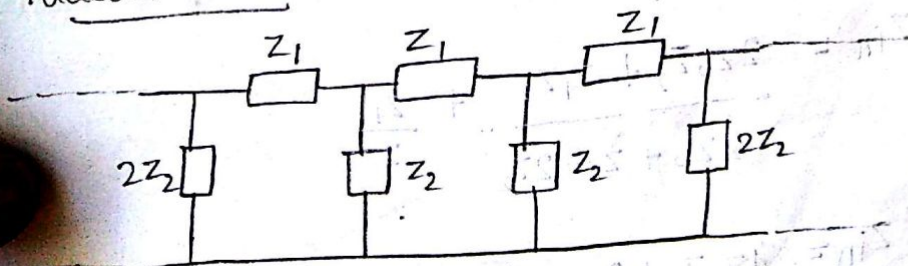
The configuration of symmetrical π -n/w is shown in fig.



Ladder T-n/w:-

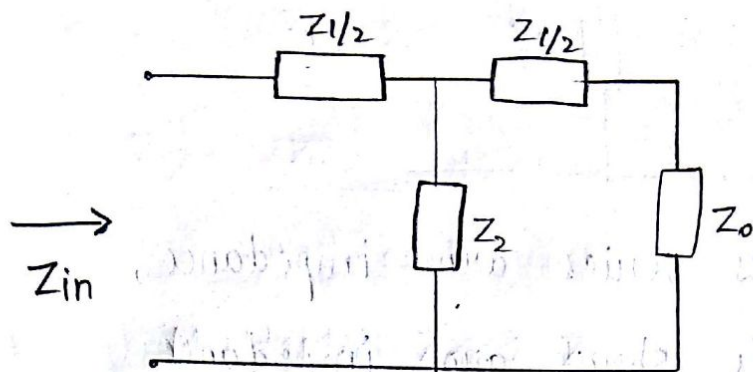


Ladder π -n/w:-



*(written at the back)

Value of characteristic impedance (Z_0):-



Consider a symmetrical T-n/w as shown in fig. The char. impedance is defined as the equivalent impedance of the 2-port n/w. at one end when the other end is terminated with its equivalent impedance.

Here Z_{in} is the equivalent impedance considered from the port 2 terminals.

$$Z_{in} = \left[\left(Z_0 + \frac{Z_1}{2} \right) \parallel Z_2 \right] + \frac{Z_1}{2}$$

$$Z_{in} = \frac{Z_0 Z_2 + \frac{Z_1}{2} Z_2}{Z_0 + \frac{Z_1}{2} + Z_2} + \frac{Z_1}{2}$$

$$Z_{in} = \frac{2Z_0 Z_2 + Z_1 Z_2}{2Z_0 + Z_1 + 2Z_2} + \frac{Z_1}{2}$$

$$Z_{in} = \frac{4Z_0 Z_2 + 2Z_1 Z_2 + 2Z_0 Z_1 + Z_1^2 + 2Z_1 Z_2}{4Z_0 + 2Z_1 + 4Z_2} \quad \text{--- ①}$$

As the n/w is symmetrical, $z_o = z_{in}$:

Sub $z_{in} = z_o$ in above equ.

$$Z_o = z_{in}$$

$$\textcircled{1} \Rightarrow 4Z_o^2 + 2Z_o Z_1 + 4Z_o Z_2 = 4Z_o Z_2 + 4Z_1 Z_2 + 2Z_o Z_1 + Z_1^2$$

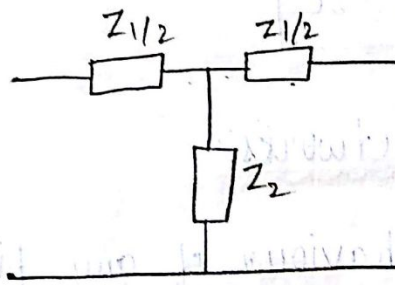
$$\Rightarrow 4Z_o^2 = 4Z_1 Z_2 + Z_1^2$$

$$\Rightarrow Z_o^2 = Z_1 Z_2 + \frac{Z_1^2}{4}$$

$$Z_{OT} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}$$

Characteristic impedance can also be found in terms of OC impedance & SC impedance.

OC port 2 terminals of T-n/w,

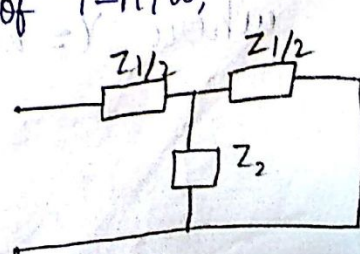


$$Z_{OC} = \frac{Z_1}{2} + Z_2$$

$$Z_{OC} = \frac{Z_1 + 2Z_2}{2}$$

SC port 2 terminals of T-n/w,

$$Z_{SC} = \left[\frac{Z_1}{2} \parallel Z_2 \right] + \frac{Z_1}{2}$$



$$Z_{SC} = \frac{Z_1 Z_2 / 2}{\frac{Z_1 + Z_2}{2}} + \frac{Z_1}{2}$$

$$Z_{SC} = \frac{Z_1 Z_2}{Z_1 + 2Z_2} + \frac{Z_1}{2}$$

Multiply Z_{OC} & Z_{SC} ,

$$Z_{OC} Z_{SC} = \left[\frac{Z_1 + 2Z_2}{2} \right] \left[\frac{Z_1 Z_2}{Z_1 + 2Z_2} + \frac{Z_1}{2} \right]$$

$$= \left[\frac{Z_1 + 2Z_2}{2} \right] \left[\frac{2Z_1 Z_2 + Z_1^2 + 2Z_1 Z_2}{2(Z_1 + 2Z_2)} \right]$$

$$Z_{OC} Z_{SC} = \frac{Z_1^2}{4} + Z_1 Z_2 = Z_{OT}^2$$

$$\therefore Z_{OT} = \sqrt{Z_{OC} \cdot Z_{SC}}$$

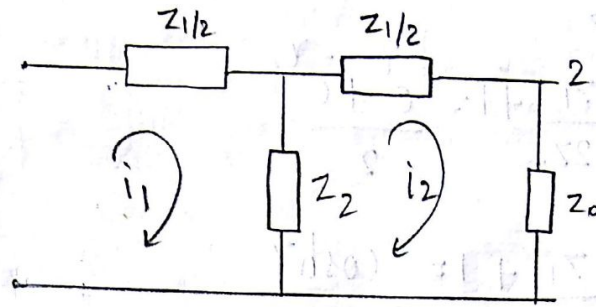
Equations of filter networks:-

The study of behaviour of any filter requires the calculation of its propagation constant (γ), phase shift (β), attenuation (α) and characteristic impedance (Z_0)

$$\text{where } \gamma = \alpha + j\beta$$

*(should be written at back)

2) Propagation constant of a symmetrical T-n/w:-



$$\gamma = \ln \left[\frac{i_1}{i_2} \right] \Rightarrow \frac{i_1}{i_2} = e^\gamma$$

Apply KVL for 2nd loop,

$$Z_0 i_2 + \frac{Z_1}{2} i_2 + Z_2 (i_2 - i_1) = 0$$

$$i_2 \left(Z_0 + \frac{Z_1}{2} + Z_2 \right) = Z_2 i_1$$

$$\Rightarrow \frac{i_1}{i_2} = \frac{Z_0 + \frac{Z_1}{2} + Z_2}{Z_2} = e^\gamma$$

$$\Rightarrow Z_0 + \frac{Z_1}{2} + Z_2 = Z_2 e^\gamma$$

$$\Rightarrow (Z_0)^2 = \left[Z_2 (e^\gamma - 1) - \frac{Z_1}{2} \right]^2$$

$$\Rightarrow Z_0^2 = Z_2^2 (e^{2\gamma} + 1 - 2e^\gamma) + \frac{Z_1^2}{4} - \cancel{\frac{Z_1}{2}} Z_2 (e^\gamma - 1)$$

$$\Rightarrow \frac{Z_1^2}{4} + \cancel{\frac{Z_1}{2}} Z_2 = Z_2^2 (e^{2\gamma} + 1 - 2e^\gamma) + \frac{Z_1^2}{4} - \cancel{\frac{Z_1}{2}} Z_2 e^\gamma + \cancel{\frac{Z_1}{2}} Z_2$$

$$\Rightarrow Z_2^2 (e^{2\gamma} + 1 - 2e^\gamma) = Z_1 Z_2 e^\gamma$$

$$\Rightarrow \frac{Z_1}{Z_2} e^\gamma = e^{2\gamma} + 1 - 2e^\gamma$$

$$\Rightarrow \frac{Z_1}{Z_2} + 2 = e^{\gamma} + e^{-\gamma}$$

$$\Rightarrow \frac{Z_1}{2Z_2} + 1 = \frac{e^{\gamma} + e^{-\gamma}}{2}$$

$$\Rightarrow \frac{Z_1}{2Z_2} + 1 = \cosh \gamma$$

$$\boxed{\cosh \gamma = 1 + \frac{Z_1}{2Z_2}}$$

$$\therefore \sinh \gamma = \sqrt{\cosh^2 \gamma - 1}$$

$$= \sqrt{\left(1 + \frac{Z_1}{2Z_2}\right)^2 - 1}$$

$$= \sqrt{1 + \frac{Z_1^2}{4Z_2^2} + \frac{2Z_1}{2Z_2} - 1}$$

$$= \sqrt{\frac{2Z_1^2 Z_2 + 8Z_1 Z_2^2}{8Z_2^3}}$$

$$= \sqrt{\frac{2Z_1 Z_2 (Z_1 + 4Z_2)}{4 \cdot 8Z_2^3}}$$

$$= \frac{1}{Z_2} \sqrt{\frac{Z_1^2}{4} + \frac{4Z_1 Z_2}{4}}$$

$$\boxed{\sinh \gamma = \frac{Z_{OT}}{Z_2}}$$

$$\therefore \tanh \gamma = \frac{\sinh \gamma}{\cosh \gamma}$$

$$\tanh \gamma = \frac{\frac{Z_{OT}}{Z_2}}{1 + \frac{Z_1}{2Z_2}} = \frac{Z_{OT}}{\frac{2Z_2 + Z_1}{2}}$$

$$= \frac{Z_{OT}}{Z_{OC}}$$

$$= \frac{\sqrt{Z_{OC} Z_{SC}}}{Z_{OC}} = \sqrt{\frac{Z_{OC} Z_{SC}}{Z_{OC}^2}}$$

$$\therefore \boxed{\tanh \gamma = \sqrt{\frac{Z_{SC}}{Z_{OC}}}}$$

From hyperbolic relations, we have

$$\cosh 2\gamma = 1 + 2\sinh^2 \gamma$$

$$\text{WKT } \cosh \gamma = 1 + \frac{Z_1}{2Z_2}$$

$$\cosh 2\left(\frac{\gamma}{2}\right) = 1 + \frac{Z_1}{2Z_2}$$

$$1 + 2\sinh^2 \frac{\gamma}{2} = 1 + \frac{Z_1}{2Z_2}$$

$$\sinh^2 \frac{\gamma}{2} = \frac{Z_1}{4Z_2}$$

$$\sinh \frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}}$$

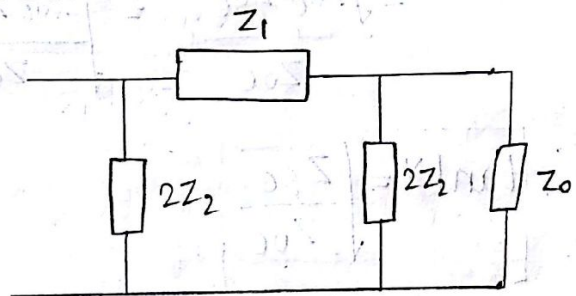
$$\text{WKT } \gamma = \alpha + j\beta$$

$$\sinh \left[\frac{\alpha + j\beta}{2} \right] = \sqrt{\frac{Z_1}{4Z_2}}$$

$$\sinh\left(\frac{\alpha}{2} + j\frac{\beta}{2}\right) = \sqrt{\frac{Z_1}{4Z_2}}$$

$$\sinh\frac{\gamma}{2} = \sinh\frac{\alpha}{2} \cos\frac{\beta}{2} + j \cdot \cosh\frac{\alpha}{2} \sin\frac{\beta}{2} = \sqrt{\frac{Z_1}{4Z_2}}$$

Char impedance of symmetrical π -n/w:-



$$Z_{in} = [(Z_0 \parallel 2Z_2) + Z_1] \parallel 2Z_2$$

$$= \left[\frac{Z_0 \times 2Z_2}{Z_0 + 2Z_2} + Z_1 \right] \parallel 2Z_2$$

$$= \left[\frac{2Z_0Z_2 + Z_1Z_0 + 2Z_1Z_2}{Z_0 + 2Z_2} \right] \parallel 2Z_2$$

$$= \frac{2Z_0Z_2 + Z_1Z_0 + 2Z_1Z_2 \times 2Z_2}{Z_0 + 2Z_2}$$

$$\frac{2Z_0Z_2 + Z_1Z_0 + 2Z_1Z_2 + 2Z_2}{Z_0 + 2Z_2}$$

$$= \frac{4Z_0Z_2^2 + 2Z_0Z_1Z_2 + 4Z_1Z_2^2}{2Z_0Z_2 + Z_1Z_0 + 2Z_1Z_2 + 2Z_0Z_2 + 4Z_2^2}$$

$$Z_{in} = \frac{4Z_0Z_2^2 + 2Z_0Z_1Z_2 + 4Z_1Z_2^2}{4Z_0Z_2 + Z_1Z_0 + 2Z_1Z_2 + 4Z_2^2}$$

$$Z_0 = Z_{in}$$

$$Z_0(4Z_0Z_2 + Z_1Z_0 + 2Z_1Z_2 + 4Z_2^2) = 4Z_0Z_2^2 + 2Z_0Z_1Z_2 + 4Z_1Z_2^2$$

$$\Rightarrow 4Z_0^2Z_2 + Z_1Z_0^2 + 2Z_0Z_1Z_2 + 4Z_0Z_2^2 = 4Z_0Z_2^2 + 2Z_0Z_1Z_2 + 4Z_1Z_2^2$$

$$\Rightarrow 4Z_0^2Z_2 + Z_1Z_0^2 - 4Z_1Z_2^2 = 0$$

$$\Rightarrow Z_0^2(4Z_2 + Z_1) = 4Z_1Z_2^2$$

$$\Rightarrow Z_0^2 = \frac{4Z_1Z_2^2}{4Z_2 + Z_1} \Rightarrow Z_0 = \frac{2Z_2\sqrt{Z_1}}{\sqrt{4Z_2 + Z_1}} \times$$

$$\Rightarrow Z_0 = \sqrt{\frac{4Z_1Z_2^2}{4Z_2 + Z_1}}$$

$$\div \text{ by } 4Z_2$$

$$Z_0 = \sqrt{\frac{4Z_1Z_2^2}{4Z_2} \cdot \frac{1}{4Z_2 + Z_1}} = \sqrt{\frac{Z_1Z_2}{1 + \frac{Z_1}{4Z_2}}}$$

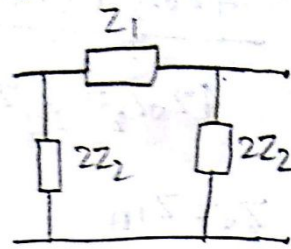
$$Z_{0x} = \sqrt{\frac{Z_1Z_2}{1 + \frac{Z_1}{4Z_2}}}$$

OC port 2 terminals of π -n/w

$$Z_{oc} = (2Z_2 + Z_1) \parallel 2Z_2$$

$$= \frac{(2Z_2 + Z_1) \times 2Z_2}{2Z_2 + Z_1 + 2Z_2}$$

$$= \frac{4Z_2^2 + 2Z_1Z_2}{Z_1 + 4Z_2}$$

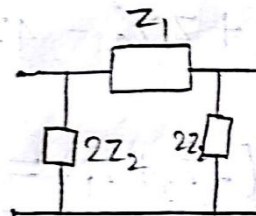


$$Z_{oc} = \frac{4Z_2^2 + 2Z_1Z_2}{Z_1 + 4Z_2}$$

SC port 2 terminals of π -n/w

$$Z_{sc} = Z_1 \parallel 2Z_2$$

$$= \frac{Z_1 \times 2Z_2}{Z_1 + 2Z_2}$$



$$\text{Now, } Z_{oc} \cdot Z_{sc} = \frac{4Z_2^2 + 2Z_1Z_2}{Z_1 + 4Z_2} \times \frac{Z_1 \times 2Z_2}{Z_1 + 2Z_2}$$

$$= \frac{4Z_1Z_2^2 + 4Z_1^2Z_2}{Z_1 + 4Z_2}$$

$$= \frac{2Z_1Z_2(4Z_2^2 + 2Z_1Z_2)}{(Z_1 + 2Z_2)(Z_1 + 4Z_2)}$$

$$= \frac{8Z_1Z_2^3 + 4Z_1^2Z_2^2}{Z_1^2 + 4Z_1Z_2 + 2Z_1Z_2 + 8Z_2^2}$$

$$= \frac{8Z_1Z_2^3 + 4Z_1^2Z_2^2}{Z_1^2 + 6Z_1Z_2 + 8Z_2^2}$$

$$= \frac{Z_1 Z_2 (8Z_2^2 + 4Z_1 Z_2)}{Z_1 Z_2 \left(\frac{Z_1}{Z_2} + 6 + 8 \frac{Z_2}{Z_1} \right)}$$

$$Z_{oc} \cdot Z_{sc} = \frac{4Z_2^2 + 2Z_1 Z_2}{Z_1 + 4Z_2} \times \frac{2Z_1 Z_2}{Z_1 + 2Z_2}$$

$$= \frac{2Z_2 (2Z_2 + Z_1)}{Z_1 + 4Z_2} \times \frac{2Z_1 Z_2}{Z_1 + 2Z_2}$$

$$Z_{oc} \cdot Z_{sc} = \frac{4Z_1 Z_2^2}{Z_1 + 4Z_2} = Z_{0\pi}^2$$

$$\therefore \boxed{Z_{0\pi} = \sqrt{Z_{oc} \cdot Z_{sc}}}$$

Classification of pass band and stopband:-

Conditions for passband & stopband,

$$\sinh \frac{\gamma}{2} = \sinh \frac{\alpha}{2} \cos \frac{\beta}{2} + j \cosh \frac{\alpha}{2} \sin \frac{\beta}{2} = \sqrt{\frac{Z_1}{4Z_2}}$$

Case: i) When Z_1 & Z_2 are of same reactances, then

$$\sqrt{\frac{Z_1}{4Z_2}} > 0 \quad \text{i.e.} \quad \sqrt{\frac{Z_1}{4Z_2}} = x + j0$$

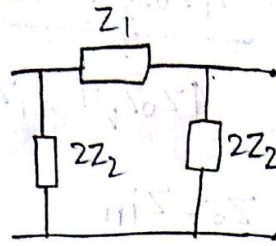
$$\sinh \frac{\alpha}{2} \cos \frac{\beta}{2} + j \cosh \frac{\alpha}{2} \sin \frac{\beta}{2} = x + j0$$

Compare real & imaginary parts.

OC port 2 terminals of π -n/w

$$Z_{oc} = (2Z_2 + Z_1) \parallel 2Z_2$$

$$= \frac{(2Z_2 + Z_1) \times 2Z_2}{2Z_2 + Z_1 + 2Z_2}$$

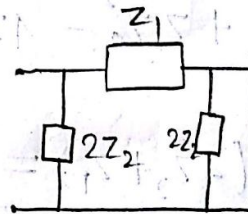


$$Z_{oc} = \frac{4Z_2^2 + 2Z_1Z_2}{Z_1 + 4Z_2}$$

SC port 2 terminals of π -n/w

$$Z_{sc} = Z_1 \parallel 2Z_2$$

$$= \frac{Z_1 \times 2Z_2}{Z_1 + 2Z_2}$$



$$\text{Now, } Z_{oc} \cdot Z_{sc} = \frac{4Z_2^2 + 2Z_1Z_2}{Z_1 + 4Z_2} \times \frac{Z_1 \times 2Z_2}{Z_1 + 2Z_2}$$

$$= \frac{4Z_1Z_2^2 + 4Z_1Z_2^2}{Z_1 + 4Z_2} = 0.5$$

$$= \frac{2Z_1Z_2(4Z_2^2 + 2Z_1Z_2)}{(Z_1 + 2Z_2)(Z_1 + 4Z_2)}$$

$$= \frac{8Z_1Z_2^3 + 4Z_1^2Z_2^2}{Z_1^2 + 4Z_1Z_2 + 2Z_1Z_2 + 8Z_2^2}$$

$$= \frac{8Z_1Z_2^3 + 4Z_1^2Z_2^2}{Z_1^2 + 6Z_1Z_2 + 8Z_2^2}$$

$$= \frac{Z_1 Z_2 (8Z_2^2 + 4Z_1 Z_2)}{Z_1 Z_2 \left(\frac{Z_1}{Z_2} + 6 + 8 \frac{Z_2}{Z_1} \right)}$$

$$Z_{oc} \cdot Z_{sc} = \frac{4Z_2^2 + 2Z_1 Z_2}{Z_1 + 4Z_2} \times \frac{2Z_1 Z_2}{Z_1 + 2Z_2}$$

$$= \frac{2Z_2 (2Z_2 + Z_1)}{Z_1 + 4Z_2} \times \frac{2Z_1 Z_2}{Z_1 + 2Z_2}$$

$$Z_{oc} \cdot Z_{sc} = \frac{4Z_1 Z_2^2}{Z_1 + 4Z_2} = Z_{0\pi}^2$$

$$\therefore \boxed{Z_{0\pi} = \sqrt{Z_{oc} \cdot Z_{sc}}}$$

Classification of pass band and stopband:-

Conditions for passband & stopband,

$$\sinh \frac{\gamma}{2} = \sinh \frac{\alpha}{2} \cos \frac{\beta}{2} + j \cosh \frac{\alpha}{2} \sin \frac{\beta}{2} = \sqrt{\frac{Z_1}{4Z_2}}$$

Case: i) When Z_1 & Z_2 are of same reactances, then

$$\sqrt{\frac{Z_1}{4Z_2}} > 0 \quad \text{i.e.} \quad \sqrt{\frac{Z_1}{4Z_2}} = x + j0$$

$$\sinh \frac{\alpha}{2} \cos \frac{\beta}{2} + j \cosh \frac{\alpha}{2} \sin \frac{\beta}{2} = x + j0$$

Compare real & imaginary parts.

$$\sinh \frac{\alpha}{2} \cos \frac{\beta}{2} = x$$

$$\cosh \frac{\alpha}{2} \sin \frac{\beta}{2} = 0$$

$$\text{if } \beta = 0 \Rightarrow \sinh \frac{\alpha}{2} = x$$

$$\Rightarrow \alpha = 2 \sinh^{-1} x$$

$$\alpha = 2 \sinh^{-1} \sqrt{\frac{Z_1}{4Z_2}} \neq 0 \quad (\text{stop band})$$

Condition : $\sqrt{\frac{Z_1}{4Z_2}} > 0$ for stop band.

Case:2) When Z_1 & Z_2 are of opposite reactances,

$$\text{then } \sqrt{\frac{Z_1}{4Z_2}} < 0 \quad \text{i.e. } \sqrt{\frac{Z_1}{4Z_2}} = 0 + jx$$

$$\sinh \frac{\alpha}{2} \cos \frac{\beta}{2} + j \cosh \frac{\alpha}{2} \sin \frac{\beta}{2} = 0 + jx$$

$$(\text{real part}) \Rightarrow \sinh \frac{\alpha}{2} \cos \frac{\beta}{2} = 0$$

$$(\text{Imag part}) \Rightarrow \cosh \frac{\alpha}{2} \sin \frac{\beta}{2} = x$$

$$\text{if } \alpha = 0 \Rightarrow \sin \frac{\beta}{2} = x$$

$$\Rightarrow \beta = 2 \sin^{-1} x$$

$$\beta = 2 \sin^{-1} \sqrt{\frac{Z_1}{4Z_2}} \quad (\text{pass band})$$

$$\text{if } \beta = \pi \Rightarrow \cosh \frac{\alpha}{2} = x$$

$$\Rightarrow \alpha = 2 \cosh^{-1} x$$

$$\Rightarrow \alpha = 2 \cosh^{-1} \sqrt{\frac{Z_1}{4Z_2}} \neq 0 \quad (\text{stop band})$$

As the limits of sine exists b/w -1 to 1 , hence the value of $\frac{Z_1}{4Z_2}$ will also lie b/w

-1 to 1 but WKT when $\frac{Z_1}{4Z_2} > 0$ it will

enter into the attenuation band. Hence to

have pass band the value of $\frac{Z_1}{4Z_2}$ should

always be b/w -1 to 0 .

Condition: $-1 < \frac{Z_1}{4Z_2} < 0$ for pass band.

From the pass band condition, we have

$$\text{LL} \Rightarrow -1 = \frac{Z_1}{4Z_2}$$

(lower limit)

$$\Rightarrow Z_1 = -4Z_2$$

$$\Rightarrow \boxed{Z_1 + 4Z_2 = 0}$$

$$\text{UL} \Rightarrow \frac{Z_1}{4Z_2} = 0$$

(upper limit)

$$\Rightarrow \boxed{Z_1 = 0}$$

Conditions for cut-off frequency

For a symmetrical T-n/w, char. impedance,

$$Z_{0T} = \sqrt{Z_1 Z_2 + \frac{Z_1^2}{4}}$$

$$= \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)}$$

Under stop band condition, $\frac{Z_1}{4Z_2} = 0$ (same type of reactance), the value of char. impedance is

$$Z_{0T} = \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)}$$

Case:1) Let $Z_1 = jx_1$, $Z_2 = jx_2$, $\frac{Z_1}{4Z_2} > 0$

$$Z_{0T} = \sqrt{j^2 x_1 x_2 \left[1 + \frac{jx_1}{4jx_2}\right]}$$

$$= \sqrt{-x_1 x_2 \left[1 + \frac{x_1}{4x_2}\right]}$$

$$= \sqrt{-k} \quad \left[\text{let } k = x_1 x_2 \left(1 + \frac{x_1}{4x_2}\right) \right]$$

$$\boxed{Z_{0T} = jk} \rightarrow \text{imaginary}$$

Case:2) Let $Z_1 = jx_1$, $Z_2 = -jx_2$, $\frac{Z_1}{4Z_2} < 0$

$$Z_{0T} = \sqrt{-j^2 x_1 x_2 \left[1 - \frac{x_1}{4x_2}\right]}$$

$$= \sqrt{x_1 x_2 \left[1 - \frac{x_1}{4x_2}\right]}$$

$$Z_{OT} = k \rightarrow \text{real}$$

Characteristics of filters

- 1) Under PB condition, the char. impedance of filter is always real.
- 2) Under SB condition, the char. n/w of the filter is always imaginary.

Filters

Active filters
(transistors)

Passive filters (L, C)

constant-k

m-derived

LPF

HPF

LPF

HPF

Band elimination filter
or
Band rejection filter

23-1-10

Constant k-type or prototype filters:-

* Filters are classified into 2 types:-

- 1) Active filters (constructed with operational amp.)
- 2) Passive filters (L, C)
- 2(i) Constant-k type or prototype (filters)
- 2(ii) M-derived filters

* A filter is said to be constant k-type filter when its elements satisfy the relationship $Z_1 Z_2 = k^2$

where Z_1 = series arm impedance

Z_2 = shunt arm impedance

k = nominal / design impedance.

Constant k-filters are classified into 2 types:- ^{const-k} LPF, constant-k HPF. (T type & π -type)

Constant-k LPF:-

When series arm impedance is inductance and shunt arm impedance is capacitance, then that filter is called lowpass filter.

$$\text{Let } Z_1 = j\omega L, \quad Z_2 = \frac{1}{j\omega C}$$

$$Z_1 Z_2 = j\omega L \times \frac{1}{j\omega C} = \frac{L}{C} = k^2$$

$$\Rightarrow k^2 = \frac{L}{C} \Rightarrow \boxed{k = \sqrt{\frac{L}{C}}}$$

From passband condition, we have,

$$-1 < \frac{Z_1}{4Z_2} < 0$$

$$-1 < \frac{j\omega L}{4j\omega C} < 0$$

$$-1 < \frac{j^2 \omega^2 LC}{4} < 0$$

$$-1 < \frac{-4\pi^2 f^2 LC}{4} < 0 \quad (\because \omega = 2\pi f)$$

$$-1 < \frac{-f^2}{\pi^2 LC} < 0 \quad \text{--- (1)}$$

From the condition of cut-off frequencies,

$$Z_1 = 0$$

$$j\omega L = 0$$

$$j2\pi fL = 0$$

$$\Rightarrow f = 0$$

$$Z_2 = -4Z_1$$

$$j\omega L = \frac{-4}{j\omega C}$$

$$j^2 \omega^2 = \frac{-4}{LC}$$

$$-4\pi^2 f^2 = \frac{-4}{LC}$$

$$f_c^2 = \frac{1}{\pi^2 LC} \Rightarrow \boxed{f_c = \frac{1}{\pi \sqrt{LC}}}$$

where f_c is called cut-off frequency which separates passband & stopband.

Now, sub. f_c in equ ①

$$-1 < -\frac{f^2}{f_c^2} < 0$$

$$-1 < -\left(\frac{f}{f_c}\right)^2 < 0$$

$$\underline{LL!} \quad -1 < -\left(\frac{f}{f_c}\right)^2$$

$$-\left(\frac{f}{f_c}\right)^2 > -1$$

$$\left(\frac{f}{f_c}\right)^2 < 1$$

$$\Rightarrow f < f_c$$

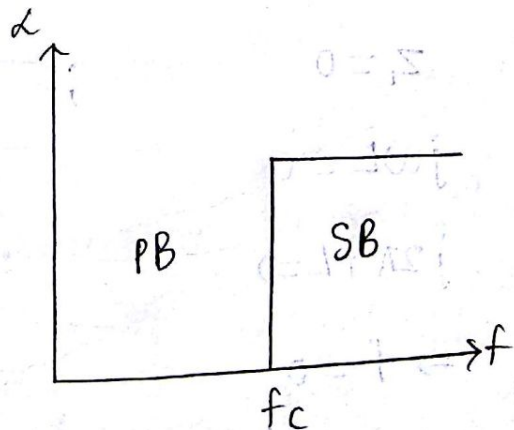
$$\underline{UL!} \quad -\left(\frac{f}{f_c}\right)^2 < 0$$

$$\left(\frac{f}{f_c}\right)^2 > 0$$

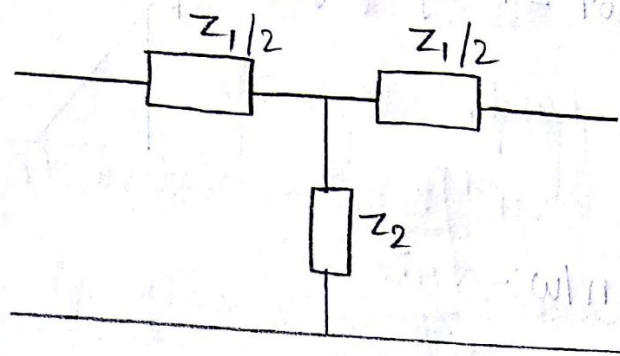
$$\Rightarrow f > 0$$

From this, $0 < f < f_c$

The range of frequencies which LPF transmits is $0 < f < f_c$



Symmetrical T-network:-

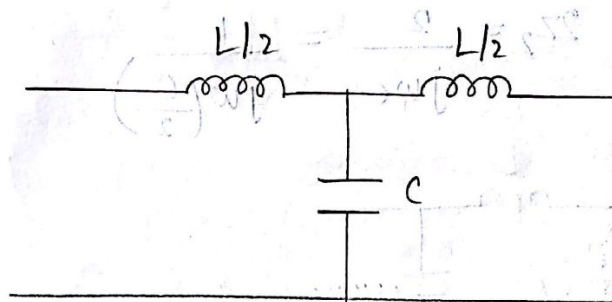


$$Z_1 = j\omega L, \quad Z_2 = \frac{1}{j\omega C}$$

Series arm impedance in T-n/w is $\frac{Z_1}{2}$

i.e. $\frac{Z_1}{2} = \frac{j\omega L}{2}$

Shunt arm impedance in T-n/w is $Z_2 = \frac{1}{j\omega C}$



Char. impedance in T-n/w is

$$Z_{OT} = \sqrt{Z_1 Z_2 + \frac{Z_1^2}{4}}$$

$$= \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)}$$

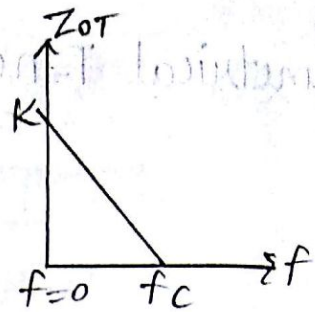
$$Z_{OT} = \sqrt{k^2 \left(1 - \left(\frac{f}{f_c}\right)^2\right)} \quad \left(\because \frac{Z_1}{4Z_2} = \left(\frac{f}{f_c}\right)^2\right)$$

at $f=0$, $Z_{OT} = k(\text{real})$

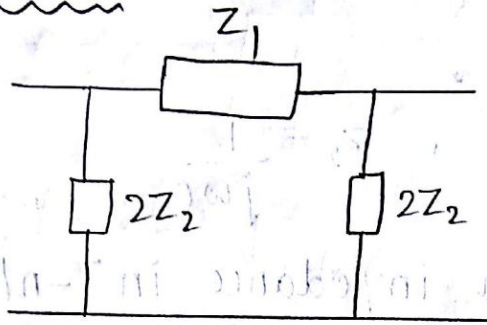
at $f=f_c$, $Z_{OT} = 0$

at $f < f_c$, $Z_{OT} = \text{real}$

at $f > f_c$, $Z_{OT} = \text{imaginary}$



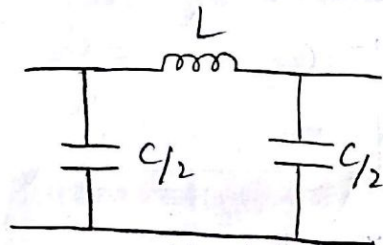
Symmetrical π -n/w:-



Series arm impedance in π -n/w is $Z_1 = j\omega L$

Shunt arm impedance in π -n/w is $2Z_2$

$$2Z_2 = \frac{2}{j\omega c} = \frac{1}{j\omega(\frac{c}{2})}$$



Char. impedance in π -n/w is

$$Z_{O\pi} = \sqrt{\frac{Z_1 Z_2}{1 + \frac{Z_1}{4Z_2}}}$$

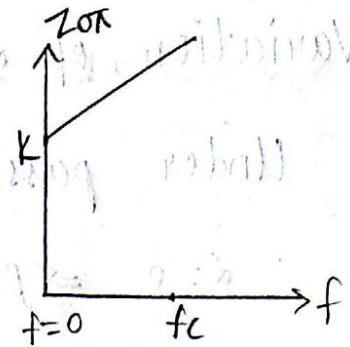
$$Z_{O\pi} = \sqrt{\frac{K^2 - 1}{1 - \left(\frac{f}{f_c}\right)^2}}$$

at $f=0$, $Z_{O\pi} = K(\text{real})$

at $f = f_c$, $Z_{0\pi} = \infty$

at $f < f_c$, $Z_{0\pi} = \text{real}$

at $f > f_c$, $Z_{0\pi} = \text{imaginary}$



Designing the LPF:-

To design LPF, we need the values of cut off frequency, inductor (L), capacitor (C).

$$k = \sqrt{\frac{L}{C}}$$

$$k^2 = \frac{L}{C} \Rightarrow L = k^2 C \quad \text{--- (1)}$$

$$f_c = \frac{1}{\pi \sqrt{LC}}$$

$$f_c^2 = \frac{1}{\pi^2 LC}$$

$$f_c^2 = \frac{1}{\pi^2 k^2 C \cdot C}$$

$$f_c^2 = \frac{1}{\pi^2 k^2 C^2}$$

$$C^2 = \frac{1}{\pi^2 k^2 f_c^2}$$

$$\boxed{C = \frac{1}{\pi k f_c}} \quad \text{F}$$

$$L = k^2 C$$

$$L = k^2 \cdot \frac{1}{\pi k f_c} \Rightarrow \boxed{L = \frac{k}{\pi f_c}} \quad \text{H}$$

Variation of α, β with frequency:-

Under pass band,

$$\alpha = 0 \Rightarrow \beta = 2 \sin^{-1} \sqrt{\frac{Z_1}{4Z_2}}$$

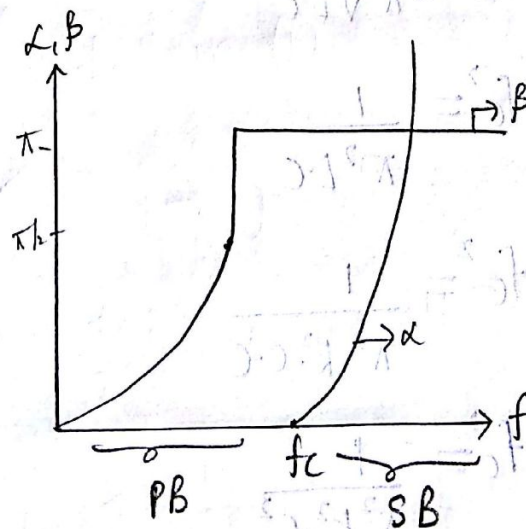
$$\beta = 2 \sin^{-1} \sqrt{j \left(\frac{f}{f_c} \right)^2}$$

$$\beta = 2 \sin^{-1} j \left(\frac{f}{f_c} \right)$$

Under stop band,

$$\beta = \pi \Rightarrow \alpha = 2 \cosh^{-1} \sqrt{\frac{Z_1}{4Z_2}}$$

$$\alpha = 2 \cosh^{-1} j \left(\frac{f}{f_c} \right)$$



Constant-k HPF:-

When series arm impedance is capacitance and shunt arm impedance is inductance, then that filter is called HPF.

$$\text{Let } Z_1 = \frac{1}{j\omega C}; \quad Z_2 = j\omega L$$

$$Z_1 \times Z_2 = \frac{1}{j\omega C} \times j\omega L$$

$$k^2 = \frac{L}{C} \Rightarrow k = \sqrt{\frac{L}{C}}$$

From passband condition, we have

$$-1 < \frac{Z_1}{4Z_2} < 0$$

$$-1 < \frac{1/j\omega C}{4j\omega L} < 0$$

$$-1 < \frac{1}{4j^2\omega^2 LC} < 0$$

$$-1 < -\frac{1}{4\omega^2 LC} < 0$$

$$-1 < -\frac{1}{4(4\pi^2 f^2)LC} < 0$$

$$-1 < -\frac{1}{16\pi^2 f^2 LC} < 0 \quad \text{--- (1)}$$

$$-1 < -\frac{f_c^2}{f^2} < 0 \Rightarrow \left(-1 < -\left(\frac{f_c}{f}\right)^2 < 0 \right)$$

From condition of cut-off frequencies,

$$Z_1 = 0$$

$$Z_1 = -4Z_2$$

$$\frac{1}{j\omega C} = 0$$

$$1/j\omega C = -4j\omega L$$

$$\frac{1}{j2\pi f C} = 0$$

$$1 = -4j^2 \omega^2 LC$$

$$1 = 4\omega^2 LC$$

$$1 = 4(4\pi^2 f^2) LC$$

$$f_c^2 = \frac{1}{16\pi^2 LC}$$

$$\Rightarrow f_c = \frac{1}{4\pi \sqrt{LC}}$$

where f_c is cut-off frequency.

Sub. f_c in ①

$$-1 < -\left(\frac{f_c}{f}\right)^2 < 0$$

$$-1 < -\left(\frac{f_c}{f}\right)^2 \quad \text{UL: } -\left(\frac{f_c}{f}\right)^2 < 0$$

$$-\left(\frac{f_c}{f}\right)^2 > -1$$

$$\left(\frac{f_c}{f}\right)^2 > 0$$

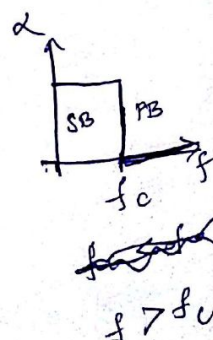
$$\left(\frac{f_c}{f}\right)^2 < 1$$

$$\Rightarrow f_c > 0$$

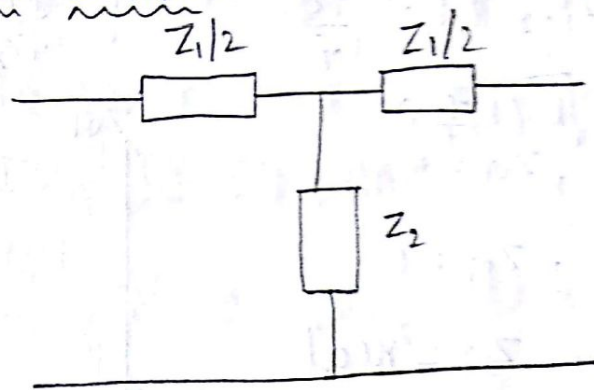
$$\Rightarrow f_c < f$$

$$\Rightarrow f > f_c$$

from this,



Symmetrical T-n/w

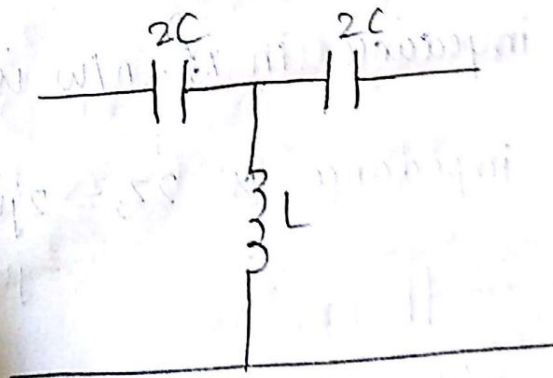


$$Z_1 = \frac{1}{j\omega C} ; Z_2 = j\omega L$$

Series arm impedance in T-n/w is $\frac{Z_1}{2}$

$$\text{i.e. } \frac{Z_1}{2} = \frac{1}{2j\omega C} = \frac{1}{j\omega(2C)}$$

Shunt arm impedance is $Z_2 = j\omega L$



Char. impedance in T-n/w is

$$\begin{aligned} Z_{OT} &= \sqrt{Z_1 Z_2 + \frac{Z_1^2}{4}} \\ &= \sqrt{\frac{1}{j\omega C} \times j\omega L + \left(\frac{1}{j\omega C}\right)^2 \times \frac{1}{4}} \\ &= \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)} \end{aligned}$$

$$Z_{OT} = \sqrt{Z_1 Z_2 \left(1 - \left(\frac{f_c}{f} \right)^2 \right)}$$

$$= \sqrt{k^2 \left(1 - \left(\frac{f_c}{f} \right)^2 \right)}$$

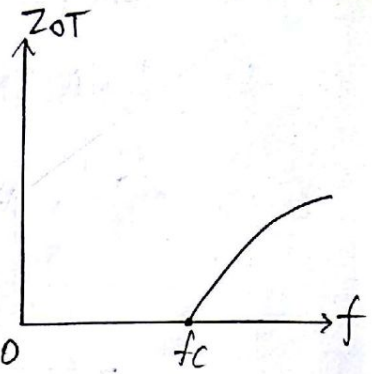
$$\left[\because \frac{Z_1}{4Z_2} = - \left(\frac{f_c}{f} \right)^2 \right]$$

at $f=0$, $Z_{OT} = \text{imaginary}$

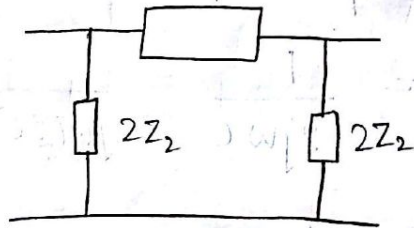
at $f=f_c$, $Z_{OT}=0$

at $f > f_c$, $Z_{OT} = \text{real}$

at $f < f_c$, $Z_{OT} = \text{imaginary}$



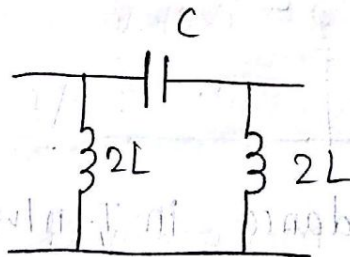
Symmetrical π -n/w:-



Series arm impedance in π -n/w is $Z_1 = \frac{1}{j\omega C}$

Shunt arm impedance is $2Z_2 = 2j\omega L$

$$= j\omega(2L)$$



Char. impedance in π -n/w is

$$Z_{O\pi} = \sqrt{\frac{Z_1 Z_2}{1 + \frac{Z_1}{4Z_2}}}$$

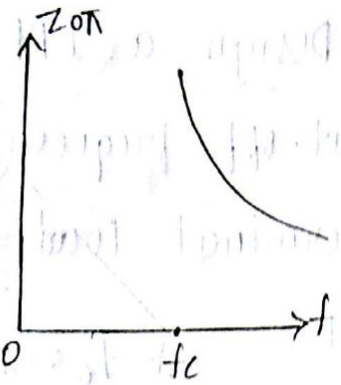
$$Z_{O\pi} = \sqrt{\frac{k^2}{1 - \left(\frac{f_c}{f} \right)^2}}$$

at $f=0$, $Z_{\infty} = \infty$ imaginary

at $f=f_c$, $Z_{\infty} = \infty$

at $f > f_c$, $Z_{\infty} = \text{real}$

at $f < f_c$, $Z_{\infty} = \text{imaginary}$



Designing the HPF:-

To design the HPF, the values of cut-off frequencies L & C should be known.

$$k = \sqrt{\frac{L}{C}} \Rightarrow k^2 = \frac{L}{C} \\ \Rightarrow L = k^2 C$$

$$f_c = \frac{1}{4\pi\sqrt{LC}}$$

$$f_c = \frac{1}{4\pi\sqrt{k^2 C \cdot C}}$$

$$f_c^2 = \frac{1}{16\pi^2 k^2 C^2}$$

$$\Rightarrow C^2 = \frac{1}{16\pi^2 k^2 f_c^2} \Rightarrow \boxed{C = \frac{1}{4\pi k f_c}}$$

$$L = k^2 \cdot C$$

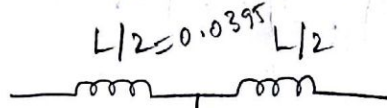
$$L = k^2 \cdot \frac{1}{4\pi k f_c} = \frac{k}{4\pi f_c} H$$

$$\boxed{L = \frac{k}{4\pi f_c} H}$$

1) Design a LPF both T & π sections having a cut-off frequency of 2 kHz to operate with a terminal load resistance of 500 Ω .

Sol

$$f_c = 2 \text{ kHz}, \quad R_L = 500 \Omega \text{ (assume it ask)}$$



$$C = \frac{1}{\pi k f_c}$$

$$L = \frac{k}{\pi f_c}$$

$$C = \frac{1}{\pi (500 \times 2 \times 10^3)}$$

$$C = 3.18 \times 10^{-7} \text{ F}$$

$$C = 0.318 \mu\text{F}$$

$$L = \frac{500}{\pi (2 \times 10^3)} = 0.079 \text{ H}$$

$$L/2 = 0.0395 \text{ H}$$

2) Design a HPF having a cut-off frequency of 1 kHz for $R_L = 600 \Omega$.

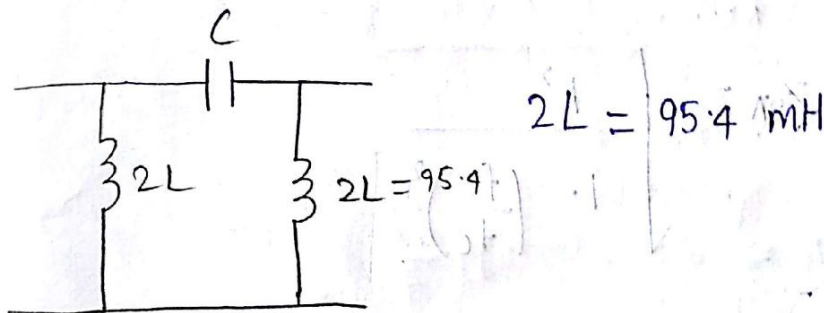
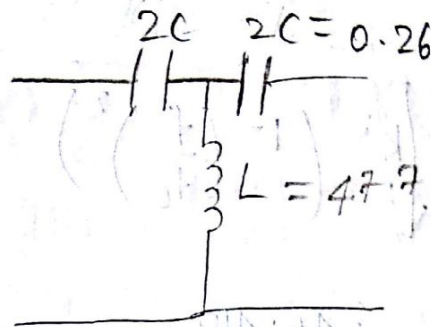
Sol

$$f_c = 1 \text{ kHz} = 1 \times 10^3 \text{ Hz}$$

$$k = R_L = 600 \Omega$$

$$C = \frac{1}{4\pi k f_c} = 1.32 \times 10^{-7} \text{ F} = 0.13 \mu\text{F}$$

$$L = \frac{k}{4\pi f_c} = \frac{600}{4\pi \times 10^3} = 0.0477 \text{ H} = 47.7 \text{ mH}$$



* 3) Design a LPF of constant k-type whose cut-off frequency is 20 KHz & for nominal impedance of 400 Ω . Also determine the values of α, β, Z_0 at a frequency of (i) 10 KHz, (ii) 30 KHz.

Sol

$$K = 400 \Omega, f_c = 20 \text{ KHz} = 20 \times 10^3$$

$$(i) f = 10 \text{ KHz} = 10 \times 10^3 \text{ Hz}$$

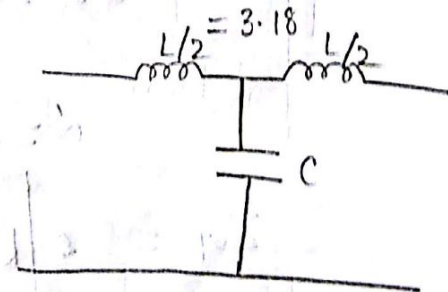
$$\alpha = 2 \cosh^{-1} j \left(\frac{f}{f_c} \right)$$

$f < f_c$

$$(PB) \alpha = 0 \Rightarrow \beta = 2 \sinh^{-1} j \left(\frac{f}{f_c} \right)$$

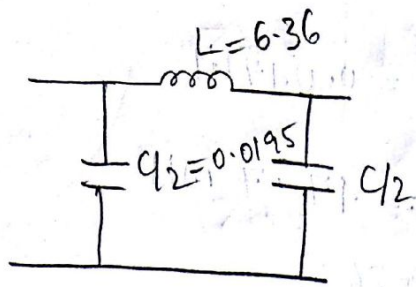
$$= 2 \sinh^{-1} \left(\frac{10}{20} \right)$$

$$\beta = 60$$



$$C = \frac{1}{\pi K f_c} = 0.039 \mu\text{F}$$

$$L = \frac{k}{\pi f_c} = 6.36 \text{ mH}$$



$$\frac{C}{2} = 0.0195$$

$$Z_{OT} = \sqrt{k^2 \left(1 - \left(\frac{f}{f_c}\right)^2\right)}$$

$$= 346.410$$

$$Z_{OT} = \sqrt{\frac{k^2}{1 - \left(\frac{f}{f_c}\right)^2}}$$

$$Z_{OT} = 461.880$$

$$(ii) f = 30 \text{ kHz} = 30 \times 10^3$$

$$f > f_c \text{ (SB)}$$

$$\beta = \pi \Rightarrow \alpha = 2 \cosh^{-1} \left(\frac{f}{f_c} \right)$$

$$\alpha = 2 \cosh^{-1} \left(\frac{30}{20} \right)$$

$$\alpha = 1.92$$

$$Z_{OT} = \sqrt{k^2 \left(1 - \left(\frac{f}{f_c}\right)^2\right)}$$

↓

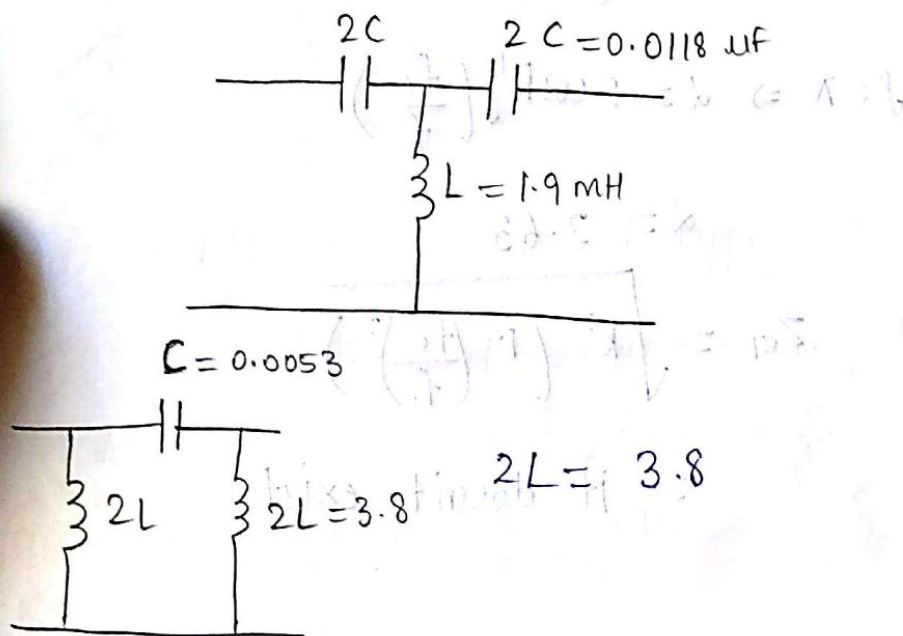
it doesn't vary in SB.

* 4) Design constant-k HPF, $f_c = 25 \text{ KHz}$ for nominal impedance of 600Ω and also determine the values of α, β, z_0 at a frequencies of 50 KHz & 12.5 KHz

Sol $f_c = 25 \times 10^3$, $k = 600 \Omega$

$$C = \frac{1}{4\pi k f_c} ; L = \frac{k}{4\pi f_c}$$

$$C = 0.0053 \mu\text{F} ; L = 1.9 \text{ mH}$$



(i) $f = 50 \text{ KHz} = 50 \times 10^3$, $f_c = 25 \text{ KHz}$

$$f > f_c \text{ (PB)}$$

$$\therefore \beta = \pi - \alpha = 2 \cosh^{-1} \sqrt{\frac{Z_1}{4Z_2}} = 2 \cosh^{-1} j \left(\frac{f_c}{f} \right)^2$$

$$\alpha = 0 \Rightarrow \beta = 2 \sin^{-1} j \left(\frac{f_c}{f} \right)$$

$$\beta = 2 \sin^{-1} j \left(\frac{25}{50} \right) = 60$$

$$Z_{OT} = \sqrt{k^2 \left(1 - \left(\frac{f_c}{f}\right)^2\right)}$$

$$Z_{OT} = 519.615$$

$$Z_{OK} = \sqrt{\frac{k^2}{1 - \left(\frac{f_c}{f}\right)^2}} = 692.820$$

$$(ii) f = 12.5 \text{ KHz} = 12.5 \times 10^3$$

$$f < f_c \text{ (SB)}$$

$$\beta = \pi \Rightarrow \alpha = 2 \cosh^{-1} \left(\frac{f_c}{f} \right)$$

$$\alpha = 2.63$$

$$Z_{OT} = \sqrt{k^2 \left(1 - \left(\frac{f_c}{f}\right)^2\right)}$$

= it doesn't exist

27-1-16

(Conti---) Unit: 3

M-derived filters:-

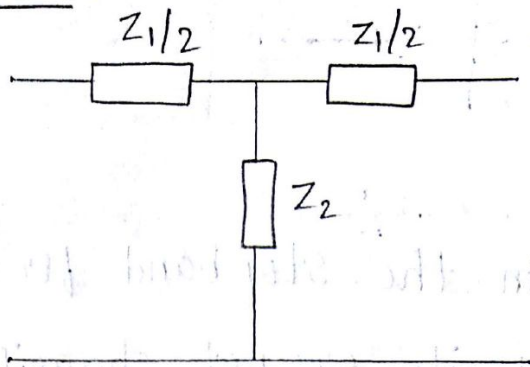
As the attenuation in the stop band for a prototype filter is not infinite or not sharp, there is a need for us to go for a new filter which will produce sharp attenuation in SB. This can be achieved in 2 ways:-

(i) By cascading no. of prototype filter and which has same characteristic impedance, same PB and same SB but this is disadvantageous since the elements are not pure. (L & C practically has internal resistance).

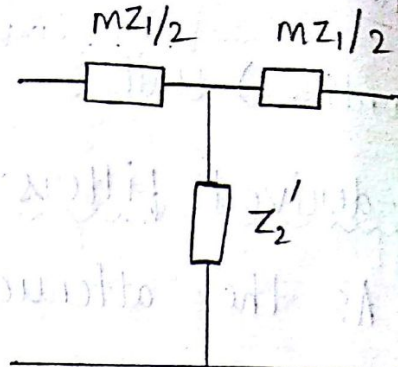
(ii) Sharp attenuation will be achieved by deriving a new filter from the prototype filter which are known as m-derived filters. The condition for deriving a filter is characteristic impedance should be same i.e. $Z_0 = Z_0'$ where Z_0 is char. impedance of prototype filter and Z_0' is char. impedance of m-derived filter.

$$Z_{OT} = Z_{OT}'$$

T-n/w:-



(constant-k)



(m-derived)

$$Z_{OT} = Z_{OT}'$$

$$\sqrt{z_1 z_2 + \frac{z_1^2}{4}} = \sqrt{m z_1 z_2' + \frac{(m z_1)^2}{4}}$$

$$z_1 z_2 + \frac{z_1^2}{4} = m z_1 z_2' + \frac{m^2 z_1^2}{4}$$

$$m z_1 z_2' = z_1 z_2 + \frac{z_1^2}{4} - \frac{m^2 z_1^2}{4}$$

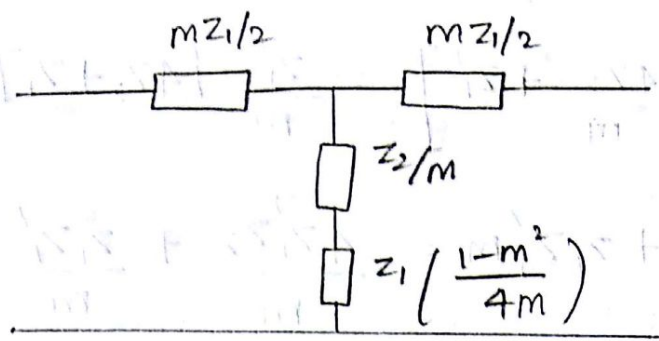
$$z_2' = \frac{z_1 z_2}{m z_1} + \frac{z_1^2}{4 m z_1} - \frac{m^2 z_1^2}{4 m z_1}$$

$$z_2' = \frac{z_2}{m} + \frac{z_1}{4m} - \frac{m z_1}{4}$$

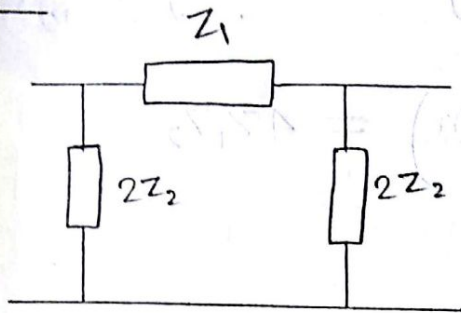
$$z_2' = \frac{z_2}{m} + \frac{z_1}{4} \left(\frac{1}{m} - m \right)$$

$$z_2' = \frac{z_2}{m} + \frac{z_1}{4} \left(\frac{1-m^2}{m} \right)$$

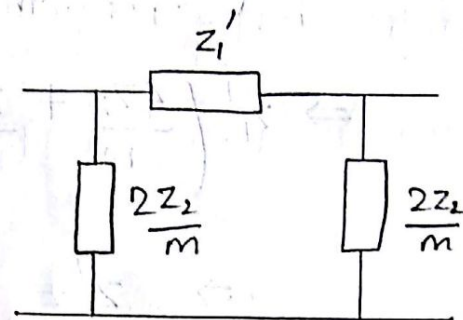
$$z_2' = \frac{z_2}{m} + z_1 \left(\frac{1-m^2}{4m} \right)$$



π -n/w:-



(constant -k)



(m-derived)

$$Z_{O\pi} = Z'_{O\pi}$$

$$\sqrt{\frac{Z_1 Z_2}{1 + \frac{Z_1}{4Z_2}}} = \sqrt{\frac{Z'_1 \frac{Z_2}{m}}{1 + \frac{Z'_1}{4(\frac{Z_2}{m})}}}$$

$$\frac{Z_1 Z_2}{\frac{4Z_2 + Z_1}{4Z_2}} = \frac{Z'_1 \frac{Z_2}{m}}{\frac{\frac{4Z_2}{m} + Z'_1}{4(\frac{Z_2}{m})}}$$

$$\frac{Z_1 Z_2 \left[\frac{4Z_2}{m} + Z'_1 \right]}{4Z_2 \frac{Z_2}{m}} = \frac{Z'_1 \frac{Z_2}{m} \left[\frac{4Z_2 + Z_1}{4Z_2} \right]}{4Z_2 \frac{Z_2}{m}}$$

$$\Rightarrow \frac{Z_1 Z_2 m}{4Z_2} \left[\frac{4Z_2}{m} + Z'_1 \right] = \frac{Z'_1 Z_2}{4m Z_2} [4Z_2 + Z_1]$$

$$\Rightarrow z_1 m \left[\frac{4z_2}{m} + z_1' \right] = \frac{z_1'}{m} [4z_2 + z_1]$$

$$\Rightarrow 4z_1 z_2 + z_1 z_1' m = \frac{4z_1' z_2}{m} + \frac{z_1 z_1'}{m}$$

$$\Rightarrow z_1' \left(z_1 m - \frac{4z_2}{m} - \frac{z_1}{m} \right) = -4z_1 z_2$$

$$\Rightarrow z_1' \left(\frac{4z_2}{m} + \frac{z_1}{m} - z_1 m \right) = 4z_1 z_2$$

$$\Rightarrow z_1' = \frac{4z_1 z_2}{\frac{4z_2 + z_1}{m} - z_1 m}$$

$$\Rightarrow z_1' = \frac{z_1 z_2}{\frac{z_2 + z_1}{m} - \frac{z_1 m}{4}}$$

$$\Rightarrow z_1' = \frac{z_1 z_2}{\frac{z_2 + z_1}{m} (1 - m^2)}$$

Multiply & divide by $\frac{4m^2}{1-m^2}$

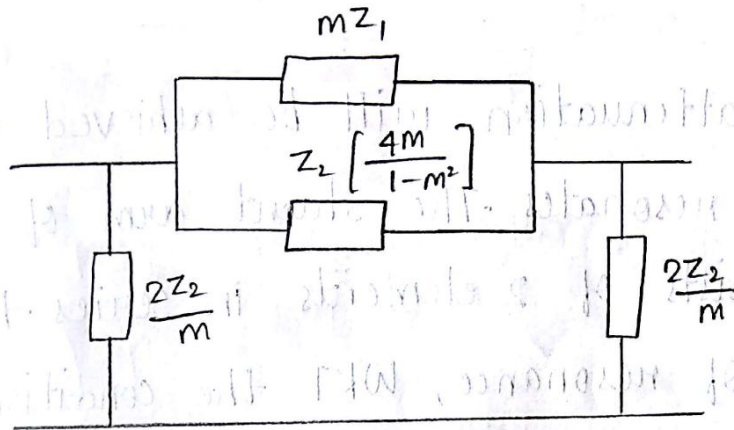
$$z_1' = z_1 z_2 \left[\frac{4m^2}{1-m^2} \right]$$

$$\left[\frac{z_2 + z_1}{m} (1-m^2) \right] \left(\frac{4m^2}{1-m^2} \right)$$

$$z_1' = \frac{z_1 z_2 \left[\frac{4m^2}{1-m^2} \right]}{\frac{z_2 + z_1}{m} (1-m^2)}$$

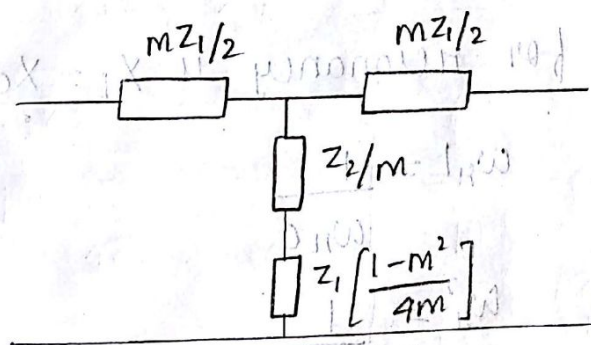
$$\frac{z_2}{m} \times \frac{4m^2}{1-m^2} + \frac{z_1}{4m} (1-m^2) \times \frac{4m}{1-m^2}$$

$$Z_1' = \frac{mZ_1 \left[\frac{4m}{1-m^2} \right] Z_2}{Z_2 \left[\frac{4m}{1-m^2} \right] + mZ_1}$$



M-derived LPF:-

T-n/w:-



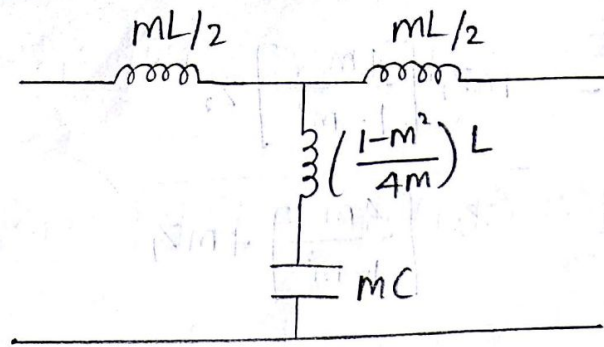
$$\text{Let } Z_1 = j\omega L ; Z_2 = \frac{1}{j\omega C}$$

$$\frac{mZ_1}{2} = m \frac{j\omega L}{2} = j\omega \left(\frac{mL}{2} \right)$$

$$\frac{Z_2}{m} = \frac{1}{j\omega Cm} = \frac{1}{j\omega (mC)}$$

$$Z_1 \left(\frac{1-m^2}{4m} \right) = j\omega \left[\frac{1-m^2}{4m} \right] L$$

→ conti. ---



Infinite attenuation will be achieved when the shunt arm resonates. The shunt arm of symmetrical T-filter consists of 2 elements in series. From the basics of resonance, WKT the condition of resonance is net resonance of the circuit is zero i.e. $X = 0$

Condition for resonancy is $X_L = X_C$

$$\omega_H L = \frac{1}{\omega_H C}$$

$$\omega_H^2 = \frac{1}{LC}$$

$$\omega_H^2 = \frac{1}{\left(\frac{1-m^2}{4m}\right)L \cdot mC}$$

$$(2\pi f_H)^2 = \frac{1}{\left(\frac{1-m^2}{4}\right)LC}$$

$$f_H^2 = \frac{1}{\left(\frac{1-m^2}{4}\right)LC \times \pi^2} = \frac{4}{\pi^2(1-m^2)LC}$$

$$f_n = \frac{1}{\pi \sqrt{(1-m^2) LC}}$$

$$f_n = \frac{f_c}{\sqrt{1-m^2}} \quad \left(\because f_c = \frac{1}{\pi \sqrt{LC}} \right)$$

$$\sqrt{1-m^2} = \frac{f_c}{f_n}$$

$$1-m^2 = \left(\frac{f_c}{f_n} \right)^2$$

$$m^2 = 1 - \left(\frac{f_c}{f_n} \right)^2 \Rightarrow m = \sqrt{1 - \left(\frac{f_c}{f_n} \right)^2}$$

$$m = \sqrt{1 - \left(\frac{f_c}{f_n} \right)^2} \quad (\because f_n = f_{\infty})$$

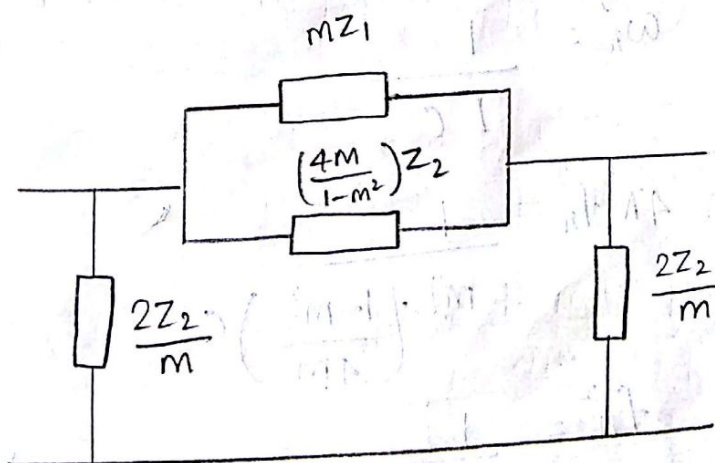
$$\rightarrow \text{If } f_c = f_{\infty} \Rightarrow m = 0$$

$$\rightarrow \text{If } f_c < f_{\infty} \Rightarrow m = \text{real}$$

$$\rightarrow \text{If } f_c = 0 \Rightarrow m = 1$$

$\therefore m$ varies from $0 \leq m < 1$

π -n/w:-

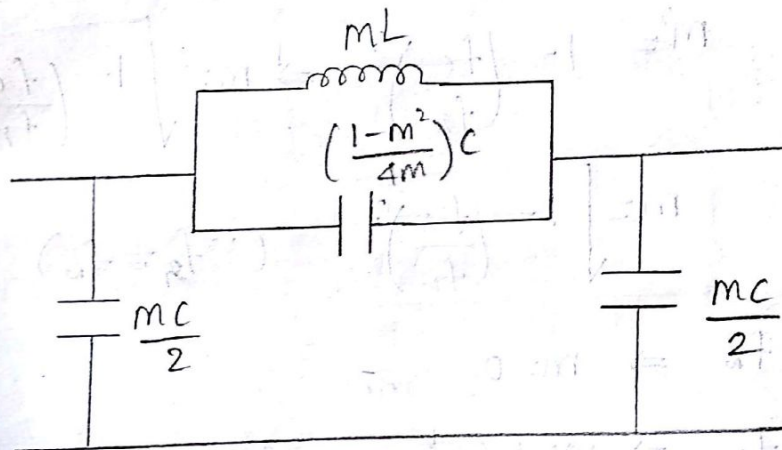


$$\text{Let } Z_1 = j\omega L ; Z_2 = \frac{1}{j\omega C}$$

$$mZ_1 = mj\omega L = j\omega(mL)$$

$$\frac{2Z_2}{m} = 2 \cdot \frac{1}{mj\omega C} = \frac{2}{j\omega(mC)} = \frac{1}{j\omega\left(\frac{mC}{2}\right)}$$

$$\left(\frac{4m}{1-m^2}\right)Z_2 = \left(\frac{4m}{1-m^2}\right) \cdot \frac{1}{j\omega C} = \frac{1}{j\omega\left(\frac{1-m^2}{4m}\right)C}$$



Condition for resonancy $X_L = X_C$

$$\omega_n L = \frac{1}{\omega_n C}$$

$$\omega_n^2 = \frac{1}{LC}$$

$$4\pi^2 f_n^2 = \frac{1}{mL \cdot \left(\frac{1-m^2}{4m}\right)C}$$

$$f_n^2 = \frac{1}{\left(\frac{1-m^2}{4}\right)LC \times 4\pi^2} = \frac{1}{\pi^2 LC (1-m^2)}$$

$$f_n = \frac{1}{\pi \sqrt{LC} (1-m^2)}$$

$$f_n = \frac{f_c}{\sqrt{1-m^2}} \quad (\because f_c = \frac{1}{\pi \sqrt{LC}})$$

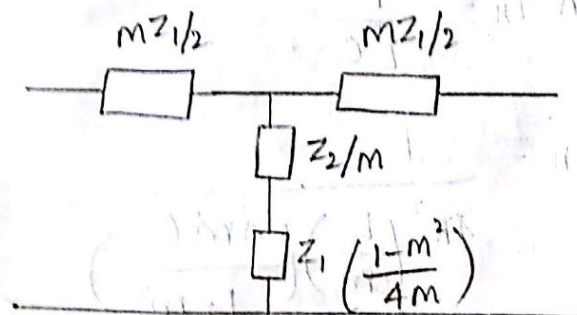
$$\sqrt{1-m^2} = \frac{f_c}{f_n}$$

$$m = \sqrt{1 - \left(\frac{f_c}{f_n}\right)^2}$$

$$* \quad m = \sqrt{1 - \left(\frac{f_c}{f_n}\right)^2} \quad (0 < m < 1)$$

M-derived HPF:-

T-n/w:-

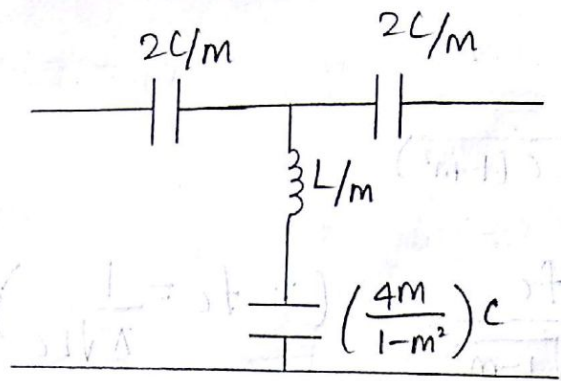


$$\text{Let } Z_1 = \frac{1}{j\omega C}, \quad Z_2 = j\omega L$$

$$\frac{mZ_1}{2} = \frac{m \cdot 1}{2j\omega C} = \frac{1 \cdot m}{j\omega (2C)} = \frac{1}{j\omega \left(\frac{2C}{m}\right)}$$

$$\frac{Z_2}{m} = \frac{j\omega L}{m} = j\omega \left(\frac{L}{m}\right)$$

$$Z_1 \left(\frac{1-m^2}{4m}\right) = \frac{1}{j\omega C} \left(\frac{1-m^2}{4m}\right) = \frac{1}{j\omega \left(\frac{4m}{1-m^2}\right) C}$$



Resonance will be occurred when $X_L = X_C$
 i.e when shunt arm impedance is resonating

$$X_L = X_C$$

$$\omega_n L = \frac{1}{\omega_n C}$$

$$\omega_n^2 = \frac{1}{LC}$$

$$4\pi^2 f_n^2 = \frac{1}{LC}$$

$$f_n^2 = \frac{1}{4\pi^2 \left(\frac{L}{m}\right) \left(\frac{4mC}{1-m^2}\right)}$$

$$f_n^2 = \frac{1-m^2}{16\pi^2 LC}$$

$$f_n = \frac{\sqrt{1-m^2}}{4\pi\sqrt{LC}}$$

$$f_n = f_c \sqrt{1-m^2} \quad \left(\because f_c = \frac{1}{4\pi\sqrt{LC}} \right)$$

$$\frac{f_n}{f_c} = \sqrt{1-m^2}$$

$$\Rightarrow m^2 = 1 - \left(\frac{f_n}{f_c}\right)^2$$

$$\Rightarrow m = \sqrt{1 - \left(\frac{f_n}{f_c}\right)^2}$$

$$m = \sqrt{1 - \left(\frac{f_\infty}{f_c}\right)^2} \quad \left(\because f_n = f_\infty\right)$$

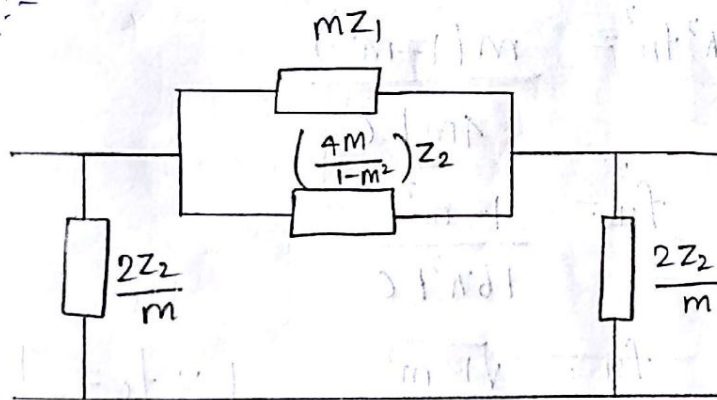
↓
resonance occurs at infinity)

$$\rightarrow \text{If } f_c = f_\infty \Rightarrow m = 0$$

$$\rightarrow \text{If } f_c = \infty \Rightarrow m = 1$$

$$\therefore 0 < m < 1$$

π -n/w:-

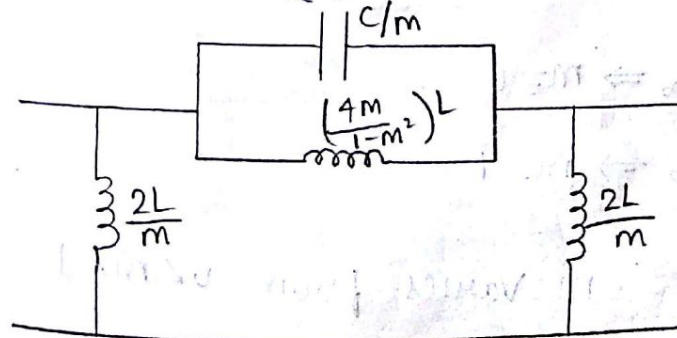


$$\text{let } z_1 = \frac{1}{j\omega c} ; z_2 = j\omega L$$

$$mZ_1 = \frac{m}{j\omega c} = \frac{1}{j\omega \left(\frac{c}{m}\right)}$$

$$\frac{2Z_2}{m} = \frac{2j\omega L}{m} = j\omega \left(\frac{2L}{m}\right)$$

$$\left(\frac{4m}{1-m^2}\right)Z_2 = \left(\frac{4m}{1-m^2}\right) \times j\omega L = j\omega \left(\frac{4m}{1-m^2}\right)L$$



Resonance occurs when $X_L = X_C$

$$\omega_a L = \frac{1}{\omega_a C}$$

$$\omega_a^2 = \frac{1}{LC}$$

$$4\pi^2 f_n^2 = \frac{1}{\left(\frac{4m}{1-m^2}\right)L \times \frac{C}{m}}$$

$$4\pi^2 f_n^2 = \frac{m(1-m^2)}{4mLC}$$

$$f_n^2 = \frac{1-m^2}{16\pi^2 LC}$$

$$f_n = \frac{\sqrt{1-m^2}}{4\pi\sqrt{LC}} \quad \left(\because f_c = \frac{1}{4\pi\sqrt{LC}} \right)$$

$$f_n = f_c \sqrt{1-m^2}$$

$$\frac{f_n}{f_c} = \sqrt{1-m^2}$$

$$m = \sqrt{1 - \left(\frac{f_n}{f_c}\right)^2}$$

$$\boxed{m = \sqrt{1 - \left(\frac{f_n}{f_c}\right)^2}}$$

$$\rightarrow \text{If } f_c = f_n \Rightarrow m = 0$$

$$\rightarrow \text{If } f_c = \infty \Rightarrow m = 1$$

$\therefore m$ varies from $0 < m < 1$

9-1-16

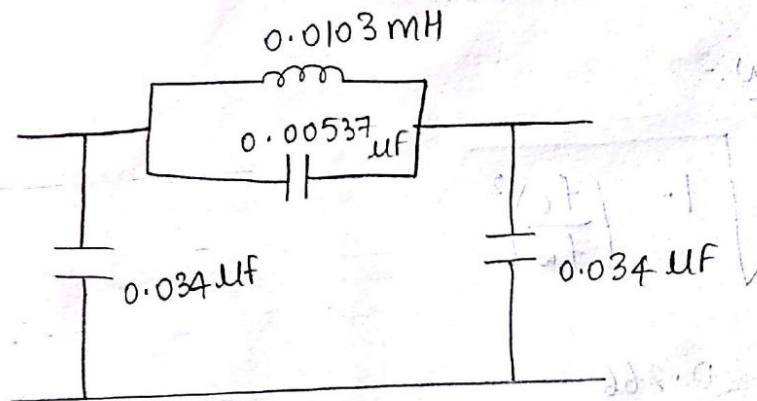
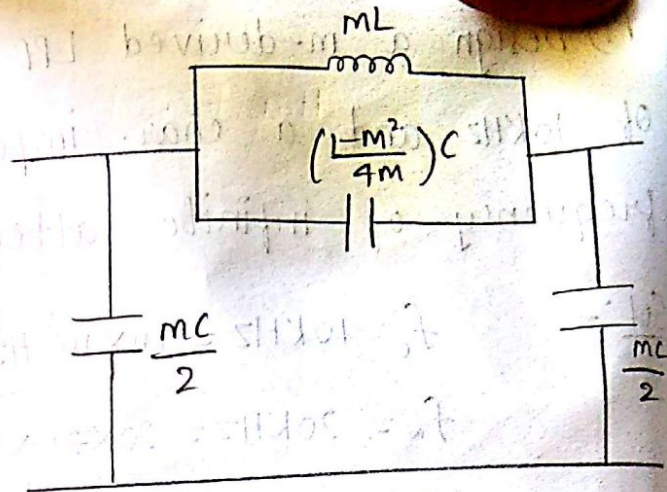
P1) Design a m -derived LPF with a cut off frequency of 10KHz and a char. impedance of 400Ω . The frequency of infinite attenuation is given as 20KHz.

For π -n/w:-

$$mL = 0.0103 \text{ mH}$$

$$\left(\frac{1-m^2}{4m}\right)C = 5.73 \times 10^{-3} \\ = 0.00573 \mu\text{F}$$

$$\frac{mC}{2} = 0.034 \mu\text{F}$$



P2) Design a m -derived HPF for a nominal impedance of 500Ω with $f_c = 25 \text{ KHz}$ to attain infinite attenuation at a frequency of 12.5 KHz .

Sol $f_c = 25 \text{ KHz}, f_\infty = 12.5 \text{ KHz}$
 $= 25 \times 10^3 \text{ Hz} \quad = 12.5 \times 10^3 \text{ Hz}$

For HPF, $m = \sqrt{1 - \left(\frac{f_\infty}{f_c}\right)^2} = 0.866$

$$L = \frac{k}{4\pi f_c} = \frac{500}{4\pi \times 25 \times 10^3} = 1.59 \text{ mH}$$

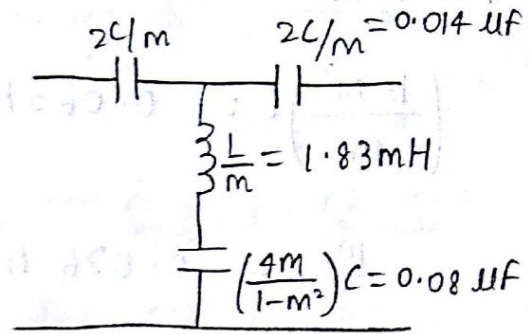
$$C = \frac{1}{4\pi k f_c} = 0.0063 \mu\text{F}$$

For T-n/w:-

$$\frac{2C}{m} = 0.0145 \mu F$$

$$\frac{L}{m} = 1.836 \text{ mH}$$

$$\left(\frac{4m}{1-m^2}\right)C = 0.0872$$

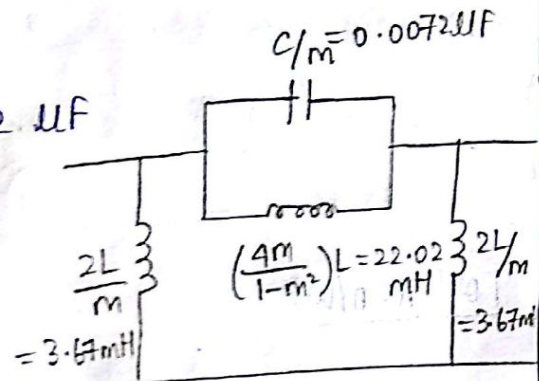


For π -n/w:-

$$\frac{C}{m} = 7.27 \times 10^{-3} = 0.0072 \mu F$$

$$\frac{2L}{m} = 3.672 \text{ mH}$$

$$\left(\frac{4m}{1-m^2}\right)L = 22.027 \text{ mH}$$



3) Design a m -derived LPF having cut off frequency of 1 kHz, design impedance of 400 Ω & the resonant frequency of 1100 Hz.

Sol $f_c = 1 \times 10^3 \text{ Hz}$, $f_r = 1100 \text{ Hz}$

$$k = 400 \Omega$$

$$m = \sqrt{1 - \left(\frac{f_c}{f_r}\right)^2} = 0.416$$

$$L = \frac{k}{\pi f_c} = 0.127 = 127 \text{ mH}$$

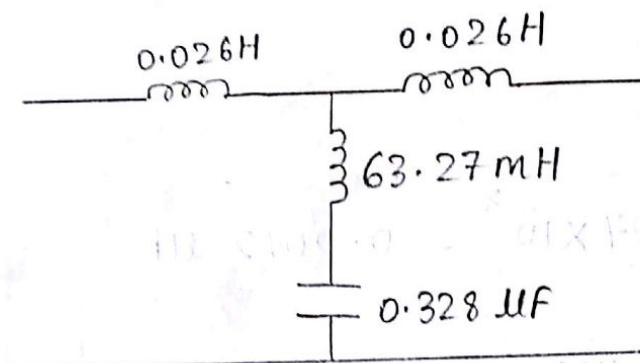
$$C = \frac{1}{\pi k f_c} = 0.796 \mu F$$

For T-n/w:-

$$\left(\frac{1-m^2}{4m} \right) L = 0.063 \text{ H} = 63.27 \text{ mH}$$

$$\frac{mL}{2} = 0.026 \text{ H}$$

$$mC = 0.328 \text{ } \mu\text{F}$$

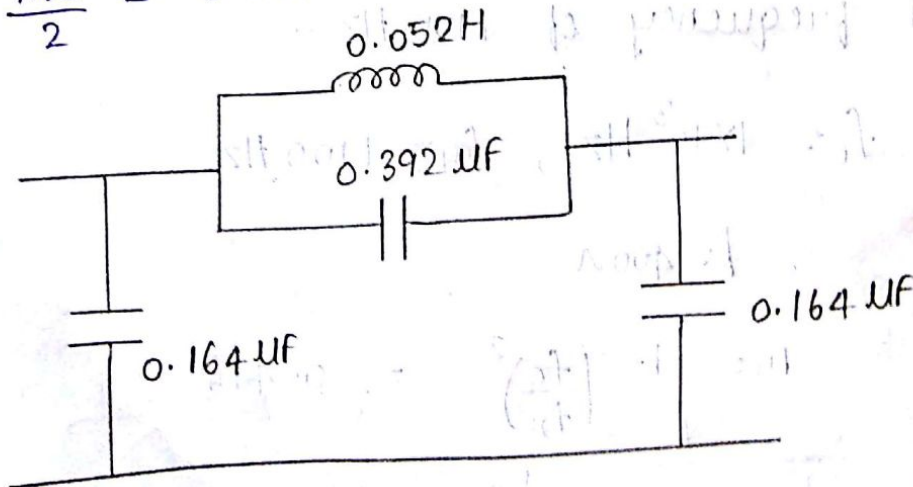


For π-n/w:-

$$mL = 0.052 \text{ H}$$

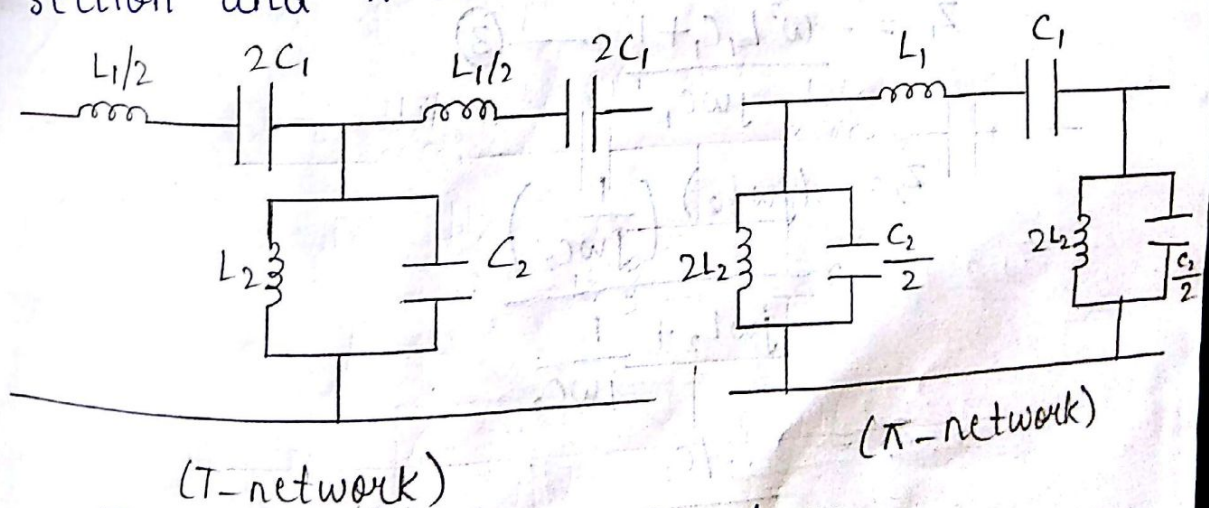
$$\left(\frac{1-m^2}{4m} \right) C = 0.392 \text{ } \mu\text{F}$$

$$\frac{mC}{2} = 0.164 \text{ } \mu\text{F}$$



Band pass filter:-

A BPF is the one which transmits certain band of frequencies and eliminates the remaining frequencies. A band pass filter can be constructed by joining a high pass filter with a low pass filter where the low pass filter cut-off frequency will be the upper cut off frequency and the cut-off frequency of HPF will be the lower cut-off frequency. But it is not possible to join 2 prototype filters of different characteristics. Hence band pass filter will be constructed by joining the elements of low pass filter and HPF together. A symmetrical T-section and π -section is shown in the fig.



(T-network)

(π -network)

For series arm impedance,

$$X_L = X_C$$

$$\omega_n \frac{L_1}{2} = \frac{1}{\omega_n 2 C_1}$$

$$\omega_H^2 = \frac{1}{\left(\frac{L_1}{2}\right)(2C_1)}$$

$$\omega_H^2 = \frac{1}{L_1 C_1} \quad \text{--- (1)}$$

For shunt arm impedance,

$$\omega_H L_2 = \frac{1}{\omega C_2}$$

$$\omega_H^2 = \frac{1}{L_2 C_2} \quad \text{--- (2)}$$

equating (1) = (2)

$$\frac{1}{L_1 C_1} = \frac{1}{L_2 C_2}$$

$$\frac{L_2}{C_1} = \frac{L_1}{C_2}$$

$$Z_1 = j\omega L_1 + \frac{1}{j\omega C_1}$$

$$Z_1 = \frac{-\omega^2 L_1 C_1 + 1}{j\omega C_1} \quad \text{--- (3)}$$

$$Z_2 = \frac{(j\omega L_2) \left(\frac{1}{j\omega C_2}\right)}{j\omega L_2 + \frac{1}{j\omega C_2}}$$

$$Z_2 = \frac{L_2 / C_2}{\frac{j^2 \omega^2 L_2 C_2 + 1}{j\omega C_2}} = \frac{j\omega L_2}{-\omega^2 L_2 C_2 + 1} \quad \text{--- (4)}$$

$$Z_1 Z_2 = k^2$$

$$Z_1 Z_2 = \left[\frac{-\omega^2 L_1 C_1 + 1}{j\omega C_1} \right] \left[\frac{j\omega L_2}{-\omega^2 L_2 C_2 + 1} \right] = \left[\frac{-\omega^2 L_2 C_2 + 1}{-\omega^2 L_2 C_2 + 1} \right] \frac{L_2}{C_1}$$

$$Z_1 Z_2 = \frac{L_2}{C_1} = \frac{L_1}{C_2} = k^2 \quad (\because L_1 C_1 = L_2 C_2)$$

Condition for cut off frequencies,

$$Z_1 = -4Z_2$$

Multiply with Z_1 ,

$$Z_1^2 = -4Z_1 Z_2$$

$$Z_1^2 = -4k^2$$

$$\boxed{Z_1 = \pm 2jk} \quad \begin{matrix} (- \text{ for LPPF}) \\ (+ \text{ for HPPF}) \end{matrix}$$

$$Z_1 = j\omega L_1 + \frac{1}{j\omega C_1} = \pm 2jk$$

at $\omega = \omega_1$,

$$Z_1 = j\omega_1 L_1 + \frac{1}{j\omega_1 C_1} = -2jk \quad \text{--- (5)}$$

at $\omega = \omega_2$,

$$Z_1 = j\omega_2 L_1 + \frac{1}{j\omega_2 C_1} = 2jk$$

$$Z_1 = - \left[j\omega_2 L_1 + \frac{1}{j\omega_2 C_1} \right] = -2jk \quad \text{--- (6)}$$

From (5) & (6)

$$j\omega_1 L_1 + \frac{1}{j\omega_1 C_1} = - \left[j\omega_2 L_1 + \frac{1}{j\omega_2 C_1} \right]$$

$$\frac{j^2 \omega_1^2 L_1 C_1 + 1}{j \omega_1 C_1} = - \left[\frac{j^2 \omega_2^2 L_1 C_1 + 1}{j \omega_2 C_1} \right]$$

$$\Rightarrow (-\omega_1^2 L_1 C_1 + 1) \omega_2 = (\omega_2^2 L_1 C_1 - 1) \omega_1$$

$$\Rightarrow -\omega_1^2 \omega_2 L_1 C_1 + \omega_2 = \omega_1 \omega_2^2 L_1 C_1 - \omega_1$$

$$\Rightarrow -\omega_1 \omega_2 L_1 C_1 (\omega_1 + \omega_2) = -(\omega_1 + \omega_2)$$

$$\Rightarrow \omega_1 \omega_2 L_1 C_1 = 1 \quad \left(\because \frac{1}{\omega_2^2} = L_1 C_1 \right)$$

$$\Rightarrow \frac{\omega_1 \omega_2}{\omega_H^2} = 1 \quad (\text{from ①})$$

$$\Rightarrow \omega_1 \omega_2 = \omega_H^2$$

$$\Rightarrow \boxed{f_H = \sqrt{f_1 f_2}}$$

Designing the bandpass filter:-

In order to design BPF, we require values of L and C.

at $\omega = \omega_1$,

$$z_1 = j \omega_1 L_1 + \frac{1}{j \omega_1 C_1} = -2jk$$

$$\textcircled{2} \quad \frac{j^2 \omega_1^2 L_1 C_1 + 1}{j \omega_1 C_1} = -2jk$$

$$\Rightarrow -\omega_1^2 L_1 C_1 + 1 = -2j^2 \omega_1 C_1 k$$

$$\Rightarrow -\omega_1^2 L_1 C_1 + 1 = 2k \omega_1 C_1$$

$$\Rightarrow \frac{-\omega_1^2}{\omega_H^2} + 1 = 2k\omega_1 C_1$$

$$\Rightarrow -\frac{(2\pi f_1)^2}{(2\pi f_H)^2} + 1 = 2k2\pi f_1 C_1$$

$$\Rightarrow 1 - \left(\frac{f_1}{f_H}\right)^2 = 4\pi k f_1 C_1$$

$$\Rightarrow 1 - \frac{f_1^2}{f_1 f_2} = 4\pi k f_1 C_1 \quad (\because f_H = \sqrt{f_1 f_2})$$

$$\Rightarrow \frac{f_2 - f_1}{f_2} = 4\pi k f_1 C_1$$

$$\Rightarrow \boxed{C_1 = \frac{f_2 - f_1}{4\pi k f_1 f_2}} \quad F$$

From equ ①, $\omega_H^2 = \frac{1}{L_1 C_1}$

$$L_1 = \frac{1}{\omega_H^2 C_1}$$

$$L_1 = \frac{(2\pi f_H)^2 \times \frac{f_2 - f_1}{4\pi k f_1 f_2}}{1}$$

$$L_1 = \frac{4\pi k f_1 f_2}{4\pi^2 f_1 f_2 \times (f_2 - f_1)} \quad (\because f_H = \sqrt{f_1 f_2})$$

$$\boxed{L_1 = \frac{k}{\pi (f_2 - f_1)}} \quad H$$

We have $\frac{L_2}{C_1} = \frac{L_1}{C_2} = k^2$

$$L_2 = C_1 k^2$$

$$L_2 = \frac{f_2 - f_1}{4\pi f_1 f_2} \times k^2$$

$$L_2 = \frac{k(f_2 - f_1)}{4\pi f_1 f_2} \quad H$$

$$\& \quad \frac{L_1}{C_2} = k^2$$

$$C_2 k^2 = L_1$$

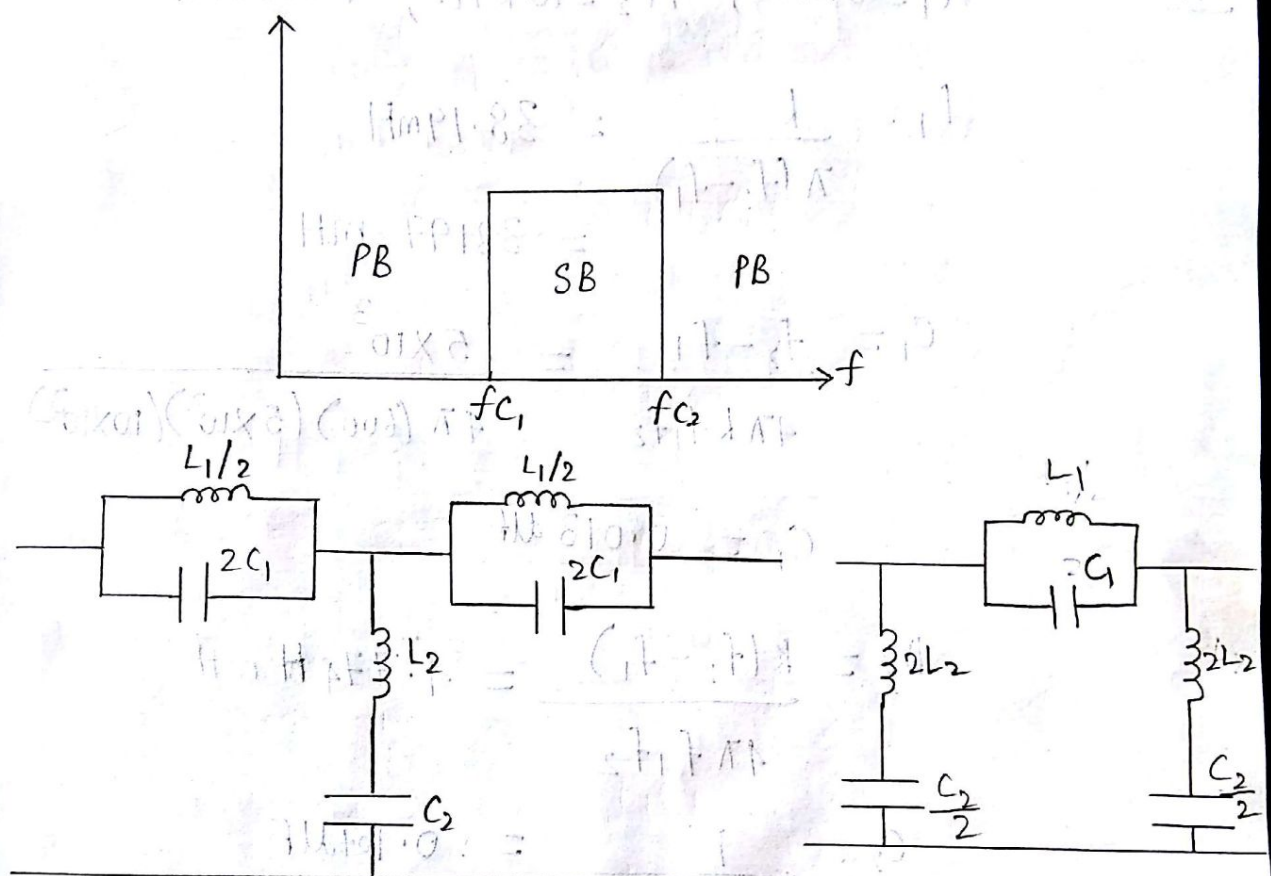
$$C_2 = \frac{L_1}{k^2}$$

$$C_2 = \frac{L_1}{\pi(f_2 - f_1) \times k^2}$$

$$C_2 = \frac{L_1}{\pi k(f_2 - f_1)} \quad F$$

Band elimination filter:-

A BEF which is the one which attenuates a band of frequencies and allows the remaining frequencies.



(T-network)

(π -network)

$$L_1 = \frac{k(f_2 - f_1)}{\pi f_1 f_2} \text{ H} ; C_1 = \frac{1}{4\pi k(f_2 - f_1)} \text{ F}$$

$$L_2 = \frac{k}{4\pi(f_2 - f_1)} \text{ H} ; C_2 = \frac{f_2 - f_1}{\pi f_1 f_2 k} \text{ F}$$

$$f_H = \sqrt{f_1 f_2}$$

11.) Design a BPF whose cutoff frequencies are $f_{c1} = 5 \text{ KHz}$, $f_{c2} = 10 \text{ KHz}$ for a nominal impedance of 600Ω . Determine values of resonant frequency

Sol

$$f_{c1} = 5 \text{ KHz}, f_{c2} = 10 \text{ KHz}, k = 600 \Omega$$

$$L_1 = \frac{k}{\pi(f_2 - f_1)} = 38.19 \text{ mH}$$

$$= \cancel{38.197} \text{ mH}$$

$$C_1 = \frac{f_2 - f_1}{4\pi k f_1 f_2} = \frac{5 \times 10^3}{4\pi (600) (5 \times 10^3) (10 \times 10^3)}$$

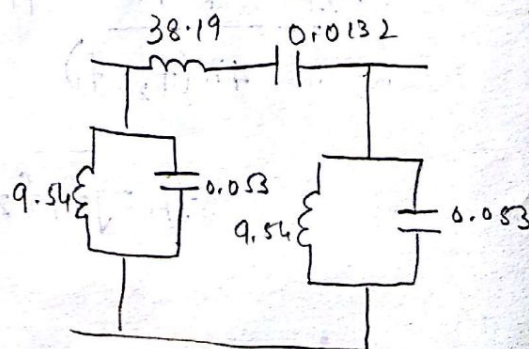
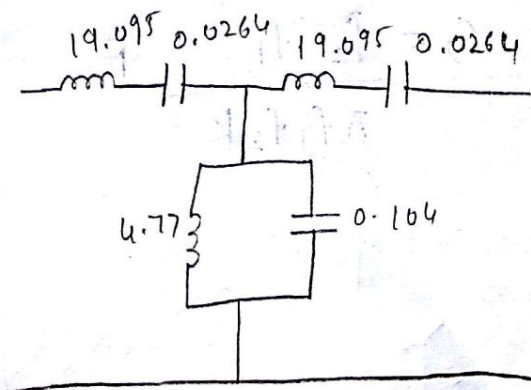
$$C_1 = 0.013 \mu\text{F}$$

$$L_2 = \frac{k(f_2 - f_1)}{4\pi f_1 f_2} = 4.774 \text{ mH}$$

$$C_2 = \frac{1}{\pi k(f_2 - f_1)} = 0.107 \mu\text{F}$$

$$\frac{L_1}{2} = 19.095 \text{ mH} ; 2C_1 = 0.026 \mu\text{F}$$

$$2L_2 = 9.54 \text{ mH} ; \frac{C_2}{2} = 0.0535 \mu\text{F}$$



2) Design a BPF whose cut-off frequency is 1 KHz, 10 KHz and nominal imp is 500Ω . Draw the symmetrical T-n/w & π -n/w.

Sol $f_{c1} = 1\text{KHz}$, $f_{c2} = 10\text{KHz}$; $K = 500\Omega$
 $= 1 \times 10^3 \text{Hz}$; $= 10 \times 10^3 \text{Hz}$

$$L_1 = \frac{K}{\pi(f_2 - f_1)} = 17.6 \text{ mH}$$

$$C_1 = \frac{f_2 - f_1}{4\pi K f_1 f_2} = 0.143 \mu\text{F}$$

$$L_2 = \frac{K(f_2 - f_1)}{4\pi f_1 f_2} = 35.8 \text{ mH}$$

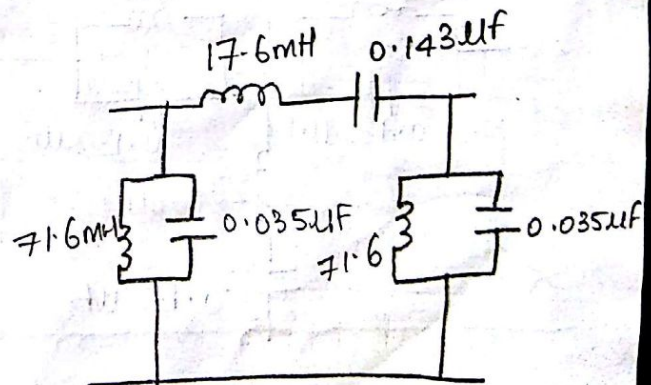
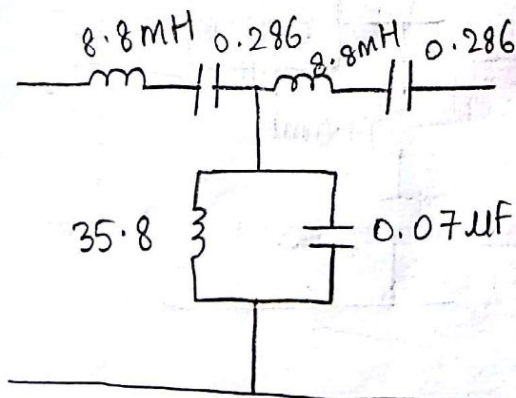
$$C_2 = \frac{1}{\pi K(f_2 - f_1)} = 0.07 \mu\text{F}$$

$$\frac{L_1}{2} = 8.8 \text{ mH}$$

$$2C_1 = 0.286 \mu\text{F}$$

$$2L_2 = 71.6 \text{ mH}$$

$$\frac{C_2}{2} = 0.035 \mu\text{F}$$



3) Design a BEF to eliminate the frequencies b/w 2KHz & 4KHz for a nominal impedance of 600 Ω .

Sol $f_{c1} = 2 \times 10^3 \text{ Hz}$; $f_{c2} = 4 \times 10^3 \text{ Hz}$

$k = 600 \Omega$

$L_1 = \frac{k(f_2 - f_1)}{\pi f_1 f_2} = 47.7 \text{ mH}$

$C_1 = \frac{1}{4\pi k(f_2 - f_1)} = 0.066 \mu\text{F}$

$L_2 = \frac{k}{4\pi(f_2 - f_1)} = 23.8 \text{ mH}$

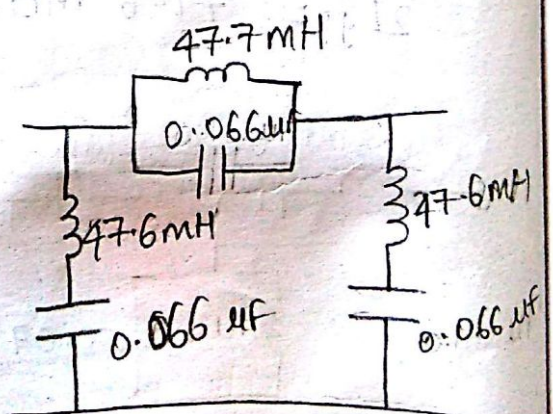
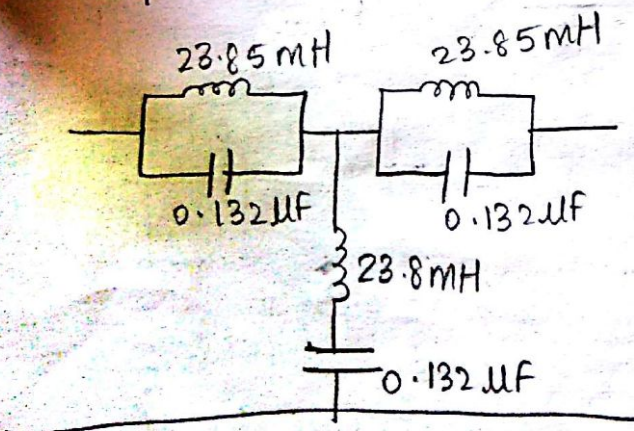
$C_2 = \frac{f_2 - f_1}{\pi k f_1 f_2} = \cancel{132.6 \mu\text{F}} \quad 0.132 \mu\text{F}$

$\frac{L_1}{2} = 23.85 \text{ mH}$;

$2L_2 = 47.6 \text{ mH}$

$2C_1 = 0.132 \mu\text{F}$;

$\frac{C_2}{2} = 0.066 \mu\text{F}$



4) Design a BEF to eliminate frequencies b/w 1 KHz & 5 KHz for a nominal imp of 400Ω .

Sol $f_{c1} = 10^3 \text{ Hz}$; $f_{c2} = 5 \times 10^3 \text{ Hz}$, $k = 400 \Omega$

$$L_1 = \frac{k(f_2 - f_1)}{\pi f_1 f_2} = 101.85 \text{ mH}$$

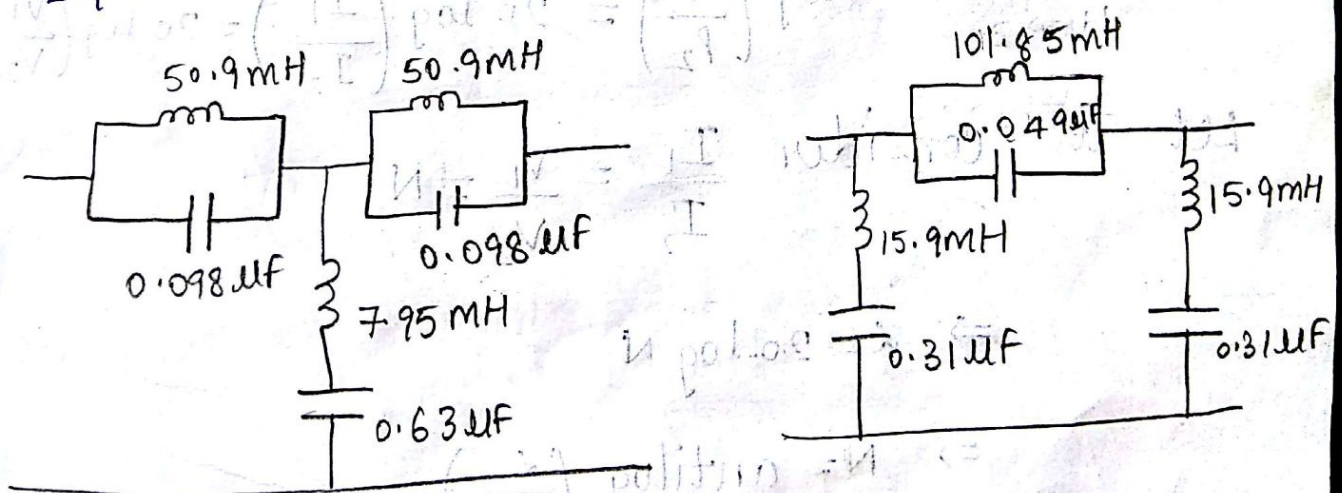
$$C_1 = \frac{1}{4\pi k(f_2 - f_1)} = 0.049 \mu\text{F}$$

$$L_2 = \frac{k}{4\pi(f_2 - f_1)} = 7.95 \text{ mH}$$

$$C_2 = \frac{f_2 - f_1}{\pi k f_1 f_2} = 0.63 \mu\text{F}$$

$$\frac{L_1}{2} = 50.925 \text{ mH}; \quad 2L_2 = 15.9 \text{ mH}$$

$$2C_1 = 0.098 \mu\text{F}; \quad \frac{C_2}{2} = 0.315 \mu\text{F}$$



15-2-16

Symmetrical Attenuators

→ Attenuator is a 2-port resistive n/w which reduces the strength of the signal. The decrease in the strength of signal is expressed in terms of attenuation. It is denoted by α & is expressed as 'decibels' or 'nepers'.

→ Attenuation in terms of nepers is given as

$$\alpha = \ln \left(\frac{I_1}{I_2} \right) = \ln \left(\frac{V_1}{V_2} \right) = \frac{1}{2} \ln \left(\frac{P_1}{P_2} \right)$$

→ Attenuation in decibel is defined as 10 times the natural logarithm of the ratio of i/p power to the o/p power.

$$\alpha = 10 \log \left(\frac{P_1}{P_2} \right) = 20 \log \left(\frac{I_1}{I_2} \right) = 20 \log \left(\frac{V_1}{V_2} \right)$$

Let us consider $\frac{I_1}{I_2} = \frac{V_1}{V_2} = N$

$$\Rightarrow \alpha = 20 \log N$$

$$\Rightarrow N = \text{antilog} \left(\frac{\alpha}{20} \right)$$

as α is expressed in terms of decibels, the above equ. can be written as

$$N = \text{antilog} \left(\frac{dB}{20} \right) \quad (\text{or}) \quad N = 10^{\left(\frac{dB}{20} \right)}$$

Classification of attenuators:

Attenuators are classified into different types based on

- 1.) Amount of attenuation $\left\{ \begin{array}{l} \rightarrow \text{Fixed attenuators} \\ \rightarrow \text{Variable attenuators} \end{array} \right.$
- 2.) Characteristic behaviour
- 3.) Type of structure $\left\{ \begin{array}{l} \rightarrow \text{T-type} \left\{ \begin{array}{l} \rightarrow \text{Symmetrical "} \\ \rightarrow \text{Asymmetrical "} \end{array} \right. \\ \rightarrow \pi\text{-type} \\ \rightarrow \text{Bridged T-type} \\ \rightarrow \text{lattice} \\ \rightarrow \text{L-type} \end{array} \right.$

Symmetrical attenuator

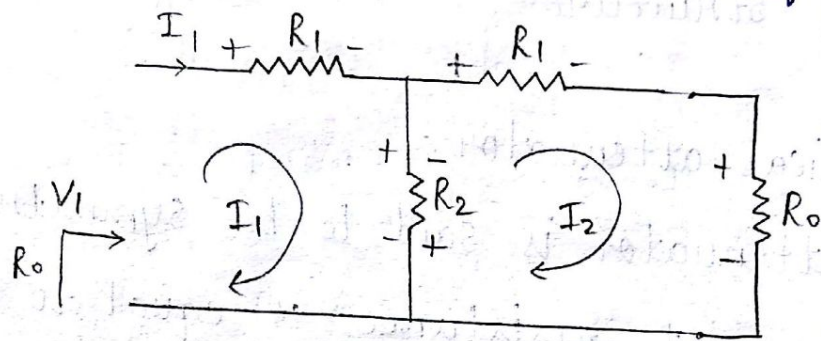
An attenuator is said to be symmetrical when i/p port resistance is equal to o/p port resistance.

→ Symmetrical attenuators are classified into 4 types:-

- 1.) Symmetrical T attenuator
- 2.) Symmetrical π attenuator
- 3.) Symmetrical bridged T attenuator
- 4.) Symmetrical lattice attenuator

1) Symmetrical T-attenuator:-

It is an attenuator whose structure looks like 'T' symbol with equal series arm resistance as shown in fig. The design of attenuator is carried through the value of characteristic resistance & amount of attenuation.



Apply KVL to loop-2

$$0 = I_2 R_1 + I_2 R_0 + (I_2 - I_1) R_2$$

$$I_2 (R_1 + R_0 + R_2) = R_2 I_1$$

$$\frac{I_1}{I_2} = \frac{R_1 + R_0 + R_2}{R_2} = N \quad \text{--- (1)}$$

$$R_0 = [(R_0 + R_1) \parallel R_2] + R_1$$

$$R_0 = \frac{(R_0 + R_1) R_2}{R_0 + R_1 + R_2} + R_1$$

$$R_0 = \frac{R_0 + R_1}{\frac{R_0 + R_1 + R_2}{R_2}} + R_1$$

$$R_0 = \frac{R_0 + R_1 + R_1}{N}$$

$$NR_0 = R_0 + R_1 + NR_1$$

$$R_0(N-1) = R_1(N+1)$$

$$\boxed{R_1 = R_0 \left(\frac{N-1}{N+1} \right)} \quad \text{--- (2)}$$

From equ (1),

$$R_1 + R_0 + R_2 = NR_2$$

Sub (2) in above equ,

$$R_0 \left(\frac{N-1}{N+1} \right) + R_0 = R_2(N-1)$$

$$R_0 \left(\frac{N-1+N+1}{N+1} \right) = R_2(N-1)$$

$$R_0 \left(\frac{2N}{(N+1)(N-1)} \right) = R_2$$

$$\boxed{R_0 \left(\frac{2N}{N^2-1} \right) = R_2} \quad \text{--- (3)}$$

P1) Design a symmetrical T-attenuator to produce an attenuation of 60 dB, char. resistance of 400 Ω

Sol

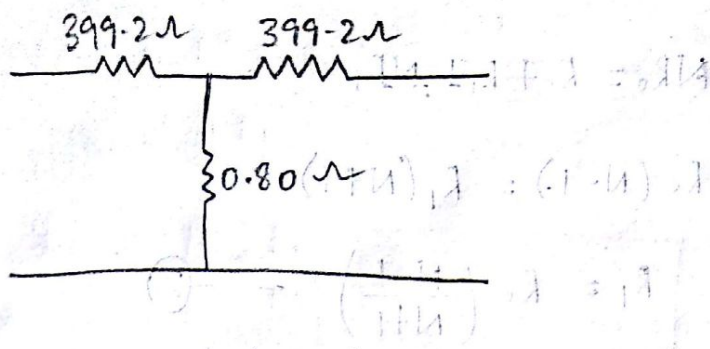
$$R_0 = 400 \Omega, \quad \alpha = 60 \text{ dB}$$

$$R_1 = R_0 \left(\frac{N-1}{N+1} \right)$$

$$N = \text{antilog} \left(\frac{\text{dB}}{20} \right) \Rightarrow N = 10^{\left(\frac{60}{20} \right)} = 1000$$

$$R_1 = 399.2 \Omega$$

$$R_2 = R_0 \left(\frac{2N}{N^2-1} \right) = 0.800 \Omega$$

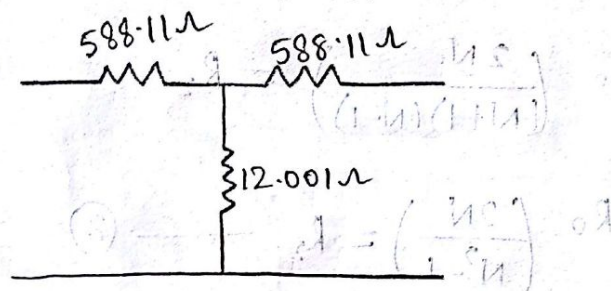


2) Design a Symmetrical T-attenuator, of 40 dB and $R_0 = 600 \Omega$

Sol $N = 10^{\left(\frac{dB}{20}\right)} = 100$

$$R_1 = R_0 \left(\frac{N-1}{N+1} \right) = 588.11 \Omega$$

$$R_2 = R_0 \left(\frac{2N}{N^2-1} \right) = 12.001 \Omega$$

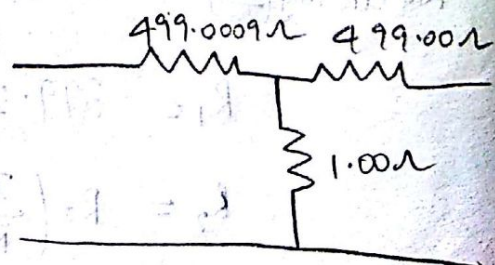


3) Design a T-pad attenuator to give an attenuation of 60 dB & to work in a line of 500Ω imp.

Sol $N = 10^{\left(\frac{dB}{20}\right)} = 1000$ $R_0 = 500; \alpha = 60$

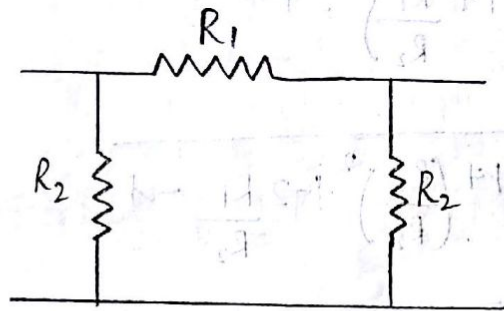
$$R_1 = 499.0009 \Omega$$

$$R_2 = 1.000 \Omega$$



2) Symmetrical π -network ^{attenuator}:-

It is an attenuator whose structure looks like a ' π ' symbol with equal shunt arm resistance as shown in fig. For a symmetrical π -configuration, char. impedance is



$$Z_{0\pi} = \sqrt{\frac{Z_1 Z_2}{1 + \frac{Z_1}{4Z_2}}}$$

$$\cosh \gamma = 1 + \frac{Z_1}{2Z_2}; \quad \sinh \frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}}$$

$$\text{where } Z_1 = R_1, \quad Z_2 = \frac{R_2}{2}$$

Attenuator is purely resistive n/w, hence the phase shift produced $\beta = 0$. Hence $\gamma = \alpha$.

The char. resistance of attenuator is

$$R_0 = \sqrt{\frac{\frac{R_1 R_2}{2}}{1 + \frac{R_1}{4\left(\frac{R_2}{2}\right)}}} = \sqrt{\frac{R_1 R_2 / 2}{\frac{2R_2 + R_1}{2R_2}}}$$

$$R_0 = \sqrt{\frac{R_1 R_2^2}{2R_2 + R_1}}$$

$$\cosh \alpha = 1 + \frac{R_1}{R_2}$$

$$\sinh \alpha = \sqrt{\cosh^2 \alpha - 1}$$

$$\sinh \alpha = \sqrt{\left(1 + \frac{R_1}{R_2}\right)^2 - 1}$$

$$\sinh \alpha = \sqrt{1 + \left(\frac{R_1}{R_2}\right)^2 + 2\frac{R_1}{R_2} - 1}$$

$$\sinh \alpha = \sqrt{\frac{R_1^2 + 2R_1 R_2}{R_2^2}}$$

Multiply & $\div$ by R_1

$$\sinh \alpha = \sqrt{\frac{R_1^3 + 2R_1^2 R_2}{R_1 R_2^2}}$$

$$\sinh \alpha = \frac{1}{\sqrt{\frac{R_1 R_2^2}{R_1^2 (R_1 + 2R_2)}}} \quad \left(\because R_0 = \sqrt{\frac{R_1 R_2^2}{2R_2 + R_1}} \right)$$

$$\sinh \alpha = \frac{1}{\frac{1}{R_1} \cdot R_0} = \frac{R_1}{R_0}$$

$$\Rightarrow R_1 = R_0 \sinh \alpha$$

$$R_1 = R_0 \left(\frac{e^\alpha - e^{-\alpha}}{2} \right)$$

$\times$ & $\div$ by e^α

$$R_1 = R_0 \left(\frac{e^{2\alpha} - 1}{2e^\alpha} \right)$$

$$\text{WKT } \alpha = \ln \left(\frac{I_1}{I_2} \right) = \ln(N)$$

$$N = e^\alpha$$

$$\Rightarrow \boxed{R_1 = R_0 \left(\frac{N^2 - 1}{2N} \right)}$$

$$\text{We know that } \sinh \frac{\alpha}{2} = \sqrt{\frac{Z_1}{4Z_2}}$$

$$\sinh \frac{\alpha}{2} = \sqrt{\frac{R_1}{2R_2}} \quad \text{--- (1)}$$

$$\cosh \frac{\alpha}{2} = \sqrt{\sinh^2 \frac{\alpha}{2} + 1}$$

$$\cosh \frac{\alpha}{2} = \sqrt{\left(\frac{R_1}{2R_2} \right)^2 + 1}$$

$$= \sqrt{\frac{R_1^2 + 2R_2^2}{4R_2^2}}$$

$$\cosh \frac{\alpha}{2} = \sqrt{\frac{R_1 + 2R_2}{2R_2}} \quad \text{--- (2)}$$

$$\coth \frac{\alpha}{2} = \frac{\cosh \alpha/2}{\sinh \alpha/2} = \sqrt{\frac{R_1 + 2R_2}{R_1}}$$

$$\coth \frac{\alpha}{2} = \sqrt{\frac{(R_1 + 2R_2) R_2^2}{R_1 R_2^2}}$$

$$\coth \frac{\alpha}{2} = \frac{1}{\sqrt{\frac{R_1 R_2^2}{(R_1 + 2R_2) R_2^2}}}$$

$$\cot \frac{h\alpha}{2} = \frac{1}{\frac{1}{R_2} + R_0}$$

$$\cot \frac{h\alpha}{2} = \frac{R_2}{R_0}$$

$$\Rightarrow R_2 = R_0 \cot \frac{h\alpha}{2}$$

$$R_2 = R_0 \left(\frac{e^{\alpha/2} + e^{-\alpha/2}}{e^{\alpha/2} - e^{-\alpha/2}} \right)$$

Multiply $\frac{1}{e^{\alpha/2} - e^{-\alpha/2}}$ by $e^{\alpha/2}$

$$R_2 = R_0 \left(\frac{e^{\alpha} + 1}{e^{\alpha} - 1} \right)$$

$$\alpha = \ln \left(\frac{I_1}{I_2} \right) = \ln(N)$$

$$\Rightarrow N = e^{\alpha}$$

$$R_2 = R_0 \left(\frac{N+1}{N-1} \right)$$

P1.) Design a symmetrical π -attenuator for an attenuation of 100 dB with char. res $R_0 = 400 \Omega$

Sol

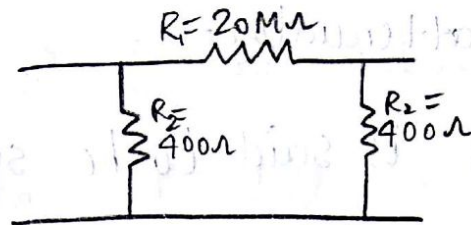
$$N = e^{\alpha} = 2.68 \times 10^4 \quad N = 10^{\left(\frac{dB}{20}\right)} = 100000$$

($\because$ given in decibels)

$$R_1 = R_0 \left(\frac{N^2 - 1}{2N} \right) = 20000000$$

$$R_1 = 20 \times 10^6 \Omega = 20 \text{ M}\Omega$$

$$R_2 = R_0 \left(\frac{N+1}{N-1} \right) = 400.008 \Omega$$



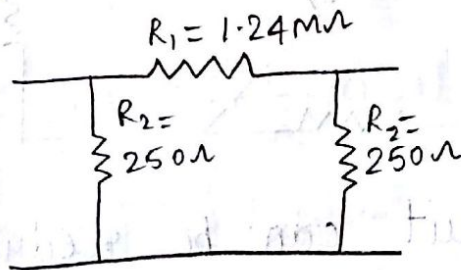
2.) Design a symmetrical π -attenuator of $\alpha = 80 \text{ dB}$ in

$$R_0 = 250 \Omega$$

Sol $N = 10^{\left(\frac{\text{dB}}{20}\right)} = 10000$

$$R_1 = 1249999.98 = 1.24 \text{ M}\Omega$$

$$R_2 = 250.05 \Omega$$



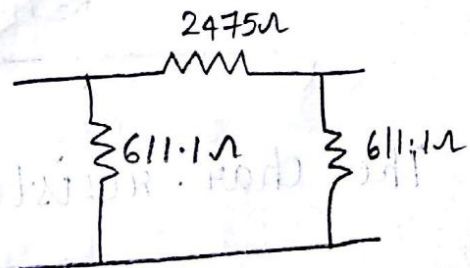
3.) Design symmetrical π -attenuator of $\alpha = 20 \text{ dB}$,

$$R_0 = 500 \Omega$$

Sol $N = 10^{\left(\frac{\text{dB}}{20}\right)} = 10$

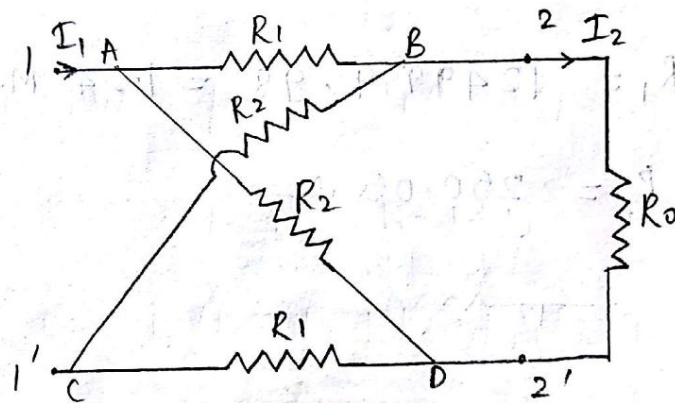
$$R_1 = 247.5 \Omega$$

$$R_2 = 611.11 \Omega$$

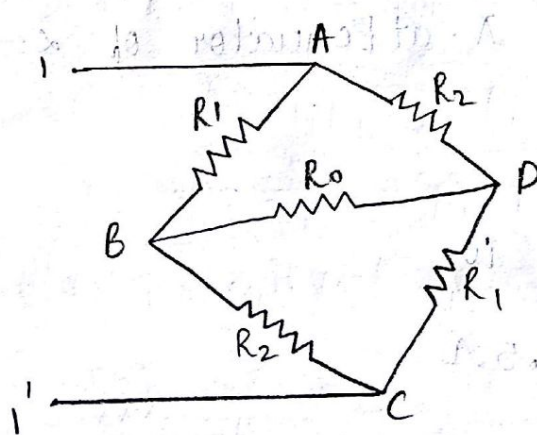


3) Symmetrical lattice attenuator:-

Lattice attenuator is said to be symmetrical when its series arm resistance is equal and diagonal arm resistance are equal. A lattice attenuator consists of 2 series arm & 2 diagonal arms. A symmetrical lattice attenuator is shown in fig



The above circuit can be redrawn as



The char. resistance is

$$Z_0 = \sqrt{Z_{oc} \cdot Z_{sc}}$$

$$R_0 = \sqrt{R_{oc} \cdot R_{sc}}$$

R_{oc} = when R_0 is open circuited.

$$R_{oc} = (R_1 + R_2) \parallel (R_1 + R_2)$$

$$R_{oc} = \frac{(R_1 + R_2)^2}{2(R_1 + R_2)}$$

$$R_{oc} = \frac{R_1 + R_2}{2}$$

R_{sc} = when R_o is SC, equivalent resistance is

$$R_{sc} = (R_1 \parallel R_2) + (R_1 \parallel R_2)$$

$$R_{sc} = \frac{R_1 R_2}{R_1 + R_2} + \frac{R_1 R_2}{R_1 + R_2}$$

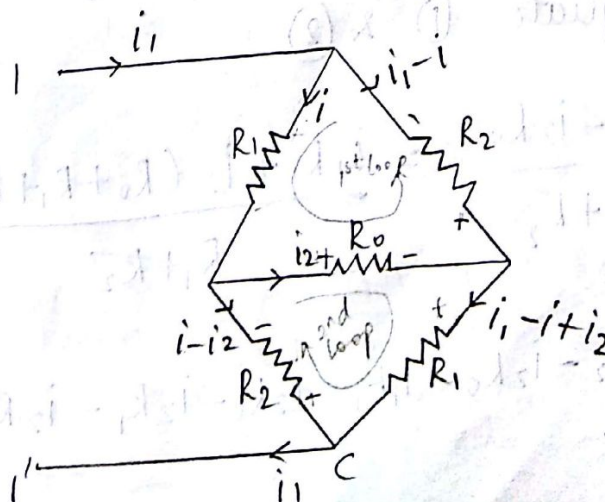
$$R_{sc} = \frac{2R_1 R_2}{R_1 + R_2}$$

$$R_o = \sqrt{\frac{R_1 + R_2}{2} \times \frac{2R_1 R_2}{R_1 + R_2}}$$

$$R_o = \sqrt{R_1 R_2}$$

$$\ln\left(\frac{I_1}{I_2}\right) = \ln(N) = \alpha$$

$$N = e^\alpha = \frac{I_1}{I_2}$$



Apply KVL for first loop,

$$0 = iR_1 + i_2 R_0 - R_2(i_1 + i)$$

$$0 = i(R_1 + R_2) + i_2 R_0 - R_2 i_1$$

$$R_2 i_1 = i(R_1 + R_2) + i_2 R_0$$

$$i_1 = \frac{i(R_1 + R_2) + i_2 R_0}{R_2} \quad \text{--- (a)}$$

Apply KVL for second loop,

$$0 = i_2 R_0 + R_1(i_1 - i + i_2) - R_2(i - i_2)$$

$$0 = i_2(R_0 + R_1 + R_2) + i_1 R_1 - i R_1 - i R_2$$

$$0 = i_2(R_0 + R_1 + R_2) + i_1 R_1 - i(R_1 + R_2)$$

$$i(R_1 + R_2) = i_1 R_1 + i_2(R_0 + R_1 + R_2)$$

$$i = \frac{i_1 R_1 + i_2(R_0 + R_1 + R_2)}{R_1 + R_2} \quad \text{--- (2)}$$

From equ (a), $i = \frac{R_2 i_1 - R_0 i_2}{R_1 + R_2}$ --- (1)

Equate (1) & (2)

$$\frac{i_1 R_2 - i_2 R_0}{R_1 + R_2} = \frac{i_1 R_1 + i_2(R_0 + R_1 + R_2)}{R_1 + R_2}$$

$$\Rightarrow i_1 R_2 - i_2 R_0 - i_1 R_1 - i_2 R_0 - i_2 R_1 - i_2 R_2 = 0$$

$$\Rightarrow i_1 (R_2 - R_1) - i_2 (R_0 + R_0 + R_1 + R_2) = 0$$

$$\Rightarrow i_1 (R_2 - R_1) - i_2 (2R_0 + R_1 + R_2) = 0$$

$$\Rightarrow i_1 (R_2 - R_1) = i_2 (2R_0 + R_1 + R_2)$$

$$\Rightarrow \frac{i_1}{i_2} = \frac{2R_0 + R_1 + R_2}{R_2 - R_1} = N$$

$$\Rightarrow 2\sqrt{R_1 R_2} + (\sqrt{R_1})^2 + (\sqrt{R_2})^2 = N (R_2 - R_1)$$

$$\Rightarrow (\sqrt{R_1} + \sqrt{R_2})^2 = N [(\sqrt{R_2})^2 - (\sqrt{R_1})^2]$$

$$(\because R_0 = \sqrt{R_1 R_2})$$

$$\Rightarrow (\sqrt{R_1} + \sqrt{R_2})^2 = N [(\sqrt{R_2} + \sqrt{R_1})(\sqrt{R_2} - \sqrt{R_1})]$$

$$\Rightarrow \sqrt{R_1} + \sqrt{R_2} = N (\sqrt{R_2} - \sqrt{R_1})$$

$$\Rightarrow N = \frac{\sqrt{R_1} + \sqrt{R_2}}{\sqrt{R_2} - \sqrt{R_1}}$$

$$\text{WKT } R_0 = \sqrt{R_1 R_2}$$

$$\sqrt{R_2} = \frac{R_0}{\sqrt{R_1}}$$

$$\Rightarrow N = \frac{\sqrt{R_1} + \frac{R_0}{\sqrt{R_1}}}{\frac{R_0}{\sqrt{R_1}} - \sqrt{R_1}}$$

$$= \frac{R_1 + R_0}{R_0 - R_1}$$

$$\Rightarrow N = \frac{R_1 + R_0}{R_0 - R_1}$$

$$\Rightarrow N(R_0 - R_1) = R_1 + R_0$$

$$NR_0 - NR_1 = R_1 + R_0$$

$$N(R_0 - R_1) = R_1 + R_0$$

$$NR_0 - R_0 = R_1 + NR_1$$

$$R_0(N-1) = R_1(N+1)$$

$$R_1 = R_0 \left(\frac{N-1}{N+1} \right)$$

$$\text{WKT } \sqrt{R_2} = \frac{R_0}{\sqrt{R_1}}$$

$$R_2 = \frac{R_0^2}{R_1}$$

$$R_2 = \frac{R_0^2}{R_0 \left(\frac{N-1}{N+1} \right)}$$

$$R_2 = R_0 \left(\frac{N+1}{N-1} \right)$$

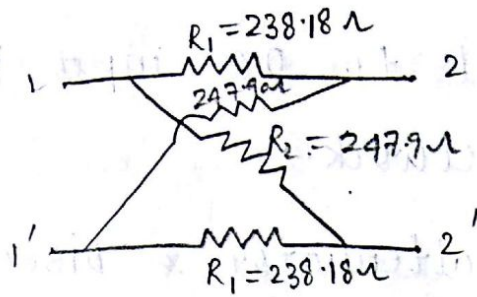
Pi) Design a symmetrical lattice attenuator to produce an attenuation of 40dB & Char. res is 243Ω.

Sol

$$N = 10^{\left(\frac{dB}{20} \right)} = 10^{\left(\frac{40}{20} \right)} = 100$$

$$R_1 = 243 \left(\frac{100-1}{100+1} \right) = 238.18 \Omega$$

$$R_2 = R_0 \left(\frac{N+1}{N-1} \right) = 247.90 \Omega$$



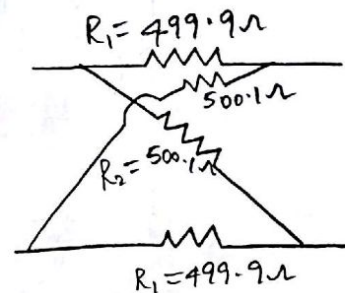
2) Design a Symmetrical lattice attenuator, $\alpha = 80\text{dB}$

$$R_o = 500\Omega$$

Sol $N = 10^{\left(\frac{dB}{20}\right)} = 10000$

$$R_1 = 499.9\Omega$$

$$R_2 = 500.1\Omega$$



9-3-16

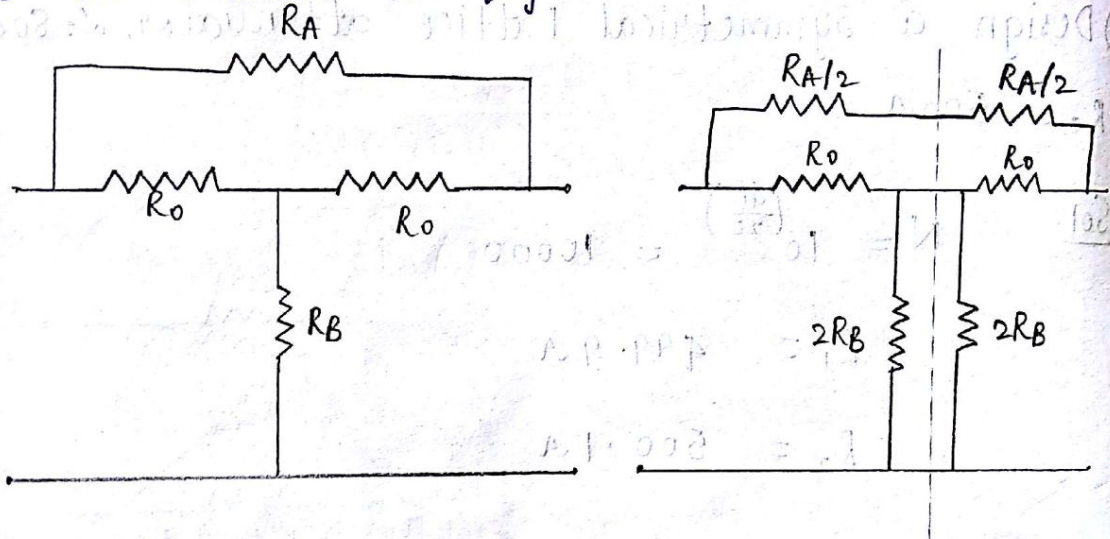
4) Symmetrical bridged-T attenuator:-

A bridged T-n/w is a T-n/w which has an extra bridge arm. The design of symmetrical bridged-T attenuator will be carried out through bisection theorem.

Bisection theorem states that the networks with mirror image symmetry can be converted into their equivalent lattice structure. Series arm impedance of equivalent lattice structure is equal to SC i/p impedance of the bisected network whereas shunt arm

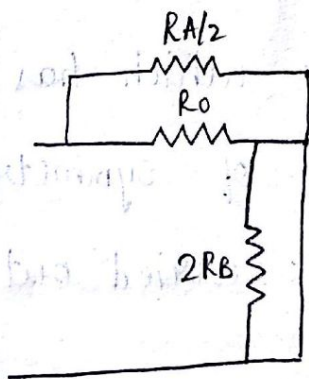
impedance is equal to OC input impedance of the bisected network.

The bridged T-attenuator & bisected attenuator are as shown in fig.



Series arm impedance $R_1 = R_{SC}$

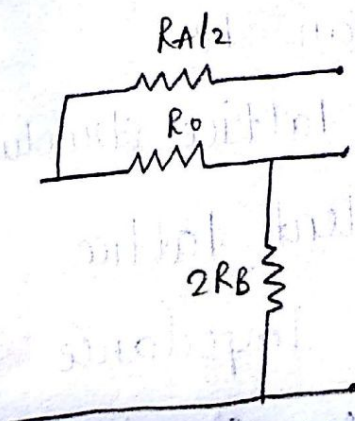
Shunt arm impedance $R_2 = R_{OC}$



$$R_1 = R_{SC} = \frac{R_A}{2} \parallel R_0$$

$$R_1 = \frac{\frac{R_A}{2} \times R_0}{\frac{R_A}{2} + R_0}$$

$$R_1 = \frac{R_A R_0}{R_A + 2R_0} \quad \text{--- (1)}$$



$$R_2 = R_{OC} = R_0 + 2R_B \quad \text{--- (2)}$$

From the symmetrical lattice attenuator structure, we have

$$R_1 = R_0 \left[\frac{N-1}{N+1} \right] \text{---(3)}, R_2 = R_0 \left[\frac{N+1}{N-1} \right] \text{---(4)}$$

Equate ① & ③

$$\frac{R_A R_0}{R_A + 2R_0} = R_0 \left[\frac{N-1}{N+1} \right]$$

$$R_A R_0 (N+1) = R_0 (N-1) (R_A + 2R_0)$$

$$R_A R_0 (N+1) = (R_A R_0 + 2R_0^2) (N-1)$$

$$R_A R_0 (N+1 - N+1) = 2R_0^2 (N-1)$$

$$\boxed{R_A = R_0 (N-1)}$$

Equate ② & ④

$$R_0 + 2R_B = R_0 \left(\frac{N+1}{N-1} \right)$$

$$(R_0 + 2R_B)(N-1) = R_0 (N+1)$$

$$R_0 (N-1 - N-1) + 2R_B (N-1) = 0$$

$$R_0 (-2) = -2R_B (N-1)$$

$$\boxed{R_B = \frac{R_0}{N-1}}$$

P1.) Design a symmetrical bridged-T attenuator to have an attenuation of 80 dB for a char. resistance of $500\ \Omega$.

Sol

$$N = 10^{\left(\frac{dB}{20}\right)} = 10^{\frac{80}{20}} = 10000$$

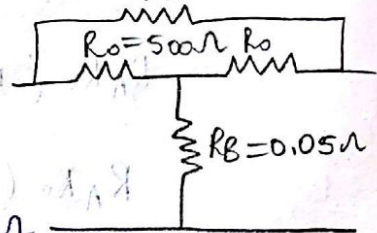
$$R_A = R_0(N-1)$$

$$R_A = 500(N-1)$$

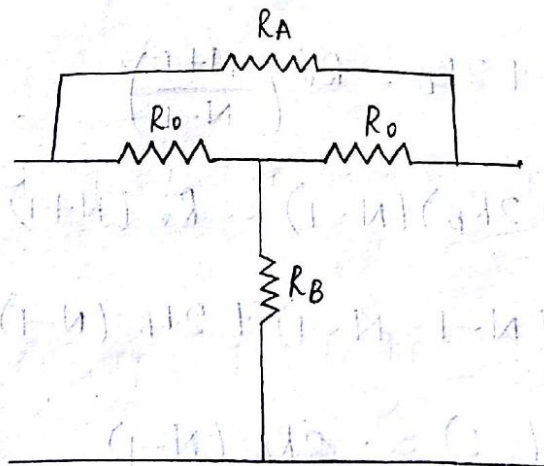
$$R_A = 4999500 = 4999.5\ \text{k}\Omega$$

$$R_B = \frac{R_0}{N-1}$$

$$R_B = \frac{500}{10000-1} = 0.05\ \Omega$$



Characteristic resistance:- (for bridged-T)



$$R_{OC} = [(R_0 + R_A) \parallel R_0] + R_B$$

$$= \frac{(R_0 + R_A) R_0}{R_0 + R_A + R_0} + R_B$$

$$R_{oc} = \frac{R_A R_o + R_o^2}{2R_o + R_A} + R_B$$

$$R_{oc} = \frac{R_A R_o + R_o^2 + 2R_o R_B + R_A R_B}{2R_o + R_A} \quad \text{--- (1)}$$

$$R_{sc} = [(R_o \parallel R_B) + R_o] \parallel R_A$$

$$= \left[\frac{R_o R_B}{R_o + R_B} + R_o \right] \parallel R_A$$

$$= \left(\frac{R_o R_B + R_o^2 + R_o R_B}{R_o + R_B} \right) \parallel R_A$$

$$= \frac{(R_o R_B + R_o^2 + R_o R_B) R_A}{R_o + R_B + R_A}$$

$$= \frac{R_o R_B + R_o^2 + R_o R_B}{R_o + R_B} + R_A$$

$$= \frac{R_o R_A R_B + R_o^2 R_A + R_o R_B R_A}{R_o R_B + R_o^2 + R_o R_B + R_A R_o + R_A R_B}$$

$$R_{sc} = \frac{2R_o R_A R_B + R_o^2 R_A}{R_o^2 + 2R_o R_B + R_o R_A + R_A R_B} \quad \text{--- (2)}$$

$$R_o = \sqrt{R_{oc} \times R_{sc}}$$

$$R_o^2 = R_{oc} \times R_{sc}$$

$$R_o^2 = R_{oc} \times R_{sc}$$

$$R_o^2 = \frac{R_A R_o + R_o^2 + \cancel{2R_o R_B} + \cancel{R_A R_B}}{2R_o + R_A} \times \frac{\cancel{2R_o R_A R_B} + R_o^2 R_A}{\cancel{R_o^2} + \cancel{2R_o R_B} + \cancel{R_o R_A} + \cancel{R_A R_B}}$$

$$R_o^2 = \frac{2R_o R_A R_B + R_o^2 R_A}{2R_o + R_A}$$

$$R_o^2 = \frac{R_o R_A (2R_B + R_o)}{2R_o + R_A}$$

$$R_o = \frac{R_A (2R_B + R_o)}{2R_o + R_A}$$

$$2R_o^2 + R_o R_A = R_A (2R_B + R_o)$$

$$\cancel{2R_o^2} + \cancel{R_o R_A} = \cancel{2R_A R_B} + \cancel{R_A R_o}$$

$$R_o^2 = R_A R_B$$

$$\boxed{R_o = \sqrt{R_A R_B}}$$

A bridged T-n/w is said to be symmetrical when its bridge arm and shunt arm satisfies the relationship $R_o^2 = R_A \cdot R_B$.

Pi.) Design a symmetrical bridged T-attenuator with $\alpha = 50 \text{ dB}$, char. resistance $R_0 = 243 \Omega$

Sol $N = 10^{\left(\frac{\alpha}{20}\right)} = 10^{\frac{50}{20}} = 316.22$

$$R_A = R_0(N-1) = 76598.46 \Omega = 76.6 \text{ k}\Omega$$

$$R_B = \frac{R_0}{N-1} = 0.77 \Omega$$

