UNIT-3

SIGNAL TRANSMISSION THROUGH LINEAR SYSTEMS

System: A system is defined as set of rules that associates an o/p time function to every i/p time function.

(or)

A system is an interconnection of elements which produces expected o/p for available i/p.

i/p excitation f(t) —— [system] —— o/p Response r(t)

$f(t) \longrightarrow r(t)$

→ System is an mathematical operator which maps i/p into o/p

Classification of system

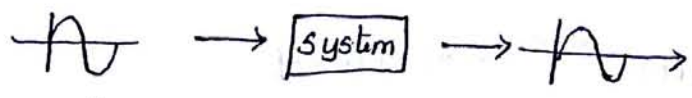
- continuous time system
- Discrete time system

continuous time system:

1. Static & Dynamic systems
2. Linear & Non-Linear
3. Time invariant & Time variant
4. Linear TIV & LTIV
5. Stable system
6. casual & non-causal systems.

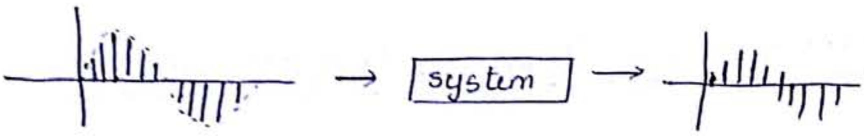
(i) continuous time systems

→ A continuous time system operates on a continuous time i/p signal to produce a continuous time o/p signal



(ii) Discrete time systems:

A discrete time system operates on a discrete time i/p signal to produce a discrete time o/p signal.



classifications :

(i) static and Dynamic Systems:

→ A static system or system is said to be static if its o/p at any instant depends only on present values of i/p.

Ex: $y(t) = ax(t)$ (ii) $y(t) = a^r x(t)$

at $t=0$ $y(0) = ax(0)$ at $t=0$ $y(0) = a^r x(0)$

at $t=1$ $y(1) = ax(1)$ at $t=1$ $y(1) = a^r x(1)$

→ A system is said to be dynamic if its o/p depends on present & past values of i/p.

Ex: $y(t) = x(t-1) + x(t-2) + x(t)$

at $t=2$

$$y(2) = x(2-1) + x(2-2) + x(2) = \underbrace{x(1) + x(0)}_{\text{past}} + \underbrace{x(2)}_{\text{present}}$$

(ii) Linear and Non Linear Systems :

→ A system is said to be linear if its satisfies the superposition principle.

→ It states that the response of the system to a weighted sum of signals be equal to the corresponding weighted sum of o/p's of the system to each of the individual i/p signal.

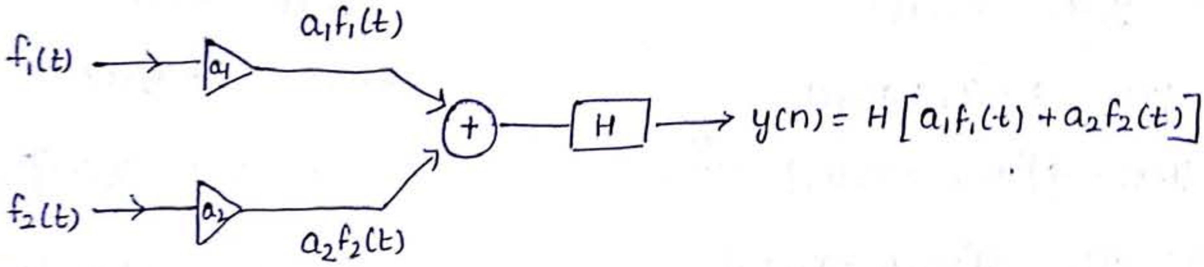
$$H[a_1 f_1(t) + a_2 f_2(t)] = a_1 H[f_1(t)] + a_2 H[f_2(t)]$$

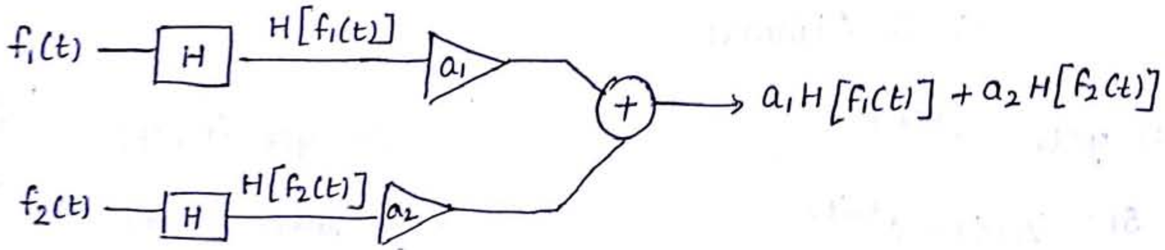
where a_1, a_2 are weighted constants.

$$a_1 f_1(t) \xrightarrow{\text{Response}} a_1 H[f_1(t)]$$

$$a_2 f_2(t) \xrightarrow{\text{Response}} a_2 H[f_2(t)]$$

$$H[a_1 f_1(t) + a_2 f_2(t)] \longrightarrow a_1 H[f_1(t)] + a_2 H[f_2(t)]$$





Block diagram.

→ Any system which does not obey the above principle is called as non-linear systems.

check for Linearity:

Procedure:

1. Apply different i/p's separately and get the o/p.
2. Apply different i/p's simultaneously and get the output.
3. If both outputs are same it is linear otherwise non-linear.

Ex:

(i) $y(t) = 4 \sin t\, x(t)$

step 1: $y_1(t) = 4 \sin t\, x_1(t)$

$y_2(t) = 4 \sin t\, x_2(t)$

$y_1(t) + y_2(t) = 4 \sin t\, [x_1(t) + x_2(t)]$

step 2: $y(t) = 4 \sin t\, [x_1(t) + x_2(t)]$

$S_1 = S_2$

(2) $y(t) = a x(t)$

Sol S_1: $y_1(t) = a\, x_1(t)$

$y_2(t) = a\, x_2(t)$

$y(t) = a\, x_1(t) + a x_2(t)$

$y(t) = a[x_1(t) + x_2(t)]$

S_2: $y(t) = a[x_1(t) + x_2(t)]$

$S_1 = S_2$ (Linear)

(3) $y(t) = x^2(t)$

Sol S_1: $y_1(t) = x_1^2(t)$

$y_2(t) = x_2^2(t)$

$y(t) = x_1^2(t) + x_2^2(t)$

S_2: $y(t) = [x_1(t) + x_2(t)]^2 \rightarrow$

$S_1 \neq S_2$ (Non-Linear)

(4) $y(t) = e^{x(t)}$

Sol S_1: $y_1(t) = e^{x_1(t)}$

$y_2(t) = e^{x_2(t)}$

$y(t) = e^{x_1(t)} + e^{x_2(t)}$

$y(t) = e^{[x_1(t) + x_2(t)]}$

S_2: $e^{x_1(t)} \cdot e^{x_2(t)}$

$S_1 \neq S_2$ (Non-Linear)

(5) $y(t) = t\, x(t)$

Sol S_1: $y_1(t) = t\, x_1(t)$

$y_2(t) = t\, x_2(t)$

$y(t) = t[x_1(t) + x_2(t)]$

S_2: $y(t) = t[x_1(t) + x_2(t)]$

$S_1 = S_2$ (Linear)

(11) $y(t) = x(t^2)$

Sol S_1: $y_1(t) = x_1(t^2)$

$y_2(t) = x_2(t^2)$

$y(t) = x_1(t^2) + x_2(t^2)$

S_2: $y(t) = x_1(t^2) + x_2(t^2)$

$S_1 = S_2$ (Linear)

(6) $y(t) = x(t - t_0)$

Sol S_1: $y_1(t) = x_1(t - t_0)$

$y_2(t) = x_2(t - t_0)$

$y(t) = x_1(t - t_0) + x_2(t - t_0)$

S_2: $y(t) = x_1(t - t_0) + x_2(t - t_0)$

$S_1 = S_2$ (Linear)

(7) $y(t) = 3x(t+3)$

Sol S_1: $y_1(t) = 3x_1(t+3)$

$y_2(t) = 3x_2(t+3)$

$y(t) = 3[x_1(t+3) + x_2(t+3)]$

S_2: $3[x_1(t+3) + x_2(t+3)]$

$S_1 = S_2$ (Linear)

(12) $y(t) = Ax(t) + B$

Sol $y_1(t) = Ax_1(t) + B$

$y_2(t) = Ax_2(t) + B$

$y(t) = A[x_1(t) + x_2(t)] + 2B$

$y(t) = A[x_1(t) + x_2(t)] + B$

$S_1 \neq S_2$ Non-linear

(8) $y(t) = x(t+1)e^{-t}$

Sol S_1: $y_1(t) = x_1(t+1)e^{-t}$; $y_2(t) = x_2(t+1)e^{-t}$

$y(t) = e^{-t}[x_1(t+1) + x_2(t+1)]$

S_2: $y(t) = e^{-t}[x_1(t+1) + x_2(t+1)]$

$S_1 = S_2$ (Linear)

(9) $y(t) = \cos[x(t)]$

Sol S_1: $y_1(t) = \cos[x_1(t)]$; $y_2(t) = \cos[x_2(t)]$

$y(t) = \cos[x_1(t)] + \cos[x_2(t)]$

S_2: $\cos[x_1(t) + x_2(t)]$

$S_1 \neq S_2$ (Non-Linear)

(10) $y(t) = k\Delta x(t)$ where $\Delta x(t) = [x(t+1) - x(t)]$

Sol S_1: $y_1(t) = k[x_1(t+1) - x_1(t)]$; $y_2(t) = k\Delta x_2(t)$

$y(t) = [x_1(t+1) - x_1(t) + x_2(t+1) - x_2(t)].k$

S_2: $y(t) = k[x_1(t+1) - x_1(t) + x_2(t+1) - x_2(t)]$

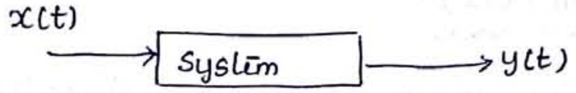
$S_1 = S_2$ (Linear)

(13) $y(t) = 4x(t) + 2\dfrac{dx(t)}{dt} \rightarrow$ Linear.

Time Variant And Time Invariant Systems

→ A system is said to be time invariant if the system does not depend on time i.e system delay is not function of time.

Ex:



$x(t) \longrightarrow y(t)$

$x(t-t_0) \longrightarrow y(t-t_0)$

→ A time shift t_0 in the input results in the same amount of time shift in the o/p but the waveshape does not change.

i.e the i/p and/ to o/p characteristics does not change with time.

→ Any system which doesnot obey the above principle is called as time varying system.

→ An electrical system is said to be time invariant if its component values (R, L, C) does not change with time.

Check for time Invariant:

1. Shift the i/p only and get the o/p.
2. Shift the entire system and get the o/p.
3. If both steps are identical for o/p then it is time invariant system.

Ex!

(1) $y(t) = 4x(t)$

Sol $S_1: y(t) = 4x(t-1)$

$S_2: y(t-1) = 4x(t-1)$ $\Big] \rightarrow S_1 = S_2$ (TIV)

(2) $y(t) = 4t\,x(t)$

Sol $S_1: y(t) = 4t\,x(t-1)$

$S_2: y(t-1) = 4(t-1)(x(t-1))$ $\Big] \rightarrow S_1 \neq S_2$ (TV)

(3) $y(t) = a x(t)$

Sol $S_1: y(t) = a x(t-1)$

$S_2\ y(t-1) = a x(t-1)$ $\Big] \rightarrow S_1 = S_2$ (TIV)

(4) $y(t) = a x(t) + b$

Sol $S_1: y(t) = a x(t-1) + b$

$S_2: y(t-1) = a x(t-1) + b$ $\Big]$ $S_1 = S_2$ (TIV)

(5) $y(t) = 5t\,[x(t)]^2$

Sol $S_1: y(t) = 5t\,[x(t-1)]^2$

$S_2: y(t-1) = 5(t-1)[x(t-1)]^2$ $\Big\}$ $S_1 \neq S_2$ (TV)

(6) $y(t) = x(t+1)e^{-t}$

Sol $S_1: y(t) = x(t+1-1)e^{-t} = x(t)e^{-t}$

$S_2: y(t-1) = x(t+1-1)e^{-(t-1)}$

$= x(t)\cdot e^{-t}\,e^{-1}$ → constant

$S_1 = S_2$ (TIV)

⑦ $y(t) = x(t+3)$

Sol $S1: y(t) = x(t+3-1)$

$S2: y(t-1) = x(t+3-1)$ $= x(t+2)$ $\Big] \rightarrow S_1 = S_2$ (TIV)

⑧ $y(t) = x^2(t)$

Sol $S1: y(t) = x^2(t-1)$

$S2: y(t-1) = x^2(t-1)$

$\therefore S_1 = S_2$ (TIV)

⑨ $y(t) = e^{x(t)}$

Sol $S1: y(t) = e^{x(t-1)}$

$S2: y(t-1) = e^{x(t-1)}$

$S_1 = S_2$ (TIV)

Linear Time Invariant System (LTI):

→ Any system which obeys the linearity and time invariant property is called as LTI system.

Linear Time Variant System (LTV):

→ Any system which obeys the linearity and does not obey time invariant property is called LTV system.

Ex: $y(t) = a x(t)$

Linearity: $y_1(t) = a x_1(t)$; $y_2(t) = a x_2(t)$

$y(t) = a x_1(t) + a x_2(t)$

$y(t) = a[x_1(t) + x_2(t)]$

$S2: y(t) = a[x_1(t) + x_2(t)]$.

$\therefore S_1 = S_2$.

T.I:

$y(t) = a x(t)$

$S1: y(t) = a x(t-1)$

$S2: y(t-1) = a x(t-1)$ $\Big] \rightarrow S_1 = S_2$ (TIV)

$\therefore$ It is a linear time invariant system (LTI)

$\|^{ly}$:

(2) $y(t) = t x(t) \rightarrow$ LTI

(3) $y(t) = a x(t) + b \rightarrow$ NLTI

(4) $y(t) = a x^2(t) \rightarrow$ NLTI

(5) $y(t) = e^{x(t)} \rightarrow$ NLTI

(6) $y(t) = x(t - t_0) \rightarrow$ LTI

Stable System:

→ System is absolutely integrable

$$\int_{-\infty}^{\infty} |f(t)|\, dt < \infty$$

Causal And Non Causal Systems:

→ A system is said to be causal if o/p $y(t_0)$ depends only on the values of i/p $x(t)$ at $t < t_0$ {present, i/p, past i/ps, past o/ps} { $x(t) = 0$, for $t < 0$; noncausal $t \le 0$, or $t \le 0$ and $t > 0$; $x(t) \neq 0$ for $t < 0$

Ex: $y(t) = 4x(t-1)$

$y(2) = 4x(2-1) \Rightarrow 4x(2)$

$y(t) = 4x(t-1) + x(t)$

$y(2) = 4x(1) + x(2)$

→ A system is said to be non-causal if the o/p depends on future values of i/p i.e future i/ps & o/ps.

Ex: $y(t) = 4x(t+1)$

$y(2) = 4x(3)$

Examples whether it is causal & Non causal:

(1) $y(t) = K[x(t+1) - x(t)]$

$y(0) = K[x(1) - x(0)] \rightarrow$ Noncausal

(2) $y(t) = 3x(t+3)$

$y(0) = 3x(3) \longrightarrow$ Non causal

(3) $y(t) = (t+3)x(t-3)$

$y(0) = (0+3)x(0-3)$

$= 3x(-3) \rightarrow$ causal

(4) $y(t) = x(t) + 3x(t+4)$

when $t=0$, $y(0) = x(0) + 3x(4)$

when $t=1$, $y(1) = x(1) + 3x(5)$

So here response at $t=0$, $y(0)$ depends on the present i/p & future i/p

hence system is noncausal.

(5) $y(t) = x(t^2)$

$t=-1$, $y(-1) = x(1) \rightarrow$ future

$t=0$, $y(0) = x(0) \rightarrow$ present

$t=1$, $y(1) = x(1) \rightarrow$ present

$t=2$, $y(2) = x(4) \rightarrow$ future

Noncausal.

Except at $t=0$, $t=1$, the response of any value of t depends on future i/p.

(6) $y(t) = x(2t) \rightarrow$ Noncausal

(7) $y(t) = x(t) - x(t-1) \rightarrow$ causal

(8) $y(t) = x(t) + \int_0^t x(\lambda)\, d\lambda$

$= x(t) + z(\lambda)\big]_0^t \Rightarrow$ Causal.

At $t=0$, $t=1$, $t=2$

Impulse Response:

The response of a system for an impulse i/p is called a impulse response of the system and it is denoted by h(t)

$$\delta(t) \longrightarrow \boxed{\text{system}} \longrightarrow h(t)$$

$$\delta(t) \rightarrow h(t)$$

→ Every system is characterised by its impulse response.

$$\underset{f(t)}{\text{i/p}} \longrightarrow \boxed{h(t)} \longrightarrow \underset{r(t)}{\text{o/p}}$$

Response of a System for an arbitary i/p:

Response of $\delta(t) \rightarrow h(t)$

$$\delta(t-t_0) \rightarrow h(t-t_0)$$

$$\delta(t) + \delta(t-t_0) = h(t) + h(t-t_0)$$

The response of a system for a given i/p f(t) is determined by using superposition principle.

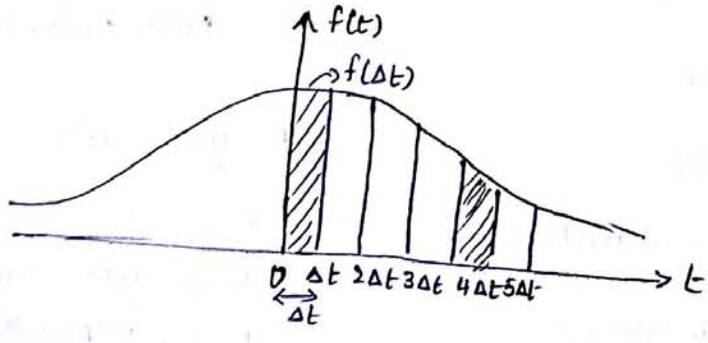
step 1: Resolve the i/p function interms of impulse functions.

step 2: Determine individually the response of LTI system for impulse function.

step 3: Find the sum of individual responses which will become the overall response r(t).

Representation of a function f(t) in terms of an impulse function:

Here the function f(t) is a impulse train function.



→ area $f(5\Delta t) \times \Delta t$

$$f(t) = \sum_{n=-\infty}^{\infty} f(n\Delta t)\,\Delta t \cdot \delta(t - n\Delta t)$$

$$f(t) = \underset{\Delta t \to 0}{Lt} \sum_{n=-\infty}^{\infty} f(n\Delta t)\, \Delta t\, \delta(t-n\Delta t)$$

The rectangular of width Δt & height $f(n\Delta t)$ and area under the rectangles is $\Delta t \cdot f(n\Delta t)$ and this n^{th} element approached a delta function of strength $f(n\Delta t)\Delta t$ located at $t = n\Delta t$. and this delta function is represented as $f(n\Delta t)\Delta t\, \delta(t-n\Delta t)$

$$\boxed{f(t) = \underset{\Delta t \to 0}{Lt} \sum_{n=-\infty}^{\infty} f(n\Delta t) \cdot \Delta t \cdot \delta(t-n\Delta t)}$$

As $\Delta t \to 0$, the n^{th} element may be considered.

2) Determination of $r(t)$ for the input $f(t)$:

Let $h(t)$ be the impulse response of the system.

$$\delta(t) \longrightarrow \boxed{\text{system}} \longrightarrow h(t)$$

then $\delta(t) \longrightarrow h(t)$

$$\delta(t-n\Delta t) \longrightarrow h(t-A\Delta t)$$

$$f(n\Delta t)\, \delta(t-n\Delta t) \longrightarrow f(n\Delta t) \cdot h(t-n\Delta t)$$

$$f(n\Delta t) \cdot \Delta t\, \delta(t-n\Delta t) \longrightarrow f(n\Delta t) \cdot \Delta t\, h(t-n\Delta t)$$

$$\underset{\Delta t \to 0}{Lt} \sum_{n=-\infty}^{\infty} f(n\Delta t) \cdot \Delta t\, \delta(t-n\Delta t) \longrightarrow \underset{\Delta t \to 0}{Lt} \sum_{n=-\infty}^{\infty} f(n\Delta t) \cdot \Delta t\, h(t-n\Delta t).$$

$$f(t) \longrightarrow \boxed{\text{system}} \longrightarrow r(t)$$

$$r(t) = \underset{\Delta t \to 0}{Lt} \sum_{n=-\infty}^{\infty} f(n\Delta t) \cdot \Delta t\, h(t-n\Delta t)$$

$\Delta t \to 0$ means summation becomes integration.

$$r(t) = \int_{-\infty}^{\infty} f(\tau) \cdot h(t-\tau)\, d\tau$$

$$r(t) = f(t) \otimes h(t)$$

$$f(t) \longrightarrow \boxed{h(t)} \longrightarrow r(t) \to f(t) \otimes h(t).$$

i.e if the response of a system is known for unit ... fn. then response to any other function f(t) can be obtained from the above eqn.

→ An unit impulse function is called as a Test function and it is used to characterise a system.

$$r(t) = f(t) \otimes h(t)$$

In frequency domain $r(t) \xleftrightarrow{FT} R(W)$

$$f(t) \xleftrightarrow{FT} F(W)$$

$$h(t) \xleftrightarrow{FT} H(W)$$

Using convolution property

$$f(t) \otimes h(t) = F(\omega) \cdot H(\omega)$$

$$R(\omega) = F(\omega) \cdot H(\omega)$$

$$\boxed{H(\omega) = \frac{R(\omega)}{F(\omega)}}$$

→ When $F(\omega) = 1$; i.e i/p is unit impulse $H(\omega) = R(\omega)$

→ Transfer function $H(\omega)$ of a system is defined as the transform of the response of a system where the i/p is unit impulse function.

$$H(\omega) = |H(\omega)| e^{j\theta(\omega)}$$

→ phase response of the system.

→ Amplitude response of the system.

$$\boxed{\begin{aligned} Ln[H(\omega)] &= Ln[|H(\omega)|] + j\theta(\omega) \\ &= L(\omega) + j\theta(\omega) \end{aligned}}$$

Gain of the system

phase shift introduced by system.

Note: An impulse function contains all frequencies in equal amount so we can use it as a test function.

$$H(\omega) = \frac{R(\omega)}{F(\omega)}$$ → Transfer fn of LTI system.

F.T $[h(t)]$

$h(t) = I.F.T[H(\omega)]$

FILTER CHARACTERISTICS OF LINEAR SYSTEMS :

IDEAL LOW PASS FILTERS :

→ It transmits all the signals below certain frequency 'B' Hz without any distortion.

→ The range of frequencies from 0Hz to 'B' Hz is called passband of lowpass filter.

→ The frequency 'B' Hz is called cut-off frequency of the ideal lowpass filter.

→ The transfer function of ideal low pass filter can be written as

$$H(f) = K e^{-j2\pi f t_0} \quad ; \; -B \leq f < B$$

$$= 0 \quad ; \; |f| > B.$$

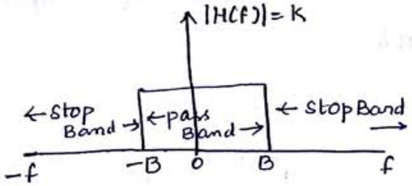
K = amplitude is assumed to be unity.

→ By K=1 in above eqn

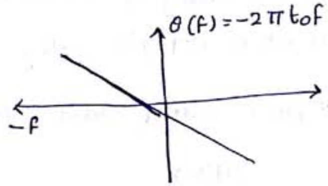
$$H(f) = e^{-j2\pi f t_0} \quad ; \; -B \leq f < B$$

$$= 0 \quad ; \; |f| > B$$

→ By inverse fourier transform, h(t) can be obtained for ideal LPF



a) magnitude response.



(b) phase response

$$h(t) = \int_{-B}^{B} e^{-j2\pi f t_0} \cdot e^{j2\pi f t} \, df$$

$$= \int_{-B}^{B} \left[e^{j2\pi f (t-t_0)} \right] df = \frac{1}{j2\pi(t-t_0)} \left[e^{j2\pi f(t-t_0)} \right]_{-B}^{B}$$

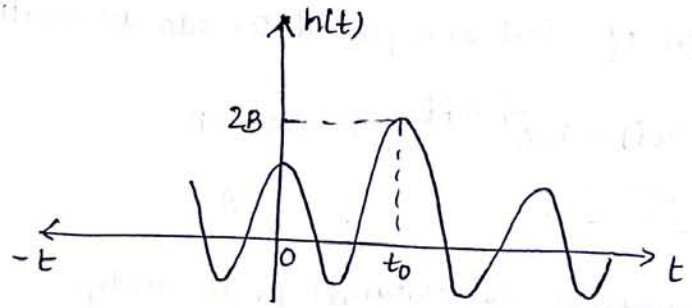
$$= \frac{1}{j2\pi(t-t_0)} \left[e^{j2\pi B(t-t_0)} - e^{-j2\pi B(t-t_0)} \right]$$

$$= \frac{1}{\pi(t-t_0)}\left[\frac{e^{j2\pi B(t-t_0)} - e^{-j2\pi B(t-t_0)}}{2j}\right]$$

$$= \frac{1}{\pi(t-t_0)} \sin\left[2\pi B(t-t_0)\right]$$

$$h(t) = 2B\left(\frac{\sin[2\pi B(t-t_0)]}{2\pi B(t-t_0)}\right) = 2B\,\text{sinc}\left[2B(t-t_0)\right]$$

Response



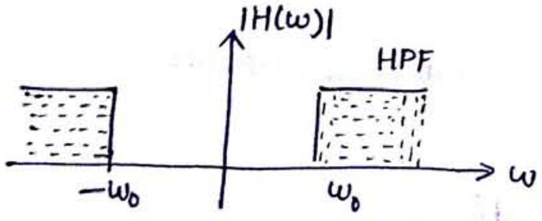
→ Figure shows that impulse response exists for negative values of 't'. But actually unit impulse is applied at t=0 always.

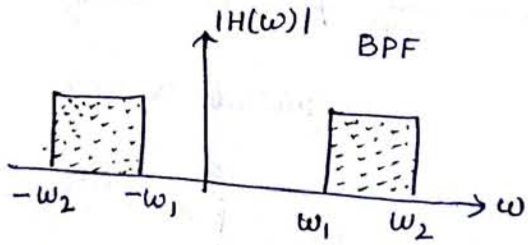
→ Practically it is impossible to implement such a system.

OTHER IDEAL FILTERS SUCH AS HPF, BPF etc.,

→ In realizability of ideal LPF its response begins before input is applied and hence it is not physically realizable.

→ $||^{ly}$ HPF, BPF ideal have frequency response as shown in figure





→ These have sharp transition in frequency response.

→ All ideal filters are physically not realizable since their impulse response is non-causal.

INTRODUCTION FOR FILTER CHARACTERISTICS:

$$f(t) \longrightarrow \boxed{h(t)} \longrightarrow r(t)$$

$F(\omega)$ $\quad$ $H(\omega)$ $\quad$ $R(\omega)$

$$R(\omega) = F(\omega).H(\omega)$$

→ The spectrum of o/p is $F(\omega).H(\omega)$ i.e the system acts as a kind of filter to various frequency components.

→ Some frequency components are boosted in strength and some are attenuated and some remain unaffected.

→ $||^{ly}$ each freq. component undergoes a different amount of phase shift i.e the modification is carried out according to $H(\omega)$.
↳ acts as waiting fn for two different frequencies.

DISTORTIONLESS TRANSMISSION THROUGH SYSTEM:

→ It means output signal is an exact replica of the i/p signal.

$$i/p \longrightarrow \boxed{\text{System}} \longrightarrow o/p$$

→ The difference between i/p and o/p of such system is that

1. Amplitude of the o/p signal may increase or decrease by some factor w.r. to i/p.
2. The o/p sgl may be delayed in time w.r. to i/p sgl because of system delay.

→ o/p sgl $y(t)$ can be written in terms of i/p $x(t)$ as

$$y(t) = k\,x(t-t_0)$$

k ↙ constant Represents change in amplitude

t_0 ↳ time delay in transmission of signal through a system.

By taking fourier transform

$$Y(f) = F[y(t)] = F\{k\,x(t-t_0)\}$$

From time shifting property of FT

$$Y(f) = k\,X(f)\,e^{-j2\pi f t_0}$$

Transfer fn $H(f) = \frac{Y(f)}{X(f)}$

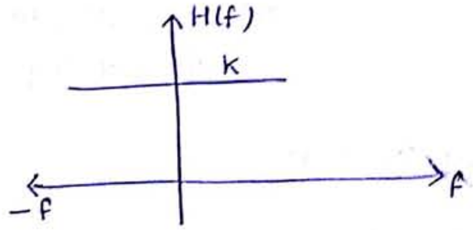
$$\therefore \quad H(f) = \frac{Y(f)}{X(f)} = k \cdot e^{-j 2\pi f t_0}.$$

Magnitude of transfer fn independent of frequency.

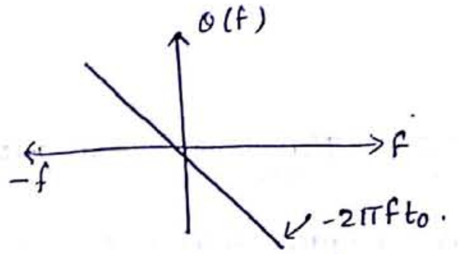
→ Transfer function has constant amplitude at all frequencies. The phase shift is

$$\theta(f) = -2\pi f t_0$$
$$= (-2\pi t_0) f.$$

→ phase shift is linearly proportional to frequency.



(a) Amplitude Spectrum



(b) phase spectrum passing through origin

→ By considering simple example

Let there be signal in time domain as

$$x(t) = \cos(2\pi f t)$$

Let the o/p sgl be same in amplitude but shifted in time by t_0 sec

$$y(t) = \cos[2\pi f (t - t_0)]$$

$$\therefore \quad y(t) = \cos(2\pi f t - 2\pi f t_0) = \cos(2\pi f t - \theta(f))$$

∴ phase shift of $y(t)$ is

$$\theta(f) = -2\pi f t_0.$$

which is proportional to frequency 'f'.

Two types :

(i) Amplitude distortion

(ii) phase distortion.

AMPLITUDE DISTORTION

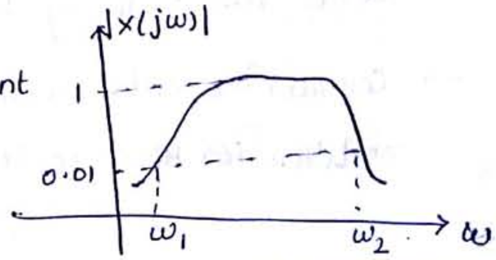
→ This distortion occurs when $|H(\omega)|$ is not constant over frequency band of interest and the frequency components present in i/p sgl are transmitted with different gain and attenuation.

PHASE DISTORTION:

→ This distortion occurs when phase of $H(\omega)$ is not linearly changing with time and different frequency components in i/p are subjected to different time delays during transmission.

SIGNAL BANDWIDTH: The band of frequencies that contains most of signal energy is called B.W of signal denoted by fm.

→ It is the range of significant signal frequencies which are present in the signal.

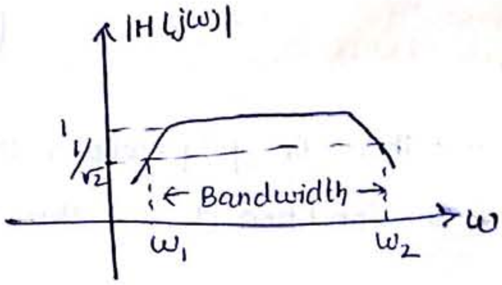


→ observe in the waveform $x(t)$ has significant frequencies from ω_1 to ω_2.

→ The B.W of this signal is $\omega_2 - \omega_1$

→ All the physically obtained signals have limited bandwidth.

SYSTEM BANDWIDTH



→ The B.W of a system is defined as range of frequencies over which $|H(\omega)|$ remains within $1/\sqrt{2}$ times of its mid-band value. for distortionless transmission the system must have infinite B.W but physical system are limited to finite BW.

→ So a system with finite BW can provide distortionless transmission for a band limited signal if $|H(\omega)|$ remains constant over BW of the signal.

→ The range of frequencies for which magnitude $|H(j\Omega)|$ of the systems remains within $\frac{1}{\sqrt{2}}$ of its maximum value

CASUAL SYSTEMS:

→ A system is said to be causal if $h(t) = 0$ if $t<0$

$h(t-t_0) = 0 \;;\; t<t_0$

i.e if i/p is zero for $t<t_0$, then o/p is also zero for $t<t_0$.

→ Any system which does not obey the above rule is non-causal system.

→ If two i/p to a causal system are equal upto some time 't_0' then corresponding o/p must we equal upto that time instant.

POLY-WIENER CRITERION

→ This gives the condition for causality in frequency domain (or) in other words the frequency domain equivalent of causal system i.e $H(\omega)$.

→ Consider a system with transfer function $H(\omega)$, the necessary and sufficient condition for $H(\omega)$ to be transfer function of causal fn is

$$\int_{-\infty}^{\infty} \frac{|Ln(H(j\omega))|}{1+\omega^2} d\omega < \infty \rightarrow (1)$$

provided $|H(j\omega)$ is square integral.

$$\int_{-\infty}^{\infty} |H(j\omega)|^2 d\omega < \infty \rightarrow (2)$$

→ This is poly-wiener criteria. If condition (2) is not satisfied then the condition (1) is neither necessary nor sufficient.

PHYSICAL REALIZABILITY:

→ A system is said to be physically realizable if it obeys the causal condition.

i.e $h(t) = 0$ for $t<0$.

Ex: $H(\omega) = \frac{1}{1+j\omega}$

$h(t) = e^{-t} u(t)$

$= 0$ for $t<0$

So the above system for transfer fn is realizable in freq. domain.

$$\int_{-\infty}^{\infty} \frac{|Ln|H(\omega)||}{H(\omega)^2} d\omega < \infty$$

→ The frequency domain statements can be interpreted as $|H(\omega)|$ of a physically realizable system may be zero for some discrete frequency but it can never be zero for a finite band of frequencies.

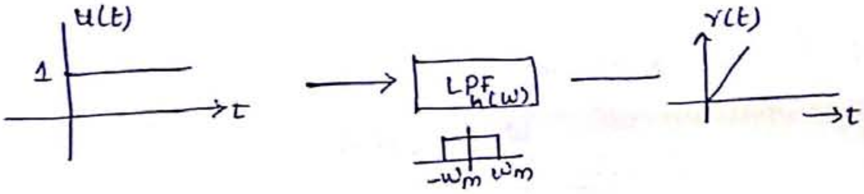
→ $H(\omega)$ for a realisable system cannot decay faster than a function of exponential order.

Ex: A system with T.F $e^{-\omega}$ is realisable whereas $e^{-\omega^2}$ is not as it decay faster.

RELATIONSHIP BETWEEN RISE TIME AND BANDWIDTH:

→ If a unit step fn u(t) is applied to an ideal LPF, the o/p will show a gradual rise instead of a sharp rise in the i/p.

→ The rise time (t_r) is the time required by the response to reach its final value from initial value.



Transfer function of ideal low pass filter is

$$H(\omega) = |H(\omega)|\, e^{j\theta(\omega)}$$

$$= G(\omega)\, e^{-j\omega t_0}$$

↓
Rectangular pulse with magnitude K.
for $-B < f \leq B$ i.e $-\omega_m \leq \omega \leq \omega_m$ where $\omega_m = 2\pi B$.

and $\theta(\omega) = -2\pi f t_0 = -\omega t_0$.

→ Fourier transform of unit step fn u(t)

$$FT\{u(t)\} \Rightarrow U(\omega) = \pi\delta(\omega) + \frac{1}{j\omega}$$

→ Fourier transform of response $R(\omega)$, input and $H(\omega)$ related as

$$R(\omega) = \left[\pi\delta(\omega) + \frac{1}{j\omega}\right] H(\omega) = \pi\delta(\omega).H(\omega) + \frac{1}{j\omega}H(\omega).$$

$\delta(\omega)$ exists only for $\omega=0$ and $H(\omega)|_{\omega=0}=1$

$$R(\omega)=\pi\delta(\omega)+\frac{1}{j\omega}H(\omega).$$

By taking IFT for above eqn

$$r(t)=\text{IFT}[R(\omega)]=\text{IFT}\left\{\pi\delta(\omega)+\frac{1}{j\omega}H(\omega)\right\}$$

$$=\text{IFT}\left\{\pi\delta(\omega)+\frac{1}{j\omega}G(\omega)e^{-j\omega t_0}\right\} \quad (\because H(\omega)=G(\omega)e^{-j\omega t_0})$$

Inverse fourier transform of $\pi\delta(\omega)$ is $\frac{1}{2}$.

$$\left\{\begin{array}{l}\because 1\rightarrow 2\pi\delta(\omega)\\ \frac{1}{2}\leftarrow\pi\delta(\omega)\end{array}\right\}$$

$$r(t)=\frac{1}{2}+\text{IFT}\left\{\frac{1}{j\omega}G(\omega)e^{-j\omega t_0}\right\}$$

$$=\frac{1}{2}+\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{j\omega}G(\omega)e^{-j\omega t_0}\cdot e^{j\omega t}d\omega$$

we know $G(\omega)=1$ for $-\omega_m\leq\omega\leq\omega_m$

$$=\frac{1}{2}+\frac{1}{2\pi}\int_{-\omega_m}^{\omega_m}\frac{e^{j\omega(t-t_0)}}{j\omega}d\omega$$

$$=\frac{1}{2}+\frac{1}{2\pi}\int_{-\omega_m}^{\omega_m}\frac{\cos\omega(t-t_0)+j\sin\omega(t-t_0)}{j\omega}d\omega$$

$$=\frac{1}{2}+\underbrace{\frac{1}{2\pi}\int_{-\omega_m}^{\omega_m}\frac{\cos\omega(t-t_0)}{j\omega}d\omega}_{=\text{zero for its odd term}}+\underbrace{\frac{1}{2\pi}\int_{-\omega_m}^{\omega_m}\frac{\sin\omega(t-t_0)}{\omega}d\omega}_{\text{even}}.$$

$$r(t)=\frac{1}{2}+\frac{1}{2\pi}\times 2\int_{0}^{\omega_m}\frac{\sin\omega(t-t_0)}{\omega}d\omega=\frac{1}{2}+\frac{1}{\pi}\int_{0}^{\omega_m}\frac{\sin\omega(t-t_0)}{\omega}d\omega$$

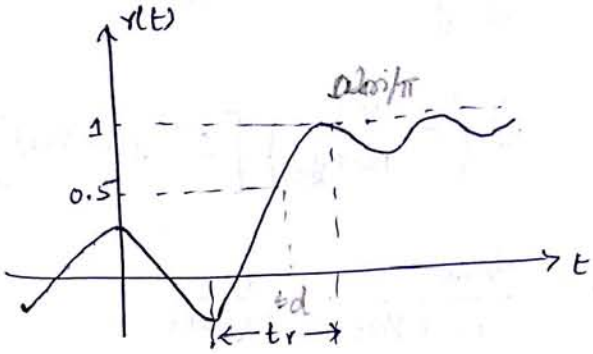
$$=\frac{1}{2}+\frac{1}{\pi}\left[\text{Si}\,\omega(t-t_0)\right]_0^{\omega_m}$$

$$=\frac{1}{2}+\frac{1}{\pi}\text{Si}\,\omega_m(t-t_0)\longrightarrow\text{sine integral}$$

$$\rightarrow\frac{dr(t)}{d}\Big|_{t=t_0}=\frac{1}{\pi}\cos[\omega_m(t-t_0)].\ \omega_m$$

The rise time is given as $t_r=\underbrace{\frac{2\pi}{\omega_m}}_{\text{cut off frequency of LPF}}=\frac{1}{B}$

$$\frac{1}{t_r}=\frac{\omega_m}{\pi}\Rightarrow t_r=\frac{\omega_m}{\pi}.$$



$w_c \to \infty$

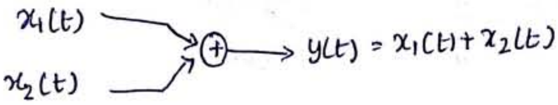
$y(t) = 1$

$w_c \to -\infty$

$y(t) = 0$

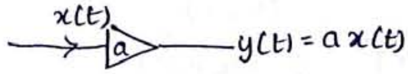
Note : { Elements of block diagram)

① Adder :



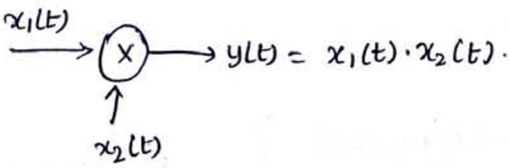
which performs the addition of two signal sequences to form sum

② constant multiplier :



It represents applying a scale factor on i/p $x(t)$.

③ Signal multiplier :



The multiplication of two signal to form product sequence.

PROBLEMS :

① The impulse response of continuous time system is given as

$$h(t) = \frac{1}{RC} e^{-t/RC} \cdot u(t)$$

Determine the frequency response & plot the magnitude phase plots .

Sol Take FT

$$H(\omega) = \int_{-\infty}^{\infty} h(t) \cdot e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} \frac{1}{RC} \cdot e^{-t/RC} \cdot u(t)\, e^{-j\omega t} dt$$

$$= \frac{1}{RC} \int_{0}^{\infty} e^{-t/RC} \cdot e^{-j\omega t} dt \quad \left(\because u(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{otherwise} \end{cases} \right)$$ step

$$= \frac{1}{RC} \int_0^{\infty} e^{-t\left(j\omega + \frac{1}{RC}\right)} dt$$

$$= \frac{1}{RC}\left(-\frac{1}{j\omega + \frac{1}{RC}}\right)\left[e^{-t\left(j\omega + 1/RC\right)}\right]_0^{\infty}$$

$$H(\omega) = \frac{1/RC}{j\omega + 1/RC} = \frac{1}{1+j\omega RC}.$$

Magnitude & phase

$$H(\omega) = \frac{1}{1+j\omega RC} \times \frac{1-j\omega RC}{1-j\omega RC} = \frac{1-j\omega RC}{1+(\omega RC)^2}$$

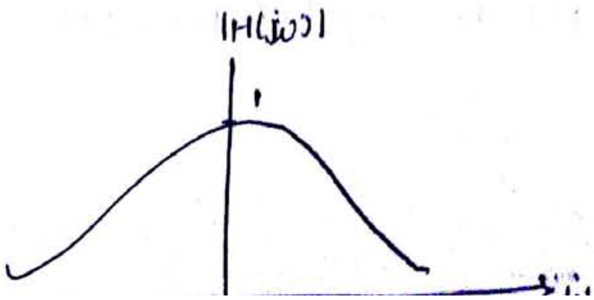
$$= \frac{1}{1+(\omega RC)^2} + j\,\frac{-\omega RC}{1+(\omega RC)^2}$$

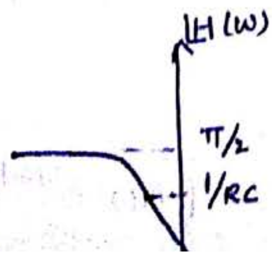
$$|H(\omega)| = \left\{\frac{1}{[1+(\omega RC)^2]^2} + \frac{(\omega RC)^2}{[1+(\omega RC)^2]^2}\right\}^{1/2}$$

$$= \frac{1}{\sqrt{1+(\omega RC)^2}}$$

$$\angle H(\omega) = \tan^{-1}\left\{\frac{(-\omega RC)/1+(\omega RC)^2}{1+[1+(\omega RC)^2]}\right\} = -\tan^{-1}(\omega RC)$$

If $RC = 1$, $|H(\omega)| = \frac{1}{\sqrt{1+\omega^2}}$; $\angle H(\omega) = -\tan^{-1}(\omega)$.





(2) For the system shown find the T.T & impulse response of the system.

$f(t) = e^{-at} \quad t>0$

$\quad 0 \quad$ elsewhere $\quad ; \; Y(\omega) = \frac{1}{\alpha + j\omega}$

Sol

$H(\omega) = \frac{R(\omega)}{F(\omega)}$

$R(\omega) = Y(\omega)$

$F(\omega) \rightarrow \boxed{} \rightarrow Y(\omega)$

$F(t) = e^{-at}$

$F(\omega) = \frac{1}{a+j\omega} \quad ; \quad Y(\omega) = \frac{1}{\alpha+j\omega}$

$H(\omega) = \frac{1/\alpha+j\omega}{1/a+j\omega} = \frac{a+j\omega}{\alpha+j\omega}$

$F^{-1}\left[\frac{a+j\omega}{\alpha+j\omega}\right] \Rightarrow \frac{a+\alpha-\alpha+j\omega}{\alpha+j\omega} = \frac{a-\alpha}{\alpha+j\omega} + \frac{\alpha+j\omega}{\alpha+j\omega}$

$= \frac{a-\alpha}{\alpha+j\omega} + 1$

$$\boxed{h(t) = (a-\alpha)\, e^{-\alpha t} u(t) + \delta(t)}$$

(3) The linear system impulse response is $[e^{-2t}+e^{-3t}]\,u(t)$ find the excitation to produce an o/p of $t \cdot e^{-2t} u(t)$?

Sol

$h(t) = [e^{-2t} + e^{-3t}]\, u(t)$

$F(\omega) \rightarrow \boxed{H(\omega)} \rightarrow R(\omega)$

$r(t) = t \cdot e^{-2t} u(t)$

$H(\omega) = \frac{R(\omega)}{F(\omega)}$

$F(\omega) = \frac{R(\omega)}{H(\omega)}$

$r(t) = t \cdot e^{-2t} u(t) \xleftrightarrow{F.T} \frac{1}{(2+j\omega)^2} \qquad \left(\because t \cdot e^{-at} u(t) \longleftrightarrow \frac{1}{(a+j\omega)^2}\right)$

$R(\omega) = \frac{1}{(2+j\omega)^2}$

$$h(t) = e^{-2t}u(t) + e^{-3t}u(t)$$

$$H(\omega) = \frac{1}{2+j\omega} + \frac{1}{3+j\omega}$$

$$R(\omega) = \frac{1/(2+j\omega)}{\frac{3+j\omega+2+j\omega}{(\cancel{2+j\omega})(3+j\omega)}} = \frac{1}{2+j\omega} \times \frac{3+j\omega}{5+2j\omega}$$

$$\frac{3+j\omega}{(2+j\omega)(5+2j\omega)} = \frac{A}{2+j\omega} + \frac{B}{5+2j\omega}$$

$$3+j\omega = A(5+2j\omega) + B(2+j\omega)$$

put $3+j\omega = 5A+2B + j\omega(2A+B)$

put $j\omega = 0$; put $j\omega(-2)$

$(3 = 5A+2B) \times 1$

$(1 = 2A+B) \times 2$

$A = 1,\ B = -1$

$$R(\omega) = \frac{1}{2+j\omega} - \frac{1}{5+2j\omega} = \frac{1}{2+j\omega} - \frac{1}{2[5/2+j\omega]}$$

$$\boxed{r(t) = e^{-2t}u(t) - \frac{1}{2}e^{-5/2\,t}u(t)}$$

DIFFERENTIAL EQUATION:

→ To obtain frequency response & impulse response.

$$\sum_{k=0}^{N} a_k \frac{d^k}{dt^k} y(t) = \sum_{k=0}^{M} b_k \frac{d^k}{dt^k} x(t).$$

differentiation property of FT is

$$\frac{d}{dt} x(t) \overset{FT}{\longleftrightarrow} j\omega X(\omega).$$

$$\sum_{K=0}^{N} a_K (j\omega)^K Y(\omega) = \sum_{K=0}^{M} b_K (j\omega)^K X(j\omega)$$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{\sum_{K=0}^{M} b_K (j\omega)^K}{\sum_{K=0}^{N} a_K (j\omega)^K}.$$

system transfer fn.

PROBLEMS:

① The differential equation of system is given as $\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = -\frac{dx(t)}{dt}$

Determine the frequency response & impulse response.

Sol

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = -\frac{dx(t)}{dt}$$

Taking F.T

$$(j\omega)^2 Y(\omega) + 5(j\omega)Y(\omega) + 6Y(\omega) = -j\omega X(\omega)$$

$$Y(\omega)\left[(j\omega)^2 + 5j\omega + 6\right] = -j\omega X(\omega)$$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{-j\omega}{(j\omega)^2 + 5j\omega + 6}$$

$$H(\omega) = \frac{-j\omega}{(j\omega+2)(j\omega+3)} = \frac{A}{j\omega+2} + \frac{B}{j\omega+3}$$

$$= \frac{2}{j\omega+2} - \frac{3}{j\omega+3}$$

$$h(t) = \left[2 \cdot e^{-2t} - 3e^{-3t}\right] u(t)$$

impulse response of the system.

$$\left\{ \because e^{-at}u(t) \xleftrightarrow{FT} \frac{1}{a+j\omega} \right\}$$

② The input voltage to the RC circuit is given by $x(t) = te^{-t/RC}u(t)$ and impulse response of this circuit is given by $h(t) = \frac{1}{RC}e^{-t/RC}u(t)$. Find output $y(t)$

Sol

output $y(t) = x(t) * h(t)$

In frequency domain

$$Y(\omega) = X(\omega)H(\omega)$$

and $H(\omega) = F[h(t)]$

$$H(\omega) = F\left\{\frac{1}{RC}e^{-t/RC}.u(t)\right\}$$

$$= \frac{1}{RC}\cdot\frac{1}{\frac{1}{RC}+j\omega} = \frac{1}{1+j\omega RC}$$

$$X(\omega) = F\left[t\cdot e^{-t/RC}u(t)\right]$$

$$= \int_{-\infty}^{\infty} t\cdot e^{-t/RC}\, e^{-j\omega t}\,dt = \frac{1}{\left(\frac{1}{RC}+j\omega\right)^2} = \frac{(RC)^2}{(1+j\omega RC)^2}$$

$$\left[\because\ te^{-at}u(t) \xleftrightarrow{FT} \left(\frac{1}{a+j\omega}\right)^2\right]$$

$$Y(\omega) = X(\omega).H(\omega)$$

$$= \frac{(RC)^2}{(1+j\omega RC)^2}\cdot\frac{1}{(1+j\omega RC)} = \frac{(RC)^2}{(1+j\omega RC)^3}$$

$$y(t) = F^{-1}\{Y(\omega)\} = F^{-1}\left\{\frac{(RC)^2}{(1+j\omega RC)^3}\right\} = F^{-1}\left\{\cancel{\frac{(RC)^2}{(RC)^3\left(\frac{1}{RC}+\right.}}\right.$$

$$\boxed{y(t) = \frac{1}{RC}\cdot\frac{t^2\cdot e^{-t/RC}}{2}u(t)}$$

1) $h(t) = e^{-5|t|}$

For stability $\int_{-\infty}^{\infty} |h(t)|\,dt < \infty$

$$\therefore \int_{-\infty}^{\infty} |h(t)|\,dt = \int_{-\infty}^{\infty} |e^{-5|t|}|\,dt = \int_{-\infty}^{\infty} e^{-5|t|}\,dt$$

$$= \int_{-\infty}^{0} e^{5t}\,dt + \int_{0}^{\infty} e^{-5t}\,dt = \left[\frac{e^{5t}}{5}\right]_{-\infty}^{0} + \left[\frac{e^{-5t}}{-5}\right]_{0}^{\infty}$$

$= \frac{2}{5}$ = constant / so system is stable.

2) $h(t) = e^{4t}u(t)$

$$= \int_{-\infty}^{\infty} |e^{4t}u(t)|\,dt = \int_{-\infty}^{\infty} e^{4t}u(t)\,dt$$

$$= \int_{0}^{\infty} e^{4t}\,dt = \frac{e^{\infty}}{4} - \frac{e^{0}}{4} = \infty - \frac{1}{4} = \infty$$ (Unstable.

3) $h(t) = e^{-4t}u(t)$ (stable)

4) $h(t) = t\cos t\,u(t)$ (unstable.

$$\int_{0}^{\infty} t\cos t\,dt$$

5) $h(t) = e^{-t}\sin t\,u(t)$ (stable.

$$= \int_{0}^{\infty} e^{-t}\sin t\,dt$$

→ The system produces the o/p of $y(t) = e^{-t}u(t)$ for an input of $x(t) = e^{-2t}u(t)$. Determine the impulse response and frequency response of the system.

Sol

$$y(t) = e^{-t}u(t)$$

$$x(t) = e^{-2t}u(t)$$

Consider standard fourier transform pair $e^{-at}u(t) \xleftrightarrow{FT} \frac{1}{a+j\omega}$

$$Y(\omega) = \frac{1}{1+j\omega} \; ; \; X(\omega) = \frac{1}{2+j\omega}.$$

From equation

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{1/1+j\omega}{1/2+j\omega} = \frac{2+j\omega}{1+j\omega}.$$

Multiply ~~and divide~~ the numerator & denominator by $1-j\omega$

$$H(\omega) = \frac{2+j\omega}{1+j\omega} \times \frac{1-j\omega}{1-j\omega} = \frac{(2+j\omega)(1-j\omega)}{(1)^2-(j\omega)^2} = \frac{2-2j\omega+j\omega+\omega^2}{1+\omega^2}$$

$$= \frac{2+\omega^2-j\omega}{1+\omega^2} \Rightarrow \frac{2+\omega^2}{1+\omega^2} + j\frac{-\omega}{1+\omega^2}.$$

Magnitude $|H(\omega)| = \left\{\sqrt{\left[\frac{2+\omega^2}{1+\omega^2}\right]^2 + \left[\frac{\omega^2}{1+\omega^2}\right]^2}\right]$

$$|H(\omega)| = \sqrt{\frac{4+\omega^2}{1+\omega^2}} \; ; \; \angle H(\omega) = \tan^{-1}\left(\frac{\frac{-\omega}{\cancel{1+\omega^2}}}{\frac{2+\omega^2}{\cancel{1+\omega^2}}}\right)$$

$$= -\tan^{-1}\left(\frac{\omega}{2+\omega^2}\right\}$$

$$\therefore H(\omega) = \frac{2+j\omega}{1+j\omega} \Rightarrow \frac{1+j\omega+1}{1+j\omega} \Rightarrow 1+\frac{1}{1+j\omega}.$$

Inverse fourier transform

$$\boxed{h(t) = IFT\{H(\omega)\} = \delta(t) + e^{-t}u(t)}$$

impulse response.

(1) The transfer function of LPF is given by

$$H(\omega) = \begin{cases} (1+k\cos\omega T)\,e^{-j\omega T} & ; \ |\omega| < 2\pi B \\ 0 & ; \ |\omega| > 2\pi B. \end{cases}$$

Determine the output $y(t)$ when a pulse $x(t)$ bandlimited in B is applied at the input.

Sol)

$$Y(\omega) = X(\omega)\,H(\omega)$$

$$= X(\omega)\left[1 + k\cos\omega T\right]e^{-j\omega \tau}$$

$$= X(\omega)\,e^{-j\omega \tau} + k\,X(\omega)\cos\omega T\,e^{-j\omega \tau}$$

we know

$$x(t-\tau) + x(t+\tau) \longleftrightarrow 2X(\omega)\cos\omega T$$

$$x(t-T) \longleftrightarrow X(\omega)\,e^{-j\omega \tau}$$

$$y(t) = F^{-1}\left[Y(\omega)\right]$$

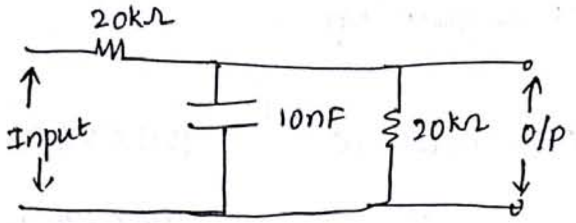
$$= F^{-1}\left[X(\omega)\,e^{-j\omega T} + k\,X(\omega)\,e^{-j\omega \tau}\cos\omega T\right]$$

$$= x(t-T) + \frac{k}{2}\left[x(t-\tau-T) + x(t-T+T)\right]$$

$$y(t) = x(t) + \frac{k}{2}\left[x(t-T) + x(t+T)\right]$$

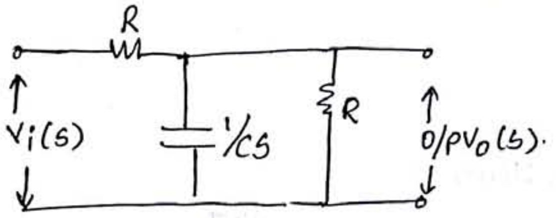
delayed by τ.

2) Determine the maximum bandwidth of signals that can be transmitted through low pass RC filter as shown in figure, if over this bandwidth, the gain variation is to be 10% and the phase variation is to be within 7% of ideal characteristics.



Sol

RC network transformed into s-domain representation.



$$H(s) = \frac{V_o(s)}{V_i(s)}$$

$$= \frac{[R \parallel (1/Cs)]}{([R \parallel (1/Cs)] + R]}$$

$$= \frac{(R/Cs) / [R + 1/Cs]}{[R/Cs] / [(R + 1/Cs)] + R}$$

$$= \frac{R/(1+sCR)}{[R/(1+sCR)] + R} = \frac{R}{R + R(1+sCR)} = \frac{\cancel{R}}{\cancel{R}(1+1+sCR)}$$

$$H(s) = \frac{1}{2+sCR}.$$

But given $R = 20k\Omega$

$C = 10nF$.

$$H(s) = \frac{1}{2+s(10\times10^{-9}\times20\times10^{3})} = \frac{10^4}{2\times10^4+2s} = \frac{1}{2+(2s/10^4)} = \frac{1}{2+s(2\times10^{-4})}$$

$$\therefore H(s) = \frac{5000}{s+10000}$$

put $s = j\omega$

$$H(w) = \frac{5000}{jw+10000}$$

$$|H(\omega)| = \frac{5000}{\sqrt{\omega^2+10000}}$$

$$\phi(w) = -\tan^{-1}\left(\frac{w}{10000}\right).$$

At $w=0$, $|H(\omega)|_{\omega=0} = \frac{5000}{10000} = 0.5$.

But there is 10% variation in gain over bandwidth B.

$$H(w) = 0.5 - 0.5\times10\% = 0.45$$

$$|H(\omega)| = \frac{5000}{\sqrt{B^2+10^8}}$$

$$B^2+10^8 = \left(\frac{5000}{0.45}\right)^2 \Rightarrow B^2 = 23.46\times10^6$$

$$B = 4.84\text{ KHZ}$$

But $B = 2\pi f$

$$f = \frac{B}{2\pi} = \frac{4.84\times10^3}{2\pi} = 770.8\text{ HZ}$$

phase at frequency, $f = 770.8$ HZ

$$\phi(\omega) = -\tan^{-1}\left(\frac{4.84}{10}\right) = -25.83\%.$$

(2) There are several possible ways of estimating an essential bandwidth of non-bandlimited signal. For a low pass signal, for example, the essential b.w may