11-12-15

Unit:1 Transient Analysis

(First & Second order circuits)

The transient behaviour is observed in an electrical circuit which has energy storing elements L and C.

Transient State:-

The state where circuit response changes from one steady state to other steady state due to sudden changes in input is called transient state.

Transient time:-

The time taken by the circuit to change from one state to another state is called transient time.

Transient behaviour will be observed under

1) When the circuit contains energy storage elements like inductors, capacitors.
2) When the input of the circuit changes suddenly total response of the circuit.
3) When an input is suddenly applied to a network, the response will undergo transient

nature before it settles to the final value. Hence the total response of the circuit can be categorised into 2 parts:-

(i) Transient (natural) response

(ii) Steady State (forced) response

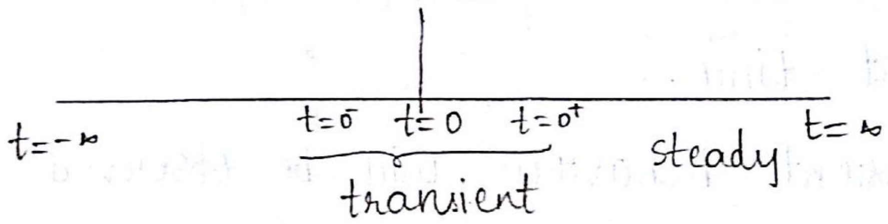
The transient response will be available for short duration where it dies out when time increases ($t \simeq \infty$).

*Total response = Transient response + steady state response

↓ complimentary fn

↳ particular solu

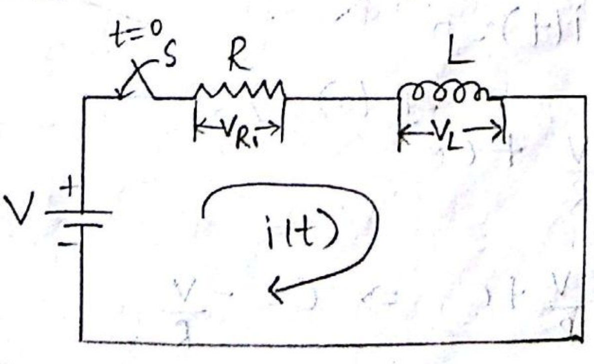
Representation of steady state & transient state on time scale:-



Behaviour of inductor & capacitor in steady state and transient state:-

Inductor	Capacitor
1) It stores energy in the form of ~~electric~~ magnetic field. Units: Henry	1) It stores energy in the form of ~~magnetic~~ electric field. Units: Farads
2) voltage across the inductor, $V_L = L \cdot \frac{di}{dt}$	2) Voltage across capacitor, $V_c = \frac{1}{C}\int i \cdot dt$
3) Current in an inductor, $I = \frac{1}{L}\int v dt$	3) Current in capacitor, $\cancel{C}$ $I = C\frac{dv}{dt}$
4) Under steady state condition, when $t=\infty \rightarrow V=0$, inductor acts as short ckt	4) Under steady state condition, when $t=\infty \rightarrow I=0$, capacitor acts as open ckt.
5) Under transient state (i.e $t=0^-, t=0, t=0^+$), $I=0$ it acts as open ckt.	5) Under transient state (i.e $t=0^-, t=0, t=0^+$), $\cancel{I}$ $V=0$ it acts as short ckt.
6) Initially charged inductor acts as current source.	6) Initially charged capacitor acts as voltage source.

DC response of series R-L ckt :-



Apply KVL to the circuit,

$$V = V_R + V_L$$

$$V = i(t)R + L\frac{di(t)}{dt}$$

$$\frac{di(t)}{dt} + \frac{R}{L}i(t) = \frac{V}{L}$$

It is in the form of $\frac{dx}{dt} + px = k$

$x = i(t)$, $p = \frac{R}{L}$, $k = \frac{V}{L}$

Sol. is $\boxed{x(t) = e^{-pt}\int ke^{pt}\,dt + ce^{-pt}}$

$$i(t) = e^{-\frac{Rt}{L}}\int \frac{V}{L}e^{+\frac{Rt}{L}}\,dt + ce^{-\frac{Rt}{L}}$$

$$i(t) = e^{-\frac{Rt}{L}}\cdot\frac{V}{L}\left[\frac{e^{+\frac{Rt}{L}}}{+R/L}\right] + ce^{-\frac{Rt}{L}}$$

$$i(t) = +e^{-\frac{Rt}{L}}\cdot\frac{V}{R}\left[e^{+\frac{Rt}{L}}\right] + ce^{-\frac{Rt}{L}}$$

$$i(t) = \frac{V}{R} + ce^{-\frac{Rt}{L}} \quad \text{—(1)}$$

Applying initial conditions,

$t = 0$, $i(t) = 0$

$$0 = \frac{V}{R} + ce^{-\frac{R}{L}(0)}$$

$$0 = \frac{V}{R} + c \Rightarrow c = -\frac{V}{R}$$

$\therefore$ (1) $\Rightarrow$ $$i(t) = \frac{V}{R} - \frac{V}{R} e^{-\frac{R}{L}t}$$

$$i(t) = \frac{V}{R}\left[1 - e^{-R/L\, t}\right]$$

$$\boxed{i(t) = \frac{V}{R}\left[1 - e^{-t/\tau}\right]}$$ $\left[\text{where}^* \ \tau = \frac{L}{R}\right]$ (time constant)

WKT voltage across resistor V_R,

$$V_R = i(t) \cdot R$$

$$V_R = V\left[1 - e^{-t/\tau}\right]$$

& voltage across inductor V_L,

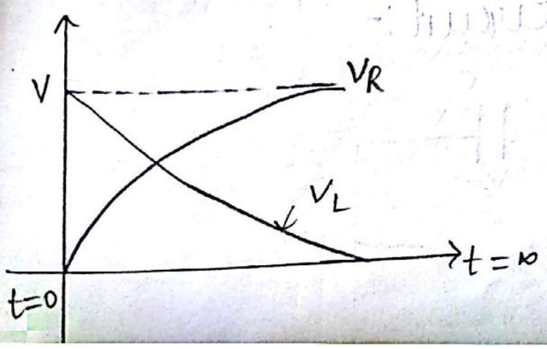
$$V_L = L \cdot \frac{di(t)}{dt}$$

$$= L \frac{d}{dt}\left[\frac{V}{R}\left(1 - e^{-t/\tau}\right)\right]$$

$$= \frac{L \cdot V}{R}\left(-\frac{1}{\tau}\left(-e^{-t/\tau}\right)\right)$$

$$= \cancel{\tau} \cdot \frac{V}{\cancel{R}} \cdot \frac{1}{\cancel{\tau}} e^{-t/\tau} \qquad \left(\because \tau = \frac{L}{R}\right)$$

$$V_L = Ve^{-t/\tau}$$



Power $P = VI = I^2R = \frac{V^2}{R}$

$$P_R = \frac{V_R^2}{R} = \frac{V^2(1-e^{-t/\tau})^2}{R}$$

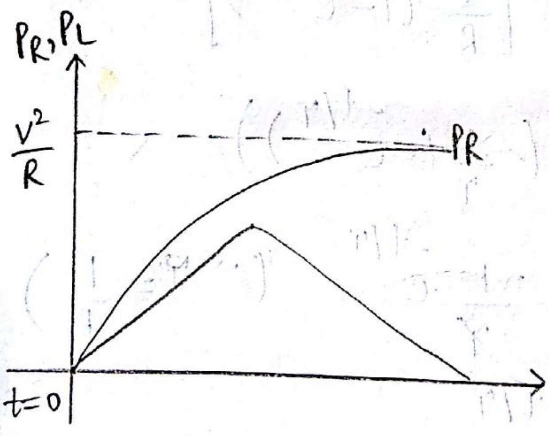
$$\boxed{P_R = \frac{V^2}{R}(1+e^{-2t/\tau} - 2e^{-t/\tau})}$$

$$P_L = V_L \cdot i(t)$$

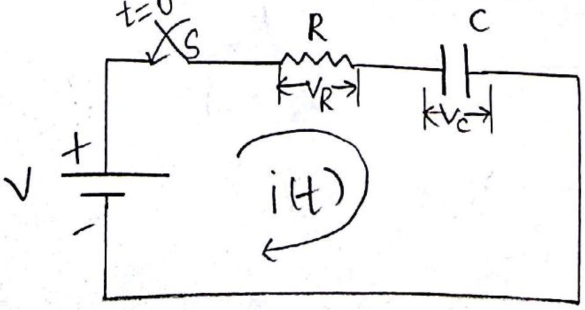
$$= Ve^{-t/\tau} \cdot \frac{V}{R}(1-e^{-t/\tau})$$

$$P_L = \frac{V^2}{R}e^{-t/\tau}(1-e^{-t/\tau})$$

$$\boxed{P_L = \frac{V^2}{R}(e^{-t/\tau} - e^{-2t/\tau})}$$



DC response of series RC circuit:-



Applying KVL to the ckt,

$$V = V_R + V_C$$

$$V = i(t)R + \frac{1}{C}\int i(t)\,dt$$

$$0 = R\frac{di(t)}{dt} + \frac{1}{C}i(t) \qquad \left(\because \frac{dv}{dt} = 0\right)$$

$$\frac{di(t)}{dt} + \frac{1}{RC}i(t) = 0$$

It is in the form of $\frac{dx}{dt} + px = k$

$$x = i(t), \quad p = \frac{1}{RC}, \quad k = 0$$

Sol. is $x(t) = e^{-pt}\int k e^{pt}\,dt + ce^{-pt}$

$$i(t) = e^{-1/RC\,t}\int 0 \cdot e^{1/RC\,t}\,dt + ce^{-1/RC\,t}$$

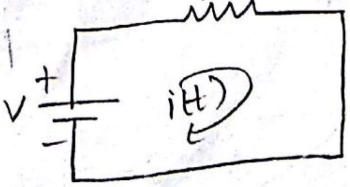
$$i(t) = ce^{-1/RC\,t}$$

$$\boxed{i(t) = ce^{-t/\tau}} \text{ — (1)} \quad (\because \tau = RC)$$

time constant

Apply initial conditions,

$t = 0$, capacitor acts as short ckt



$$V = i(t) \cdot R$$

$$i(t) = \frac{V}{R}$$

$$\frac{V}{R} = ce^{0/\tau} \Rightarrow c = \frac{V}{R}$$

Sub $c = \frac{V}{R}$ in $i(t)$, we have

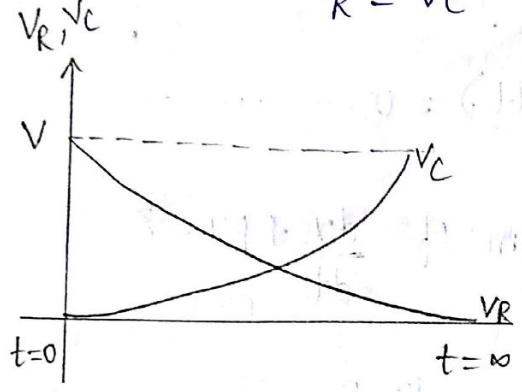
$$i(t) = \frac{V}{R} e^{-t/\tau}$$

we know $V_R = i(t)R$; $V_C = V - V_R$ $(V = V_R + V_C)$

$$V_R = V e^{-t/\tau}$$

$$= V - V e^{-t/\tau}$$

$$V_C = V(1 - e^{-t/\tau})$$



Power $P = VI = \frac{V^2}{R}$

$$P_R = \frac{V_R^2}{R} = \frac{(V e^{-t/\tau})^2}{R}$$

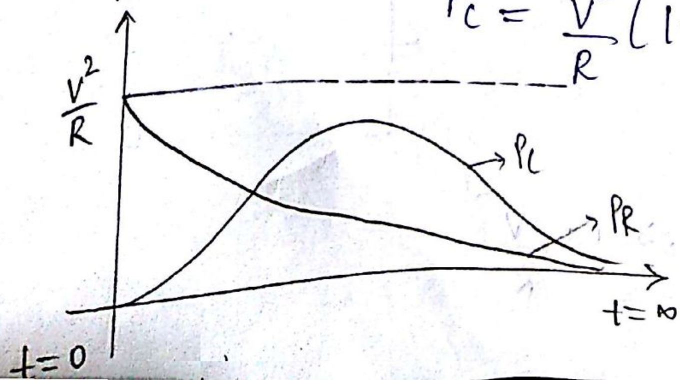
$$P_R = \frac{V^2}{R} e^{-2t/\tau}$$

$$P_C = \frac{V_C^2}{R} \neq \cancel{\frac{V^2(1-e^{-t/\tau})^2}{R}}$$

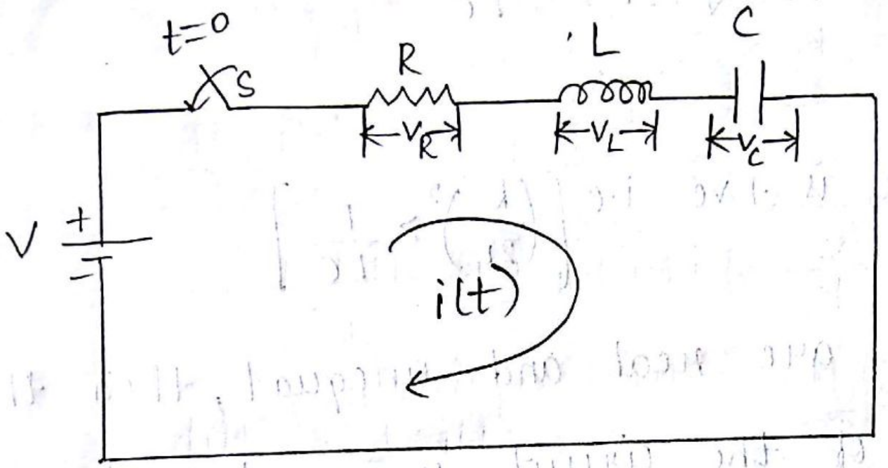
~~(or)~~

$$P_C = V_C\, i(t) = V(1 - e^{-t/\tau}) \cdot \frac{V}{R} e^{-t/\tau}$$

$$P_C = \frac{V^2}{R}(1 - e^{-t/\tau}) e^{-t/\tau}$$



DC response of series RLC ckt:-



Apply KVL to the ckt,

$$V = V_R + V_L + V_C$$

$$V = i(t)R + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)\,dt$$

$$0 = R\cdot\frac{di(t)}{dt} + L\frac{d^2 i(t)}{dt^2} + \frac{1}{C}\, i(t)$$

$$L\frac{d^2 i(t)}{dt^2} + \frac{1}{C}\, i(t) + R\frac{di(t)}{dt} = 0$$

$$\frac{d^2 i(t)}{dt^2} + \frac{R}{L}\frac{di(t)}{dt} + \frac{1}{LC}\, i(t) = 0$$

It is in the form of second order diff equ.

i.e $\left(D^2 + \frac{R}{L}D + \frac{1}{LC}\right) i(t) = 0$

$$k_1, k_2 = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - 4\left(\frac{1}{LC}\right)}}{2}$$

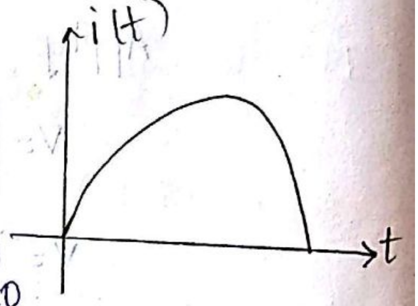
$$\Rightarrow \underbrace{-\frac{R}{2L}}_{K_1} \pm \underbrace{\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}}_{K_2}$$

Case:1) K_2 is +ve i.e $\left[\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}\right]$

Roots are real and unequal, then the response of the circuit is overdamped.

Sol. of this equation is

$$\boxed{x(t) = C_1 e^{(K_1+K_2)t} + C_2 e^{(K_1-K_2)t}}$$



i.e $[D-(K_1+K_2)][D-(K_1-K_2)]\, i(t) = 0$

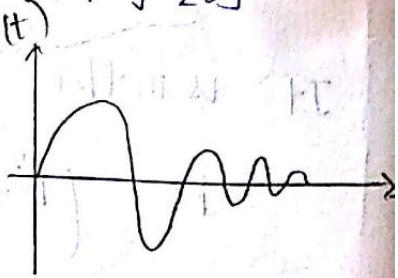
Case:2) K_2 is -ve i.e $\left[\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}\right]$

Roots are imaginary, response of ckt is underdamp

The roots are $[D-(K_1+jK_2)][D-(K_1-jK_2)]\, i(t) = 0$

Solution for above equ is

$$\boxed{x(t) = e^{-K_1 t}(C_1 \cos K_2 t + C_2 \sin K_2 t)}$$



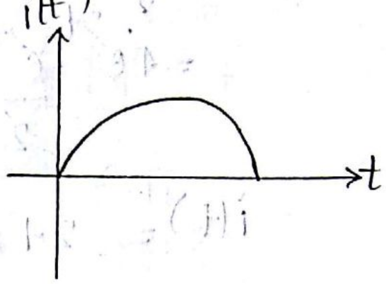
Case:3) $K_2 = 0$ i.e $\left[\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}\right]$

Roots are real and equal, then the response of the ckt is critically damped

The roots are $D - k_1^2 = 0$

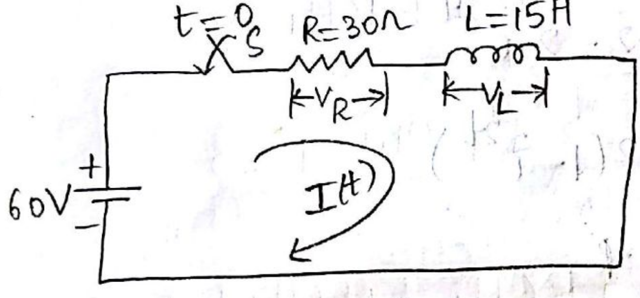
Solution of above equ is

$$\boxed{x(t) = e^{k_1 t}(c_1 + c_2 t)}$$



P1) A series R-L ckt with $R = 30\Omega$, $L = 15H$ has a constant voltage $V = 60V$ applied at $t=0$. Determine the current I, the voltage across resistor & voltage across inductor.

Sol



Apply KVL to ckt,

$$V = V_R + V_L$$

$$60 = V_R + V_L$$

$$60 = i(t)R + L\frac{di(t)}{dt}$$

$$60 = i(t)30 + 15\frac{di(t)}{dt}$$

$$\frac{di(t)}{dt} + 2i(t) = 4 \rightarrow \frac{dx}{dt} + px = k$$

$$x = i(t),\quad p = 2,\quad k = 4$$

$$i(t) = e^{-2t} \int 4e^{2t}\,dt + ce^{-2t}$$

$$= \overset{2}{\cancel{4}} e^{-2t} \frac{e^{2t}}{\cancel{2}} + ce^{-2t}$$

$$i(t) = 2 + ce^{-2t}$$

Apply initial conditions,

$$t = 0, \quad i(t) = 0$$

$$0 = 2 + c$$

$$c = -2$$

$$\therefore i(t) = 2 - 2e^{-2t}$$

$$i(t) = 2(1 - e^{-2t})$$

Now, $V_R = i(t) \cdot R$

$$V_R = 2(1 - e^{-2t}) \cdot 30$$

$$V_R = 60(1 - e^{-2t})$$

$$V_L = L \frac{di(t)}{dt}$$

$$V_L = 15(-2(-2e^{-2t}))$$

$$V_L = 60e^{-2t}$$

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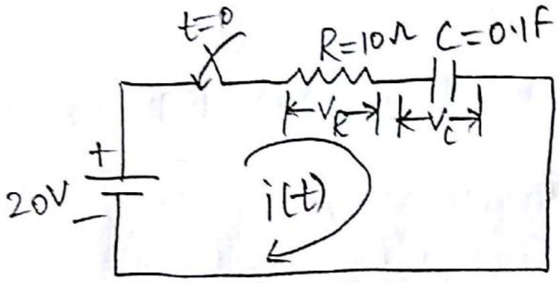
2) A series RC ckt consistor of resistor of 10Ω, capacitance of 0.1F. A constant voltage of 20V is applied to ckt at t=0. Obtain the current equ & determine the voltages across resistor & capacitor.

Sol



Apply KVL,

$$V = V_R + V_C$$

$$20 = i(t)\,10 + \frac{1}{0.1}\int i(t)\,dt$$

$$0 = 10\,\frac{di(t)}{dt} + \frac{1}{0.1}\,i(t)$$

$$\frac{di(t)}{dt} + i(t) = 0$$

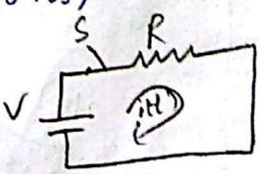
$$x = i(t)\quad,\ p=1,\ k=0$$

$$i(t) = e^{-t}\int 0.e^{t}\,dt + ce^{-t}$$

$$i(t) = ce^{-t}$$

Apply initial conditions,

$$V = i(t)\cdot R$$



$$i(t) = \frac{V}{R}$$

$$\frac{V}{R} = Ce^{0}$$

$$C = \frac{V}{R} = \frac{20}{10} = 2$$

$$\therefore i(t) = \frac{V}{R}e^{-t}$$

$$V_R = i(t) \cdot R$$

$$= Ve^{-t}$$

$$V_R = 20e^{-t}$$

& $\cancel{V_c = \frac{1}{C}\int i(t)\,dt}$

$\cancel{= \frac{1}{0.1}\int \frac{V}{R} e^{-t}\,dt}$

$\cancel{= \frac{20}{0.1 \times 10} \frac{e^{-t}}{-1}}$

$\cancel{V_c = -20e^{-t}}$

$$V = V_R + V_C$$

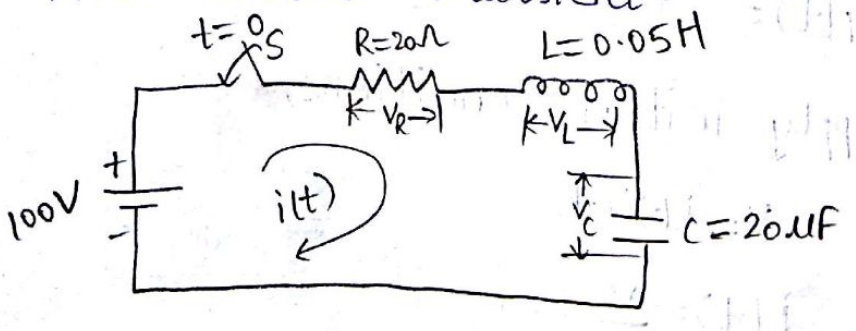
$$V_C = V - V_R$$

$$= 20 - 20e^{-t}$$

$$V_C = 20(1 - e^{-t})$$

3) The ckt shown in fig consists of resistance, inductance & capacitance in series with 100V constant source when the switch is closed at t=0. Find current transient.

Sol



Apply KVL,

$V = V_R + V_L + V_C$

$$100 = i(t)\cdot 20 + 0.05\,\frac{di(t)}{dt} + \frac{1}{20\times10^{-6}}\int i(t)\,dt$$

$$0 = 20\cdot\frac{di(t)}{dt} + 0.05\,\frac{d^2 i(t)}{dt^2} + \frac{1}{2\times10^{-5}}\,i(t)$$

$$\frac{di^2(t)}{dt^2} + \frac{10^6}{\cancel{10^6}}\,i(t) + 400\,\frac{di(t)}{dt} = 0$$

$$(D^2 + 400D + 10^6)\,i(t) = 0$$

$$K_1, K_2 = \frac{-400 \pm \sqrt{(400)^2 - 4\times10^6}}{2} = \underbrace{-200}_{K_1} \pm \underbrace{j979.79}_{K_2}$$

$$= \frac{-20}{0.1} \pm \sqrt{\left(\frac{20}{0.1}\right)^2 - \frac{1}{10^{-6}}}$$

$$= \underbrace{\frac{-20}{0.1}}_{K_1} \pm \underbrace{\sqrt{\left(\frac{20}{0.1}\right)^2 - 10^6}}_{K_2}$$

Here $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$

$\therefore$ Roots are imaginary

$\cancel{[D-(-200+j96\times10^4)]\,[D-(-200-j96\times10^4)]\,i(t)=0}$

$$x(t) = e^{-k_1 t}\,[C_1 \cos k_2 t + C_2 \sin k_2 t]$$

$$i(t) = e^{200t}\,[C_1 \cos 979.79t + C_2 \sin 979.79t] \quad \text{—(1)}$$

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Applying initial conditions, $i(t)=0$

$t=0$, $V_L = L\dfrac{di(t)}{dt}$ $\quad$ $V_L = 100$ since $V_L = V + v_R$

$= 100 + i(t)$

$= 100 + 0$

$100 = 0.05\dfrac{di(t)}{dt} \Rightarrow \dfrac{di(t)}{dt} = 2000$ $\quad$ $V_L = 100$

Sub $t=0$ & $i(t)=0$ in ①

$C_1 = 0$

$\therefore i(t) = e^{200t}[C_2 \sin 979.79t]$

$V_L = L\dfrac{di(t)}{dt}$

$V_L = 0.05 \cdot \dfrac{d}{dt}[e^{200t}(C_2 \sin 979.79t)]$

$= 0.05[200e^{200t}(C_2 \sin 979.79t) + e^{200t}(C_2 (+\cos 979.79t)$

$\times 979.79$

$V_L = 0.05[200e^{200t}(C_2 \sin 979.79t) + 979.79e^{200t}$

$(C_2 \cos 979.79t)]$

V_L at $t=0$

$V_L = 0 + 48.9895(C_2 \cdot 1)$

$\dfrac{100}{48.9895} = C_2$

$C_2 = 2.04$

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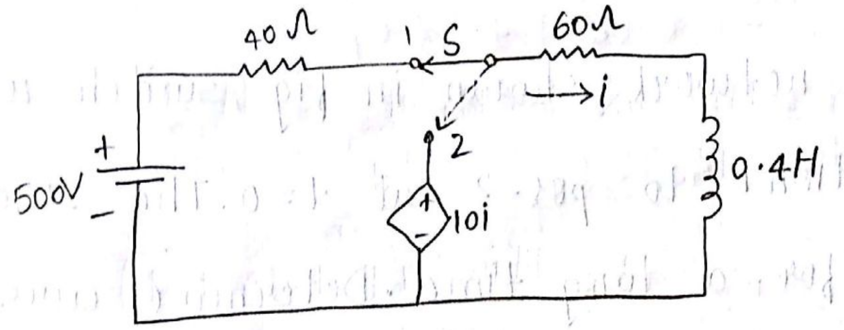
$\therefore \; i(t) = e^{200t} \left[2.04 \sin 979.79t \right]$

4) For the ckt shown in fig, find current equ when switch is changed from position 1 to pos 2 at $t=0$. The switch is at pos-1 for a long time.



Sol Apply KVL when switch is at pos-2.

$$10i = 60i + 0.4\frac{di(t)}{dt}$$

$$\frac{di(t)}{dt} + \frac{50i}{0.4} = 0$$

$$\frac{di(t)}{dt} + 125i = 0$$

$$x = i(t), \; P = 125, \; K = 0$$

Sol. is $\; i(t) = e^{-125t} \int 0 \cdot e^{125t} dt + Ce^{-125t}$

$$i(t) = Ce^{-125t}$$

inductor doesn't allow sudden changes in the current

$$500 = i(40+60)$$

$$i(t) = \frac{500}{100} = 5A$$

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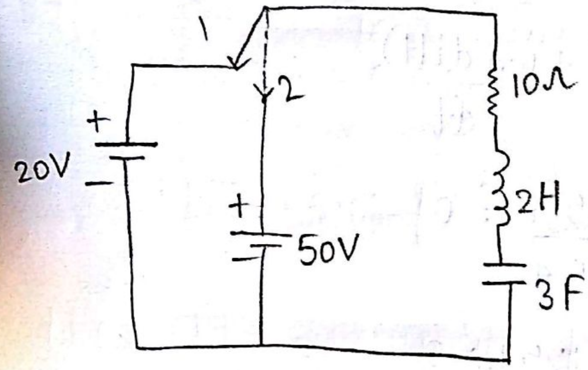
at $t=0, i(t)=5$

$$5 = Ce^{0}$$

$$C = 5$$

$$\therefore i(t) = 5e^{-125t}$$

5) In the network shown in fig, switch is moved from position 1 to pos-2 at $t=0$. The switch is in pos-1 for a long time. Determine current expression $i(t)$



Sol

Apply KVL when switch is at pos-2,

$$50 = i(t)\cdot 10 + 2\frac{di(t)}{dt} + \frac{1}{3}\int i(t)\,dt$$

$$0 = 10\frac{di(t)}{dt} + 2\frac{d^2 i(t)}{dt^2} + \frac{i(t)}{3}$$

$$\frac{d i^2(t)}{dt^2} + 5\frac{di(t)}{dt} + \frac{i(t)}{6} = 0$$

$$\left(D^2 + 5D + \frac{1}{6}\right)i(t) = 0$$

$$D = \frac{-15 \pm \sqrt{219}}{6}$$

$$= -\frac{5}{2} \pm 2.46$$

$$= \underset{K_1}{-2.5} \pm \underset{K_2}{2.46}$$

$$\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$$

∴ Roots are real & unequal

$$i(t) = C_1 e^{(-2.5+2.46)t} + C_2 e^{(-2.5-2.46)t} \quad -(1)$$

Apply initial conditions, $i(t)=0$

$$t=0, \quad V_L = L\frac{di(t)}{dt}$$

$$\overset{50}{\cancel{20}} = 2 \cdot \frac{di(t)}{dt} \Rightarrow \frac{di(t)}{dt} = \cancel{10}\ 25$$

Sub $t=0$ & $i(t)=0$ in (1)

$$0 = C_1 + C_2$$

$$C_1 = -C_2$$

$$\therefore i(t) = C_1\left[e^{(-2.5+2.46)t} - e^{-2.5-2.46t}\right]$$

$$\frac{di(t)}{dt} = C_1\left[(-2.5+2.46)e^{(-2.5+2.46)t} - (-2.5-2.46)e^{(-2.5-2.46)t}\right]$$

$$= C_1\left[-0.04 e^{-0.04t} + 4.96 e^{-4.96t}\right]$$

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$$V_L = L\frac{di(t)}{dt}$$

$$\cancel{5}0 = 2\,C_1\left[0.04\,e^{-0.04t} + 4.96\,e^{-4.96t}\right]$$

at $t = 0$

$$\overset{25}{\cancel{10}} = -C_1\,0.04 + 4.96\,C_1$$

$$\overset{25}{\cancel{10}} = 4.92\,C_1$$

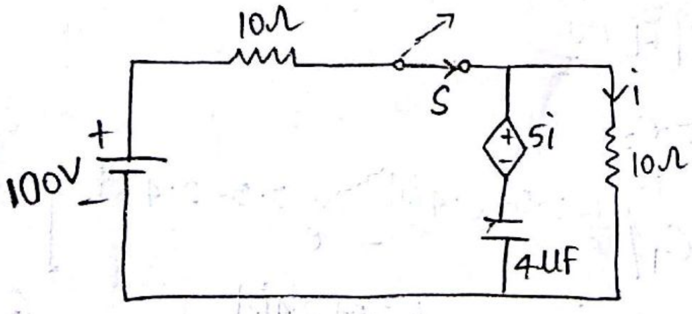
$$C_1 = \cancel{2.032}\ \frac{25}{4.92}$$

$$C_1 = 5.08$$

$$\therefore C_2 = -5.08$$

$$\therefore i(t) = 5.08\left[e^{-0.04t} - e^{-4.96t}\right]$$

6.) For the ckt shown in fig, find current eq when switch is opened at t=0



Sol

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Laplace transforms:-

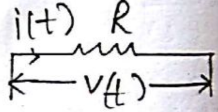
LT is a mathematical tool which transfor[ms] a function in time domain into a fn in frequency domain. The LT is given by

$$L[f(t)] = \int_{-\infty}^{\infty} f(t)e^{-st}\,dt$$

Transform impedance elements:-

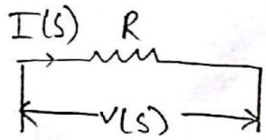
1.) Resistor in t-domain & S-domain:

From ohms law, $v(t) = i(t)\cdot R$



writing the above equ in s-domain, we have

$$V(S) = I(S)\cdot R$$



2.) Inductor in time domain & freq domain:-

For inductor, V-I char in time domain are

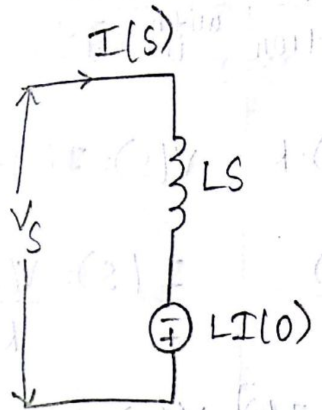
$$V_L = L\frac{di(t)}{dt} \quad ; \quad i(t) = \frac{1}{L}\int v(t)\,dt + i(0)$$

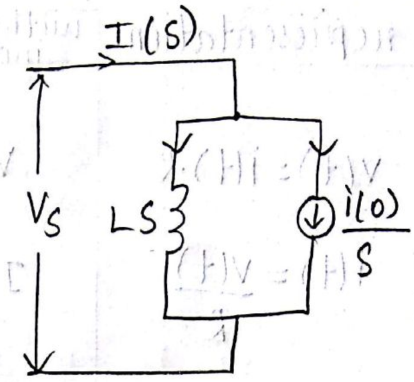
The corresponding frequency domain relations are given as

$$V(S) = L[SI(S) - I(0)]$$

$$V(S) = LSI(S) - LI(0)$$

$$I(s) = \frac{1}{Ls} V(s) + \frac{i(0)}{s}$$





3) Capacitor in t-domain & freq domain:-

Voltage across capacitor is

$$V_C(t) = \frac{1}{C} \int i(t)\,dt + V(0)$$

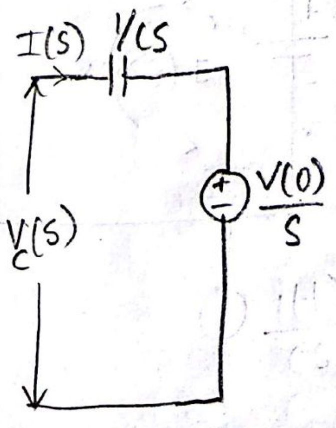
& current is $i(t) = C \frac{dv}{dt}$

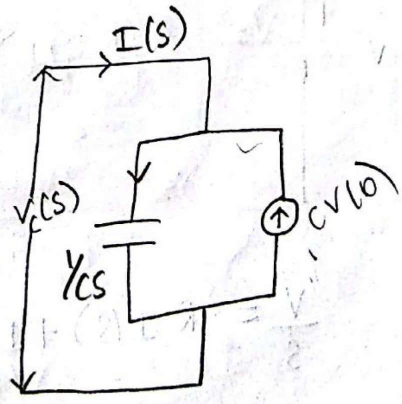
The corresponding frequency domain relations are

$$V_C(s) = \frac{1}{Cs} I(s) + \frac{V(0)}{s}$$

$$I(s) = C\,[sV(s) - V(0)]$$

$$I(s) = CsV(s) - CV(0)$$





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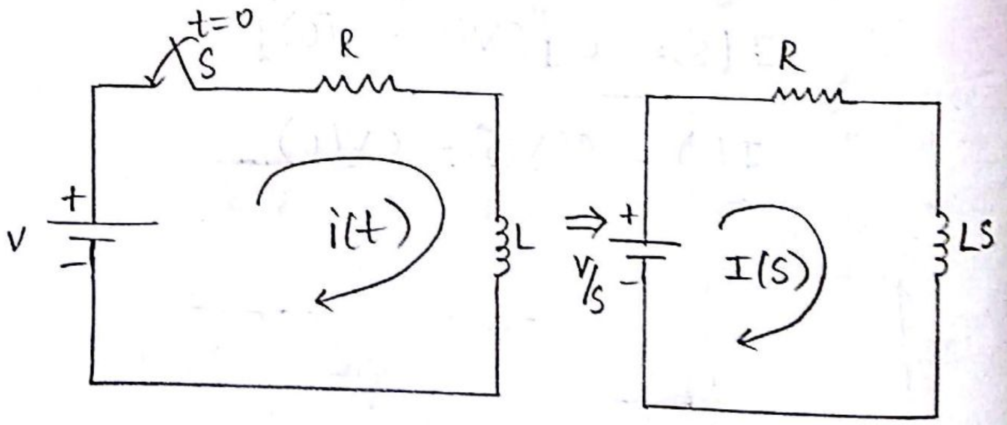
Element	Time domain representation	S-domain representation without zero initial condition	S-domain representation with zero initial condition
Resistor	$v(t)=i(t)\cdot R$	$V(S)=I(S)\cdot R$	$V(S)=I(S)\cdot R$
	$i(t)=\frac{v(t)}{R}$	$I(S)=\frac{V(S)}{R}$	$I(S)=\frac{V(S)}{R}$
Inductor	$V_L=L\frac{di(t)}{dt}$	$V_L(S)=L[SI(S)-I(0)]$	$V(S)=LSI(S)$
	$i(t)=\frac{1}{L}\int v(t)dt+i(0)$	$I(S)=\frac{1}{LS}V(S)+\frac{i(0)}{S}$	$I(S)=\frac{1}{LS}V(S)$
Capacitor	$V_C=\frac{1}{C}\int i(t)dt+v(0)$	$V_C(S)=\frac{1}{CS}I(S)+\frac{v(0)}{S}$	$V_C(S)=\frac{1}{CS}I(S)$
	$i(t)=C\frac{dv}{dt}$	$I(S)=CSV(S)-CV(0)$	$I(S)=CSV(S)$

DC Response of series RL ckt using LT :-



$$\frac{V}{S}=RI(S)+I(S)Ls$$

$$\frac{V}{S}=I(S)(R+LS)$$

$$I(s) = \frac{V}{s(R+Ls)}$$

$$I(s) = \frac{V}{Ls\left(s+\frac{R}{L}\right)}$$

$$I(s) = \frac{V/L}{s\left(s+\frac{R}{L}\right)}$$

$$\frac{V/L}{s(s+R/L)} = \frac{A}{s} + \frac{B}{s+\frac{R}{L}}$$

$$\frac{V}{L} = A\left(s+\frac{R}{L}\right) + Bs$$

s-coeff $\Rightarrow$ $0 = A+B$

$$A = -B$$

const $\Rightarrow$ $\frac{V}{\cancel{L}} = A\frac{R}{\cancel{L}}$

$$A = \frac{V}{R} \Rightarrow B = \frac{-V}{R}$$

$$I(s) = \frac{V/R}{s} - \frac{V/R}{s+R/L}$$

$$\boxed{I(s) = \frac{V}{Rs} - \frac{V}{R\left(s+\frac{R}{L}\right)}}$$

$$i(t) = \frac{V}{R} - \frac{V}{R}e^{-R/L\,t}$$

$$i(t) = \frac{V}{R}\left[1 - e^{-R/L\,t}\right] \quad \text{or} \quad \tau = \frac{L}{R}$$

$$i(t) = \frac{V}{R}\left(1 - e^{-t/\tau}\right)$$

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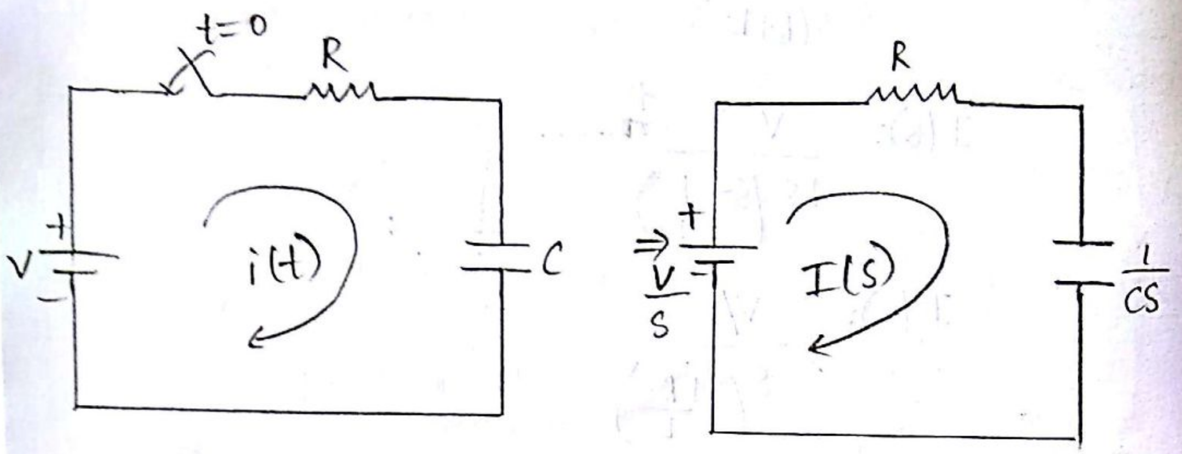
DC response of series RC ckt using LT:-



$$\frac{V}{S} = R \cdot I(S) + I(S) \cdot \frac{1}{CS}$$

$$\frac{V}{S} = I(S)\left(R + \frac{1}{CS}\right)$$

$$I(S) = \frac{V/S}{R + \frac{1}{CS}}$$

$$I(S) = \frac{V}{S\left(R + \frac{1}{CS}\right)} = \frac{V}{SR + \frac{1}{C}} \Rightarrow \frac{V}{R\left[S + \frac{1}{RC}\right]}$$

$$\cancel{\frac{V}{S\left(R+\frac{1}{CS}\right)} = \frac{A}{S} + \frac{B}{R+\frac{1}{CS}}} \qquad \frac{V/R}{S + 1/RC}$$

$$\cancel{V = A\left(R + \frac{1}{CS}\right) + BS}$$

$$I(S) = \frac{V}{R\left(S + \frac{1}{RC}\right)}$$

$$\boxed{I(S) = \frac{V/R}{S + \frac{1}{RC}}}$$

$$i(t) = \frac{V}{R} e^{-1/RC\, t} \quad \text{or} \quad \tau = LC$$

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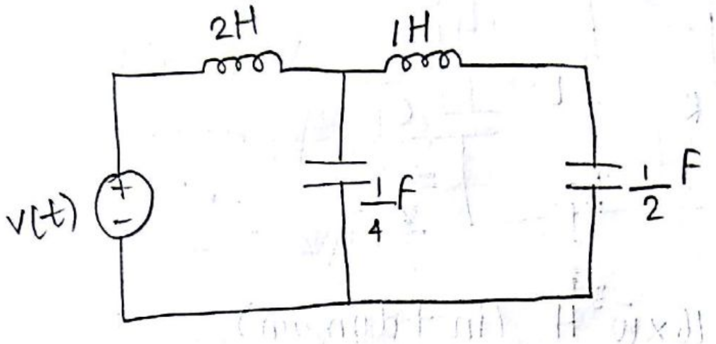
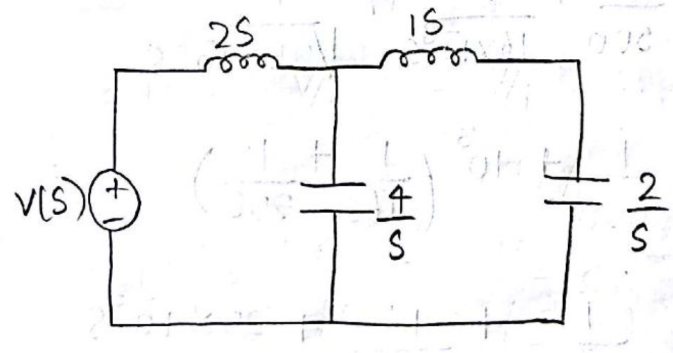
17-12-15

1) Draw the s-domain ckt for ckt shown in fig. Assume initial conditions are 0



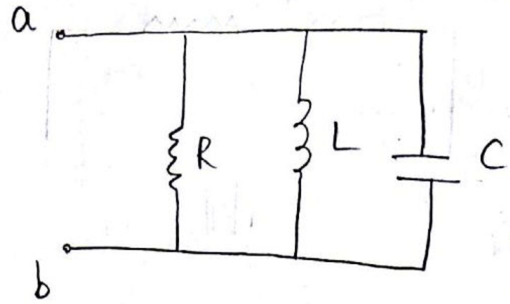
Sol

	Time domain	Freq domain
voltage source	$v(t)$	$V(s)$
Inductor	$L\frac{di(t)}{dt}$	$LS = 2S$
Inductor	$L\frac{di(t)}{dt}$	$LS = 1S$
capacitor	$\frac{1}{C}\int i(t)dt$	$\frac{1}{CS} = \frac{1}{\left(\frac{1}{4}\right)S} = \frac{4}{S}$
capacitor	$\frac{1}{C}\int i(t)dt$	$\frac{1}{CS} = \frac{1}{\left(\frac{1}{2}\right)S} = \frac{2}{S}$



2) A 500Ω resistor, a 16mH inductor and a 25mf capacitor are connected in $\parallel^{d}$. Express the admittance of this $\parallel^{l}$ combination of the elements as the fn of s.

Sol $R = 50\Omega$, $L = 16mH$, $C = 25mF$



$L = 16\times10^{-3}H$ (in t domain)

$= 16\times10^{-3}s$ (in freq-domain)

$C = 25\times10^{-3}F$ (in t)

$= \frac{1}{25\times10^{-3}s}$ (in s)

$$Y(s) = \frac{1}{Z(s)}$$

$$= \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}$$

$$= \frac{1}{500} + \frac{1}{16\times10^{-3}s} + \frac{1}{1/25\times10^{-3}s}$$

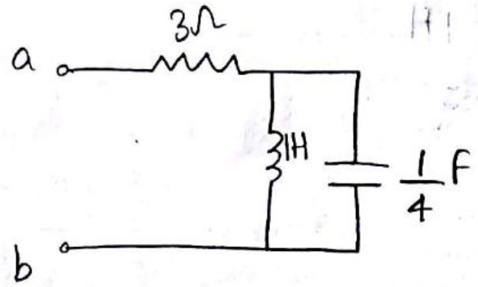
$$= \cancel{\frac{1}{500} + 10^3\left(\frac{1}{16s} + \frac{1}{25s}\right)}$$

$$= \frac{1}{500} + \frac{1}{16\times10^{-3}s} + 25\times10^{-3}s$$

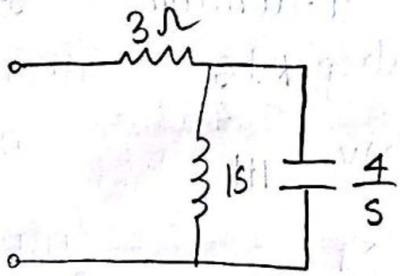
$$= 2\times10^{-3} + \frac{6\times10^5}{s} + 25\times10^{-3}s$$

$$= (2\times10^{-3}s + 6\times10^5 + 25\times10^{-3}s^2)/s$$

3) Transform the ckt shown to the s-domain & determine the laplace impedance



Sol s-domain ⇒



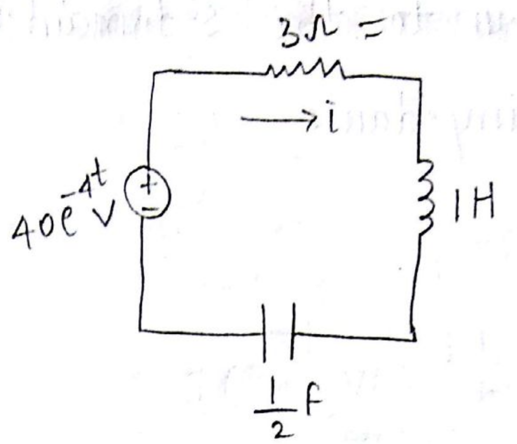
$$1S \,//\, \frac{4}{S} = \frac{4}{1S + \frac{4}{S}} = \frac{4S}{S^2+4}$$

3Ω series with $\frac{4S}{S^2+4}$

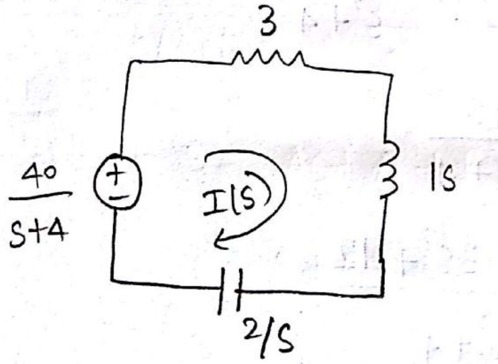
$$= 3 + \frac{4S}{S^2+4}$$

$$= \frac{4S + 3S^2 + 12}{S^2+4}$$

4) Determine the current i if the ckt is driven by a voltage source as shown in fig. The initial value of the voltage across the capacitor & the initial current through the inductor are both zero.



	t-domain	s-domain
Resistor	$R=3$	$R=3$
Inductor	$1H$	$Ls - \underbrace{Li(0)}_{0} = 1s$
capacitor	$\frac{1}{2}F$	$\frac{1}{Cs} + \underbrace{\frac{v(0)}{s}}_{0} = \frac{2}{s}$
voltage source	$40e^{-4t}$	$\frac{40}{s+4}$



$$I(s) = \frac{V(s)}{Z(s)}$$

$$Z(s) = 3 + 1s + \frac{2}{s}$$

$$Z(s) = \frac{s^2+3s+2}{s}$$

$$\therefore I(s) = \frac{40s}{(s+4)(s^2+3s+2)}$$

$$\cancel{40S = A(S^2+3S+2)+}$$

$$I(S) = \frac{40S}{(S+4)(S+1)(S+2)}$$

$$\frac{40S}{(S+4)(S+1)(S+2)} = \frac{A}{S+4} + \frac{B}{S+1} + \frac{C}{S+2}$$

$$40S = A(S+1)(S+2) + B(S+4)(S+2) + C(S+4)(S+1)$$

S^2 coeff $\Rightarrow 0 = A+B+C$

S coeff $\Rightarrow 40 = 3A+6B+5C$

const $\Rightarrow 0 = 2A+8B+4C$

$$A = \frac{-80}{3}, \quad B = \frac{-40}{3}, \quad C = 40$$

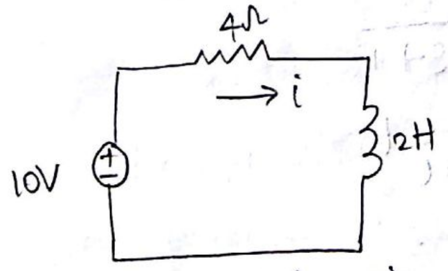
$$I(S) = \frac{-80}{3(S+4)} - \frac{40}{3(S+1)} + \frac{40}{S+2}$$

$\rightarrow$ ILT

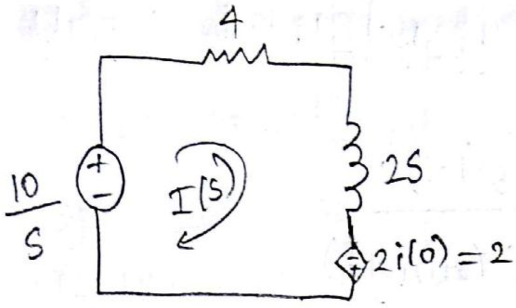
$$i(t) = -\frac{80}{3} e^{-4t} - \frac{40}{3} e^{-t} + 40 e^{-2t}$$

5) Determine the current i for $t \geq 0$ if initial current $i(0) = 1$ for the ckt shown in fig.

Sol



	t-domain	s-domain
resistor	4	4
inductor	2	$Ls - Li(0)$ $2S - 2 = 2(S-1)$



$$I(s) = \frac{V(s)}{Z(s)}$$

$$= \frac{10/s}{4+2s-2}$$

$$= \frac{10}{s(2+2s)}$$

$$I(s) = \frac{10}{2s^2+2s} = \frac{5}{s^2+s} = \frac{5}{s(s+1)}$$

$$\frac{5}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$$

$$5 = As + A + Bs$$

$$0 = A + B$$

$$5 = A$$

$$B = -5$$

$$I(s) = \frac{5}{s} - \frac{5}{s+1}$$

$$i(t) = 5 - 5e^{-t}$$

$$\frac{10}{s} = 4I(s) + 2sI(s) - 2$$

$$\frac{10}{s} + 2 = (4+2s)I(s)$$

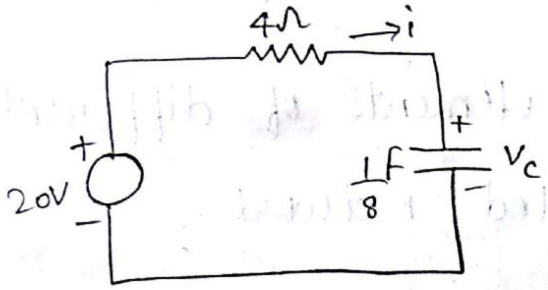
$$I(s) = \frac{\frac{10}{s}+2}{4+2s}$$

$$= \frac{10+2s}{2s^2+4s}$$

$$= \frac{5+s}{s^2+2s}$$

$$= \frac{5+s}{s(s+2)}$$

6) Determine current i for $t \geq 0$ if $V_C(0) = 4V$ for ckt shown in fig.



Sol

$R = 4$

$C = \frac{8}{s} + \frac{4}{s}$; $V = \frac{20}{s}$

Apply KVL to ckt,

$\frac{20}{s} = 4I(s) + \cancel{\left(\frac{8}{s} + \frac{4}{s}\right) I(s)}$ $\frac{8}{s} I(s) + \frac{4}{s}$

$\cancel{\frac{20}{s} = \left(4 + \frac{12}{s}\right) I(s)}$ $\frac{20}{s} - \frac{4}{s} = \left(4 + \frac{8}{s}\right) I(s)$

$\cancel{I(s) = \frac{20/s}{\frac{4s+12}{s}}} = \cancel{\frac{5/8}{\frac{s+3}{8}}}$ $I(s) = \frac{16/s}{\frac{4s+8}{s}}$

$\cancel{\frac{5s}{s+3}} \neq \cancel{\frac{A}{s+3}}$ $I(s) = \frac{4}{s+2}$

$\cancel{I(s) = \frac{5}{s+3}}$ $i(t) = 4e^{-2t}$

~~Apply ILT,~~

$\cancel{i(t) = 5e^{-3t}}$