

12-12-15

Unit: 2 Twoport network parameters

Network:-

Interconnection of elements of different characteristics is called network.

Circuit:-

A circuit is a network with a closed path.

Classification of networks:-

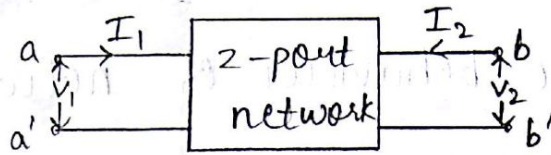
- 1) Active & passive networks
- 2) Linear & nonlinear networks
- 3) Unilateral & bilateral networks
- 4) Lumped & distributed networks

Port:-

It is the pair of terminals of the network from where the i/p is supplied & o/p is extracted.

Two-port network:-

A network which contains 2 pairs of accessible terminals and is represented as



Twoport network parameters:-

To describe the behaviour of an electrical network from 4 variables of 2 port network (V_1, I_1, V_2, I_2) are used using the four variables. Mathematically equations will be constructed from which the network behaviour is determined through certain constants values known as '2-port network parameters'.

There are 4 types of parameters:-

- 1) Z-parameters / Impedance / oc parameters
- 2) Y-parameters / Admittance / SC parameters
- 3) ABCD parameters / Transmission parameters / general klt
- 4) h-parameters / hybrid parameters

1) Z-parameters:-

They define the behaviour of network in terms of impedance values.

From ohms law, $V=IZ$

For a 2-port n/w, there are 2 voltages in matrix form
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [Z] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

where V_1 = port 1 voltage

V_2 = o/p voltage at port 2

I_1 = port 1 current

I_2 = o/p current at port 2

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \quad \text{--- (1)}$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 \quad \text{--- (2)}$$

Determination of Z-parameters:-

Case: i) Let $I_1=0$ i.e port 1 terminals are OC.

$$V_1 = Z_{12}I_2$$

$$\Rightarrow Z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1=0} \quad (\text{OC reverse transfer impedance})$$

$$V_2 = Z_{22} I_2$$

$$\Rightarrow Z_{22} = \frac{V_2}{I_2} \bigg|_{I_1=0} \text{ (oc o/p impedance)}$$

Case: 2) Let $I_2 = 0$ i.e port 2 terminals are oc

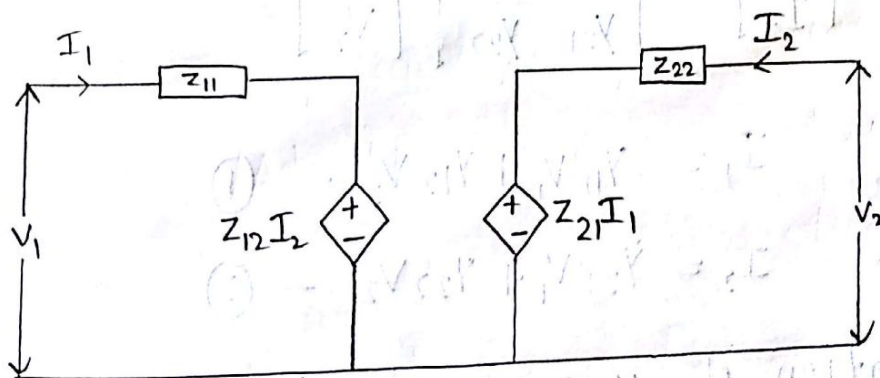
$$V_1 = Z_{11} I_1$$

$$\Rightarrow Z_{11} = \frac{V_1}{I_1} \bigg|_{I_2=0} \text{ (oc i/p impedance)}$$

$$V_2 = Z_{21} I_1$$

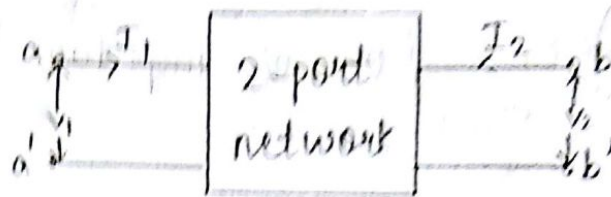
$$\Rightarrow Z_{21} = \frac{V_2}{I_1} \text{ (oc forward transfer impedance)}$$

Z-parametric equivalent ckt:-



As z-parameters are determined by open circuiting each port at a time, these are called open circuited parameters.

2) y-parameters :-



$$[V] = [Z][I]$$

$$\Rightarrow I = \frac{V}{Z} = Y \cdot V$$

$$[I] = [Y][V]$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [Y] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \text{--- (1)}$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad \text{--- (2)}$$

Determination of y-parameters:-

Case: i) let $V_1 = 0$ i.e. SC port 1 terminals

$$I_1 = Y_{12} V_2$$

$$\Rightarrow Y_{12} = \frac{I_1}{V_2} \bigg|_{V_1=0} \quad (\text{SC reverse transfer admittance})$$

$$I_2 = Y_{22} V_2$$

$$\Rightarrow Y_{22} = \frac{I_2}{V_2} \bigg|_{V_1=0} \quad (\text{SC o/p admittance})$$

Case:2) Let $V_2 = 0$ i.e SC port 2 terminals

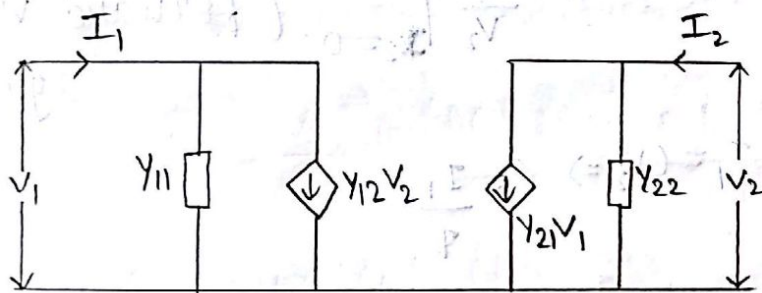
$$I_1 = Y_{11} V_1$$

$$\Rightarrow Y_{11} = \frac{I_1}{V_1} \bigg|_{V_2=0} \quad (\text{SC i/p admittance})$$

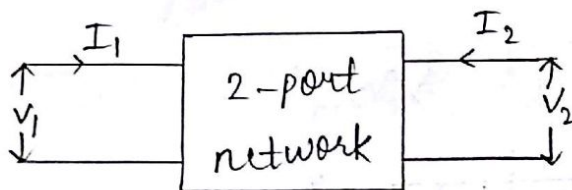
$$I_2 = Y_{21} V_1$$

$$\Rightarrow Y_{21} = \frac{I_2}{V_1} \bigg|_{V_2=0} \quad (\text{SC forward transfer admittance})$$

Y-parametric equivalent ckt:-



3) ABCD parameters:-



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix}_{2 \times 1} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{2 \times 2} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}_{2 \times 1}$$

$$V_1 = AV_2 - BI_2 \quad \text{--- ①}$$

$$I_1 = CV_2 - DI_2 \quad \text{--- ②}$$

ABCD parameters are normally used to analyse the ckt $\therefore$ they are called general ckt parameters

Case:1) Let $V_2 = 0$ (SC port 2 terminals)

$$V_1 = -BI_2 \Rightarrow B = -\frac{V_1}{I_2} \bigg|_{V_2=0} \quad (\text{SC reverse transfer impedance})$$

$$I_1 = DI_2 \Rightarrow D = \frac{I_1}{I_2} \bigg|_{V_2=0} \quad (\text{reverse current gain})$$

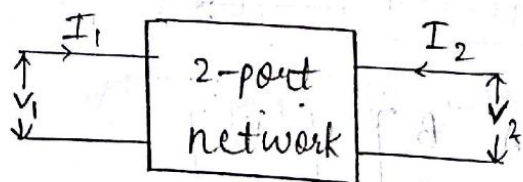
Case:2) Let $I_2 = 0$ (OC port 2 terminals)

$$V_1 = AV_2 \Rightarrow A = \frac{V_1}{V_2} \bigg|_{I_2=0} \quad (\text{reverse forward voltage gain})$$

$$V_2 = \frac{I_1}{C} \Rightarrow C = \frac{I_1}{V_2} \bigg|_{I_2=0}$$

$$I_1 = CV_2 \Rightarrow C = \frac{I_1}{V_2} \bigg|_{I_2=0} \quad (\text{OC reverse transfer admittance})$$

4) h-parameters:-



$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$V_1 = h_{11}I_1 + h_{12}V_2 \quad \text{--- ①}$$

$$I_2 = h_{21}I_1 + h_{22}V_2 \quad \text{--- ②}$$

Case:1) Let $V_2 = 0$ (SC port 2 terminals)

$$V_1 = h_{11} I_1 \Rightarrow h_{11} = \frac{V_1}{I_1} \bigg|_{V_2=0} \text{ (SC i/p impedance)}$$

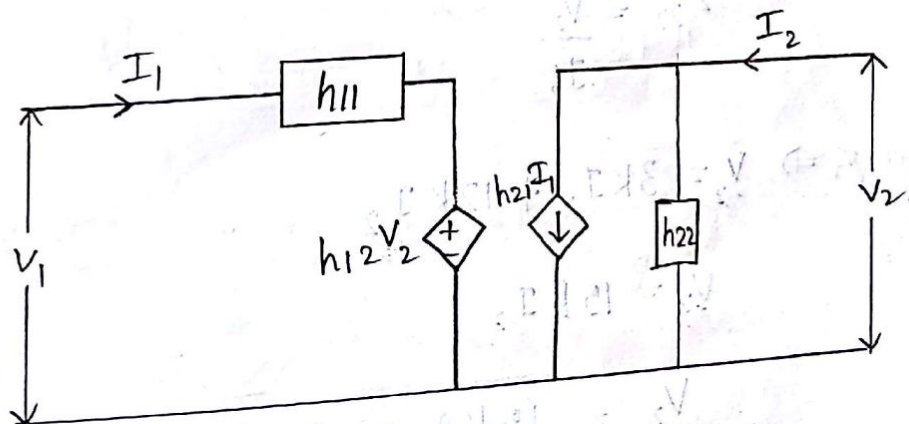
$$I_2 = h_{21} I_1 \Rightarrow h_{21} = \frac{I_2}{I_1} \bigg|_{V_2=0} \text{ (SC current gain)}$$

Case:2) Let $I_1 = 0$ (OC port 1 terminals)

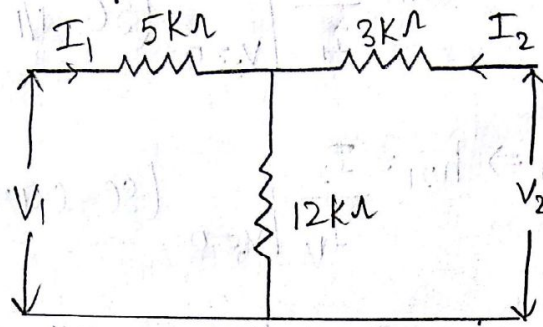
$$V_1 = h_{12} V_2 \Rightarrow h_{12} = \frac{V_1}{V_2} \bigg|_{I_1=0} \text{ (OC reverse voltage gain)}$$

$$I_2 = h_{22} V_2 \Rightarrow h_{22} = \frac{I_2}{V_2} \bigg|_{I_1=0} \text{ (OC o/p admittance)}$$

h-parametric equivalent ckt :-



P1) Determine z-parameters for given 2-port n/w & hence draw equivalent ckt.



Sol

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{--- (1)}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{--- (2)}$$

Case: 1) $I_1 = 0$

$$Z_{12} = \frac{V_1}{I_2}$$

$$Z_{22} = \frac{V_2}{I_2}$$

$$\text{At } V_2 \Rightarrow V_2 = 3k I_2 + 12k I_2$$

$$V_2 = 15k I_2$$

$$\frac{V_2}{I_2} = 15k\Omega = Z_{22}$$

$$V_1 = V_{5k\Omega} + V_{12k\Omega}$$

$$V_1 = 0 + V_{12k\Omega}$$

$$V_1 = V_{12k\Omega} = I_2 (12k)$$

$$\frac{V_1}{I_2} = 12k\Omega = Z_{12}$$

Case: 2) $I_2 = 0$

$$Z_{11} = \frac{V_1}{I_1}$$

$$Z_{21} = \frac{V_2}{I_1}$$

KVL at $V_1 \Rightarrow V_1 = 5kI_1 + 12kI_1$

$$V_1 = 17kI_1$$

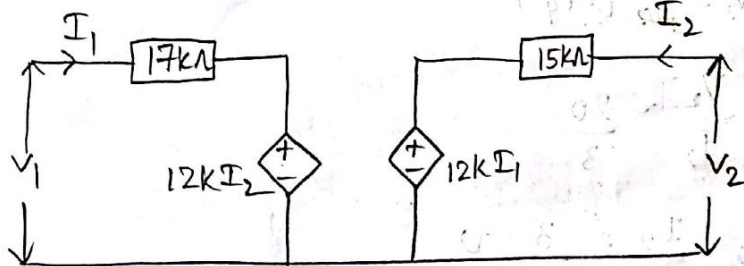
$$\frac{V_1}{I_1} = \boxed{17k\Omega = Z_{11}}$$

$$V_2 = V_{3k\Omega} + V_{12k\Omega}$$

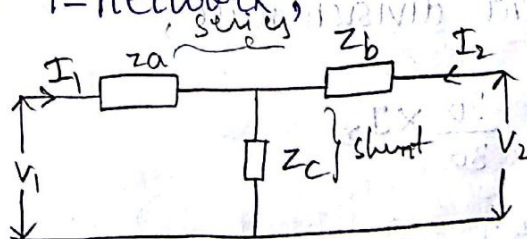
$$V_2 = 0 + V_{12k\Omega}$$

$$V_2 = V_{12k\Omega} = I_1 (12k)$$

$$\frac{V_2}{I_1} = \boxed{12k\Omega = Z_{21}}$$



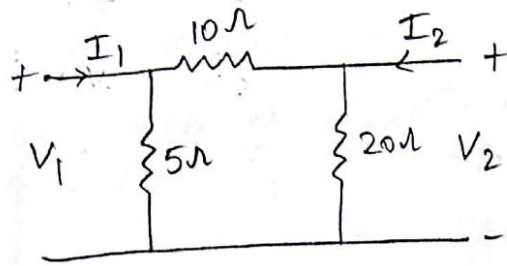
→ For a T-network,



$$Z_{11} = Z_a + Z_c \quad ; \quad Z_{22} = Z_b + Z_c$$

$$Z_{12} = Z_c \quad , \quad Z_{21} = Z_c$$

2) Determine y-parameters of the network Show



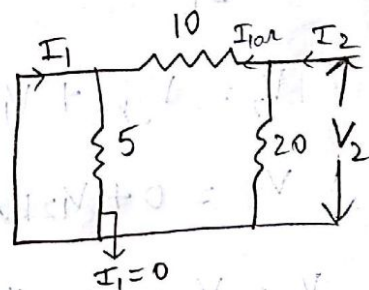
$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$

Case: i) $V_1 = 0$ (port 1 terminals are SC)

$$y_{12} = \frac{I_1}{V_2}$$

$$y_{22} = \frac{I_2}{V_2}$$



$$Z_{eq} = \frac{10 \times 20}{10 + 20} = \frac{200}{30} = \frac{20}{3}$$

$$V_2 = I_2 (Z_{eq})$$

$$\frac{V_2}{I_2} = \frac{20}{3}$$

$$y_{22} = \frac{I_2}{V_2} = \frac{3}{20} \text{ S}$$

Apply current division rule,

$$I_1 = I_{10\Omega} = \frac{20}{30} \times I_2$$

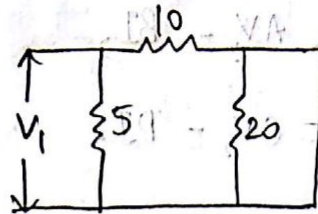
$$I_1 = \frac{2}{3} I_2$$

$$y_{12} = \frac{I_1}{V_2} = \frac{\frac{2}{3} I_2}{\frac{20}{3} I_2} \Rightarrow y_{12} = \frac{1}{10} = 0.1 \text{ S}$$

Case: 2) $V_2 = 0$

$$y_{11} = \frac{I_1}{V_1}$$

$$y_{21} = \frac{I_2}{V_1}$$



$$Z_{eq} = \frac{5 \times 10}{5 + 10} = \frac{50}{15} = \frac{10}{3} \Omega$$

$$V_1 = I_1 (Z_{eq})$$

$$\frac{V_1}{I_1} = \frac{10}{3}$$

$$y_{11} = \frac{I_1}{V_1} = \frac{3}{10} \text{ S} = 0.3 \text{ S}$$

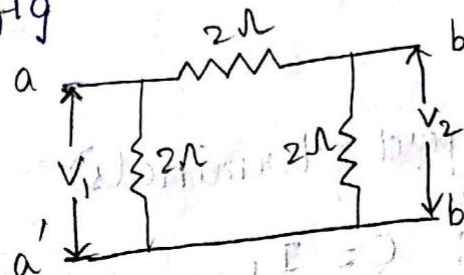
Apply current division,

$$I_2 = I_{10\Omega} = \frac{5}{15} \times I_1$$

$$I_2 = \frac{1}{3} I_1$$

$$y_{21} = \frac{I_2}{V_1} = \frac{\frac{1}{3} I_1}{\frac{10}{3} I_1} = \frac{1}{10} = 0.1 \text{ S}$$

3) Find transmission parameters for the ckt shown in fig



Sol

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

Case:1) $V_2 = 0$ (SC port 2 terminals)

$$B = - \frac{V_1}{I_2}$$

$$D = - \frac{I_1}{I_2}$$

$$Z_{eq} = \frac{2 \times 2}{2+2} = 1$$

$$V_2 = I_2 (Z_{eq})$$

$$V_1 = I_1 (Z_{eq})$$

$$V_1 = I_1 (1)$$

$$I_2 = I_{2n} = \frac{2}{4} I_1$$

$$I_2 = \frac{1}{2} I_1$$

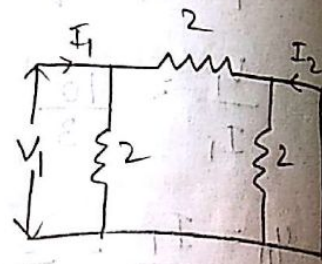
$$\therefore B = - \frac{V_1}{I_2} = -2 \text{ (Opp direction)}$$

$$= - \frac{1}{2} I_1$$

$$D = - \frac{I_1}{\frac{1}{2} I_1} = -2$$

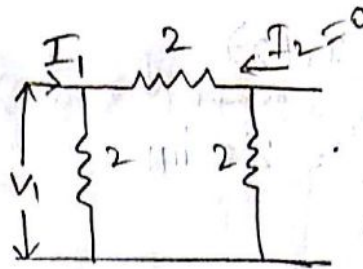
Case:2) $I_2 = 0$ (OC port 2 terminals)

$$A = \frac{V_1}{V_2} ; C = \frac{I_1}{V_2}$$



$$V_1 = \frac{2+2}{3} I_1$$

$$V_1 = \frac{4}{3} I_1$$



$$V_2 = V_{2\Omega} \Rightarrow I_{2\Omega} \times 2$$

$$I_{2\Omega} = \frac{2}{6} \times I_1 \Rightarrow \frac{1}{3} I_1$$

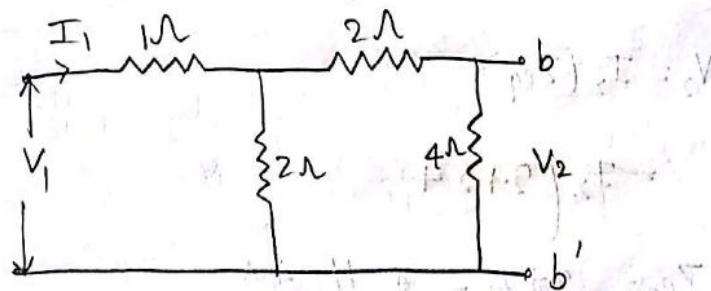
$$V_2 = 2 \times \frac{1}{3} I_1$$

$$C = \frac{I_1}{V_2} = \frac{3}{2} = 1.5 \text{ V}$$

$$A = \frac{\frac{4}{3} I_1}{\frac{2}{3} I_1} = 2$$

$$A = 2$$

4) Find h-parameters of the network shown in fig.



Sol

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

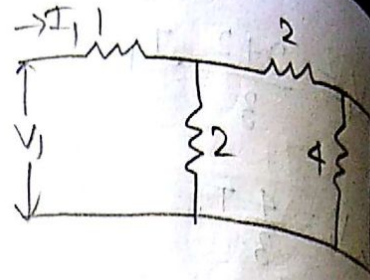
Case: i) $I_1 = 0$ $V_2 = 0$

$$h_{11} = \frac{V_1}{I_1} ; h_{21} = \frac{I_2}{I_1}$$

$$V_1 = I_1(2)$$

$$Z_{eq} = \frac{2 \times 2 + 1}{2 + 2} \quad (\because V_3 = V_{4\Omega} = 0)$$

$$\frac{V_1}{I_1} = 2 = h_{11}$$



$$I_2 = \frac{V_1}{2}$$

$$I_2 = I_{2\Omega} = \frac{2}{4} \times I_1$$

$$I_2 = -\frac{1}{2} I_1$$

$$\frac{I_2}{I_1} = -\frac{1}{2}$$

$$h_{21} = \frac{I_2}{I_1} = -\frac{1}{2}$$

Case: 2) $I_1 = 0$

$$h_{12} = \frac{V_1}{V_2} ; \quad h_{22} = \frac{I_2}{V_2}$$

$$V_2 = I_2(Z_{eq})$$

$$= I_2(2 + 2 + \dots)$$

$$Z_{eq} = 2 + 2 = 4 \parallel 4\Omega$$

$$= \frac{4 \times 4}{4 + 4} = 2 \Omega$$

$$V_2 = 2I_2$$

$$h_{22} = \frac{I_2}{V_2} = \frac{1}{2}$$

$$V_{2\Omega} = V_{1\Omega} + V_{2\Omega} = 0 + V_{2\Omega}$$

$$V_1 = V_{2\Omega} = I_{2\Omega} \times 2$$

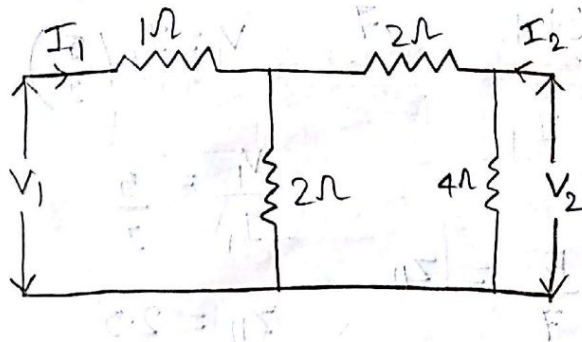
$$I_{2\Omega} = \frac{4}{8} I_2$$

$$V_1 = \frac{1}{2} I_2 \times 2$$

$$V_1 = I_2$$

$$h_{12} = \frac{V_1}{V_2} = \frac{I_2}{2I_2} = \frac{1}{2}$$

5) Find z-parameters for ckt shown in fig.



Sol

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

Case: i) $I_1 = 0$

$$Z_{12} = \frac{V_1}{I_2} ; \quad Z_{22} = \frac{V_2}{I_2}$$

$$V_2 = 2 I_2$$

$$\frac{V_2}{I_2} = 2 = Z_{22}$$

$$V_1 = V_{2\Omega} = I_{2\Omega} \times 2$$

$$I_{2\Omega} = \frac{4}{8} I_2 = \frac{1}{2} I_2$$

$$V_1 = \frac{2}{2} I_2$$

$$Z_{12} = \frac{I_2}{I_1} = 1$$

Case: 2) $I_2 = 0$

$$Z_{11} = \frac{V_1}{I_1} ; Z_{21} = \frac{V_2}{I_1}$$

$$\cancel{Z_{eq} = 2 + 1} \\ = 3 \parallel 4 \Omega$$

$$R_{eq} = \frac{6 \times 2}{6 + 2} = \frac{12}{8} = \frac{3}{2}$$

$$\cancel{= \frac{3 \times 4}{3 + 4}} = \frac{12}{7}$$

$$V_1 = I_1 \times R_{eq}$$

$$V_1 = I_1 \left(\frac{5}{2} \right)$$

$$\cancel{V_1 = \frac{12}{7} I_1}$$

$$\frac{V_1}{I_1} = \frac{5}{2}$$

$$\cancel{\frac{V_1}{I_1} = \frac{12}{7} = Z_{11}}$$

$$Z_{11} = 2.5$$

$$V_2 = I_{4\Omega} \times 4$$

$$I_{4\Omega} = \frac{2 \times I_1}{8}$$

$$I_{4\Omega} = \frac{1}{4} I_1$$

$$V_2 = \frac{1}{4} I_1 \times 4$$

$$V_2 = I_1$$

$$Z_{21} = \frac{V_2}{I_1} = \frac{I_1}{I_1}$$

$$Z_{21} = 1$$

23-12-15

Conversion of one parameter to another:-

Express in terms of z :-

1) Y-parameters in terms of z :-

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{--- (1)}$$

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{--- (1)}$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 \quad \text{--- (2)}$$

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \text{ --- (a)}$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad \text{--- (b)}$$

From eqn (2), we have

$$I_2 = \frac{1}{Z_{22}} V_2 - \frac{Z_{21}}{Z_{22}} I_1 \quad \text{--- (3)}$$

Sub (3) in equ (1)

$$V_1 = Z_{11} I_1 + Z_{12} \left[\frac{1}{Z_{22}} V_2 - \frac{Z_{21}}{Z_{22}} I_1 \right]$$

$$V_1 = Z_{11} I_1 + \frac{Z_{12}}{Z_{22}} V_2 - \frac{Z_{21} Z_{12}}{Z_{22}} I_1$$

$$V_1 = I_1 \left[Z_{11} - \frac{Z_{21}Z_{12}}{Z_{22}} \right] + \frac{Z_{12}}{Z_{22}} V_2$$

$$V_1 = I_1 \left[\frac{Z_{11}Z_{22} - Z_{21}Z_{12}}{Z_{22}} \right] + \frac{Z_{12}}{Z_{22}} V_2$$

$$V_1 = T_1 \left[\frac{\Delta Z}{Z_{22}} \right] + \frac{Z_{12}}{Z_{22}} V_2$$

$$I_1 = \frac{Z_{22}}{\Delta Z} V_1 - \frac{Z_{12}}{Z_{22}} V_2 \times \frac{Z_{22}}{\Delta Z}$$

$$I_1 = \left[\frac{Z_{22}}{\Delta Z} \right] V_1 - \left[\frac{Z_{12}}{\Delta Z} \right] V_2 \quad \text{--- (4)}$$

Compare equ (4) with equ (a)

$$Y_{11} = \frac{Z_{22}}{\Delta Z} \quad ; \quad Y_{12} = -\frac{Z_{12}}{\Delta Z}$$

Sub. (4) in equ (2)

$$V_2 = Z_{21} \left[\left(\frac{Z_{22}}{\Delta Z} \right) V_1 - \left(\frac{Z_{12}}{\Delta Z} \right) V_2 \right] + Z_{22} I_2$$

$$V_2 = \frac{Z_{21} Z_{22}}{\Delta Z} V_1 - \frac{Z_{21} Z_{12}}{\Delta Z} V_2 + Z_{22} I_2$$

$$V_2 \left[1 + \frac{Z_{21} Z_{12}}{\Delta Z} \right] - \frac{Z_{21} Z_{22}}{\Delta Z} V_1 = Z_{22} I_2$$

$$V_2 \left[\frac{\Delta Z + Z_{21} Z_{12}}{\Delta Z} \right] - \frac{Z_{21} Z_{22}}{\Delta Z} V_1 = Z_{22} I_2$$

$$\Rightarrow V_2 \left[\frac{\Delta Z + Z_{11} Z_{22} - Z_{12} Z_{21} + Z_{21} Z_{12}}{\Delta Z} \right] - \frac{Z_{21} Z_{22}}{\Delta Z} V_1 = Z_{22} I_2$$

$$\Rightarrow V_2 \left[\frac{Z_{11} Z_{22}}{\Delta Z} \right] - \left[\frac{Z_{21} Z_{22}}{\Delta Z} \right] V_1 = Z_{22} I_2$$

$$\Rightarrow I_2 = -\left[\frac{Z_{21}}{\Delta Z} \right] V_1 + \left[\frac{Z_{11}}{\Delta Z} \right] V_2 \quad \text{--- (5)}$$

Compare equ (5) with (b),

$$Y_{21} = -\frac{Z_{21}}{\Delta Z} \quad , \quad Y_{22} = \frac{Z_{11}}{\Delta Z}$$

$$Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{Z_{22}}{\Delta Z} & -\frac{Z_{12}}{\Delta Z} \\ -\frac{Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{bmatrix}$$

2.) ABCD parameters in terms of z parameters:-

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{--- (1)}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{--- (2)}$$

$$V_1 = A V_2 - B I_2 \quad \text{--- (a)}$$

$$I_1 = C V_2 - D I_2 \quad \text{--- (b)}$$

From equ (2), we have

$$I_1 = \frac{V_2}{Z_{21}} - \frac{Z_{22}}{Z_{21}} I_2 \quad \text{--- (3)}$$

Compare (3) with equ (b)

$$C = \frac{1}{Z_{21}} ; \quad D = \frac{Z_{22}}{Z_{21}}$$

Sub (3) in (1)

$$V_1 = Z_{11} \left[\frac{V_2}{Z_{21}} - \frac{Z_{22}}{Z_{21}} I_2 \right] + Z_{12} I_2$$

$$V_1 = \frac{Z_{11}}{Z_{21}} V_2 - \frac{Z_{11} Z_{22}}{Z_{21}} I_2 + Z_{12} I_2$$

$$V_1 = \frac{Z_{11}}{Z_{21}} V_2 - \left[\frac{Z_{11} Z_{22}}{Z_{21}} - Z_{12} \right] I_2$$

$$V_1 = \frac{Z_{11}}{Z_{21}} V_2 - \left[\frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{21}} \right] I_2$$

$$V_1 = \frac{Z_{11}}{Z_{21}} V_2 - \left[\frac{\Delta Z}{Z_{21}} \right] I_2 \text{ --- (4)}$$

Compare (4) with equ (a)

$$A = \frac{Z_{11}}{Z_{21}} ; B = \frac{\Delta Z}{Z_{21}}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{Z_{11}}{Z_{21}} & \frac{\Delta Z}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{11}} \end{bmatrix}$$

3) h-parameters in terms of z parameters

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \text{ --- (1)}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \text{ --- (2)}$$

$$V_1 = h_{11} I_1 + h_{12} V_2 \text{ --- (a)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \text{ --- (b)}$$

From (2), we have

$$I_2 = \frac{V_2 - Z_{21} I_1}{Z_{22}}$$

$$I_2 = -\frac{Z_{21}}{Z_{22}} I_1 + \frac{1}{Z_{22}} V_2 \text{ --- (3)}$$

Compare (3) with (b)

$$h_{21} = -\frac{Z_{21}}{Z_{22}} ; h_{22} = \frac{1}{Z_{22}}$$

Sub. ③ in ①

$$V_1 = Z_{11} I_1 + Z_{12} \left[\frac{-Z_{21}}{Z_{22}} I_1 + \frac{1}{Z_{22}} V_2 \right]$$

$$V_1 = Z_{11} I_1 + \frac{Z_{12}}{Z_{22}} V_2 - \frac{Z_{12} Z_{21}}{Z_{22}} I_1$$

$$V_1 = \left[Z_{11} - \frac{Z_{12} Z_{21}}{Z_{22}} \right] I_1 + \frac{Z_{12}}{Z_{22}} V_2$$

$$V_1 = \left[\frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{22}} \right] I_1 + \frac{Z_{12}}{Z_{22}} V_2$$

$$V_1 = \left[\frac{\Delta Z}{Z_{22}} \right] I_1 + \left[\frac{Z_{12}}{Z_{22}} \right] V_2 \quad \text{--- ④}$$

Compare ④ with ①

$$h_{11} = \frac{\Delta Z}{Z_{22}} ; \quad h_{12} = \frac{Z_{12}}{Z_{22}}$$

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \frac{\Delta Z}{Z_{22}} & \frac{Z_{12}}{Z_{22}} \\ \frac{-Z_{21}}{Z_{22}} & \frac{1}{Z_{22}} \end{bmatrix}$$

4) Z parameters in terms of y parameters:-

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \text{--- ①}$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad \text{--- ②}$$

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{--- a}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{--- b}$$

From (2), we have

$$V_2 = \frac{I_2}{Y_{22}} - \frac{Y_{21} V_1}{Y_{22}} \quad \text{--- (3)}$$

Sub (3) in (1)

$$I_1 = Y_{11} V_1 + Y_{12} \left[\frac{I_2}{Y_{22}} - \frac{Y_{21} V_1}{Y_{22}} \right]$$

$$I_1 = Y_{11} V_1 + \frac{Y_{12}}{Y_{22}} I_2 - \frac{Y_{12} Y_{21}}{Y_{22}} V_1$$

$$I_1 = \left[Y_{11} - \frac{Y_{12} Y_{21}}{Y_{22}} \right] V_1 + \frac{Y_{12}}{Y_{22}} I_2$$

$$I_1 = \left[\frac{Y_{11} Y_{22} - Y_{12} Y_{21}}{Y_{22}} \right] V_1 + \frac{Y_{12}}{Y_{22}} I_2$$

$$I_1 = \left[\frac{\Delta Y}{Y_{22}} \right] V_1 + \left[\frac{Y_{12}}{Y_{22}} \right] I_2$$

$$V_1 = \frac{I_1 \times Y_{22}}{\Delta Y} - \frac{Y_{12}}{Y_{22}} I_2 \times \frac{Y_{22}}{\Delta Y}$$

$$V_1 = \left[\frac{Y_{22}}{\Delta Y} \right] I_1 - \left[\frac{Y_{12}}{\Delta Y} \right] I_2 \quad \text{--- (4)}$$

Compare (4) with (a)

$$Z_{11} = \frac{Y_{22}}{\Delta Y}; \quad Z_{12} = -\frac{Y_{12}}{\Delta Y}$$

Sub ④ in equ ②

$$I_2 = Y_{21} \left[\frac{Y_{22}}{\Delta Y} I_1 - \frac{Y_{12}}{\Delta Y} I_2 \right] + Y_{22} V_2$$

$$I_2 = \frac{Y_{21} Y_{22}}{\Delta Y} I_1 - \frac{Y_{21} Y_{12}}{\Delta Y} I_2 + Y_{22} V_2$$

$$I_2 \left[1 + \frac{Y_{21} Y_{12}}{\Delta Y} \right] = \frac{Y_{21} Y_{22}}{\Delta Y} I_1 + Y_{22} V_2$$

$$I_2 \left[\frac{\Delta Y + Y_{21} Y_{12}}{\Delta Y} \right] = \frac{Y_{21} Y_{22}}{\Delta Y} I_1 + Y_{22} V_2$$

$$I_2 \left[\frac{Y_{11} Y_{22} - Y_{12} Y_{21} + Y_{21} Y_{12}}{\Delta Y} \right] = \frac{Y_{21} Y_{22}}{\Delta Y} I_1 + Y_{22} V_2$$

$$I_2 \left[\frac{Y_{11} Y_{22}}{\Delta Y} \right] - \left[\frac{Y_{21} Y_{22}}{\Delta Y} \right] I_1 = Y_{22} V_2$$

$$V_2 = \left[\frac{Y_{11}}{\Delta Y} \right] I_2 - \left[\frac{Y_{21}}{\Delta Y} \right] I_1 \quad \text{--- (5)}$$

Compare ⑤ with ⑥

$$Z_{21} = -\frac{Y_{21}}{\Delta Y} ; \quad Z_{22} = \frac{Y_{11}}{\Delta Y}$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{Y_{22}}{\Delta Y} & -\frac{Y_{12}}{\Delta Y} \\ -\frac{Y_{21}}{\Delta Y} & \frac{Y_{11}}{\Delta Y} \end{bmatrix}$$

5) ABCD parameters in terms of Y parameters

$$\textcircled{1} \quad I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \text{--- (1)}$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad \text{--- (2)}$$

$$V_1 = A V_2 - B I_2 \quad \text{--- (a)}$$

$$I_1 = C V_2 - D I_2 \quad \text{--- (b)}$$

From (2), we have

$$V_1 = \frac{I_2 - Y_{22} V_2}{Y_{21}} \quad \text{--- (3)}$$

Compare (3) with (a)

$$A = -\frac{Y_{22}}{Y_{21}}; \quad B = -\frac{1}{Y_{21}}$$

Sub. (3) in (1)

$$I_1 = Y_{11} \left[-\frac{Y_{22}}{Y_{21}} V_2 + \frac{1}{Y_{21}} I_2 \right] + Y_{12} V_2$$

$$I_1 = -\frac{Y_{11} Y_{22}}{Y_{21}} V_2 + \frac{Y_{11}}{Y_{21}} I_2 + Y_{12} V_2$$

$$I_1 = \left[Y_{12} - \frac{Y_{11} Y_{22}}{Y_{21}} \right] V_2 + \frac{Y_{11}}{Y_{21}} I_2$$

$$I_1 = \left[\frac{Y_{12} Y_{21} - Y_{11} Y_{22}}{Y_{21}} \right] V_2 + \frac{Y_{11}}{Y_{21}} I_2$$

$$I_1 = -\left[\frac{\Delta Y}{Y_{21}}\right] V_2 + \left[\frac{Y_{11}}{Y_{21}}\right] I_2 \quad \text{--- (4)}$$

Compare equ (4) with (b)

$$C = \frac{\Delta Y}{Y_{21}} ; D = -\frac{Y_{11}}{Y_{21}}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} -\frac{Y_{22}}{Y_{21}} & -\frac{1}{Y_{21}} \\ -\frac{\Delta Y}{Y_{21}} & -\frac{Y_{11}}{Y_{21}} \end{bmatrix}$$

h parameters in terms of y parameters:-

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \text{--- (1)}$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad \text{--- (2)}$$

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{--- (a)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{--- (b)}$$

From equ (1),

$$V_1 = \frac{I_1}{Y_{11}} - \frac{Y_{12} V_2}{Y_{11}} \quad \text{--- (3)}$$

compare (3) with (a)

$$h_{11} = \frac{1}{Y_{11}} ; h_{12} = -\frac{Y_{12}}{Y_{11}}$$

Sub. (3) in (2)

$$I_2 = Y_{21} \left[\frac{1}{Y_{11}} I_1 - \frac{Y_{12} V_2}{Y_{11}} \right] + Y_{22} V_2$$

③

$$I_2 = \frac{Y_{21}}{Y_{11}} I_1 - \frac{Y_{21} Y_{12}}{Y_{11}} V_2 + Y_{22} V_2$$

$$I_2 = \frac{Y_{21}}{Y_{11}} I_1 + \left[Y_{22} - \frac{Y_{21} Y_{12}}{Y_{11}} \right] V_2$$

$$I_2 = \frac{Y_{21}}{Y_{11}} I_1 + \left[\frac{Y_{22} Y_{11} - Y_{21} Y_{12}}{Y_{11}} \right] V_2$$

$$I_2 = \left[\frac{Y_{21}}{Y_{11}} \right] I_1 + \left[\frac{\Delta Y}{Y_{11}} \right] V_2 \quad \text{--- (4)}$$

Compare ④ with ⑥

$$h_{21} = \frac{Y_{21}}{Y_{11}} \quad ; \quad h_{22} = \frac{\Delta Y}{Y_{11}}$$

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{Y_{11}} & -\frac{Y_{12}}{Y_{11}} \\ \frac{Y_{21}}{Y_{11}} & \frac{\Delta Y}{Y_{11}} \end{bmatrix}$$

7) Z parameters in terms of ABCD parameters:-

$$V_1 = A V_2 - B I_2 \quad \text{--- (1)}$$

$$I_1 = C V_2 - D I_2 \quad \text{--- (2)}$$

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{--- (a)}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{--- (b)}$$

From ②,

$$V_2 = \frac{I_1}{C} + \frac{D I_2}{C} \quad \text{--- (3)}$$

Compare (3) with (b)

$$Z_{21} = \frac{1}{C} ; Z_{22} = \frac{D}{C}$$

Sub. (3) in (1)

$$V_1 = A \left[\frac{I_1}{C} + \frac{D}{C} I_2 \right] - B I_2$$

$$V_1 = \frac{A}{C} I_1 + \frac{AD}{C} I_2 - B I_2$$

$$V_1 = \frac{A}{C} I_1 + \left[\frac{AD}{C} - B \right] I_2$$

$$V_1 = \frac{A}{C} I_1 + \left[\frac{AD - BC}{C} \right] I_2 \quad \left[\begin{matrix} A & B \\ C & D \end{matrix} \right] = (AD - BC) = 1 \Rightarrow$$

$$V_1 = \frac{A}{C} I_1 + \frac{\Delta A}{C} I_2 \quad \text{--- (4)}$$

Compare (4) with (a)

$$Z_{11} = \frac{A}{C} ; Z_{12} = \frac{\Delta A}{C}$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{A}{C} & \frac{\Delta A}{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix}$$

8) y parameters in terms of ABCD parameters:-

$$V_1 = A V_2 - B I_2 \quad \text{--- (1)}$$

$$I_1 = C V_2 - D I_2 \quad \text{--- (2)}$$

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \text{--- (a)}$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad \text{--- (b)}$$

(6)

From (1),

$$I_2 = \frac{A}{B} V_2 - \frac{V_1}{B}$$

$$I_2 = -\frac{1}{B} V_1 + \frac{A}{B} V_2 \quad \text{--- (3)}$$

Compare (3) with (b)

$$Y_{21} = -\frac{1}{B}, \quad Y_{22} = \frac{A}{B}$$

Sub (3) in (2)

$$I_1 = C V_2 - D \left[-\frac{1}{B} V_1 + \frac{A}{B} V_2 \right]$$

$$I_1 = C V_2 + \frac{D}{B} V_1 - \frac{AD}{B} V_2$$

$$I_1 = \left[C - \frac{AD}{B} \right] V_2 + \frac{D}{B} V_1$$

$$I_1 = -\left[\frac{AD - BC}{B} \right] V_2 + \frac{D}{B} V_1$$

$$I_1 = -\left[\frac{AD - BC}{B} \right] V_2 + \left[\frac{D}{B} \right] V_1 \quad \text{--- (4)}$$

Compare (4) with (a)

$$Y_{12} = -\frac{AD - BC}{B}; \quad Y_{11} = \frac{D}{B}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} -\frac{AD - BC}{B} & \frac{D}{B} \\ -\frac{1}{B} & \frac{A}{B} \end{bmatrix}$$

h parameters in terms of ABCD parameters:-

$$V_1 = AV_2 - BI_2 \quad \text{--- (1)}$$

$$I_1 = CV_2 - DI_2 \quad \text{--- (2)}$$

$$V_1 = h_{11}I_1 + h_{12}V_2 \quad \text{--- (a)}$$

$$I_2 = h_{21}I_1 + h_{22}V_2 \quad \text{--- (b)}$$

From (2)

$$I_2 = \frac{CV_2 - I_1}{D}$$

$$I_2 = -\frac{1}{D}I_1 + \frac{C}{D}V_2 \quad \text{--- (3)}$$

Compare (3) with (b)

$$h_{21} = -\frac{1}{D}, \quad h_{22} = \frac{C}{D}$$

Sub (3) in (1)

$$V_1 = AV_2 - B \left[-\frac{1}{D}I_1 + \frac{C}{D}V_2 \right]$$

$$V_1 = AV_2 + \frac{B}{D}I_1 - \frac{BC}{D}V_2$$

$$V_1 = \left[A - \frac{BC}{D} \right] V_2 + \frac{B}{D}I_1$$

$$V_1 = \left[\frac{\Delta A}{D} \right] V_2 + \left[\frac{B}{D} \right] I_1 \quad \text{--- (4)}$$

Compare (4) with (a)

$$h_{11} = \frac{B}{D}, \quad h_{12} = \frac{\Delta A}{D}$$

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \frac{B}{D} & \frac{\Delta A}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{bmatrix}$$

10) z parameters in terms of h parameters:-

(7)

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{--- (1)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{--- (2)}$$

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{--- (a)}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{--- (b)}$$

From (2),

$$V_2 = \frac{I_2}{h_{22}} - \frac{h_{21} I_1}{h_{22}}$$

$$V_2 = -\frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2 \quad \text{--- (3)}$$

Compare (3) with (b)

$$Z_{21} = -\frac{h_{21}}{h_{22}} ; \quad Z_{22} = \frac{1}{h_{22}}$$

Sub. (3) in (1)

$$V_1 = h_{11} I_1 + h_{12} \left[-\frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2 \right]$$

$$V_1 = h_{11} I_1 + \frac{h_{12}}{h_{22}} I_2 - \frac{h_{12} h_{21}}{h_{22}} I_1$$

$$V_1 = \left[h_{11} - \frac{h_{12} h_{21}}{h_{22}} \right] I_1 + \frac{h_{12}}{h_{22}} I_2$$

$$V_1 = \left[\frac{h_{11} h_{22} - h_{12} h_{21}}{h_{22}} \right] I_1 + \left[\frac{h_{12}}{h_{22}} \right] I_2$$

$$V_1 = \left[\frac{\Delta h}{h_{22}} \right] I_1 + \left[\frac{h_{12}}{h_{22}} \right] I_2 \quad \text{--- (4)}$$

Compare ④ with ②

$$Z_{11} = \frac{\Delta h}{h_{22}} ; Z_{12} = \frac{h_{12}}{h_{22}}$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{\Delta h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ -\frac{h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$$

Parameters in terms of h parameters:-

$$V_1 = h_{11} I_1 + h_{12} V_2 \text{ --- (1)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \text{ --- (2)}$$

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \text{ --- (a)}$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \text{ --- (b)}$$

From ①, $V_1 = h_{11} I_1 + h_{12} V_2$

$$I_1 = \frac{V_1}{h_{11}} - \frac{h_{12}}{h_{11}} V_2 \text{ --- (3)}$$

Compare ③ with ②

$$Y_{11} = \frac{1}{h_{11}} ; Y_{12} = -\frac{h_{12}}{h_{11}}$$

Sub ③ in ②

$$I_2 = h_{21} \left[\frac{V_1}{h_{11}} - \frac{h_{12}}{h_{11}} V_2 \right] + h_{22} V_2$$

$$I_2 = \frac{h_{21}}{h_{11}} V_1 - \frac{h_{21} h_{12}}{h_{11}} V_2 + h_{22} V_2$$

$$I_2 = \frac{h_{21}}{h_{11}} V_1 + \left[h_{22} - \frac{h_{21} h_{12}}{h_{11}} \right] V_2$$

$$I_2 = \frac{h_{21}}{h_{11}} V_1 + \frac{\Delta h}{h_{11}} V_2 \quad \text{--- (4)}$$

(a)

Compare (4) with (b)

$$Y_{21} = \frac{h_{21}}{h_{11}} ; \quad Y_{22} = \frac{\Delta h}{h_{11}}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{h_{11}} & -\frac{h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta h}{h_{11}} \end{bmatrix}$$

12) ABCD parameters interms of h parameters:-

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{--- (1)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{--- (2)}$$

$$V_1 = A V_2 - B I_2 \quad \text{--- (a)}$$

$$I_1 = C V_2 - D I_2 \quad \text{--- (b)}$$

from (2),

$$I_1 = \frac{I_2}{h_{21}} - \frac{h_{22}}{h_{21}} V_2$$

$$I_1 = -\frac{h_{22}}{h_{21}} V_2 + \frac{1}{h_{21}} I_2 \quad \text{--- (3)}$$

Compare (3) with (b)

$$C = -\frac{h_{22}}{h_{21}} ; \quad D = -\frac{1}{h_{21}}$$

Sub (3) in (1)

$$V_1 = h_{11} \left[-\frac{h_{22}}{h_{21}} V_2 + \frac{1}{h_{21}} I_2 \right] + h_{12} V_2$$

(10)

$$V_1 = -\frac{h_{11} h_{22}}{h_{21}} V_2 + \frac{h_{11}}{h_{21}} I_2 + h_{12} V_2$$

$$V_1 = \left[h_{12} - \frac{h_{11} h_{22}}{h_{21}} \right] V_2 + \frac{h_{11}}{h_{21}} I_2$$

$$V_1 = \left[\frac{h_{12} h_{21} - h_{11} h_{22}}{h_{21}} \right] V_2 + \frac{h_{11}}{h_{21}} I_2$$

$$V_1 = -\left[\frac{\Delta h}{h_{21}} \right] V_2 + \left[\frac{h_{11}}{h_{21}} \right] I_2 \quad \text{--- (4)}$$

Compare (4) with (a)

$$A = -\frac{\Delta h}{h_{21}} \quad ; \quad B = -\frac{h_{11}}{h_{21}}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} -\frac{\Delta h}{h_{21}} & -\frac{h_{11}}{h_{21}} \\ -\frac{h_{22}}{h_{21}} & -\frac{1}{h_{21}} \end{bmatrix}$$

	Z	Y	ABCD	h
Z	$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{Y_{22}}{\Delta Y} & -\frac{Y_{12}}{\Delta Y} \\ -\frac{Y_{21}}{\Delta Y} & \frac{Y_{11}}{\Delta Y} \end{bmatrix}$	$\begin{bmatrix} \frac{A}{C} & \frac{\Delta A}{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix}$	$\begin{bmatrix} \frac{\Delta h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ -\frac{h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$
Y	$\begin{bmatrix} \frac{Z_{22}}{\Delta Z} & -\frac{Z_{12}}{\Delta Z} \\ -\frac{Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{bmatrix}$	$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$	$\begin{bmatrix} -\frac{\Delta A}{B} & \frac{D}{B} \\ -\frac{1}{B} & \frac{A}{B} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{h_{11}} & -\frac{h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta h}{h_{11}} \end{bmatrix}$
ABCD	$\begin{bmatrix} \frac{Z_{11}}{Z_{21}} & \frac{\Delta Z}{Z_{21}} \\ 1 & \frac{Z_{22}}{Z_{21}} \end{bmatrix}$	$\begin{bmatrix} -\frac{Y_{22}}{\Delta Y} & -\frac{1}{Y_{21}} \\ -\frac{\Delta Y}{Y_{21}} & \frac{Y_{11}}{Y_{21}} \end{bmatrix}$	$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$	$\begin{bmatrix} -\frac{\Delta h}{h_{21}} & -\frac{h_{11}}{h_{21}} \\ -\frac{h_{22}}{h_{21}} & \frac{1}{h_{21}} \end{bmatrix}$
h	$\begin{bmatrix} \frac{\Delta Z}{Z_{22}} & \frac{Z_{12}}{Z_{22}} \\ -\frac{Z_{21}}{Z_{22}} & \frac{1}{Z_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{Y_{11}} & -\frac{Y_{12}}{Y_{11}} \\ \frac{Y_{21}}{Y_{11}} & \frac{\Delta Y}{Y_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{B}{D} & \frac{\Delta A}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{bmatrix}$	$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$

11) A 2 port n/w has the following z parameters.
 $z_{11} = 40 \Omega$, $z_{22} = 30 \Omega$, $z_{12} = z_{21} = 20 \Omega$, Compute the transmission parameters for n/w. Hence write n/w equations with the use of parameters.

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} \frac{A}{C} & \frac{\Delta A}{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix}$$

$$\begin{bmatrix} 40 & 20 \\ 20 & 30 \end{bmatrix} = \begin{bmatrix} \frac{A}{C} & \frac{\Delta A}{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix}$$

$$\frac{A}{C} = 40 \quad ; \quad \frac{\Delta A}{C} = 20$$

$$A = 40C$$

$$\Delta A = 20C = \frac{20}{20} = 1$$

$$\frac{1}{C} = 20 \Rightarrow C = \frac{1}{20}$$

$$\therefore A = 2$$

$$\frac{D}{C} = 30 \Rightarrow D = 600$$

$$\Delta A = AD - BC$$

$$1 = 120 - \frac{B}{20}$$

$$\frac{B}{20} = 119$$

$$B = 2380$$

X Solution done at the back

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{Z_{11}}{Z_{21}} & \frac{\Delta Z}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{21}} \end{bmatrix}$$

$$\frac{Z_{11}}{Z_{21}} = \frac{40}{20} = 2$$

$$\Delta Z = Z_{11}Z_{22} - Z_{12}Z_{21}$$

$$= 40(30) - (20)(20)$$

$$= 800$$

$$\frac{\Delta Z}{Z_{21}} = \frac{800}{20} = 40$$

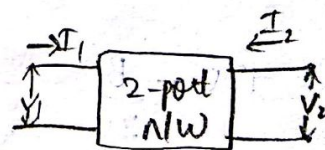
$$\frac{Z_{22}}{Z_{21}} = \frac{30}{20} = \frac{3}{2}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 2 & 40 \\ \frac{1}{20} & 3/2 \end{bmatrix} \Rightarrow \begin{matrix} V_1 = 2 \\ I_1 = 1 \end{matrix}$$

2) For a 2port network shown in fig, the currents I_1 & I_2 are entering at port 1 & 2 resp, and are given by the eqn

$$I_1 = 0.5V_1 - 0.2V_2$$

$$I_2 = -0.2V_1 + V_2$$



V_1 & V_2 are port voltages at port 1, 2 resp. Find

$Y, Z, ABCD$

given:- $Y_{11} = 0.5$; $Y_{12} = -0.2$

$Y_{21} = -0.2$, $Y_{22} = 1$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{Y_{22}}{\Delta Y} & -\frac{Y_{12}}{\Delta Y} \\ -\frac{Y_{21}}{\Delta Y} & \frac{Y_{11}}{\Delta Y} \end{bmatrix}$$

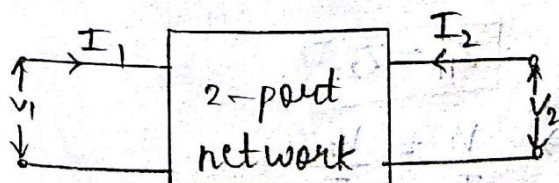
$$= \begin{bmatrix} 2.173 & 0.434 \\ 0.434 & 1.086 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} -\frac{Y_{22}}{Y_{21}} & -\frac{1}{Y_{21}} \\ -\frac{\Delta Y}{Y_{21}} & -\frac{Y_{11}}{Y_{21}} \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 5 \\ 2.3 & 1.08 \end{bmatrix}$$

Symmetrical & reciprocal networks:-

Symmetry:- A n/w is said to be symmetrical if it offers same amount of impedance on either sides that is, i/p impedance of the n/w at one port is equal to impedance of the n/w at the other port.



To be symmetrical, the given 2-port n/w should satisfy the condition,

$$\boxed{\frac{V_1}{I_1} = \frac{V_2}{I_2}}$$

Reciprocity:- For a given n/w to be reciprocal it should satisfy the condition,

$$\boxed{\frac{V_1}{I_2} = \frac{V_2}{I_1}}$$

Since reciprocity means ratio of excitation-response should be constant.

→ Conditions for symmetry & reciprocity in terms of

1) Z-parameters:-

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$Z_{11} = \frac{V_1}{I_1} ; Z_{12} = \frac{V_1}{I_2}$$

$$Z_{21} = \frac{V_2}{I_1} ; Z_{22} = \frac{V_2}{I_2}$$

cond. for symmetry $\Rightarrow \frac{V_1}{I_1} = \frac{V_2}{I_2}$

cond. for reciprocity

$$\Rightarrow \frac{V_1}{I_2} = \frac{V_2}{I_1}$$

$$\boxed{Z_{11} = Z_{22}}$$

$$\boxed{Z_{12} = Z_{21}}$$

Y-parameters:-

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

$$Y_{11} = \frac{I_1}{V_1} \quad ; \quad Y_{12} = \frac{I_1}{V_2}$$

$$Y_{21} = \frac{I_2}{V_1} \quad ; \quad Y_{22} = \frac{I_2}{V_2}$$

cond. for
symmetry

$$\Rightarrow \frac{V_1}{I_1} = \frac{V_2}{I_2}$$

$$\Rightarrow \frac{1}{(I_1/V_1)} = \frac{1}{(I_2/V_2)}$$

$$\Rightarrow \frac{1}{Y_{11}} = \frac{1}{Y_{22}}$$

$$\Rightarrow \boxed{Y_{22} = Y_{11}}$$

cond. for
reciprocity

$$\Rightarrow \frac{V_1}{I_2} = \frac{V_2}{I_1}$$

$$\Rightarrow \frac{1}{(I_2/V_1)} = \frac{1}{(I_1/V_2)}$$

$$\Rightarrow \frac{1}{Y_{21}} = \frac{1}{Y_{12}}$$

$$\Rightarrow \boxed{Y_{12} = Y_{21}}$$

3) ABCD parameters:-

cond. for symmetry $\Rightarrow \frac{V_1}{I_1} = \frac{V_2}{I_2}$

$$\Rightarrow Z_{11} = Z_{22}$$

$$\Rightarrow \frac{A}{C} = \frac{D}{C}$$

$$\Rightarrow \boxed{A = D}$$

cond. for reciprocity $\Rightarrow \frac{V_1}{I_2} = \frac{V_2}{I_1}$

$$\Rightarrow Z_{12} = Z_{21}$$

$$\Rightarrow \frac{AD - BC}{C} = \frac{1}{C}$$

$$\Rightarrow \boxed{AD - BC = 1}$$

4) h-parameters:-

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$h_{11} = \frac{V_1}{I_1} \quad ; \quad h_{12} = \frac{V_1}{V_2}$$

$$h_{21} = \frac{I_2}{I_1} \quad ; \quad h_{22} = \frac{I_2}{V_2}$$

cond. for symmetry $\Rightarrow \frac{V_1}{I_1} = \frac{V_2}{I_2} \Rightarrow Z_{11} = Z_{22}$

$$\Rightarrow \frac{\Delta h}{h_{22}} = \frac{1}{h_{22}}$$

$$\Rightarrow \frac{1}{h_{22}} = \frac{1}{h_{22}}$$

$$\Rightarrow \Delta h = 1$$

$$\Rightarrow h_{11}h_{22} = 1$$

$$\Delta h = 1$$

cond. for reciprocity

$$\Rightarrow \frac{V_1}{I_2} = \frac{V_2}{I_1}$$

$$\Rightarrow Z_{12} = Z_{21}$$

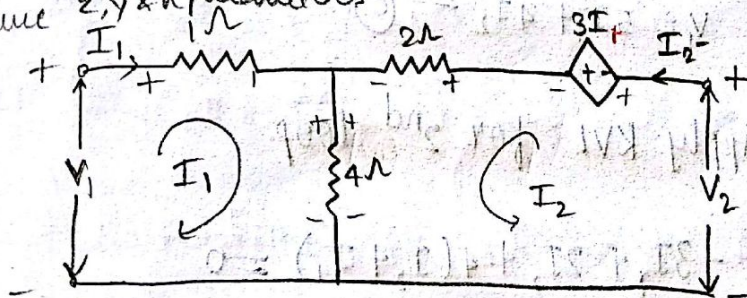
$$\Rightarrow \frac{h_{12}}{h_{22}} = -\frac{h_{21}}{h_{22}}$$

$$\Rightarrow h_{12} = -h_{21}$$

Condition table:-

	Z	Y	ABCD	h
Symmetrical	$Z_{11} = Z_{22}$	$Y_{11} = Y_{22}$	$A = D$	$\Delta h = 1$
Reciprocity	$Z_{12} = Z_{21}$	$Y_{12} = Y_{21}$	$AD - BC = 1$	$h_{12} = -h_{21}$

Q) Find transmission parameters for ckt shown in fig.
Hence deduce z, y & h parameters.



Sol

$$V_1 = AV_2 = BI_2 \quad \text{--- (1)}$$

$$I_1 = CV_2 - DI_2 \quad \text{--- (2)}$$

Case: i) $I_2 = 0$ (OC port 2 terminals)

$$A = \frac{V_1}{V_2}, \quad C = \frac{I_1}{V_2}$$

3)

$$V_1 = 5I_1 \text{ --- (a)}$$

$$V_2 = -3I_1 + 2I_2 + 4(I_1 + I_2)$$

$$V_2 = -3I_1 + 4I_1 \quad (\because I_2 = 0)$$

$$V_2 = I_1 \text{ --- (b)}$$

$$A = \frac{V_1}{V_2} = \frac{(a)}{(b)} = 5$$

$$C = \frac{I_1}{V_2} \Rightarrow \frac{I_1}{I_1} = 1$$

Case: 2) $V_2 = 0$ (SC port 2 terminals)

$$B = -\frac{V_1}{I_2}, \quad D = -\frac{I_1}{I_2}$$

Apply KVL for 1st loop

$$V_1 = I_1 + 4(I_1 + I_2)$$

$$V_1 = 5I_1 + 4I_2 \text{ --- (c)}$$

Apply KVL for 2nd loop

$$-3I_1 + 2I_2 + 4(I_2 + I_1) = 0$$

$$-3I_1 + 2I_2 + 4I_2 + 4I_1 = 0$$

$$I_1 = -6I_2 \text{ --- (d)}$$

$$D = -\frac{I_1}{I_2} = 6$$

Sub. (d) in (c)

$$V_1 = 5(-6I_2) + 4I_2$$

$$V_1 = -26I_2$$

$$B = \frac{26I_2}{I_2} = 26$$

$$A = 5, B = 26, C = 1, D = 6$$

$$\therefore V_1 = 5V_2 - 26I_2$$

$$I_1 = V_2 - 6I_2$$

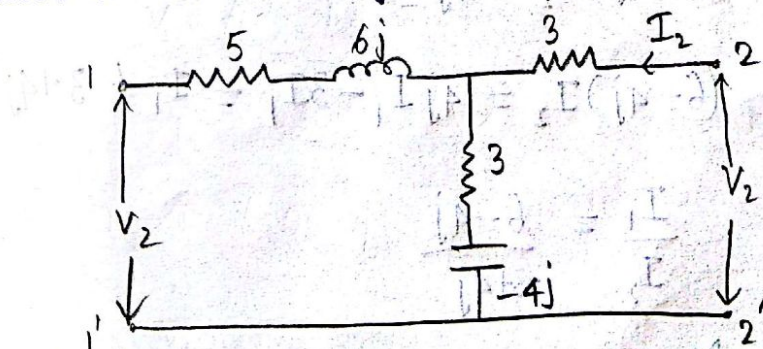
$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 5 & 4/1 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 1 & 6 \end{bmatrix}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 6/26 & -1/26 \\ -1/26 & 5/26 \end{bmatrix} = \begin{bmatrix} 0.23 & -1 \\ -0.03 & 0.19 \end{bmatrix}$$

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} 1/6 & 26/6 \\ -1/6 & 1/6 \end{bmatrix} = \begin{bmatrix} 0.16 & 4.3 \\ -0.16 & 0.16 \end{bmatrix}$$

2) Obtain the transmission parameters for foll.

T-network & verify reciprocity theorem.



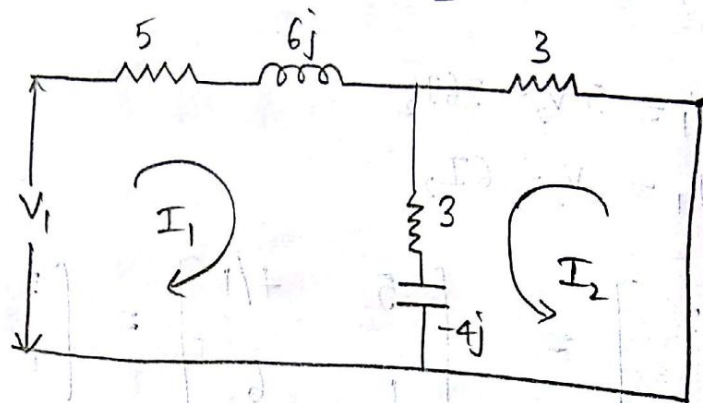
3) Sol

$$V_1 = AV_2 - BI_2 \quad \text{--- (1)}$$

$$I_1 = CV_2 - DI_2 \quad \text{--- (2)}$$

Case: i) $V_2 = 0$ (SC port 2 terminals)

$$B = -\frac{V_1}{I_2}, \quad D = -\frac{I_1}{I_2}$$



KVL at 1 loop $\Rightarrow V_1 = (5 + 6j)I_1 + (3 - 4j)(I_2 + I_1)$

$$V_1 = 8I_1 + 6jI_1 + 3I_2 - 4jI_2 - 4jI_1$$

$$V_1 = 8I_1 + 2jI_1 + 3I_2 - 4jI_2$$

$$V_1 = (8 + 2j)I_1 + (3 - 4j)I_2$$

KVL at 2 loop $\Rightarrow 0 = 3I_2 + (3 - 4j)(I_2 + I_1)$

$$0 = 6I_2 - 4jI_2 - 4jI_1 + 3I_1$$

$$(6 - 4j)I_2 - 4jI_1 + 3I_1 = 0$$

$$(6 - 4j)I_2 = 4jI_1 - 3I_1 = I_1(-3 + 4j)$$

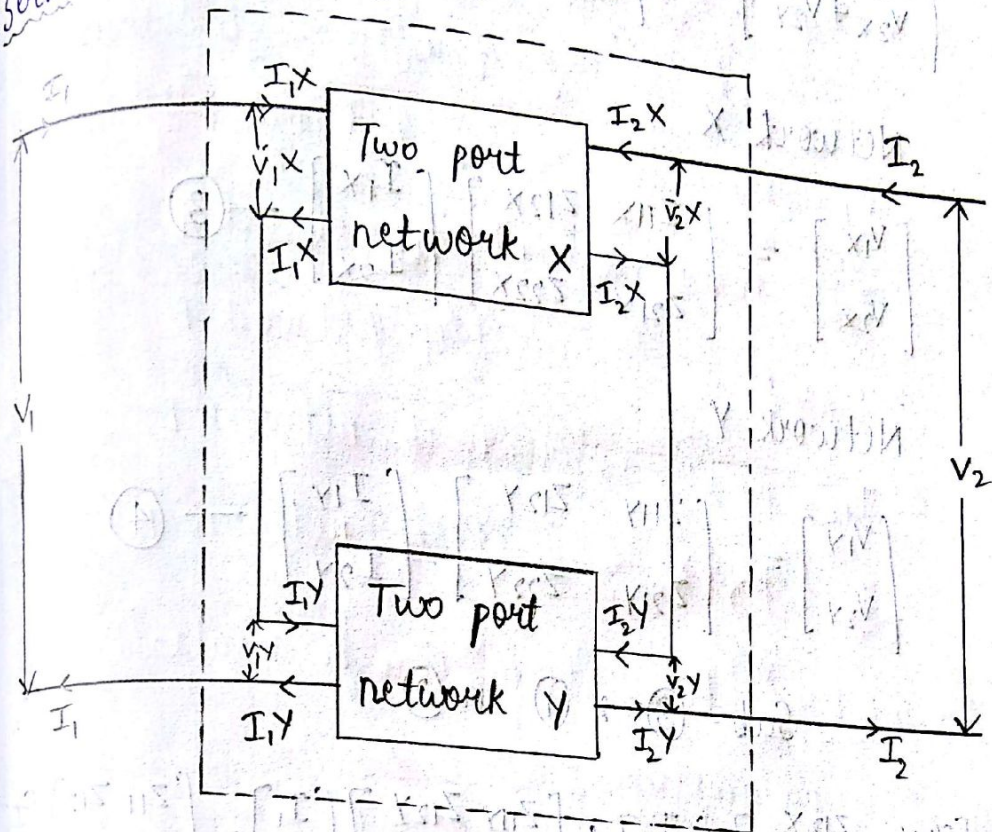
$$\frac{I_1}{I_2} = \frac{6 - 4j}{-3 + 4j}$$

$$D = -\frac{I_1}{I_2} = -\frac{6 - 4j}{-3 + 4j} = \frac{6 - 4j}{-3 + 4j}$$

SL-12-15

Interconnection of networks:-

Series configuration:-



Two port networks are said to be series when current through each network is same where as voltages are different.

$$I_1 = I_{1X} = I_{1Y}$$

$$I_2 = I_{2X} = I_{2Y}$$

$$V_1 = V_{1X} + V_{1Y} \text{ --- (a)}$$

$$V_2 = V_{2X} + V_{2Y} \text{ --- (b)}$$

$$V = [Z] [I]$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \text{ --- (1)}$$

Sub. (a) & (b) in (1)

$$\begin{bmatrix} V_{1X} + V_{1Y} \\ V_{2X} + V_{2Y} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad \text{--- (2)}$$

Network X

$$\begin{bmatrix} V_{1X} \\ V_{2X} \end{bmatrix} = \begin{bmatrix} Z_{11X} & Z_{12X} \\ Z_{21X} & Z_{22X} \end{bmatrix} \begin{bmatrix} I_{1X} \\ I_{2X} \end{bmatrix} \quad \text{--- (3)}$$

Network Y

$$\begin{bmatrix} V_{1Y} \\ V_{2Y} \end{bmatrix} = \begin{bmatrix} Z_{11Y} & Z_{12Y} \\ Z_{21Y} & Z_{22Y} \end{bmatrix} \begin{bmatrix} I_{1Y} \\ I_{2Y} \end{bmatrix} \quad \text{--- (4)}$$

Sub. (3) & (4) in (2)

$$\begin{bmatrix} Z_{11X} & Z_{12X} \\ Z_{21X} & Z_{22X} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} + \begin{bmatrix} Z_{11Y} & Z_{12Y} \\ Z_{21Y} & Z_{22Y} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

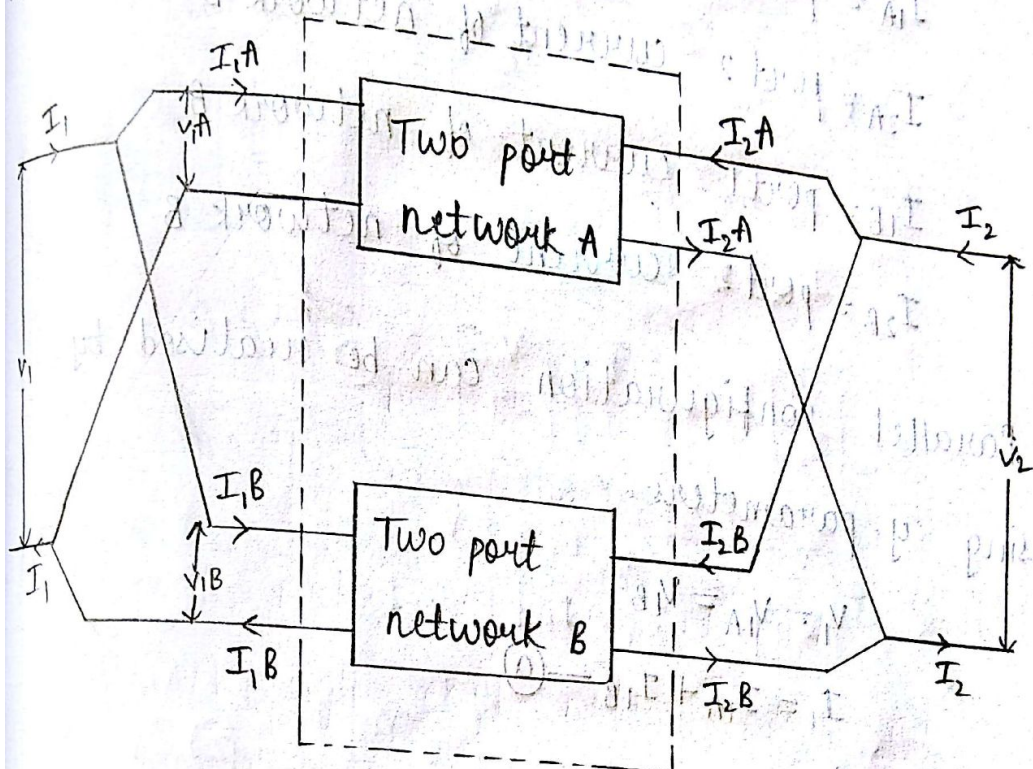
$$\left[\begin{bmatrix} Z_{11X} & Z_{12X} \\ Z_{21X} & Z_{22X} \end{bmatrix} + \begin{bmatrix} Z_{11Y} & Z_{12Y} \\ Z_{21Y} & Z_{22Y} \end{bmatrix} \right] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} Z_{11X} & Z_{12X} \\ Z_{21X} & Z_{22X} \end{bmatrix} + \begin{bmatrix} Z_{11Y} & Z_{12Y} \\ Z_{21Y} & Z_{22Y} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

$$Z_T = [Z_X] + [Z_Y] + \dots$$

If 'n' 2 port networks are connected in series, the z-parameters of the total configuration is algebraic sum of individual z-parameters.

Parallel configuration:-



Two 2-port networks are said to be in parallel when each port of each network have same voltage whereas currents are different.

V_1 = port 1 voltage

V_2 = port 2 voltage

I_1 = port 1 current

I_2 = port 2 current

V_{1A} = port 1 voltage of network A

V_{2A} = port 2 voltage of network A

V_{1B} = port 1 voltage of network B

V_{2B} = port 2 voltage of network B

I_{1A} = port 1 current of network A

I_{2A} = port 2 current of network A

I_{1B} = port 1 current of network B

I_{2B} = port 2 current of network B

Parallel configuration can be realised by using y-parameters.

$$V_1 = V_{1A} = V_{1B}$$

$$I_1 = I_{1A} + I_{1B} \text{ --- (a)}$$

$$V_2 = V_{2A} = V_{2B}$$

$$I_2 = I_{2A} + I_{2B} \text{ --- (b)}$$

$$[I] = [Y][V]$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \text{ --- (1)}$$

Sub (a) & (b) in (1)

$$\begin{bmatrix} I_{1A} + I_{1B} \\ I_{2A} + I_{2B} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad \text{--- (2)}$$

Network A

$$\begin{bmatrix} I_{1A} \\ I_{2A} \end{bmatrix} = \begin{bmatrix} Y_{11A} & Y_{12A} \\ Y_{21A} & Y_{22A} \end{bmatrix} \begin{bmatrix} V_{1A} \\ V_{2A} \end{bmatrix} \quad \text{--- (3)}$$

Network B

$$\begin{bmatrix} I_{1B} \\ I_{2B} \end{bmatrix} = \begin{bmatrix} Y_{11B} & Y_{12B} \\ Y_{21B} & Y_{22B} \end{bmatrix} \begin{bmatrix} V_{1B} \\ V_{2B} \end{bmatrix} \quad \text{--- (4)}$$

Sub. (3) & (4) in (2)

$$\begin{bmatrix} Y_{11A} & Y_{12A} \\ Y_{21A} & Y_{22A} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + \begin{bmatrix} Y_{11B} & Y_{12B} \\ Y_{21B} & Y_{22B} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

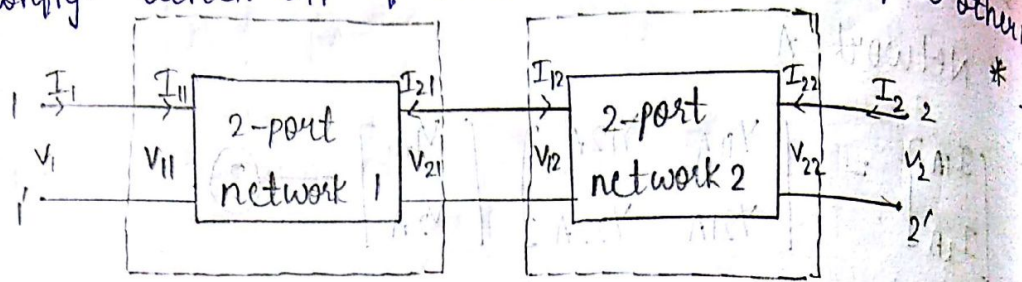
$$\begin{bmatrix} Y_{11A} & Y_{12A} \\ Y_{21A} & Y_{22A} \end{bmatrix} + \begin{bmatrix} Y_{11B} & Y_{12B} \\ Y_{21B} & Y_{22B} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

$$Y_T = Y_A + Y_B + \dots$$

If n 2 port n/w's are connected in //,
the y-parameters of total configuration is
equal to ^{algebraic} individual sum of individual
y-parameters:

3) Cascaded configuration:- /Ladder/ Side-By Side

Two 2 port n/w's are said to be in cascaded config. when o/p of 1 network is given as i/p to other.



$$V_{21} = V_{12} ; I_{21} = -I_{12}^*$$

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \text{--- (1)}$$

$$\begin{bmatrix} V_{11} \\ I_{11} \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_{21} \\ -I_{21} \end{bmatrix}$$

$$\begin{bmatrix} V_{11} \\ I_{11} \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_{12} \\ I_{12} \end{bmatrix} \text{--- (2)}$$

$$\begin{bmatrix} V_{12} \\ I_{12} \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_{22} \\ -I_{22} \end{bmatrix} \text{--- (3)}$$

Sub. (3) in (2)

$$\begin{bmatrix} V_{11} \\ I_{11} \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_{22} \\ -I_{22} \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \text{--- (4)}$$

Compare ④ with ①

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

cascaded connection of 2 2-port networks is used by using transmission (ABCD) parameters. From the configuration,

V_1 = port 1 voltage of entire configuration

I_1 = port 1 current of entire configuration

V_2 = port 2 voltage of entire configuration

I_2 = port 2 current of entire configuration.

V_{11} = port 1 voltage of n/w 1

V_{21} = port 2 voltage of n/w 1

I_{11} = port 1 current of n/w 1

I_{21} = port 2 current of n/w 1

V_{12} = port 1 voltage of n/w 2

I_{12} = port 1 current of n/w 2

V_{22} = port 2 voltage of n/w 2

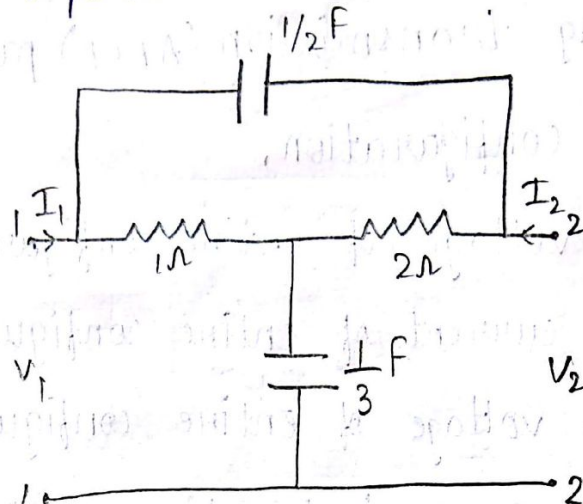
I_{22} = port 2 current of n/w 2

If n 2-port networks are connected in cascaded configuration, the ABCD parameters of

3) final configuration is the product of ABCD matrices of individual n/w i.e

$$[A_T] = [A_1][A_2] \dots [A_n]$$

11) Find z-parameters of n/w shown in fig.

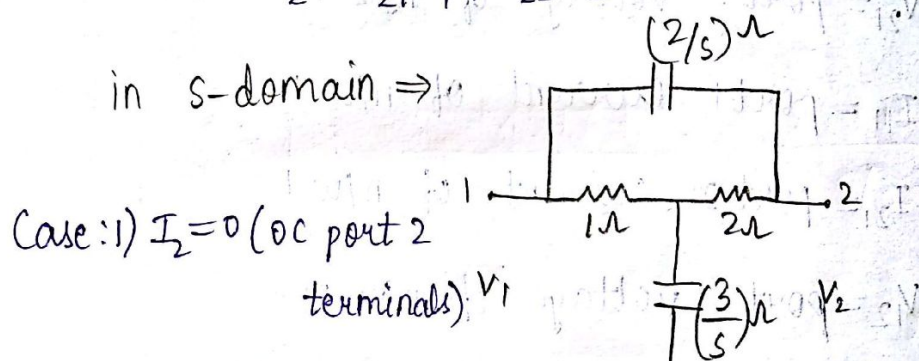


Sol

$$V_1 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = z_{21}I_1 + z_{22}I_2$$

in s-domain $\Rightarrow$



Case: 1) $I_2 = 0$ (oc port 2

terminals); V_1

$$z_{11} = \frac{V_1}{I_1} ; z_{21} = \frac{V_2}{I_1}$$

$$Z_{eq} = \left[\left(2 + \frac{2}{s} \right) \parallel 1 \right] + \frac{3}{s}$$

$$= \frac{2s+2}{s+2} + \frac{3}{s}$$

$$Z_{eq} = \frac{2s^2 + 11s + 6}{s(3s+2)}$$

$$V_1 = \left[\frac{2s^2 + 11s + 6}{s(3s+2)} \right] I_1$$

$$Z_{11} = \frac{V_1}{I_1} = \frac{2s^2 + 11s + 6}{s(3s+2)}$$

$$V_2 = V_{2L} + V_{(3/s)L}$$

$$V_{2L} = I_{2L} \times 2$$

$$I_{2L} = \frac{1}{1 + \frac{2}{s} + 2} \times I_1$$

$$V_{2L} = \frac{2s}{2+3s} I_1$$

$$V_{(3/s)L} = I_{(3/s)} \times \frac{3}{s} \Rightarrow \frac{3}{s} I_1$$

$$V_2 = \frac{2s}{3s+2} I_1 + \frac{3}{s} I_1$$

$$V_2 = \left[\frac{2s}{3s+2} + \frac{3}{s} \right] I_1$$

$$V_2 = \frac{2s^2 + 9s + 6}{s(3s+2)} I_1$$

$$Z_{21} = \frac{V_2}{I_1} = \frac{2s^2 + 9s + 6}{s(3s+2)}$$

Case:2) $I_1 = 0$ (OC port 1 terminals)

$$Z_{12} = \frac{V_1}{I_2} \quad ; \quad Z_{22} = \frac{V_2}{I_2}$$

$$Z_{eq} = \left[\left(1 + \frac{2}{s} \right) \parallel 2 \right] + \frac{3}{s}$$

$$= \frac{\left(1 + \frac{2}{s} \right) \times 2}{1 + \frac{2}{s} + 2} + \frac{3}{s}$$

$$= \frac{2 + \frac{4}{s}}{\frac{s + 2 + 2s}{s}} + \frac{3}{s}$$

$$= \frac{2s + 4}{2s + 3s} + \frac{3}{s}$$

$$Z_{eq} = \frac{2s^2 + 4s + 6 + 9s}{s(2 + 3s)} = \frac{2s^2 + 13s + 6}{s(3s + 2)}$$

$$V_2 = Z_{eq} I_2$$

$$Z_{22} = \frac{V_2}{I_2} = \frac{2s^2 + 13s + 6}{s(3s + 2)}$$

$$V_1 = V_{1n} + V_{(3/s)n} = \frac{V}{s} = 10V$$

$$V_{1n} = I_{1n} \times 1$$

$$I_{1n} = \frac{2}{1 + \frac{2}{s} + 2} \times I_2 = \frac{2}{3 + \frac{2}{s}} \times I_2$$

$$V_{1L} = \frac{2}{3 + \frac{2}{s}} \times I_2$$

$$V(3/s) = I(3/s) \times \frac{3}{s} \Rightarrow \frac{3}{s} I_2$$

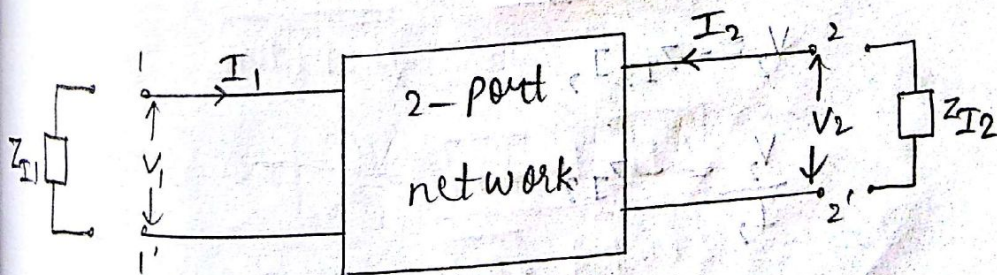
$$\therefore V_1 = \frac{2}{3 + \frac{2}{s}} \times I_2 + \frac{3}{s} I_2$$

$$V_{01} = \left(\frac{2s}{3s+2} + \frac{3}{s} \right) I_2$$

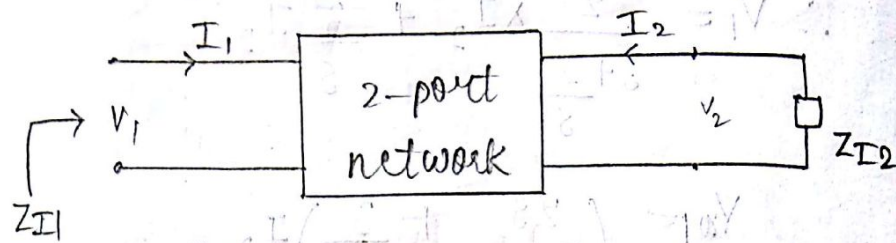
$$Z_{12} = \frac{V_1}{I_2} = \frac{2s}{3s+2} + \frac{3}{s}$$

Image parameters:-

They are the impedance parameters which are used to determine the behaviour of the n/w. These parameters are denoted by Z_{I1} & Z_{I2} . So, these image parameters are defined as impedance at one port when other port is terminated by its image impedance $\left(\frac{V}{I} \right)_A = Z_{I1}$



3, Case: i) Z_{I1} is the i/p impedance at port 1 when port 2 terminals are terminated by its image impedance Z_{I2} .



$$V_1 = AV_2 - BI_2 \quad (a)$$

$$I_1 = CV_2 - DI_2 \quad (b)$$

$$Z_{I1} = \frac{V_1}{I_1}$$

$$= \frac{AV_2 - BI_2}{CV_2 - DI_2}$$

$$= \frac{I_2 \left(A \frac{V_2}{I_2} - B \right)}{I_2 \left(C \frac{V_2}{I_2} - D \right)}$$

$$Z_{I1} = \frac{A \left(\frac{V_2}{I_2} \right) - B}{C \left(\frac{V_2}{I_2} \right) - D}$$

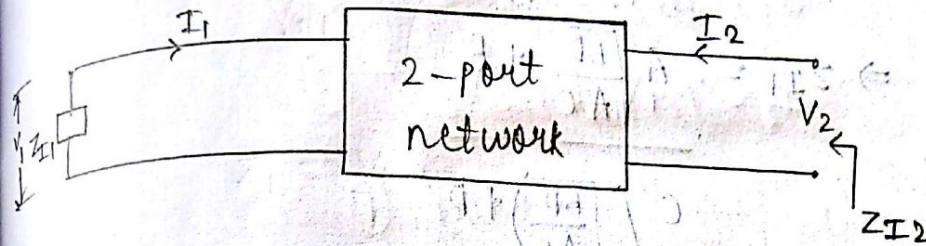
$$V_2 = -Z_{I2} I_2$$

$$\frac{V_2}{I_2} = -Z_{I2}$$

$$Z_{I1} = \frac{A(-Z_{I2}) - B}{C(-Z_{I2}) - D}$$

$$Z_{I1} = \frac{AZ_{I2} + B}{CZ_{I2} + D} \quad \text{--- (1)}$$

Case: 2) Z_{I2} is the impedance at port 2 when port 1 terminals are terminated by its image impedance Z_{I1} . From fig, voltage across Z_{I1} is V_1 ,



$$V_1 = -Z_{I1} I_1 \quad \text{and} \quad Z_{I2} = \frac{V_2}{I_2}$$

$$Z_{I1} = -\left(\frac{V_1}{I_1}\right)$$

$$Z_{I1} = -\left[\frac{AV_2 + BI_2}{CV_2 + DI_2}\right] = -\left[\frac{I_2 \left(A \cdot \frac{V_2}{I_2} + B\right)}{I_2 \left(C \cdot \frac{V_2}{I_2} + D\right)}\right]$$

$$Z_{I1} = -\left[\frac{A(Z_{I2}) + B}{C(Z_{I2}) + D}\right] \quad \text{--- (2)}$$

From equ (1) & (2),

$$\frac{AZ_{I2} + B}{CZ_{I2} + D} = -\left[\frac{AZ_{I2} + B}{C(Z_{I2}) + D}\right]$$

$$\Rightarrow (AZ_{I2} + B)(CZ_{I2} + D) = -(AZ_{I2} + B)(CZ_{I2} + D)$$

$$\Rightarrow ACZ_{I2}^2 + BCZ_{I2} + ADZ_{I2} + BD = -ACZ_{I2}^2 - AZ_{I2}^2 D$$

$$= (ACZ_{I2}^2 + BCZ_{I2} + BD) + (ADZ_{I2} + BD)$$

$$\Rightarrow 2ACZ_{I2}^2 - 2BD = 0$$

$$\Rightarrow ACZ_{I2}^2 - BD = 0$$

$$\Rightarrow Z_{I2}^2 = \frac{BD}{AC}$$

$$\Rightarrow Z_{I2} = \sqrt{\frac{BD}{AC}}$$

Sub. Z_{I2} in ①

$$\Rightarrow Z_{I1} = \frac{A \sqrt{\frac{BD}{AC}} + B}{C \left(\sqrt{\frac{BD}{AC}} \right) + D}$$

$$= \frac{\sqrt{\frac{A^2 BD}{AC}} + B}{\sqrt{\frac{C^2 BD}{AC}} + D}$$

$$= \frac{\sqrt{\frac{ABD}{C}} + B}{\sqrt{\frac{BCD}{A}} + D}$$

$$= \frac{\sqrt{ABD} + B\sqrt{C}}{\sqrt{C}} \times \frac{\sqrt{A}}{\sqrt{BCD} + DA}$$

$$= \frac{\sqrt{A^2 BD} + \sqrt{B^2 AC}}{\sqrt{BC^2 D} + \sqrt{ACD^2}}$$

$$Z_{I1} = \frac{\sqrt{AB} (\sqrt{AD} + \sqrt{BC})}{\sqrt{CD} (\sqrt{BC} + \sqrt{AD})} = \frac{\sqrt{AB}}{\sqrt{CD}}$$

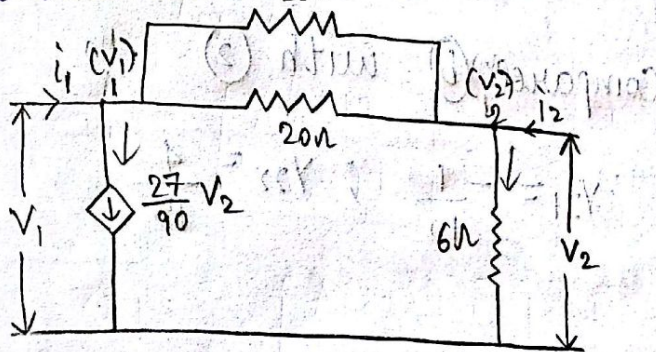
$$Z_{I1} = \sqrt{\frac{AB}{CD}}$$

If the n/w is symmetrical, then the condition for ABCD parameters to be symmetry is $A=D$,

$$\text{if } A=D, Z_{I1} = \sqrt{\frac{DB}{BC}} = \sqrt{\frac{B}{C}}$$

$$Z_{I2} = \sqrt{\frac{B}{C}} \Rightarrow Z_{I1} = Z_{I2}$$

1b) Determine y-parameters for n/w shown in fig.
Hence determine z-parameters



Sol

Apply KCL at node A (1)

$$i_1 = \frac{27}{90} V_2 + \frac{V_1 - V_2}{10}$$

$$i_1 = \frac{27V_2}{90} + \frac{V_1}{10} - \frac{V_2}{10}$$

$$i_1 = \frac{V_1}{10} + \frac{V_2}{5} \quad \text{--- (1)}$$

Apply KCL at node (2)

$$\frac{V_1 - V_2}{10} + i_2 = \frac{V_2}{6}$$

$$i_2 = \frac{V_2}{6} - \frac{V_1}{10} + \frac{V_2}{10}$$

$$i_2 = \frac{4V_2}{15} - \frac{V_1}{10} \quad \text{--- (b)}$$

Y-parametric equ's are

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \text{--- (1)}$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad \text{--- (2)}$$

Compare (a) with (1)

$$Y_{11} = \frac{1}{10} ; Y_{12} = \frac{1}{5}$$

Compare (b) with (2)

$$Y_{21} = -\frac{1}{10} ; Y_{22} = \frac{4}{15}$$

(1) A short for 10K WPA

$$\frac{3V}{0.1} - \frac{4V}{0.1} + \frac{5V}{0.1} = 1$$

$$\frac{3V}{0.1} - \frac{4V}{0.1} + \frac{5V}{0.1} = 1$$

$$(1) \rightarrow \frac{3V}{0.1} + \frac{4V}{0.1} = 1$$