FOURIER SERIES:

It is a mathematical tool that allows the representation of any periodic signal as the sum of harmonically related sinusoids.

Purpose:

→ Used to analyse periodic signals.
→ Harmonic constant of the signals is analyzed with the help of fourier series
→ It can be developed for continuous time as well as discrete time signals.

TYPES OF FOURIER SERIES:

(1) Trigonometric fourier series
(2) Compact trigonometric fourier series or polar fourier series
(3) Exponential Fourier series.

Trigonometric fourier series: (quadrature fourier series).

It is expressed as

$$x(t) = a_0 + \sum_{K=1}^{\infty}\left[a_K \cos(K\omega_0 t)\right] + \sum_{K=1}^{\infty} b_K \sin(K\omega_0 t)\Big] \rightarrow ①$$

where a_0, a_K, b_K are the trigometric fourier series coefficients.

The coefficients may be obtained from $x(t)$ using

$$a_0 = \frac{1}{T}\int_0^T x(t)\,dt$$

$$a_K = \frac{2}{T}\int_0^T x(t)\cos(K\omega_0 t)\,dt$$

$$b_K = \frac{2}{T}\int_0^T x(t)\sin(K\omega_0 t)\,dt.$$

where $\int$ indicates integration over one time period.

$\omega_0 = \frac{2\pi}{T}$ → where T = period of signal $x(t)$.

Compact Trigonometric or polar fourier series:

There are two ways in representation in polar form.

Case 1: Assume $a_n = c_n \cos(\theta_n)$ and $b_n = -c_n \sin\theta_n$ where c_n and θ_n are related to a_n and b_n as

$$c_0 = a_0 \quad \text{and} \quad c_n = \sqrt{a_n^2 + b_n^2} \quad \text{for } n \geq 1$$

$$\theta_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right)$$

Substituting $a_n = c_n \cos(\theta_n)$ and $b_n = -c_n \sin(\theta_n)$ in eqn ① gives

$$x(t) = a_0 + \sum_{n=1}^{\infty}\left[c_n \cos(\theta_n)\cos(n\omega_0 t) - c_n \sin\theta_n \sin(n\omega_0 t)\right]$$

$$= a_0 + \sum_{n=1}^{\infty}\left[c_n \cos(n\omega_0 t + \theta_n)\right]$$

$$x(t) = c_0 + \sum_{n=1}^{\infty} c_n \cos(n\omega_0 t + \theta_n)$$

↳ Amplitude & ↳ phase of n^{th} harmonic.

Case 2: Assume $a_n = c_n \sin\phi_n$ and $b_n = c_n \cos\phi_n$ where c_n and ϕ_n are related to a_n and b_n as

$$c_0 = a_0 \quad \text{and} \quad c_n = \sqrt{a_n^2 + b_n^2}, \quad \text{for } n \geq 1$$

$$\phi_n = \tan^{-1}\left(\frac{a_n}{b_n}\right)$$

Substituting $a_n = c_n \sin(\phi_n)$ and $b_n = c_n \cos(\phi_n)$ in eq ① gives

$$x(t) = a_0 + \sum_{n=1}^{\infty}\left[c_n \sin(\phi_n)\cos(n\omega_0 t) + c_n \cos(\phi_n)\sin(n\omega_0 t)\right]$$

$$= a_0 + \sum_{n=1}^{\infty} c_n \sin(n\omega_0 t + \phi_n)$$

$$x(t) = c_0 + \sum_{n=1}^{\infty} c_n \sin(n\omega_0 t + \phi_n).$$

Exponential Fourier Series:

It is expressed as

$$x(t) = \sum_{n=-\infty}^{\infty} x_n e^{jn\omega_0 t} \quad \text{(synthesis eqn)}$$

$$\text{where } x_n = \frac{1}{T}\int_0^T x(t)\, e^{-jn\omega_0 t}\, dt. \quad \text{(analysis eqn).}$$

→ Fourier series coefficients are known as frequency domain representation of $x(t)$

→ $X(n)$ or X_n are called

→ $x(t)$ and $X(n)$ are represented by fourier series (FS) pair as

$$x(t) \xleftrightarrow{FS} X(k) \text{ or } X_n.$$

Note:

(i) $\int_0^T \sin(m\omega_0 t)\,dt = 0$, for all m and $\int_0^T \cos(n\omega_0 t)\,dt = 0$ for all $n \neq 0$ → ②

because average value of a sinusoid over m and n complete cycles in the period T is zero.

(ii) $\int_0^T \sin(m\omega_0 t)\cos(n\omega_0 t)\,dt = 0$ for all m, n, → ③

$$\int_0^T \sin(m\omega_0 t)\sin(n\omega_0 t)\,dt = \begin{cases} 0, & m \neq n \\ \frac{T}{2}, & m = n \end{cases} \rightarrow ④$$

$$\int_0^T \cos(m\omega_0 t)\cos(n\omega_0 t)\,dt = \begin{cases} 0, & m \neq n \\ \frac{T}{2}, & m = n \end{cases} \rightarrow ⑤$$

Proof to get the coefficients:

(i) $a_0 = \frac{1}{T}\int_0^T x(t)\,dt$

proof From eq (1), we can say that

$$x(t) = a_0 + [a_1\cos(\omega_0 t) + b_1\sin(\omega_0 t)] + [a_2\cos(2\omega_0 t) + b_2\sin(2\omega_0 t)] + \cdots$$
$$\cdots + [a_n\cos(n\omega_0 t) + b_n\sin(n\omega_0 t)] + \cdots$$

Integrating above eqn over the interval of time 0 to T gives

$$\int_0^T x(t)\,dt = \int_0^T a_0\,dt + \int_0^T a_1\cos(\omega_0 t)\,dt + \int_0^T b_1\sin(\omega_0 t)\,dt + \int_0^T a_2\cos 2\omega_0 t\,dt$$
$$+ \int_0^T b_2\sin 2\omega_0 t\,dt + \cdots + \int_0^T a_n\cos(n\omega_0 t)\,dt + \int_0^T b_n\sin(n\omega_0 t)\,dt + \cdots$$

Using eq (2) we know that all terms on RHS of above eqn are found to have zero value except the first term.

$$\int_0^T x(t)\,dt = \int_0^T a_0\,dt + 0 + 0 + \cdots$$

$$\int_0^T x(t)\,dt = a_0 T$$

$$\boxed{a_0 = \frac{1}{T}\int_0^T x(t)\,dt}$$ → Average value of $x(t)$ over a period or dc value of the signal.

proof 2: $a_n = \frac{2}{T}\int_0^T x(t)\cos(n\omega_0 t)\,dt$

From eq (1), we have

$$x(t) = a_0 + [a_1\cos(\omega_0 t) + b_1\sin(\omega_0 t)] + a_2[\cos(2\omega_0 t) + b_2(\sin(2\omega_0 t)] + \cdots [a_n\cos(n\omega_0 t) + b_n\sin(n\omega_0 t)]]$$

By multiplying both sides by $\cos n\omega_0 t$ and integrating over the interval of time 0 to T, we get

$$\int_0^T x(t)\cos(n\omega_0 t)\,dt = \int_0^T a_0\cos(n\omega_0 t)\,dt + \int_0^T a_1\cos\omega_0 t\cos(n\omega_0 t)\,dt + \int_0^T b_1\sin(\omega_0 t)\cos(n\omega_0 t)\,dt + \int_0^T a_2\cos(2\omega_0 t)\cos(n\omega_0 t)\,dt + \cdots + \int_0^T a_n\cos^2(n\omega_0 t)\,dt + \int_0^T b_n\sin(n\omega_0 t)\cos(n\omega_0 t)\,dt + \cdots$$

Using (4), (5), (6) eqns all the terms on RHS is having zero value except the integration of $\cos^2(n\omega_0)\,dt$ which has value $\frac{T}{2}$.

$$\int_0^T x(t)\cos(n\omega_0 t)\,dt = a_n \cdot \frac{T}{2}$$

$$\boxed{a_n = \frac{2}{T}\int_0^T x(t)\cos(n\omega_0 t)\,dt}$$

(Alternate). 1

a_0 : $x(t)\,dt = a_0 + \sum_{k=1}^{\infty} a_k \cos k\omega_0 t + \sum_{k=1}^{\infty} b_k \sin k\omega_0 t$

$$\rightarrow \int_0^T x(t)\,dt = \int_0^T a_0\,dt + \int_0^T \sum_{k=1}^{\infty} a_k \cos k\omega_0 t\,dt + \int_0^T \sum_{k=1}^{\infty} b_k \sin k\omega_0 t\,dt$$

$$= a_0 [t]_0^T + \sum_{k=1}^{\infty} a_k \int_0^T \cos k\omega_0 t\,dt + \sum_{k=1}^{\infty} b_k \int_0^T \sin k\omega_0 t\,dt$$

$$= a_0[T] + \sum_{k=1}^{\infty} a_k \left[\frac{\sin k\omega_0 t}{k\omega_0}\right]_0^T + \sum_{k=1}^{\infty} b_k \left[\frac{-\cos k\omega_0 t}{k\omega_0}\right]_0^T$$

$$= a_0 \cdot T + \sum_{k=1}^{\infty} a_k \left[\frac{\sin k\omega_0 T}{k\omega_0}\right] + \sum_{k=1}^{\infty} b_k \left[\frac{-\cos k\omega_0 T}{k\omega_0} + \frac{\cos 0}{k\omega_0}\right]$$

$$= a_0 \cdot T + \sum_{k=1}^{\infty} a_k \left[\frac{\sin k \cdot \frac{2\pi}{\cancel{T}} \cdot \cancel{T}}{k \frac{2\pi}{T}}\right] + \sum_{k=1}^{\infty} b_k \left[\frac{-\cos k \frac{2\pi}{\cancel{T}} \cdot \cancel{T}}{k \frac{2\pi}{T}} + \frac{1}{k \cdot \frac{2\pi}{T}}\right]$$

$$= a_0 T + \sum_{k=1}^{\infty} a_k T \left[\frac{\sin k2\pi}{k2\pi}\right] + \sum_{k=1}^{\infty} b_k T \left[\frac{-\cos k2\pi}{k2\pi} + \frac{1}{k2\pi}\right]$$

$$= a_0 T + \sum_{k=1}^{\infty} a_k \cdot T(0) + \sum_{k=1}^{\infty} b_k \cdot T \left[\frac{-1}{k2\pi} + \frac{1}{k2\pi}\right] \qquad \left[\because \sin k2\pi = 0,\ \cos k2\pi = 1 \text{ for integer } k\right]$$

$$\int_0^T x(t)\,dt = a_0 T$$

$$\boxed{\therefore\ a_0 = \frac{1}{T} \int_0^T x(t)\,dt}$$

Evaluation of a_n:

$$a_n = \frac{2}{T} \int_{-T/2}^{T/2} x(t) \cos k\omega_0 t\,dt \ \text{(or)}\ a_n = \frac{2}{T} \int_0^T x(t) \cos k\omega_0 t\,dt.$$

proof:

$$x(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos k\omega_0 t + \sum_{k=1}^{\infty} b_k \sin k\omega_0 t.$$

$$= a_0 + a_1 \cos w_0 t + a_2 \cos 2w_0 t + \cdots + a_n \cos n w_0 t + \cdots$$
$$b_1 \sin w_0 t + b_2 \sin 2w_0 t + \cdots + b_n \sin n w_0 t + \cdots$$

Let us multiply the above eqn by $\cos n w_0 t$

$$\therefore x(t) \cos n w_0 t = a_0 \cos n w_0 t + a_1 \cos w_0 t \cos n w_0 t + a_2 \cos 2 w_0 t \cos n w_0 t + \cdots$$
$$\cdots + a_n \cos^2 n w_0 t + \cdots + b_1 \sin w_0 t \cos n w_0 t + b_2 \sin 2 w_0 t \cos n w_0 t$$
$$+ \cdots + b_n \sin n w_0 t \cdot \cos n w_0 t + \cdots$$

Let us integrate the above eqn between limits 0 to T

$$\therefore \int_0^T x(t) \cos n w_0 t \, dt = \int_0^T a_0 \cos n w_0 t \, dt + \int_0^T a_1 \cos w_0 t \cos n w_0 t \, dt + \cdots$$

$\cdots\cdots$

{ ∵ all definite integrals will be zero }

$$\therefore \int_0^T x(t) \cos n w_0 t \, dt = \int_0^T a_n \cos^2 n w_0 t \, dt = a_n \int_0^T \frac{1 + \cos 2 n w_0 t}{2} dt$$

$$= \frac{a_n}{2} \int_0^T 1 + \cos 2 n w_0 t \, dt$$

$$= \frac{a_n}{2} \left[t + \frac{\sin 2 n w_0 t}{2 w_0 n} \right]_0^T$$

$$= \frac{a_n}{2} \left[T + \frac{\sin 2 n w_0 T}{2 w_0 n} - 0 - \frac{\sin 0}{2 w_0 n} \right] = \frac{a_n}{2} \left[T + \frac{\sin 2n \cdot \frac{2\pi}{T} \cdot T}{2 \cdot \frac{2\pi}{T} \cdot n} \right]$$

$$= \frac{a_n}{2} [T]$$

$$a_n = \frac{2}{T} \int_0^T x(t) \cos n w_0 t \, dt \quad \{ \text{gives the } n^{th} \text{ coefficient of } a_n \}$$

Hence then K^{th} coefficient a_K is given by

$$a_K = \frac{2}{T} \int_0^T x(t) \cos K \Omega_0 t$$

b_n: $= \frac{2}{T}\int_{-T/2}^{T/2} x(t)\sin k\omega_0 t\,dt$ (or) $b_n = \frac{2}{T}\int_0^T x(t)\sin k\omega_0 t\,dt$

proof

$$x(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos k\omega_0 t + \sum_{k=1}^{\infty} b_k \sin k\omega_0 t$$

$$= a_0 + a_1\cos\omega_0 t + a_2\cos 2\omega_0 t + \cdots + a_n\cos n\omega_0 t + \cdots + b_1\sin\omega_0 t$$

$$+ b_2\sin 2\omega_0 t + \cdots b_n\sin n\omega_0 t + \cdots$$

Multiply with eq by $\sin n\omega_0 t$

$$x(t)\sin n\omega_0 t = a_0\sin n\omega_0 t + a_1\cos\omega_0 t\sin n\omega_0 t + a_2\sin n\omega_0 t\cos 2\omega_0 t$$

$$+ \cdots + a_n\cos n\omega_0 t\sin n\omega_0 t + \cdots + b_1\sin\omega_0 t\sin n\omega_0 t$$

$$+ b_2\sin 2\omega_0 t\sin n\omega_0 t + \cdots + b_n\sin^2 n\omega_0 t + \cdots$$

Integratio the above eqn between limits 0 to T

$$\int_0^T x(t)\sin n\omega_0 t = \int_0^T a_0\sin n\omega_0 t + \int_0^T a_1\cos\omega_0 t\sin n\omega_0 t + \cdots$$

$$\int_0^T x(t)\sin n\omega_0 t = \int_0^T b_n \sin^2 n\omega_0 t$$

$$= \int_0^T b_n \cdot \frac{1-\cos 2n\omega_0 t}{2}\,dt$$

$$= \frac{b_n}{2}\int_0^T [1-\cos 2n\omega_0 t]\,dt$$

$$= \frac{b_n}{2}\left\{[t]_0^T - \left[\frac{\sin 2n\omega_0 t}{2n\omega_0}\right]_0^T\right.$$

$$= \frac{b_n}{2}\left[T - \frac{\sin 2n\omega_0 \cdot T}{2n\omega_0}\right]$$

$$= \frac{b_n}{2}\left[T - \frac{\sin 2n\cdot\frac{2\pi}{\cancel{T}}\cdot\cancel{T}}{2n\omega_0}\right] = \frac{T}{2}\cdot b_n$$

$$\therefore b_n = \frac{2}{T}\int_0^T x(t)\sin n\omega_0 t\, dt$$

gives n^{th} coefficient b_n. Hence the K^{th} coefficient b_K is given by

$$\boxed{b_K = \frac{2}{T}\int_0^T x(t)\sin K\omega_0 t}\ .$$

Complex Fourier Exponential series: (Alternate):

$$x(t) = a_0 + \sum_{n=1}^{\infty}\left[a_n \cos n\omega_0 t + b_n \sin n\omega_0 t\right] \rightarrow (1)$$

we know Euler's identity

$$e^{j\theta} = \cos\theta + j\sin\theta \rightarrow (2) \quad \text{and} \quad e^{-j\theta} = \cos\theta - j\sin\theta \rightarrow (3)$$

Adding 2 & (3)

$$e^{j\theta} + e^{-j\theta} = 2\cos\theta$$

$$\Rightarrow \cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

Subtracting (2) & (3)

$$e^{j\theta} - e^{-j\theta} = 2j\sin\theta$$

$$\Rightarrow \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

So $$\cos(n\omega_0 t) = \frac{e^{jn\omega_0 t} + e^{-jn\omega_0 t}}{2}$$

$$\sin(n\omega_0 t) = \frac{e^{jn\omega_0 t} - e^{-jn\omega_0 t}}{2}$$

Substituting the values of $\cos n\omega_0 t$ and $\sin n\omega_0 t$

$$x(t) = a_0 + \sum_{n=1}^{\infty}\left\{a_n\left[\frac{e^{jn\omega_0 t} + e^{-jn\omega_0 t}}{2}\right] + b_n\left[\frac{e^{jn\omega_0 t} + e^{-jn\omega_0 t}}{2j}\right]\right\}$$

$$x(t) = a_0 + \sum_{n=1}^{\infty}\left[\frac{(a_n - jb_n)e^{jn\omega_0 t}}{2} + \frac{(a_n + jb_n)e^{-jn\omega_0 t}}{2}\right]$$

Let $C_n = \frac{1}{2}(a_n - jb_n)$

Complex conjate of C_n $\quad C_{-n} = \frac{1}{2}(a_n + jb_n)$

$$C_0 = a_0$$

$$\therefore\ x(t) = C_0 + \sum_{n=1}^{\infty} C_n \cdot e^{jn\omega_0 t} + \sum_{n=1}^{\infty} C_{-n} e^{-jn\omega_0 t}$$

$$x(t) = C_0 + \sum_{n=1}^{\infty} C_n \cdot e^{jn\omega_0 t} + \sum_{n=-\infty}^{-1} C_n e^{jn\omega_0 t}$$

$$= \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}.$$

$\longrightarrow$ Substitute in eq. (1)

$$\therefore\ C_n = \frac{1}{2}(a_n - jb_n) = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\left[\cos n\omega_0 t - j\sin n\omega_0 t\right] dt$$

(or) $$C_n = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-jn\omega_0 t}\, dt$$

$\|^{ly}$ $$C_{-n} = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{jn\omega_0 t}\, dt.$$

Proof 3: $b_n = \frac{2}{T}\int_0^T x(t)\sin(n\omega_0 t)\,dt$

From eq(1), we have

$$x(t) = a_0 + [a_1\cos(\omega_0 t) + b_1\sin(\omega_0 t)] + [a_2\cos(2\omega_0 t) + b_2\sin(2\omega_0 t)] + \cdots$$
$$+ [a_n\cos(n\omega_0 t) + b_n\sin(n\omega_0 t)] + \cdots$$

By multiplying both sides by $\sin(n\omega_0 t)$ and integrating over interval 0 to T, we get

$$\int_0^T x(t)\sin(n\omega_0 t)\,dt = \int_0^T a_0\sin(n\omega_0 t)\,dt + \int_0^T a_1\sin(\omega_0 t)\sin(n\omega_0 t)\,dt$$
$$+ \int_0^T b_1\sin(\omega_0 t)\sin(n\omega_0 t)\,dt + \cdots + \int_0^T a_n\cos(n\omega_0 t)\sin(n\omega_0 t)\,dt +$$
$$\int_0^T b_n\sin^2(n\omega_0 t)\,dt + \cdots$$

Using the eqns (2), (3), (4), (5), we get

$$\int_0^T x(t)\sin(n\omega_0 t)\,dt = b_n\cdot\frac{T}{2}$$

$$\boxed{b_n = \frac{2}{T}\int_0^T x(t)\sin(n\omega_0 t)\,dt}$$

Symmetry Conditions:

→ Signal can be represented as a sum of even and odd functions.

$$x(t) = x_e(t) + x_o(t)$$

$x_o(t)$ → odd function: $[x(t) - x(-t)] = x_o(t)$.

$x_e(t)$ → Even function: $x_e(t) = \frac{1}{2}[x(t) + x(-t)]$

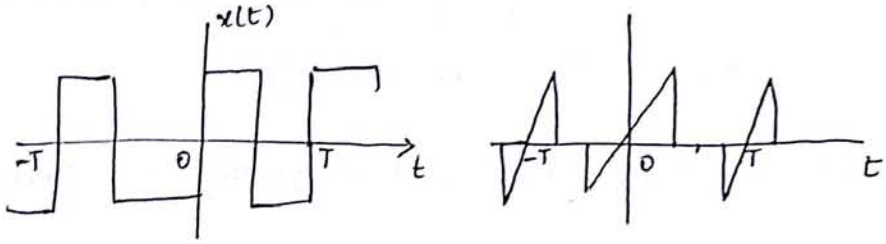
→ odd Functions have only sine terms:

Before we go through the concept, first lets know $x(t)$ is an odd signal i.e $x(t) = -x(-t)$.

Eg: $x(t) = t + t^3 + t^5$ is odd signal since the value of the signal for t and $-t$ are of opposite sign. $x(-t) = -t - t^3 - t^5 = -(t + t^3 + t^5) = -x(t)$

→ The sum of two or more odd signals is odd signal but addition of constant removes the odd nature.

→ The product of two odd signal is an even signal.



→ If $x(t)$ has odd symmetry, then fourier series coefficients a_0, a_n and b_n are defined as $a_0 = 0,\ a_n = 0,\ b_n = \frac{4}{T}\int_0^{T/2} x(t)\sin(n\omega_0 t)\,dt.$

Proof: From eqn $a_0 = \frac{1}{T}\int_0^T x(t)\,dt$

$$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\,dt$$

$$= \frac{1}{T}\left[\int_{-T/2}^{0} x(t)\,dt + \int_0^{T/2} x(t)\,dt\right] = \frac{1}{T}\left[\int_0^{T/2} x(-t)\,dt + \int_0^{T/2} x(t)\,dt\right]$$

Since $x(t)$ is odd signal, $x(t) = -x(-t)$

$$a_0 = \frac{1}{T}\left[-\int_0^{T/2} x(t)\,dt + \int_0^{T/2} x(t)\,dt\right] = 0$$

From eqn $a_n = \frac{2}{T}\int_{-T/2}^{T/2} x(t)\cos(n\omega_0 t)\,dt$

$$= \frac{2}{T}\left(\int_{-T/2}^{0} x(t)\cos(n\omega_0 t)\,dt + \int_0^{T/2} x(t)\cos(n\omega_0 t)\,dt\right.$$

$$= \frac{2}{T}\left[\int_0^{T/2} x(-t)\cos(n\omega_0 t)\,dt + \int_0^{T/2} x(t)\cos(n\omega_0 t)\,dt\right]$$

∴ $x(t)$ is odd signal and $\cos(n\omega_0 t)$ is even signal. The product of odd and even results in odd signal.

$$a_n = \frac{2}{T}\left(-\int_0^{T/2} x(t)\cos(n\omega_0 t)\,dt + \int_0^{T/2} x(t)\cos(n\omega_0 t)\,dt\right) = 0$$

Proof 3: $b_n = \frac{2}{T}\int_{-T/2}^{T/2} \sin(n\omega_0 t)\cdot x(t)\,dt$

$$= \frac{2}{T}\left(\int_{-T/2}^{0} x(t)\sin(n\omega_0 t)\,dt + \int_0^{T/2} x(t)\sin(n\omega_0 t)\,dt\right]$$

$$= \frac{2}{T}\left(\int_0^{T/2} x(-t)\sin(-n\omega_0 t)\,dt + \int_0^{T/2} x(t)\sin(n\omega_0 t)\,dt\right]$$

Both $x(t)$ & $\sin(n\omega_0 t)$ are odd signals. The product of odd signals is even signal

$$b_n = \frac{2}{T}\left(\int_0^{T/2} x(t)\sin(n\omega_0 t)\,dt + \int_0^{T/2} x(t)\sin(n\omega_0 t)\,dt\right) = \frac{4}{T}\int_0^{T/2} x(t)\sin n\omega_0 t\,dt.$$

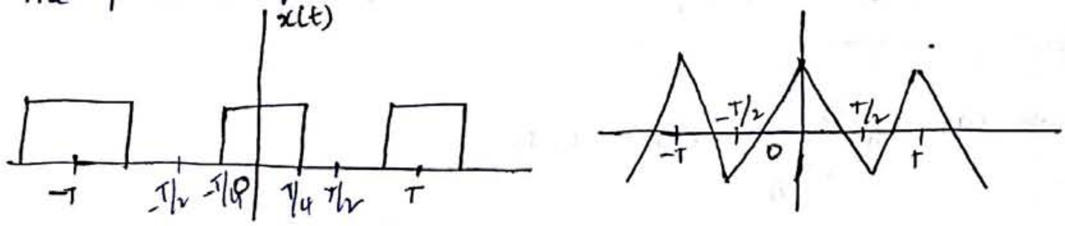
(ii) Even Function contains only cosine terms:

→ $x(t)$ is an even signal i.e $x(t) = x(-t)$

→ $x(t) = 2 + t^2 + t^4$ is an example of even signal because $x(-t) = 2 + t + t^4 = x(t)$.

→ The sum of or product of two or more even signals is an even signal.

→ The product of odd and even results in odd signal.



→ If $x(t)$ has even symmetry, then fourier series coefficients a_0, a_n, b_n are defined as (interval from $-T/2, T/2$)

$$a_0 = \frac{2}{T}\int_0^{T/2} x(t)\,dt$$

$$b_n = 0$$

$$a_n = \frac{4}{T}\int_0^{T/2} x(t)\cos(n\omega_0 t)\,dt.$$

Proof: From eqn $a_0 = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\,dt$

$$= \frac{1}{T}\left(\int_{-T/2}^{0} x(t)\,dt + \int_{0}^{T/2} x(t)\,dt\right) = \frac{1}{T}\left[\int_{0}^{T/2} x(-t)\,dt + \int_{0}^{T/2} x(t)\,dt\right]$$

$$= \frac{2}{T}\int_{0}^{T/2} x(t)\,dt \quad \left(\because \text{Since } x(t) \text{ is even sgl } x(t) = x(-t)\right]$$

(b) From eqn $a_n = \frac{2}{T}\int_{-T/2}^{T/2} x(t)\cos(n\omega_0 t)\,dt$

$$= \frac{2}{T}\left[\int_{-T/2}^{0} x(t)\cos(n\omega_0 t)\,dt + \int_{0}^{T/2} x(t)\cos(n\omega_0 t)\,dt\right]$$

$$= \frac{2}{T}\left[\int_{0}^{T/2} x(-t)\cos(-n\omega_0 t)\,dt + \int_{0}^{T/2} x(t)\cos(n\omega_0 t)\,dt\right]$$

Both $x(t)$ & $\cos(n\omega_0 t)$ are even signals

$$a_n = \frac{2}{T}\left[\int_{0}^{T/2} x(t)\cos(n\omega_0 t)\,dt + \int_{0}^{T/2} x(t)\cos(n\omega_0 t)\,dt\right]$$

$$\boxed{a_n = \frac{4}{T}\left[\int_{0}^{T/2} x(t)\cos(n\omega_0 t)\,dt\right]}$$

(c) From eqn $b_n = \frac{2}{T}\int_{-T/2}^{T/2} x(t)\sin(n\omega_0 t)\,dt$

$$= \frac{2}{T}\left(\int_{-T/2}^{0} x(t)\sin(n\omega_0 t)\,dt + \int_{0}^{T/2} x(t)\sin(n\omega_0 t)\,dt\right)$$

$$= \frac{2}{T}\left(\int_{0}^{T/2} x(-t)\sin(-n\omega_0 t)\,dt + \int_{0}^{T/2} x(t)\sin(n\omega_0 t)\,dt\right)$$

Since $x(t)$ is ~~odd~~ even signal & ~~sin~~ $\sin(n\omega_0 t)$ ~~are~~ odd signals. The product of odd & even results in odd signal

$$= \frac{2}{T}\left(-\int_{0}^{T/2} x(t)\sin(n\omega_0 t)\,dt + \int_{0}^{T/2} x(t)\sin(n\omega_0 t)\,dt\right)$$

$$\boxed{b_n = 0}$$

Half wave symmetry i.e $x(t) = -x\left(t \pm \frac{T}{2}\right)$

If $x(t)$ has a half wave symmetry, the fourier series coefficients a_0, a_n & b_n are defined as

$$a_0 = 0,\ b_n = a_n = 0 \text{ for n even}$$

$$a_n = \frac{4}{T}\int_0^{T/2} x(t)\cos(n\omega_0 t)\,dt \quad \text{'n' odd}$$

$$b_n = \frac{4}{T}\int_0^{T/2} x(t)\sin(n\omega_0 t)\,dt \quad \text{'n' odd.}$$

It contains only odd harmonics.

proof: From eqn $a_0 = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\,dt$

$$= \frac{1}{T}\left(\int_{-T/2}^{0} x(t)\,dt + \int_0^{T/2} x(t)\,dt\right) = \frac{1}{T}\left[\int_0^{T/2} x(t - T/2)\,dt + \int_0^{T/2} x(t)\,dt\right)$$

$x(t)$ has half symmetry $-x(t) = x\left(t - \frac{T}{2}\right)$

$$a_0 = \frac{1}{T}\left[-\int_0^{T/2} x(t)\,dt + \int_0^{T/2} x(t)\,dt\right] = 0$$

(b) From eqn

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} x(t)\cos(n\omega_0 t)\,dt$$

$$= \frac{2}{T}\left(\int_{-T/2}^{0} x(t)\cos(n\omega_0 t)\,dt + \int_0^{T/2} x(t)\cos(n\omega_0 t)\,dt\right]$$

$$\cancel{= \frac{2}{T}\left(\int_0^{T/2} x(t)\cos(n\omega_0(t - T/2)\right.}$$

$$= \frac{2}{T}\left[\int_0^{T/2} x(t - T/2)\cos(n\omega_0(t - T/2))\,dt + \int_0^{T/2} x(t)\cos n\omega_0 t\,dt\right]$$

$$= \frac{2}{T}\left(-\int_0^{T/2} x(t)\cos(n\omega_0(t - T/2))\,dt + \int_0^{T/2} x(t)\cos n\omega_0 t)\,dt\right]$$

Since $x(t)$ is half wave symmetry, $-x(t) = x(t - T/2)$

$$\cos(n\omega_0(t - T/2)) = \cos(n\omega_0 t - n\omega_0 T/2) = \cos\left(n\omega_0 t - n\cdot\frac{2\pi}{T}\cdot\frac{T}{2}\right)$$

$$= \cos(n\omega_0 t - n\pi)$$

$$\begin{cases} \cos(n\omega_0 t), & n \text{ even} \\ -\cos(n\omega_0 t), & n \text{ odd.} \end{cases}$$

$$a_n = \begin{cases} \frac{2}{T}\left(-\int_0^{T/2} x(t)\cos(n\omega_0 t)\,dt + \int_0^{T/2} x(t)\cos(n\omega_0 t)\,dt\right), & n \text{ even} \\ \frac{2}{T}\left(\int_0^{T/2} x(t)\cos(n\omega_0 t)\,dt + \int_0^{T/2} x(t)\cos(n\omega_0 t)\,dt\right), & n \text{ odd} \end{cases}$$

$$a_n = \begin{cases} 0 & , n \text{ even.} \\ \frac{4}{T}\int_0^{T/2} x(t)\cos(n\omega_0 t)\,dt, & n \text{ odd} \end{cases}$$

(c) From eqn $b_n = \frac{2}{T}\int_{-T/2}^{T/2} x(t)\sin(n\omega_0 t)\,dt$

$$= \frac{2}{T}\left[\int_{-T/2}^{0} x(t)\sin(n\omega_0 t)\,dt + \int_0^{T/2} x(t)\sin(n\omega_0 t)\,dt\right]$$

$$= \frac{2}{T}\left[\int_0^{T/2} x(t-T/2)\sin(n\omega_0(t-T/2))\,dt + \int_0^{T/2} x(t)\sin(n\omega_0 t)\,dt\right]$$

$$= \frac{2}{T}\left[-\int_0^{T/2} x(t)\sin(n\omega_0(t-T/2))\,dt + \int_0^{T/2} x(t)\sin(n\omega_0 t)\,dt\right)$$

Since $x(t)$ is half wave symmetry, $-x(t) = x(t-T/2)$

$$\sin(n\omega_0(t-T/2)) = \sin\left(n\omega_0 t - n\omega_0\tfrac{T}{2}\right) = \sin\left(n\omega_0 t - n\cdot\tfrac{2\pi}{T}\cdot\tfrac{T}{2}\right)$$

$$= \sin(n\omega_0 t - n\pi)$$

$$= \begin{cases} \sin n\omega_0 t & , n \text{ even} \\ -\sin(n\omega_0 t), & n \text{ odd} \end{cases}$$

We have

$$b_n = \begin{cases} \frac{2}{T}\left(-\int_0^{T/2} x(t)\sin(n\omega_0 t)\,dt + \int_0^{T/2} x(t)\sin(n\omega_0 t)\,dt\right), & n \text{ even} \\ \frac{2}{T}\left(\int_0^{T/2} x(t)\sin(n\omega_0 t)\,dt + \int_0^{T/2} x(t)\sin(n\omega_0 t)\,dt\right), & n \text{ odd} \end{cases}$$

$$b_n = \begin{cases} 0 & , n \text{ even.} \\ \frac{4}{T}\int_0^{T/2} x(t)\sin(n\omega_0 t)\,dt, & n \text{ odd} \end{cases}$$

Properties of fourier Series:

① Linearity: If $x(t) \xleftrightarrow{FS} x(n)$ and $y(t) \xleftrightarrow{FS} Y(n)$ then

$$z(t) = a\,x(t) + b\,y(t) \xleftrightarrow{FS} z(n) = a\,x(n) + b\,Y(n)$$

Proof: From exponential fourier series coefficient of $z(t)$ is

$$z(n) = \frac{1}{T}\int_0^T z(t)\cdot e^{-jnw_0t}\,dt$$

$$= \frac{1}{T}\int_0^T \left[a\,x(t) + b\,y(t)\right] e^{-jnw_0t}\,dt$$

$$= \frac{a}{T}\left[\int_0^T x(t) + b.\frac{1}{T}\int_0^T y(t)\right] e^{-jnw_0t}\,dt$$

$$= \frac{a}{T}\int_0^T x(t)e^{-jnw_0t}\,dt + \frac{b}{T}\left[\int_0^T y(t)e^{-jnw_0t}\,dt\right]$$

$$\boxed{z(n) = a\,x(n) + b\,y(n)}$$

→ This property is used to analyze signals which are represented as linear combination of other signals.

(2) Time Shifting or Translation:

If $x(t) \xleftrightarrow{FS} x(n)$ then $z(t) = x(t-t_0) \xleftrightarrow{FS} Z(n) = e^{-jn w_0 t_0}\, x(n)$

proof Fourier coefficients of $x(t-t_0)$ will be $\quad z(n) = \frac{1}{T}\int_0^T z(t)\cdot e^{-jnw_0t}\,dt$

$$z(n) = \frac{1}{T}\int_0^T x(t-t_0)\,e^{-jnw_0t}\,dt$$

put $t - t_0 = m$, which also yields $dm = dt$, $m \to -t_0$

as $t \to 0$ and $m \to T - t_0$ as $t \to T$

$$z(n) = \frac{1}{T}\int_{-t_0}^{T-t_0} x(m)\,e^{-jn\,w_0(m+t_0)}\,dm$$

$$z(n) = \frac{1}{T}\int_{-t_0}^{T-t_0} x(m)\,e^{-jnw_0m}\,dm \cdot e^{-jnw_0t_0} = x(n)\,e^{-jnw_0t_0}$$

Frequency shift:

(3) If $x(t) \xleftrightarrow{FS} X(n)$ then $z(t) = e^{jm\omega_0 t} x(t) \xleftrightarrow{FS} Z(n) = X(n-m)$

proof

$$Z(n) = \frac{1}{T}\int_0^T z(t) e^{-jn\omega_0 t} dt = \frac{1}{T}\int_0^T e^{jm\omega_0 t} \cdot x(t) e^{-jn\omega_0 t} dt$$

$$= \frac{1}{T}\int_0^T x(t) \cdot e^{+j\omega_0 t(m-n)} dt$$

$$= \frac{1}{T}\int_0^T x(t) e^{-j(n-m)\omega_0 t} dt = X(n-m)$$

(4) **Time Scaling:**

If $x(t) \xleftrightarrow{FS} X(n)$ then $z(t) = x(at) \xleftrightarrow{FS} Z(n) = X(n)$

proof: An operation that in general changes the period of the under lying signal

$$\rightarrow \quad X(n) = \frac{1}{T}\int_0^T x(t) e^{-jn\omega_0 t} dt$$

Since $x(t)$ is periodic, then $z(t) = x(at)$ is also periodic. And if 'T' is the period of $x(t)$, then period of $z(t)$ will be T/a.

→ $||^{ly}$ if frequency of $x(t)$ is ω_0. The frequency of $z(t) = x(at)$ will be $a\omega_0$, since 't' is multiplied by factor 'a'.

$$Z(n) = \frac{1}{T/a}\int_0^{(T/a)} z(t) \cdot e^{-jna\omega_0 t} dt$$

$$= \frac{1}{T/a}\int_0^{T/a} x(at) \cdot e^{-jna\omega_0 t} dt$$

By letting $\tau = at$ which also yields $d\tau = a\,dt$, As $t \to 0$ as $\tau \to 0$ and $t \to T/a$ as $\tau \to T$.

$$Z(n) = \frac{1}{T}\int_0^T x(\tau) e^{-jn\omega_0 \tau} d\tau = X(n)$$

(5) **Time Differentiation:**

If $x(t) \xleftrightarrow{FS} X(n)$ then $\frac{dx(t)}{dt} \xleftrightarrow{FS} jn\omega_0 X(n)$

proof $\quad x(t) = \sum_{n=-\infty}^{\infty} X(n) e^{jn\omega_0 t}$ By definition

Time Differentiation:

If $x(t) \xleftrightarrow{FS} X(K)$ then $\frac{dx(t)}{dt} \xleftrightarrow{FS} jk\omega_0 X(K)$

proof

$$x(t) = \sum_{K=-\infty}^{\infty} X(K) e^{jk\omega_0 t}$$ { By definition of Exponential fourier series $\longrightarrow$ 1

differentiating w. r. to 't'

$$\frac{dx(t)}{dt} = \sum_{K=-\infty}^{\infty} X(k)\, jk\omega_0 e^{jk\omega_0 t}$$

$$\therefore \frac{dx(t)}{dt} = \sum_{K=-\infty}^{\infty} [jk\omega_0 X(k)] e^{jk\omega_0 t}$$

we know $x(t) \xleftrightarrow{FS} X(k)$. Comparing above eqn with eq 1

$$\frac{dx(t)}{dt} \longleftrightarrow jk\omega_0 X(k).$$

Convolution in Time:

If $x(t) \xleftrightarrow{FS} X(K)$ and $y(t) \xleftrightarrow{FS} Y(K)$ then $z(t) = x(t) * y(t) \xleftrightarrow{FS} Z(K) = T X(K) Y(K)$

proof we know that

$$Z(K) = \frac{1}{T}\int_{\langle T\rangle} z(t) e^{-jk\omega_0 t} dt$$

$$= \frac{1}{T}\int_{\langle T\rangle} [x(t) * y(t)] e^{-jk\omega_0 t} dt$$

we know that convolution $x(t) * y(t) = \int x(\tau) y(t-\tau) d\tau$.

$$= \frac{1}{T}\left[\int_{\langle T\rangle}\int_{\langle T\rangle} x(\tau) y(t-\tau) d\tau\, e^{-jk\omega_0 t} dt d\tau\right]$$

put $t-\tau = m$ then $dt = dm$.

$$Z(K) = \frac{1}{T}\int_{\langle T\rangle} x(\tau)\int_{\langle T\rangle} y(m) e^{-jk\omega_0(\tau+m)} d\tau dm]$$

$$= \frac{1}{T}\int_{\langle T\rangle} x(\tau)\int_{\langle T\rangle} y(m) e^{-jk\omega_0\tau} . e^{-jk\omega_0 m} d\tau dm.$$

$$= \frac{1}{T} \int x(\tau) e^{-jk\omega_0 \tau} d\tau \int y(m) e^{-jk\omega_0 m} dm$$

$$= \frac{1}{T} \left[T\, X(K) \right] \cdot \left[T \cdot Y(K) \right] = T\, X(K) Y(K)$$

Multiplication or Modulation Theorem:

If $x(t) \xleftrightarrow{FS} X(K)$ and $y(t) \xleftrightarrow{FS} Y(K)$ then $z(t) = x(t) \cdot y(t) \xleftrightarrow{FS} Z(K) = X(K) * Y(K)$

proof

$$Z(K) = \frac{1}{T} \int z(t) e^{-jk\omega_0 t} dt$$

$$= \frac{1}{T} \int \left[x(t) \cdot y(t) \right] e^{-jk\omega_0 t} dt$$

By putting $x(t) = \sum_{K=-\infty}^{\infty} X(K) e^{jk\omega_0 t} dt$

$$Z(K) = \frac{1}{T} \int \sum_{m=-\infty}^{\infty} X(m) e^{jm\omega_0 t} y(t) e^{-jk\omega_0 t} dt]$$

$$Z(K) = \sum_{m=-\infty}^{\infty} X(m) \left[\frac{1}{T} \int_{\langle T \rangle} y(t) e^{-j(k-m)\omega_0 t} dt \right]$$

$$Z(K) = \sum_{m=-\infty}^{\infty} X(m)\, Y(K-m)$$

$$Z(K) = X(K) * Y(K)$$

Parseval's Theorem:

If $x(t)$ is periodic power signal with fourier coefficients $X(K)$, then average power in the signal is given by $P = \sum_{K=-\infty}^{\infty} |X(K)|^2$

proof

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

$$= \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt \text{ for periodic sgl}$$

$$= \frac{1}{T} \int_{-T/2}^{T/2} x(t) \cdot x^*(t)\, dt$$

we know $x(t) = \sum_{K=-\infty}^{\infty} X(K) e^{+jk\omega_0 t}$

$$x^*(t) = \left[\sum_{K=-\infty}^{\infty} X(K) e^{jk\omega_0 t} \right]^*$$

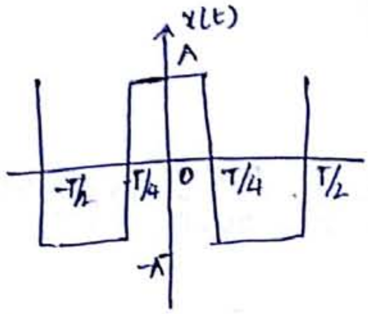
$$= \sum_{K=-\infty}^{\infty} X^* K\, e^{-jk\omega_0 t}$$

putting in eq

$$P = \frac{1}{T} \int_{\langle T \rangle} x(t) \sum_{K=-\infty}^{\infty} X^* K\, e^{-jk\omega_0 t} dt$$

$$P = \sum_{K=-\infty}^{\infty} X^*(K) \cdot \frac{1}{T} \int_{\langle T \rangle} x(t) e^{-jk\omega_0 t} dt = \sum_{K=-\infty}^{\infty} X^*(K) X(K) = \sum_{K=-\infty}^{\infty} |X(K)|^2$$

① Figure below shows a periodic square wave signal which is symmetrical with respect to vertical axis. Obtain its fourier series representation



Sol F.S. Representation

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t.$$

Since the given waveform is symmetrical about vertical axis i.e $x(t) = x(-t)$

$\therefore$ F.S representation only cosine terms are present. i.e $b_n = 0$.

$$a_0 = 0.$$

$$\therefore x(t) = \sum_{n=1}^{\infty} a_n \cos n\omega_0 t$$

$x(t)$ from figure

$$x(t) = -A \text{ for } -T/2 < t < -T/4, \; +A \text{ for } -T/4 < t < +T/4$$

$$= -A \text{ for } +T/4 < t < T/2.$$

$$\therefore a_n = \frac{2}{T}\int_{-T/2}^{T/2} x(t) \cos n\omega_0 t \, dt$$

$$a_n = \frac{2}{T}\left[\int_{-T/2}^{-T/4} -A\cos n\omega_0 t \, dt + \int_{-T/4}^{T/4} A\cos n\omega_0 t \, dt + \int_{T/4}^{T/2} -A\cos n\omega_0 t \, dt\right]$$

$$a_n = \frac{2A}{T}\int_{-T/2}^{-T/4}(-\cos n\omega_0 t)\,dt + \int_{-T/4}^{T/4}\cos n\omega_0 t \, dt + \int_{T/4}^{T/2}(-\cos n\omega_0 t)\,dt$$

$$a_n = \frac{2A}{T}\left[\left.\frac{-\sin n\omega_0 t}{n\omega_0}\right|_{-T/2}^{-T/4} + \left[\frac{\sin n\omega_0 t}{n\omega_0}\right]_{-T/4}^{T/4} + \left[\frac{-\sin n\omega_0 t}{n\omega_0}\right]_{T/4}^{T/2}\right]$$

$$\Rightarrow a_n = \frac{2A}{n\omega_0 T}\left[-\sin\left(\frac{-n\omega_0 T}{4}\right) + \sin\left(\frac{-n\omega_0 T}{2}\right) + \sin\left(\frac{n\omega_0 T}{4}\right)\right]$$

$$a_n = \frac{8A}{n\omega_0 T}\sin\left(\frac{n\omega_0 T}{4}\right) - \frac{4A}{4\omega_0 T}\sin\left(\frac{n\omega_0 T}{2}\right)$$

we know that $\omega_0 = \frac{2\pi}{T}$ or $\omega_0 T = 2\pi$

$$\therefore \quad a_n = \frac{8A}{2\pi n}\sin\left(\frac{n\pi}{2}\right) = \frac{4A}{n\pi}\sin\left(\frac{n\pi}{2}\right)$$

putting the values of an in eq (ii)

$$x(t) = \sum_{n=1}^{\infty} \frac{4A}{n\pi}\cdot\sin\left(\frac{n\pi}{2}\right)\cos n\omega_0 t$$

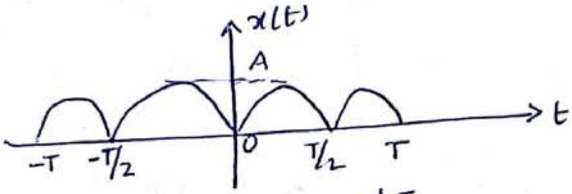
$n = 1, 2, 3 \cdot \cdot$

$$x(t) = \frac{4A}{\pi}\sin\left(\frac{\pi}{2}\right)\cos(\omega_0 t) + \frac{4A}{3\pi}\sin\left(\frac{3\pi}{2}\right)\cos 3\omega_0 t + \cdot\cdot$$

$$x(t) = \frac{4A}{\pi}\cdot 1\cos\omega_0 t + 0 + \frac{4A}{3\pi}(-1)\cos 3\omega_0 t + \cdot\cdot$$

$$x(t) = \frac{4A}{\pi}\left[\cos\omega_0 t - \frac{1}{3}\cos 3\omega_0 t + \frac{1}{5}\cos(5\omega_0 t) - - - \right]$$

(1) . Determine the trigonometric form of fourier series of full wave rectified sine wave as shown in fig



Sol The waveform shown and it has even symmetry.

$$\therefore b_n = 0, \quad a_0 = \frac{4}{T}\int_0^{T/2} x(t)\,dt \;; \quad a_n = \frac{4}{T}\int_0^{T/2} x(t)\cos n\Omega_0 t\,dt.$$

The mathematical eqn of full wave rectified o/p is

$$x(t) = A\sin\Omega_0 t \;; \text{ for } t = 0 \text{ to } T/2 \text{ and } \Omega_0 = \frac{2\pi}{T}$$

Evaluation of a_0:

$$a_0 = \frac{4}{T}\int_0^{T/2} x(t)\,dt = \frac{4}{T}\int_0^{T/2} A\sin\Omega_0 t\,dt = \frac{4A}{T}\left[-\frac{\cos\Omega_0 t}{\Omega_0}\right]_0^{T/2}$$

$$= \frac{4A}{T}\left[-\frac{\cos\frac{2\pi}{T}\cdot\frac{T}{2}}{\frac{2\pi}{T}} + \frac{\cos 0}{\frac{2\pi}{T}}\right] = \frac{2A}{\pi}\left[-\cos\pi + \cos 0\right]$$

$$= \frac{2A}{\pi}\left[1+1\right] = \frac{4A}{\pi} \quad \left\{ \because \cos\pi = -1,\ \cos 0 = 1 \right.$$

Evaluation of a_n:

$$a_n = \frac{4}{T}\int_0^{T/2} x(t)\cos n\Omega_0 t\,dt$$

$$= \frac{4}{T}\int_0^{T/2} A\sin\Omega_0 t\cos n\Omega_0 t\,dt$$

$$= \frac{4A}{T}\int_0^{T/2} \frac{\sin(\Omega_0 t + n\Omega_0 t) + \sin(\Omega_0 t - n\Omega_0 t)}{2}\,dt \quad \left\{ \because \sin(A+B) + \sin(A-B) = 2\sin A\cos B \right.$$

$$= \frac{2A}{T}\int_0^{T/2} \sin(1+n)\Omega_0 t\,dt + \frac{2A}{T}\int_0^{T/2} \sin(1-n)\Omega_0 t\,dt$$

$$= \frac{2A}{T}\left[\frac{-\cos(1+n)\Omega_0 t}{(1+n)\Omega_0}\right]_0^{T/2} + \frac{2A}{T}\left[\frac{-\cos(1-n)\Omega_0 t}{(1-n)\Omega_0}\right]_0^{T/2}$$

$$= \frac{2A}{T}\left[\frac{-\cos(1+n)\frac{2\pi}{T}.t}{(1+n)\frac{2\pi}{T}}\right]_0^{T/2} + \frac{2A}{T}\left[\frac{-\cos(1-n)\frac{2\pi}{T}.t}{(1-n)\frac{2\pi}{T}}\right]_0^{T/2}$$

$$= \frac{2A}{T}\left[\frac{-\cos(1+n)\frac{2\pi}{T}.\frac{T}{2}}{(1+n)\frac{2\pi}{T}} + \frac{\cos(1+n).0}{(1+n)\frac{2\pi}{T}}\right] + \frac{2A}{T}\left[\frac{-\cos(1-n)\frac{2\pi}{T}.\frac{T}{2}}{(1-n)\frac{2\pi}{T}} + \frac{\cos(1-n).0}{(1-n)\frac{2\pi}{T}}\right]$$

$$a_n = \frac{2A}{\pi}\left[\frac{-\cos(1+n)\pi}{(1+n)}\right] + \frac{A}{\pi(1+n)}$$

$\{\because \cos 0 = 1$

$$-\frac{A\cos(1-n)\pi}{(1-n)\pi} + \frac{A}{(1-n)\pi}$$

→ a_n can be evaluated for all values of n except $n=1$, For $n=1$, a_n has to be estimated sepearately

$$a_1 = \frac{4}{T}\int_0^{T/2} x(t)\cos\Omega_0 t\,dt = \frac{4}{T}\int_0^{T/2} A\sin\Omega_0\cos\Omega_0 t\,dt$$

$$= \frac{4}{T}\int_0^{T/2} A\,\frac{\sin 2\Omega_0 t}{2}dt = \frac{2A}{T}\int_0^{T/2}\sin 2\Omega_0 t\,dt \qquad \{\because \sin 2\theta = 2\sin\theta\cos\theta$$

$$= \frac{2A}{T}\left[\frac{-\cos 2\Omega_0 t}{2\Omega_0}\right]_0^{T/2} = \frac{2A}{T}\left[\frac{-\cos 2\Omega_0 T/2}{2\Omega_0} + \frac{\cos 0}{2\Omega_0}\right]$$

$$= \frac{2A}{T}\left[\frac{-\cos 2\times\frac{2\pi}{T}\times T/2}{2\times\frac{2\pi}{T}} + \frac{\cancel{\cos 0}}{2.\frac{2\pi}{T}}\right]$$

$$= \frac{2A}{T}\left[\frac{-T}{4\pi}\cos 2\pi + \frac{T}{4\pi}\right]$$

$$= \frac{2A}{T}\left[\frac{-T}{4\pi} + \frac{T}{4\pi}\right] = 0 \qquad \{\because \cos 2\pi = \cos 0 = 1\}$$

For values of $n>1$

$$\therefore a_n = \frac{-A\cos(1+n)\pi}{(1+n)\pi} + \frac{A}{(1+n)\pi} - \frac{A\cos(1-n)\pi}{(1-n)\pi} + \frac{A}{(1-n)\pi}$$

when n is even integer, $(1+n)$ & $(1-n)$ will be odd.

$$\therefore \cos(1+n)\pi = -1 \; ; \; \cos(1-n)\pi = -1$$

when n is odd integer, $(1+n)$ & $(1-n)$ will be even

$$\therefore \cos(1+n)\pi = 1 \; , \; \cos(1-n)\pi = 1$$

$\therefore a_n = 0$; for odd values of n

$$a_n = \frac{A}{(1+n)\pi} + \frac{A}{(1+n)\pi} + \frac{A}{(1-n)\pi} + \frac{A}{(1-n)\pi} \; ; \text{for even value}$$

$$= \frac{2A}{(1+n)\pi} + \frac{2A}{(1-n)\pi} = \frac{2A(1-n)+2A(1+n)}{(1+n)(1-n)\pi} = \frac{4A}{(1-n^2)\pi}$$

$$\therefore a_2 = \frac{4A}{(1-2^2)\pi} = \frac{-4A}{3\pi}$$

$$a_4 = \frac{4A}{(1-4^2)\pi} = -\frac{4A}{15\pi}$$

$$a_6 = \frac{4A}{(1-6^2)\pi} = \frac{-4A}{35\pi} \text{ and so on.}$$

Fourier series:

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\Omega_0 t$$

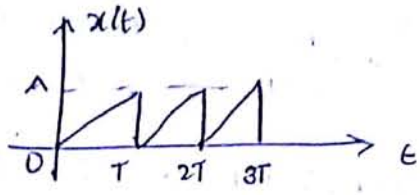
$$x(t) = a_0 + \sum_{n=\text{even}} a_n \cos n\omega_0 t$$

$$x(t) = \frac{4A}{\pi} + a_2\cos 2\Omega_0 t + a_4 \cos 4\Omega_0 t + a_6 \cos 6\Omega_0 t + \cdots$$

$$= \frac{4A}{\pi} - \frac{4A}{3\pi}\cos 2\Omega_0 t - \frac{4A}{15\pi}\cos 4\Omega_0 t - \frac{4A}{35\pi}\cos 6\Omega_0 t + \cdots$$

$$= \frac{4A}{\pi} - \frac{4A}{\pi}\left[\frac{\cos 2\Omega_0 t}{3} + \frac{\cos 4\Omega_0 t}{15} + \frac{\cos 6\Omega_0 t}{35} + \cdots\right]$$

(2) Determine the trigonometric form of fourier series of the ramp signal.



Sol

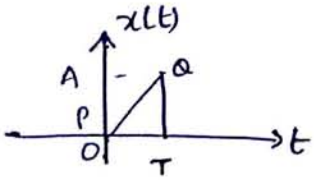
$$a_0 = \frac{1}{T}\int_0^T x(t)\,dt \; ; \; a_n = \frac{2}{T}\int_0^T x(t)\cos n\Omega_0 t\,dt \; ; \; b_n = \frac{2}{T}\int_0^T x(t)\sin n\Omega_0 t\,dt$$

To find the mathematical Eqn

Consider eqn of straight line, $\dfrac{y-y_1}{y_1-y_2} = \dfrac{x-x_1}{x_1-x_2}$

$y = x(t)$; $x = t$

$$\frac{x(t) - x_1(t_1)}{x_1(t_1) - x_2(t_2)} = \frac{t - t_1}{t_1 - t_2}$$



Consider points P & Q

coordinates of $P = \overset{t_1,\, x(t_1)}{[0, 0]}$

coordinates of $Q = [t_2, x_2(t_2)] = [T, A]$

$$\frac{x(t) - 0}{0 - A} = \frac{t - 0}{0 - T}$$

$$\Rightarrow x(t) = \frac{-At}{-T} = \frac{At}{T}$$

$\therefore x(t) = \dfrac{At}{T}$ for $t = 0$ to T.

Evaluation of a_0

$$a_0 = \frac{1}{T}\int_0^T x(t)\,dt = \frac{1}{T}\int_0^T \frac{At}{T}\,dt = \frac{A}{T^2}\int_0^T t\,dt = \frac{A}{T^2}\left[\frac{t^2}{2}\right]_0^T$$

$$= \frac{A}{2}.$$

$$\left\{ \because \int uv = u\int v - \left[\int \left[du \int v\right]\right] \right\}$$

Evaluation of a_n:

$$a_n = \frac{2}{T}\int_0^T x(t)\cos n\Omega_0 t\, dt = \frac{2}{T}\int_0^T \frac{At}{T}\cos n\Omega_0 t\, dt$$

$$= \frac{2A}{T^2}\left[\int_0^T t\cdot\cos n\Omega_0 t\, dt\right] = \frac{2A}{T^2}\left[t\int_0^T \cos n\Omega_0 t\, dt - \int 1\cdot\frac{\sin n\Omega_0 t}{n\Omega_0}dt\right]_0^T$$

$$a_n = \frac{2A}{T^2}\left[t\cdot\frac{\sin n\Omega_0 t}{n\Omega_0} - \left(\frac{-\cos n\Omega_0 t}{n^2\Omega_0^2}\right)\right]_0^T$$

$$= \frac{2A}{T^2}\left[\frac{t\cdot\sin n\frac{2\pi}{T}\cdot t}{n\Omega_0} + \frac{\cos n\cdot\frac{2\pi}{T}\cdot t}{n^2\cdot\frac{4\pi^2}{T^2}}\right]_0^T$$

$$= \frac{2A}{T^2}\left[\frac{T\sin n\frac{2\pi}{T}\cdot T}{n\cdot\frac{2\pi}{T}} + \frac{\cos n\frac{2\pi}{T}\cdot T}{n^2\cdot\frac{4\pi^2}{T^2}} - 0 - \frac{\cos 0}{n^2\frac{4\pi^2}{T^2}}\right]$$

$$= \frac{2A}{T^2}\left[\frac{T^2}{2\pi n}\sin 2\pi n + \frac{T^2}{4\pi^2 n^2}\cdot\cos n2\pi - 0 - \frac{T^2}{n^2 4\pi^2}\right]$$

$$\left\{\begin{array}{l}\sin n2\pi = 0\\ \cos n2\pi = 1\end{array}\right.$$

$$= \frac{2A}{T^2}\left[\frac{T^2}{4\pi^2 n^2} - \frac{T^2}{n^2 4\pi^2}\right] = 0$$

Evaluation of b_n:

$$b_n = \frac{2}{T}\int_0^T x(t)\sin n\Omega_0 t\, dt$$

$$= \frac{2}{T}\int_0^T \frac{At}{T}\sin \Omega_0 n t\, dt = \frac{2A}{T^2}\int_0^T t\cdot\sin n\Omega_0 t\, dt$$

$$= \frac{2A}{T^2}\left[t\cdot\left(\frac{-\cos n\Omega_0 t}{n\Omega_0}\right) - \int\left(\frac{-\cos n\Omega_0 t}{n\Omega_0}\right)dt\right]_0^T$$

$$= \frac{2A}{T^2}\left[-t\,\frac{\cos n\Omega_0 t}{n\Omega_0} + \frac{\sin n\Omega_0 t}{n^2\Omega_0^2}\right]_0^T$$

$$= \frac{2A}{T^2}\left[\frac{-T\cos n\cdot\frac{2\pi}{T}\cdot T}{n\cdot\frac{2\pi}{T}} + \frac{\sin n\cdot\frac{2\pi}{T}\cdot T}{\frac{n^2 4\pi^2}{T^2}} + 0 - \frac{\sin 0}{\frac{n^2\cdot 4\pi^2}{T^2}}\right]$$

$$= \frac{2A}{T^2}\left[\frac{-T^2}{2\pi n}\cos n2\pi + \frac{T^2}{4\pi^2 n^2}\sin n2\pi\right]$$

$$b_n = \frac{-A}{n\pi}$$

$$\therefore b_1 = \frac{-A}{\pi} \ ; \ b_2 = \frac{-A}{2\pi} \ ; b_3 = \frac{-A}{3\pi} \ \ldots$$

Fourier Series

$$\therefore x(t) = a_0 + \sum_{n=1}^{\infty} b_n \sin n\Omega_0 t \, dt$$

$$= \frac{A}{2} + b_1 \sin \Omega_0 t + b_2 \sin 2\Omega_0 t + b_3 \sin 3\Omega_0 t + \ldots$$

$$= \frac{A}{2} - \frac{A}{\pi}\sin\Omega_0 t - \frac{A}{2\pi}\sin 2\Omega_0 t - \frac{A}{3\pi}\sin 3\Omega_0 t + \ldots$$

$$= \frac{A}{2} - \frac{A}{\pi}\left[\sin\Omega_0 t + \frac{\sin 2\Omega_0 t}{2} + \frac{\sin 3\Omega_0 t}{3} + \ldots\right]$$

Atun 28/7/14

2, 3, 4, 10, 12, 13, 15, 23, 26, 27,
29, 40, ~~43~~, 46, 48, 55.

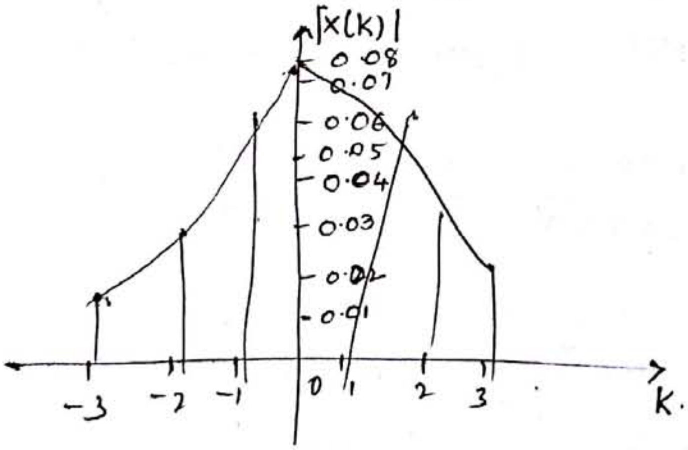
phase spectrum

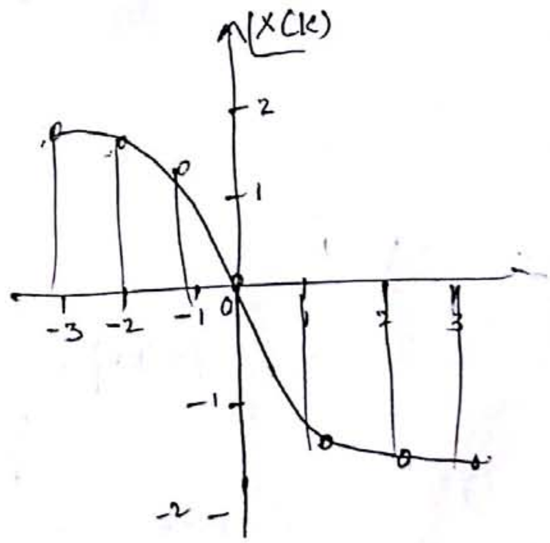
$$\angle X(K) = \tan^{-1}\left[\frac{\text{Imaginary part}}{\text{Real part}}\right]$$

$$= -\tan^{-1}(4\pi K)$$

Table:

K	$\lvert X(K)\rvert$	$\angle X(K)$
-3	0.0208	1.5442
-2	0.0312	1.5310
-1	0.0624	1.491
0	0.7869	0
1	0.0624	-1.491
2	0.0312	-1.5310
3	0.0208	-1.5442





$$= \frac{-2}{1+j4\pi K}\left[e^{-(1+j4\pi k)0.5} - e^{0}\right]$$

$$= \frac{-2}{1+j4\pi k}\left[e^{-0.5}\cdot e^{-j2\pi k} - 1\right]$$

$\because\ e^{-j2\pi k} = \cos 2\pi k - j\sin 2\pi k = 1$ always

$$x(k) = \frac{-2}{1+j4\pi k}\left[0.606-1\right] = \frac{0.7869}{1+j4\pi k}.$$

step 2: To express exponential Fourier series

$$x(t) = \sum_{K=-\infty}^{\infty} \frac{0.7869}{1+j4\pi K} e^{jK\omega_0 t}.$$

step 3: To obtain magnitude & phase spectrum of $x(K)$

$$x(K) = \frac{0.7869}{1+j4\pi K}$$

$$= \frac{0.7869}{1+j4\pi K} \times \frac{1-j4\pi k}{1-j4\pi k} = \frac{0.7869(1-j4\pi k)}{1+(4\pi k)^2}$$

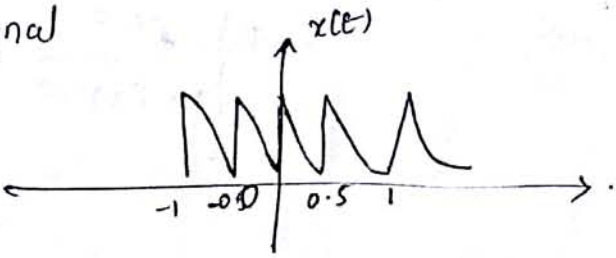
$$= \frac{0.7869}{1+(4\pi k)^2} - j\,\frac{0.7869\times 4\pi k}{1+(4\pi k)^2}.$$

$$|x(k)| = \sqrt{\frac{(0.7869)^2}{(1+(4\pi k)^2)^2} + \frac{(0.7869\times 4\pi k)^2}{[1+(4\pi k)^2]^2}}$$

$$= \sqrt{\frac{(0.7869)^2 + (0.7869\times 4\pi k)^2}{(1+(4\pi k)^2]^2}}$$

$$= \sqrt{\frac{(0.7869)^2(1+(4\pi k)^2)}{[1+(4\pi k)^2]^2}} = \frac{0.7869}{\sqrt{1+(4\pi k)^2}}$$

(2) Obtain the polar fourier series of signal



Sol

$C_0 = a_0 = 0.7869$

$C_n = \sqrt{a_n^2 + b_n^2}$

$= \frac{(1.576)^2}{[1+4\pi k)^2]^2} + \frac{6.32\pi k)^2}{[1+(4\pi k)^2]^2}$

$= \frac{\sqrt{(1.576)^2 + (6.32\pi k)^2}}{\sqrt{(1+(4\pi k)^2)^2}}$

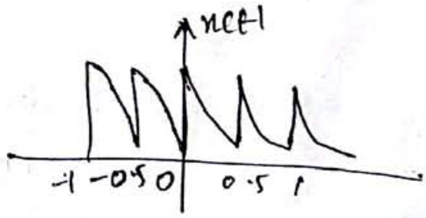
$= \sqrt{\frac{(1.576)^2[1+4\pi k)^2]}{(1+(4\pi k)^2)^2}}$

$\phi(n) = -\tan^{-1}\left(\frac{b_n}{a_n}\right) = -\tan^{-1}$

$= -\tan^{-1}(4\pi k).$

$\therefore\ x(t) = 0.7869 + \sum_{k=1}^{\infty} \frac{1.576}{1+(4\pi k)^2} \cos[k\omega_0 t - \tan^{-1}(4\pi k)]$

(3) Obtain Exponential F.S and plot the magnitude & phase spectrum also



Sol

$X(K) = \frac{1}{T}\int_0^T x(t)\, e^{-jk\omega_0 t}\, dt$

$x(t) = e^{-t}$. for 0 to 0.5 $\omega_0 = \frac{2\pi}{T} = 4\pi$

$X(K) = \frac{1}{0.5}\int_0^{0.5} e^{-t} e^{-jk4\pi t} dt = 2\int_0^{0.5} e^{-(1+jk4\pi)t} dt$

$= 2 \frac{1}{-(1+j4\pi k)}\left[e^{-(1+jk4\pi)t}\right]_0^{0.5}$

$$= 4\left\{ \frac{e^{-0.5}}{1+(4\pi k)^2}\left[-\cos(4\pi k)0.5 + 4\pi k\sin(4\pi k)(0.5)\right] - \frac{e^{0}}{1+(4\pi k)^2}\left[-\cos(4\pi k)0 + 4\pi k\sin(4\pi k)\cdot 0\right]\right\}$$

$$= \frac{4}{1+(4\pi k)^2}\left[0.606\left[-\cos(2\pi k) + 4\pi k\sin(2\pi k)\right] - \left[-\cos(0) + 4\pi k\sin(0)\right]\right]$$

$$= \frac{4}{1+(4\pi k)^2}\left\{-0.606+0+1+0\right\}$$

$$= \frac{1.576}{1+(4\pi k)^2}.$$

(iii) To calculate $b(k)$

$$\left\{\int e^{ax}\sin bx\, dx = \frac{e^{ax}}{a^2+b^2}\left[a\sin bx - b\cos bx\right]\right.$$

$$b_K = \frac{2}{T}\int_0^T x(t)\sin k\omega_0 t$$

$$= \frac{2}{0.5}\int_0^{0.5} e^{-t}\sin(K\cdot 4\pi t)\,dt = 4\int_0^{0.5} e^{-t}\sin(4\pi kt)\,dt.$$

$$= 4\left[\frac{e^{-t}}{1+(4\pi k)^2}\left[(-1)\sin(4\pi n)t - (4\pi n)\cos(4\pi n)t\right]\right]_0^{0.5}$$

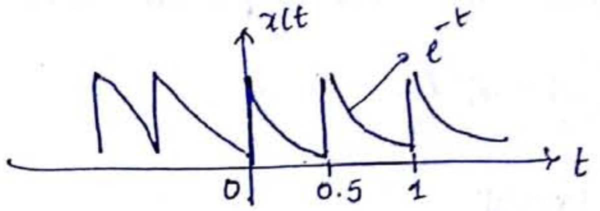
$$= \frac{6.32\pi K}{1+(4\pi K)^2}.$$

(iv) To obtain fourier series

$$x(t) = 0.7869 + \sum_{k=1}^{\infty}\frac{1.576}{1+(4\pi k)^2}\cos k\omega_0 t + \sum_{k=1}^{\infty}\frac{6.32\pi K}{1+(4\pi K)^2}\sin k\omega_0 t.$$

Problems:

(1) Find trigonometric fourier series for the periodic signal



Sol

$$T = 0.5 \qquad x(t) = e^{-t}$$

(i) To calculate a_0:

$$a_0 = \frac{1}{T}\int_0^T x(t)\,dt$$

$$= \frac{1}{0.5}\int_0^{0.5} e^{-t}\,dt = \frac{1}{0.5}\left[\frac{e^{-t}}{-1}\right]_0^{0.5} = \frac{1}{0.5}\left[e^{-0.5}+1\right]$$

$$= 0.7869$$

(ii) To calculate a_K:

$$a_K = \frac{2}{T}\int_0^T x(t)\cos k\omega_0 t\,dt$$

$$= \frac{2}{0.5}\int_0^{0.5} e^{-t}\cos k.\frac{2\pi}{T}.t\,dt$$

$$= \frac{2}{0.5}\int_0^{0.5} e^{-t}\cos k.\frac{2\pi}{0.5}.t\,dt$$

$$= 4\int_0^{0.5} e^{-t}\cos 4\pi k t\,dt$$

we use $\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$

with $a = -1$, $b = 4\pi K$

The eqn will be

$$a_K = 4\left\{\frac{e^{-t}}{1+(4\pi k)^2}\left[(-1)\cos(4\pi k)t + (4\pi k)\sin(4\pi k)t\right\}_0^{0.5}\right.$$

Parseval's Theorem:

If $x(t)$ is periodic power signal with Fourier coefficients $x(K)$, then average in the signal is given by $\sum_{K=-\infty}^{\infty} |x(K)|^2$ i.e

$$P = \sum_{K=-\infty}^{\infty} |x(K)|^2.$$

proof

The power in the signal is given as

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt \longrightarrow ①$$

$$= \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt \text{ for periodic signal}$$

$$= \frac{1}{T} \int_{-T/2}^{T/2} | x(t) \cdot x^*(t) \, dt \longrightarrow ②$$

$$x(t) = \sum_{K=-\infty}^{\infty} x(K) e^{jK\omega_0 t}$$

$$x^*(t) = \left[\sum_{K=-\infty}^{\infty} x(K) e^{jK\omega_0 t} \right]^*$$

$$= \sum_{K=-\infty}^{\infty} x^*(K) e^{-jK\omega_0 t}$$

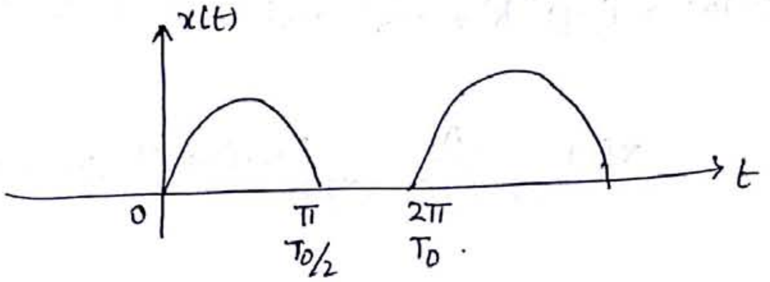
putting $x^*(t)$ in eq ③

$$P = \frac{1}{T} \int_{-T/2}^{T/2} x(t) \cdot \sum_{K=-\infty}^{\infty} x^*(K) e^{-jK\omega_0 t} dt$$

$$P = \sum_{K=-\infty}^{\infty} x^*(K) \cdot \frac{1}{T} \int x(t) e^{-jK\omega_0 t} dt$$

$$= \sum_{K=-\infty}^{\infty} x^*(K) \cdot X(K) = \sum_{K=-\infty}^{\infty} | X(K)|^2.$$

1) Find the exponential Fourier series and plot the magnitude & phase spectrums of half wave rectified sine wave.



Sol

step 1: $x(t) = \begin{cases} A \sin \omega_0 t & \text{for } 0 \leq t \leq T_0/2 \\ 0 & \text{for } \frac{T_0}{2} \leq t < T_0 . \end{cases}$

$T_0 = 2\pi = T \quad \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1$

step 2:

$$X(K) = \frac{1}{T} \int_0^T x(t) e^{-jk\omega_0 t} dt$$

$$= \frac{1}{2\pi} \int_0^{\pi} A \sin \omega_0 t \, e^{-j\omega_0 t k} dt$$

$$= \frac{A}{2\pi} \int_0^{\pi} \sin t \, e^{-jkt} dt$$

$$\int e^{ax} \sin (bx+c) \, dx = \frac{e^{ax}}{a^2+b^2} \left[a \sin (bx+c) - b \cos (bx+c) \right]$$

with $a = -jk$, $b=1$, $c=0$ and $x=t$.

$$X(K) = \frac{A}{2\pi} \left\{ \frac{e^{-jkt}}{(-jk)^2+1} \left[-jk \sin t - \cos t \right] \right\}_0^{\pi}$$

$$= \frac{A}{2\pi} \left\{ \frac{e^{-j\pi k}}{(-jk)^2+1} \left[-jk \sin \pi - \cos \pi \right] - \frac{e^0}{(-jk)^2+1} \left[-jk \sin 0 - \cos 0 \right] \right\}$$

$$= \frac{A}{2\pi}\left\{\frac{e^{-j\pi k}}{(-jk)^2+1} + \frac{1}{(-jk)^2+1}\right\} = \frac{A}{2\pi[(-jk)^2+1]}\left\{e^{-j\pi k}+1\right\}$$

here $(-jk)^2 = (-j)^2 \cdot k^2 = -k^2$ and $e^{-j\pi k} = (-1)^k$

$$x(k) = \frac{A}{2\pi(1-k^2)}\left[(-1)^k+1\right] \text{ for } k \neq \pm 1$$

$$= \begin{cases} \frac{A}{\pi(1-k^2)} & \text{for } k = 0, \pm 2, \pm 4, \pm 6 \dots \\ 0 & \text{for } k = \pm 1, \pm 3 \dots \end{cases}$$

putting for $k=1$

$$x(k) = \frac{A}{2\pi}\int_0^{\pi} \sin t \cdot e^{-jt}\, dt$$

$$= \frac{A}{2\pi}\int_0^{\pi} \frac{e^{jt}-e^{-jt}}{2j} \cdot e^{-jt}\, dt$$

$$= \frac{A}{4j\pi}\int_0^{\pi} (1-e^{-j2t})\, dt = \frac{A}{4j\pi}\left\{(t)_0^{\pi} - \frac{1}{j2}\left[e^{-j2t}\right]_0^{\pi}\right\}$$

$$= \frac{A}{4j\pi}\left\{\pi - 0 + \frac{1}{j2}\left[e^{-j2t} - e^0\right]\right\}$$

$$= \frac{A}{4j}$$

Similarly putting $k=-1$ in eq.

$$x(k) = \frac{A}{2\pi}\int_0^{\pi} \sin t \cdot e^{jt}\, dt$$

$$= \frac{A}{2\pi}\int_0^{\pi} \frac{e^{jt}-e^{-jt}}{2j} e^{jt}\, dt$$

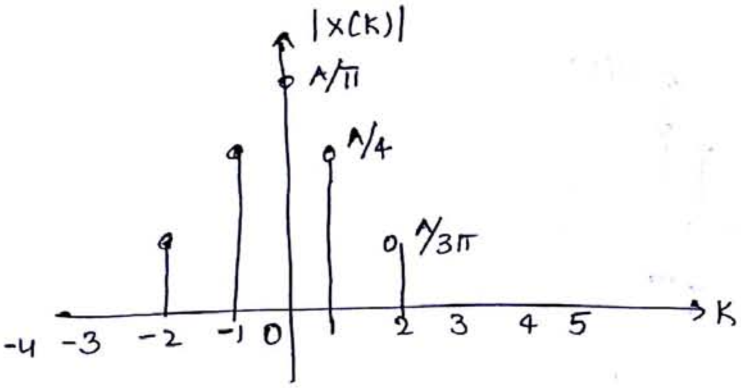
$$= \frac{A}{4\pi j}\int_0^{\pi} (e^{j2t}-1)\, dt = \frac{A}{4\pi j}\left\{\frac{1}{j2}\left[e^{j2t}\right]_0^{\pi} - (t)_0^{\pi}\right\}$$

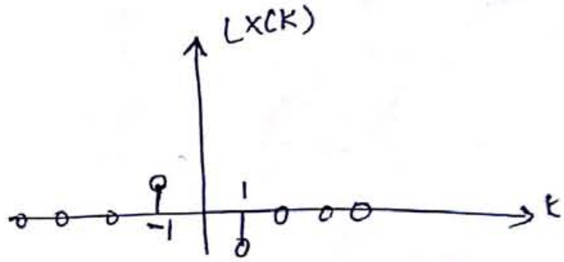
$$= -\frac{A}{4j}$$

$$X(K) = \begin{cases} \dfrac{A}{\pi(1-K^2)} & \text{for } k = 0, \pm 2, \pm 4 \ldots \\ 0 & \text{for } K = \pm 1, \pm 3, \pm 5 \ldots \\ j\dfrac{A}{4} & \text{for } k = 1 \\ -jA/4 & \text{for } K = -1 \end{cases}$$

$$|X(K)| = \begin{cases} \left| \dfrac{A}{\pi(1-K^2)} \right| & \text{for } 0, \pm 2, \pm 4 \ldots \\ \dfrac{A}{4} & \text{for } K = \pm 1 \\ 0 & \text{for } K = \pm 1, \pm 3, \pm 5 \end{cases}$$

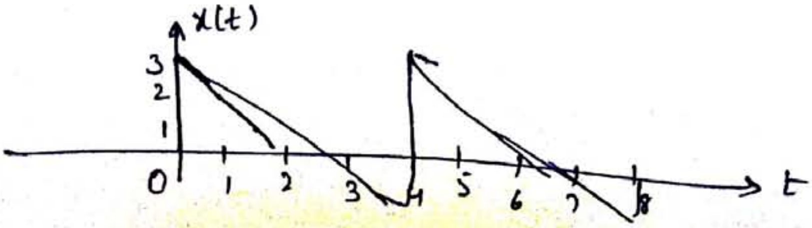
$$\angle X(K) = \begin{cases} \tan^{-1}\left(\dfrac{-A/4}{0}\right) = \dfrac{-\pi}{2} & \text{for } K = 1 \\ \tan^{-1}\left(\dfrac{A/4}{0}\right) = \dfrac{\pi}{2} & \text{for } K = -1 \\ 0 & \text{for } K \neq \pm 1 \end{cases}$$





2) A periodic signal with a period of 4 sec is described over one fundamental period by $x(t) = 3 - t$, $0 \leq t \leq 4$. Plot the signal & find the exponential fourier sers. plot the amplitude & phase spectrum

Sol



$x(t) = 3 - t, \; 0 \le t \le 4.$

step 2:

$$X(K) = \frac{1}{T}\int_0^T x(t)\, e^{-jK\omega_0 t}\, dt = \frac{1}{T}\int_0^T (3-t)\, e^{-jK\omega_0 t}\, dt$$

$$= \frac{1}{T}\int_0^T 3\, e^{-jK\omega_0 t}\, dt - \frac{1}{T}\int_0^T t\, e^{-jK\omega_0 t}\, dt.$$

$=$

$$= \frac{1}{jK\omega_0} = -j\frac{1}{K\omega_0}.$$

$$X(0) = \frac{1}{T}\int_0^T (3-t)\, dt = 1.$$

$$\therefore \; |X(K)| = \sqrt{\left(\frac{1}{K\omega_0}\right)^2} = \frac{1}{K\omega_0}.$$

$$\angle X(K) = \tan^{-1}\left(\frac{1/K\omega_0}{0}\right) = \pm 90^\circ \begin{cases} -90^\circ & \text{for } K > 0 \\ 90^\circ & \text{for } K < 0 \end{cases}$$