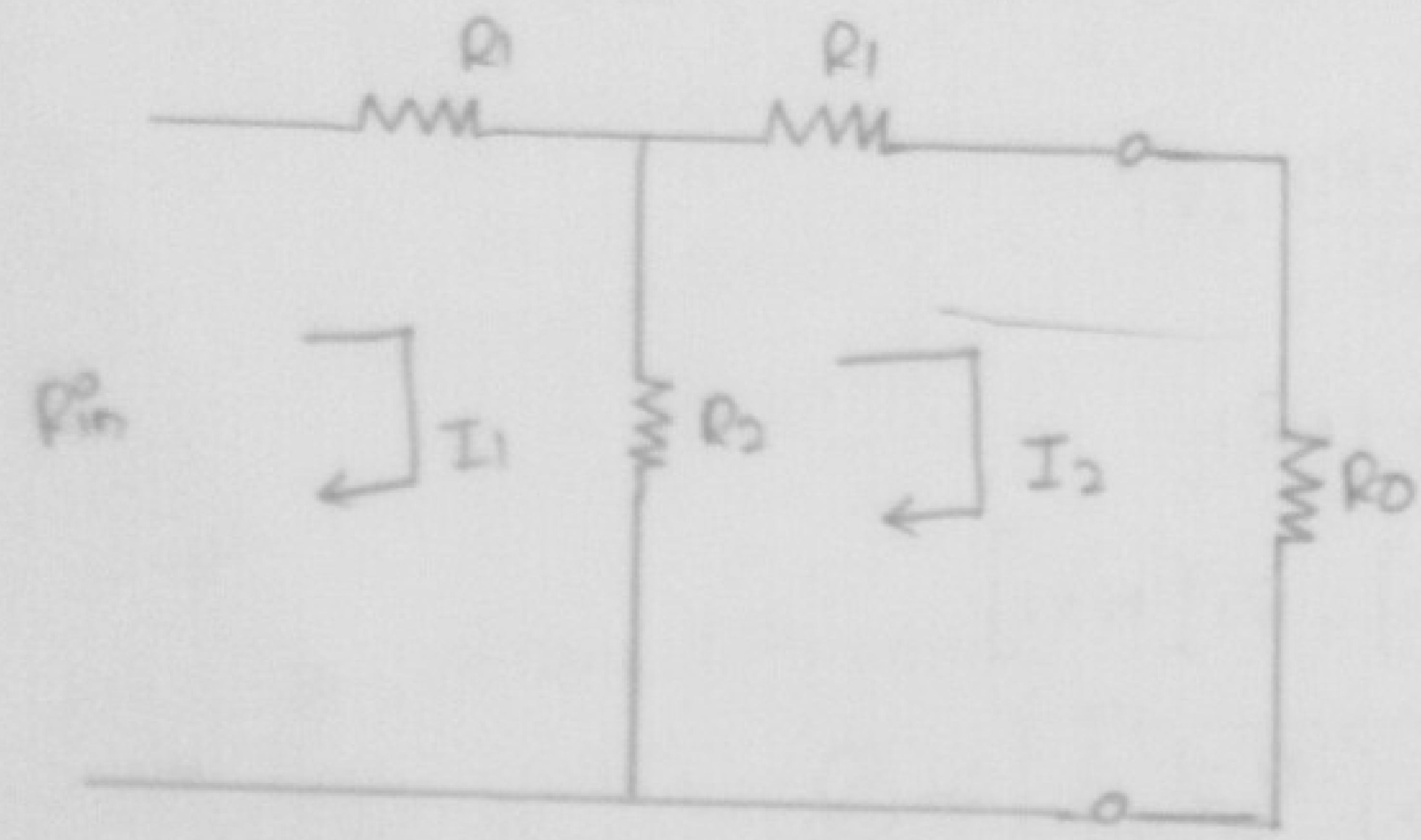
1 explain a T-type attenuator and also design a T-type attenuator to give an attenuation of 60 dB and to work in a line of 500 Ω impedance.

Symmetrical T-type attenuator



It is an attenuator whose structure look like a T symbol with equal series arm resistance. The symmetrical T attenuator is shown in above fig.

Let R_0 be the characteristic resistance of the attenuator

we have $\frac{I_1}{I_2} = N$

Apply KVL to loop 2 we have

$$R_1 I_2 + R_0 I_2 + R_2 [I_2 - I_1] = 0$$

$$I_2 [R_1 + R_0 + R_2] = R_2 I_1$$

$$\frac{I_1}{I_2} = \frac{R_1 + R_0 + R_2}{R_2} = N \quad \text{--- (1)}$$

$R_{in} = [R_1 + R_0] \parallel R_2 + R_1$

$$\Rightarrow \frac{[R_1+R_0]\,R_2}{R_1+R_0+R_2} + R_1$$

For a symmetrical attenuator $R_{in} = R_0$

$R_0 [R_1 + R_0$

$$R_0 = \frac{[R_1+R_0]\,R_2}{R_1+R_0+R_2} + R_1$$

$$R_0 = \frac{R_1+R_0}{N} + R_1$$

$NR_0 = R_1 + R_0 + NR_1$

$R_0[N-1] = R_1[N+1]$

$$\boxed{R_1 = R_0\left[\frac{N-1}{N+1}\right]} \quad \text{— (2)}$$

From (1)

$R_1 + R_2 + R_0 = NR_2$

$R_2[N-1] = R_1 + R_0$

$$R_2[N-1] = R_0\left[\frac{N-1}{N+1}\right] + R_0$$

$$\Rightarrow R_0\left[\frac{N-1+N+1}{N+1}\right]$$

$$R_2[N-1] = R_0\left[\frac{2N}{N+1}\right]$$

$$\boxed{R_2 = R_0\left[\frac{2N}{N^2-1}\right]}$$

3

Given data

attenuation in dB = 60

$R_0 = 500\ \Omega$

$$N = \text{antilog}\left[\frac{dB}{20}\right]$$

$$N = 10^{\left[\frac{dB}{20}\right]}$$

$$= 10^{\left[\frac{60}{20}\right]}$$

$$N = 10^3 = 1000$$

$$R_1 = R_0\left[\frac{N-1}{N+1}\right]$$

$$= 500\left[\frac{1000-1}{1000+1}\right]$$

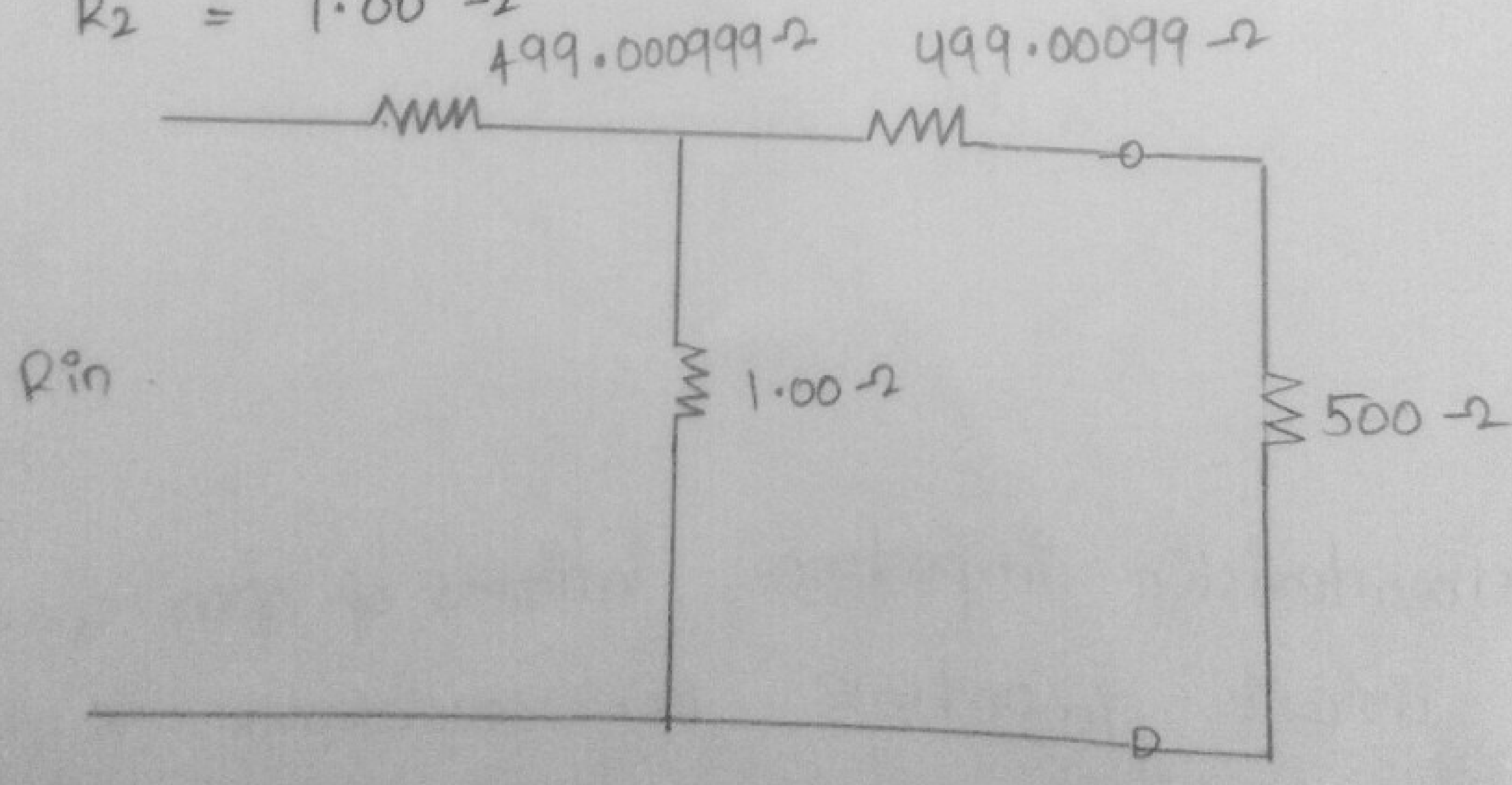
$$= 500 \times 0.998001$$

$$= 499.000999$$

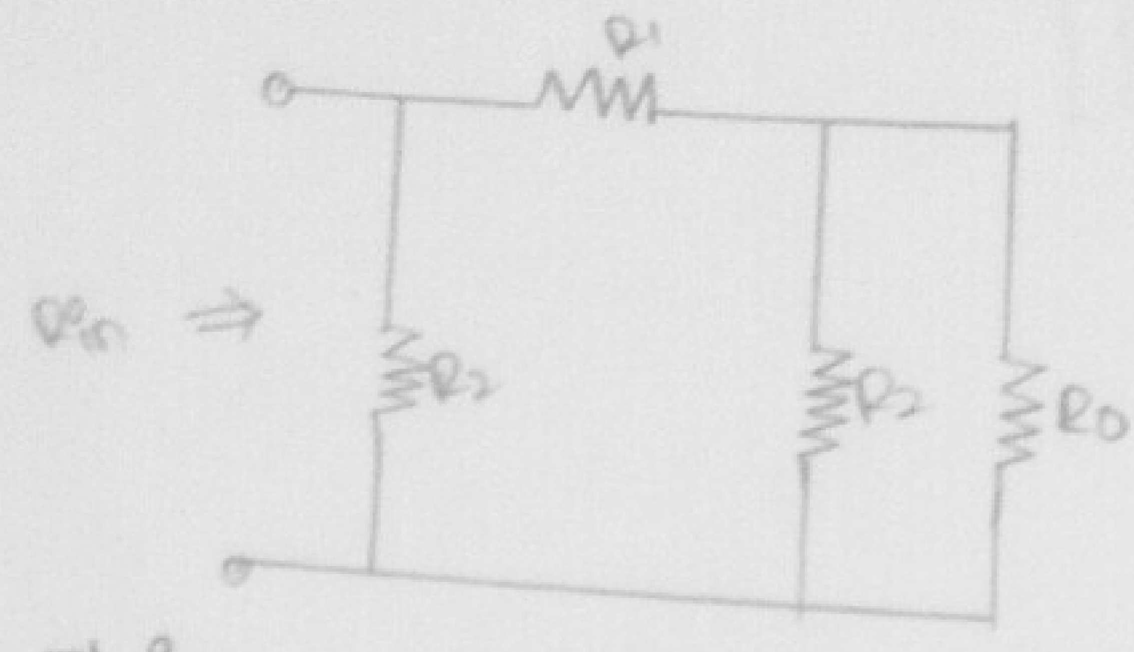
$$R_2 = R_0\left[\frac{2N}{N^2-1}\right]$$

$$= 500\left[\frac{2 \times 1000}{(1000)^2-1}\right]$$

$$R_2 = 1.00\ \Omega$$



3) explain π-type attenuator and also design it to gain 20 dB attenuation and to have characteristic impedance of 100 Ω



It is an attenuator whose structure look like π-symbol with equal shunt arm resistance R_2. A symmetrical π-configuration is shown in above fig.

From the propogation constant we have

$$\gamma = \alpha + j\beta = \ln\left[\frac{I_1}{I_2}\right]$$

From the above eqn we have

$$\frac{I_1}{I_2} = e^{\gamma} = N$$

As attenuator is purely resistive we have $\gamma = \alpha$

$$e^{\gamma} = e^{\alpha}$$

From the basics we have R_1 and R_2

$$R_1 = R_0 \sinh\alpha \quad -(1)$$

$$R_2 = R_0 \coth\frac{\alpha}{2} \quad -(2)$$

$$R_1 = R_0\left[\frac{e^{\alpha} - e^{-\alpha}}{2}\right]$$

$$\Rightarrow R_0\left[\frac{e^{2\alpha} - 1}{2e^{\alpha}}\right]$$

$$\boxed{R_1 = R_0\left[\frac{N^2 - 1}{2N}\right]}$$

$$R_2 = R_0\left[\frac{\cosh \alpha/2}{\sinh\alpha/2}\right]$$

$$= R_0\left[\frac{e^{\alpha/2}+e^{-\alpha/2}}{e^{\alpha/2}-e^{-\alpha/2}}\right]$$

$$= R_0\left[\frac{e^{\alpha}+1}{e^{\alpha}-1}\right]$$

$$\boxed{R_2 = R_0\left[\frac{N+1}{N-1}\right]}$$

Given data

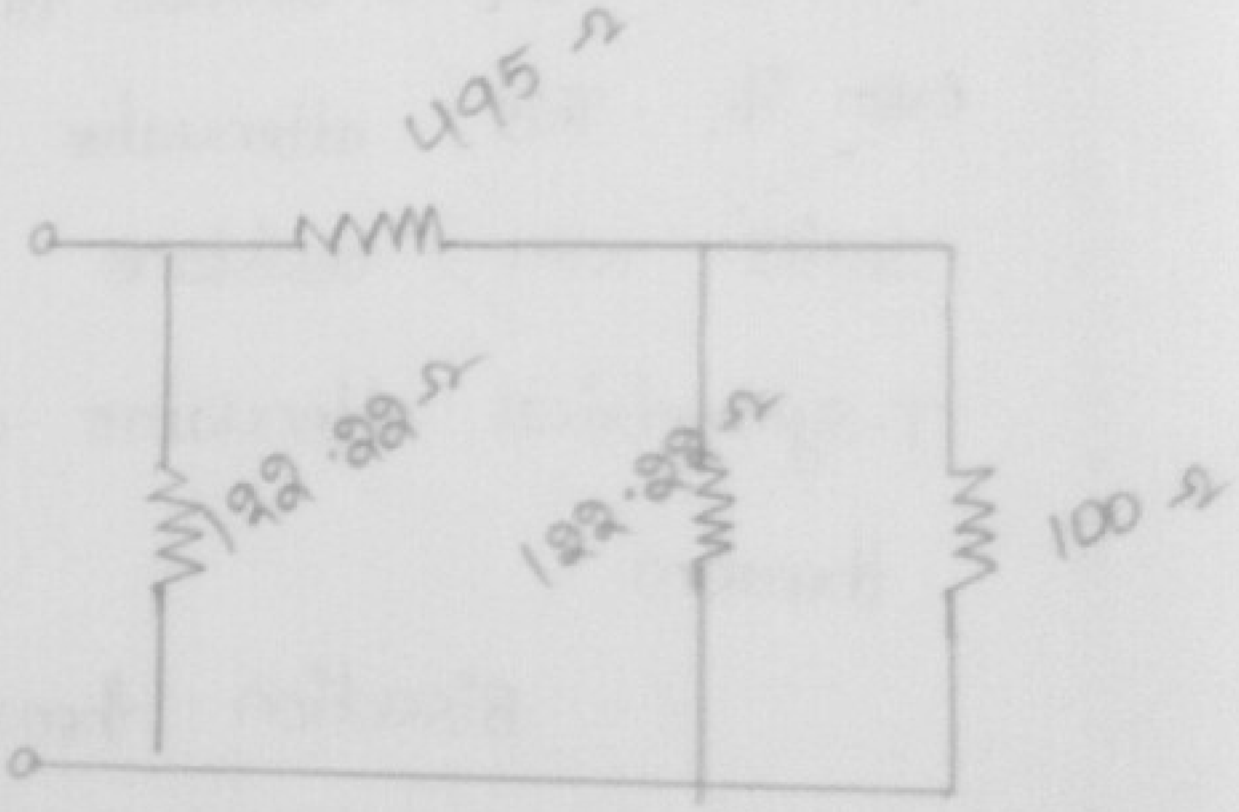
$dB = 20$

$R_0 = 100\,\Omega$

$N = 10^{\left[\frac{dB}{20}\right]}$

$= 10^{\left[\frac{20}{20}\right]}$

$N = 10$



$$R_1 = R_0\left[\frac{N^2-1}{2N}\right]$$

$$= 100\left[\frac{10^2-1}{20}\right]$$

$$= 100\left[\frac{\cdot 99}{20}\right] = 495\,\Omega$$

$$R_2 = R_0\left[\frac{N+1}{N-1}\right]$$

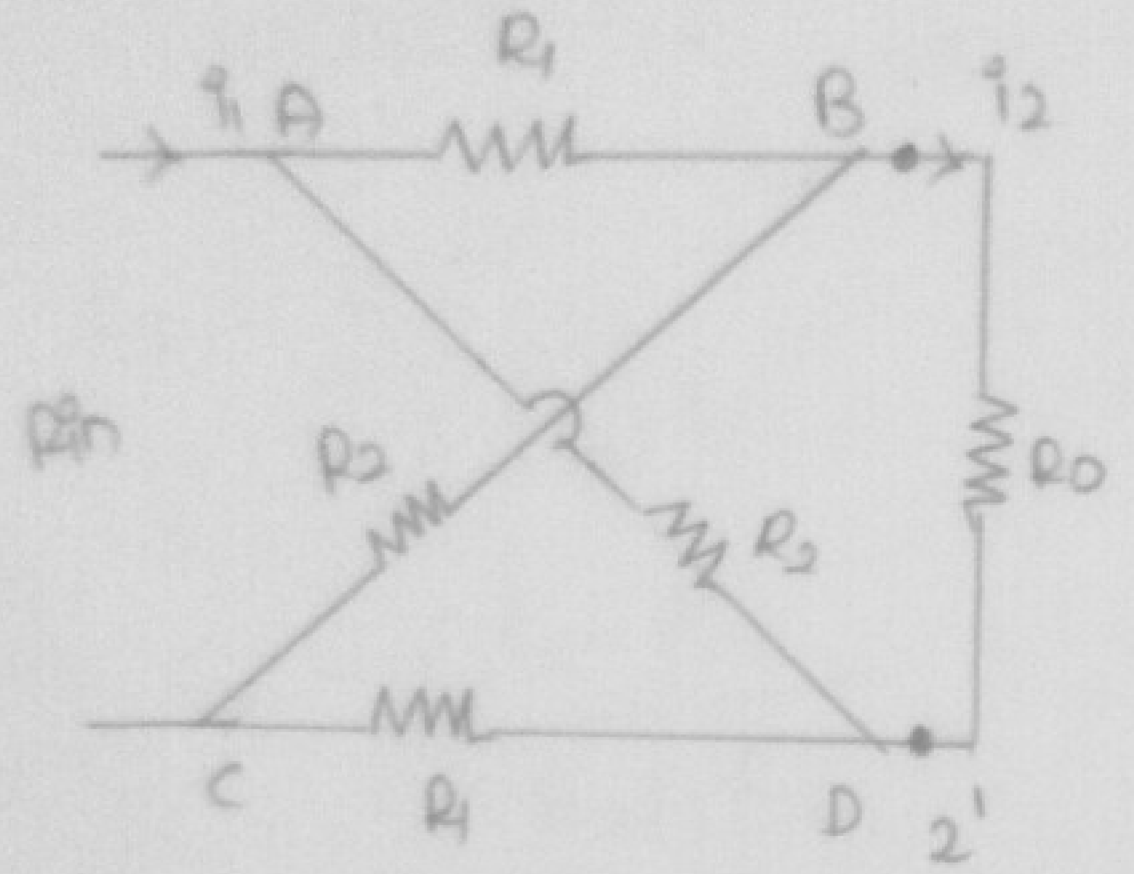
$$= 100\left[\frac{10+1}{10-1}\right]$$

$$= 100\left[\frac{11}{9}\right]$$

$$= 122.22\,\Omega$$

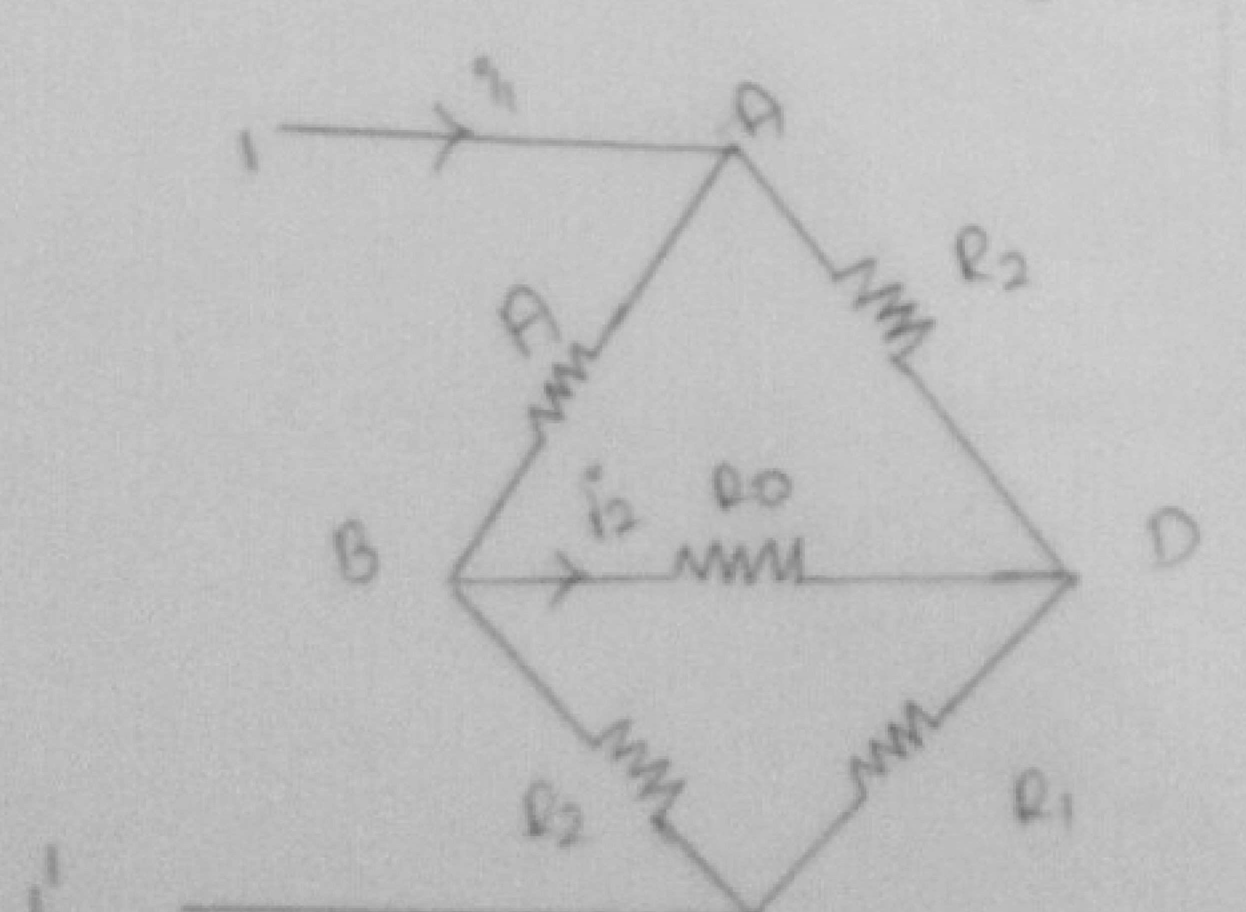
2) Explain the lattice attenuator and also design a lattice attenuator to have a characteristic impedance of 800 Ω and attenuation of 20dB.

(6)



A attenuator is said to be a lattice attenuator if its is consists of two series arm impedance two diagonal arm impedance. A lattice attenuator is said to be symmetrical lattice attenuator when its series arm impedance are equal and diagonal arm impedance are equal. A lattice attenuator is as shown in figure.

The above n/w is redrawn as shown in figure



The characteristic impedance interms of open circuit and short circuit impedance (or) resistance is given as

$$Z_0 = \sqrt{Z_{oc} \cdot Z_{sc}}$$

$$R_0 = \sqrt{R_{oc} \cdot R_{sc}} \quad — ⓐ$$

where R_{oc} is the equivalent resistance at Port 1-1' when the port 2-2' is open circuited.

R_{sc} is the equivalent resistance at port 1-1' when Port 2 terminals are short circuited.

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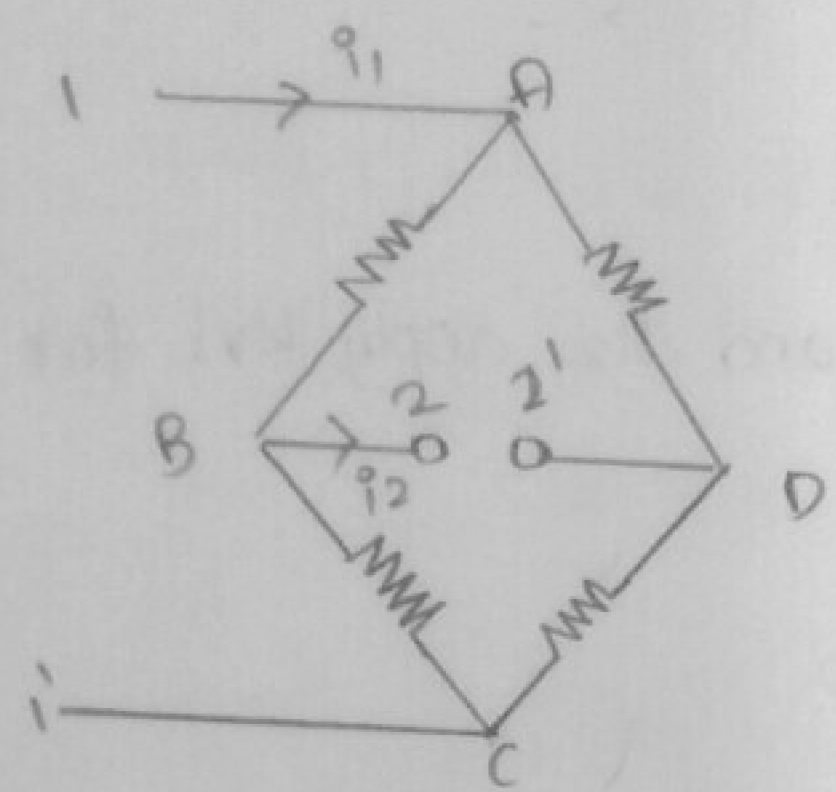
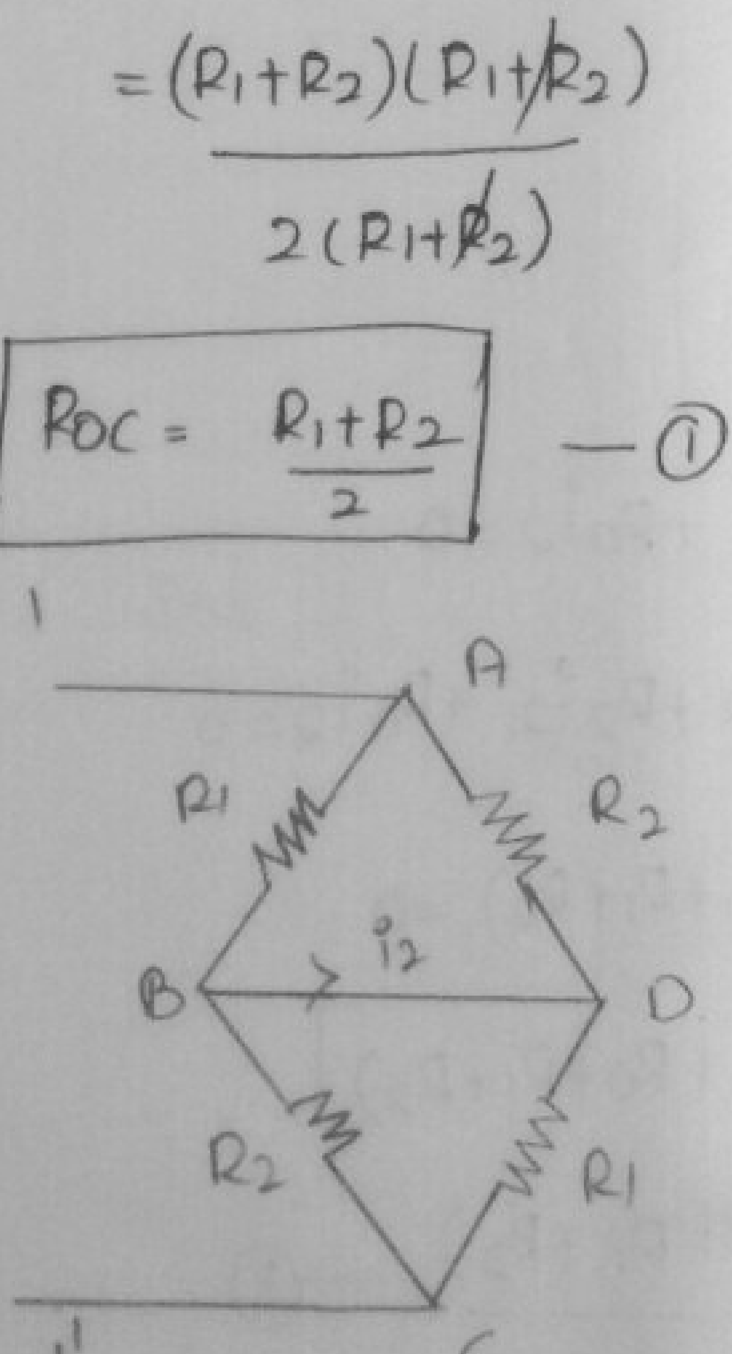


$$R_{oc} = (R_1+R_2) \parallel R_1+R_2$$

$$= \frac{(R_1+R_2)(R_1+\cancel{R_2})}{2(R_1+\cancel{R_2})}$$

$$\boxed{R_{oc} = \frac{R_1+R_2}{2}} \quad — ①$$



$$R_{sc} = [R_1 \parallel R_2] + [R_1 \parallel R_2]$$

$$\frac{R_1R_2}{R_1+R_2} + \frac{R_1R_2}{R_1+R_2}$$

$$\boxed{R_{sc} = \frac{2R_1R_2}{R_1+R_2}} \quad -②$$

Substitute ① and ② in eq ⓐ

$$R_0 = \sqrt{\left(\frac{R_1+R_2}{\cancel{2}}\right)\left(\frac{\cancel{2}R_1R_2}{R_1+R_2}\right)}$$

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$$\boxed{R_0 = \sqrt{R_1R_2}}$$

$$\sqrt{R_2} = \frac{R_0}{\sqrt{R_1}}$$

In order to find R_1 and R_2 apply KVL for upper loop we have

$$R_2(i_1-i) - R_0 i_2 - R_1 i = 0$$

$$-i[R_2+R_1] = R_0 i_2 - R_2 i_1$$

$$i = \frac{R_2 i_1 - R_0 i_2}{R_2+R_1}$$

Apply KVL to 2nd loop

$$R_1(i_1 - i + i_2) - R_2(i - i_2) + R_0 i_2 = 0$$

$$-i(R_1+R_2) + R_1 i_1 + R_1 i_2 + R_2 i_2 + R_0 i_2 = 0$$

$$-i(R_1+R_2) + R_1 i_1 + i_2(R_0+R_1+R_2) = 0$$

$$-i(R_1+R_2) = -[R_1 i_1 + i_2(R_0+R_1+R_2)]$$

$$i = \frac{R_1 i_1 + i_2(R_0+R_1+R_2)}{R_1+R_2} \quad -③$$

equating ③ & ④

$$\frac{R_2 i_1 - R_0 i_2}{\cancel{R_1+R_2}} = \frac{R_1 i_1 + i_2(R_0+R_1+R_2)}{\cancel{R_1+R_2}}$$

Given attenuation in dB = 20 ②

$R_0 = 800\,\Omega$

$N = \text{antilog}\left[\frac{dB}{20}\right]$

$= 10^{\left[\frac{dB}{20}\right]}$

$= 10^{\left[\frac{20}{20}\right]}$

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$N = 10$

$R_1 = R_0\left[\frac{N-1}{N+1}\right]$

$= 800\left[\frac{10-1}{10+1}\right]$

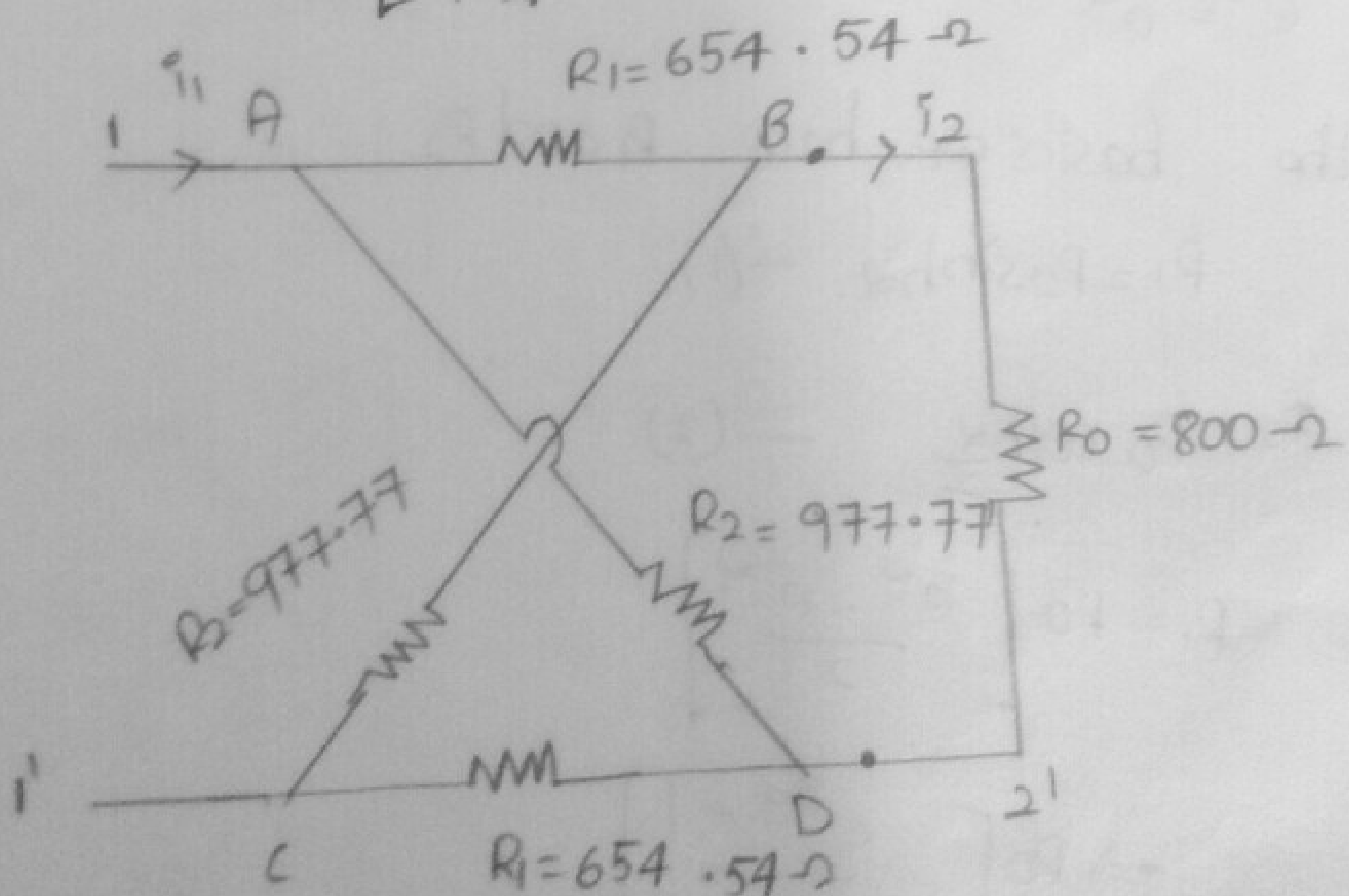
$= 800\left[\frac{9}{11}\right]$

$= 654.5454\,\Omega$

$R_2 = R_0\left[\frac{N+1}{N-1}\right]$

$= 800\left[\frac{10+1}{10-1}\right]$

$= 800\left[\frac{11}{9}\right] = 977.777\,\Omega$

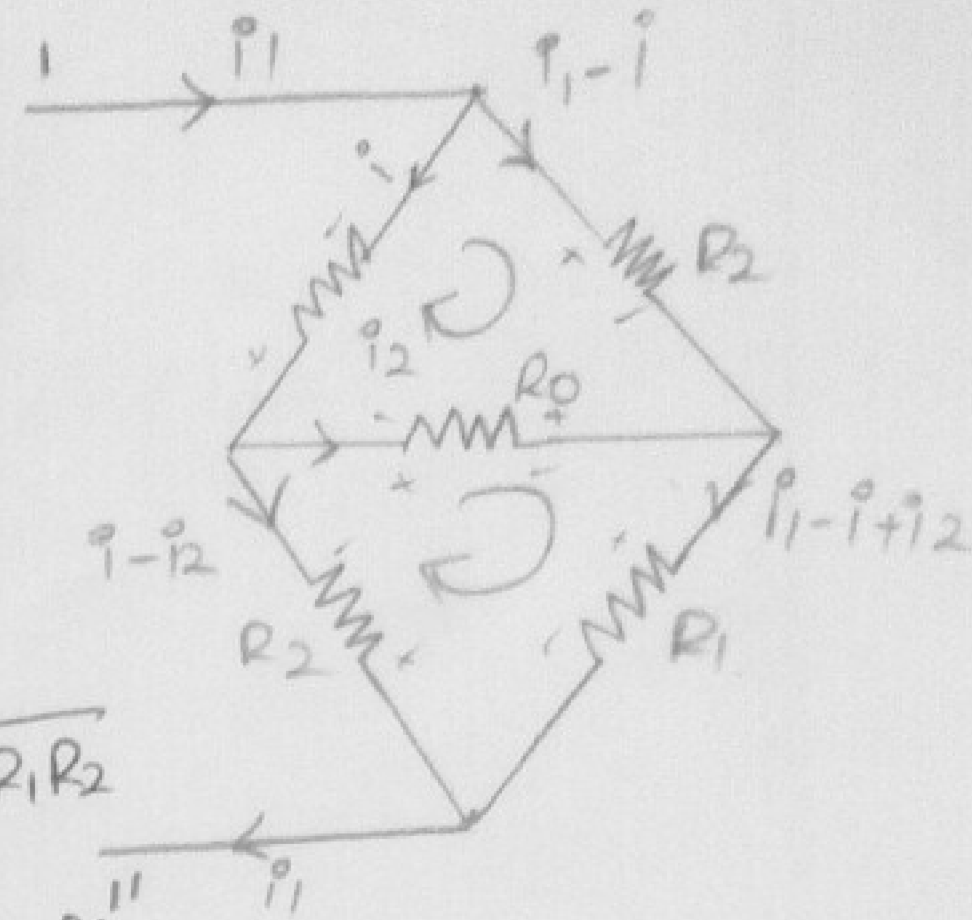


3

$$i_1(R_2-R_1) - i_2(2R_0+R_1+R_2)=0$$

$$i_1(R_2-R_1) = i_2(2R_0+R_1+R_2)$$

$$\frac{i_1}{i_2} = \frac{2R_0+R_1+R_2}{R_2-R_1} = N$$



$$\frac{i_1}{i_2} = N = e^{\gamma} \to e^{\alpha}$$

$$(R_2-R_1)N = R_1+R_2+2\sqrt{R_1 R_2}$$

$$N(R_2-R_1) = (\sqrt{R_1})^2+(\sqrt{R_2})^2+2\sqrt{R_1R_2}$$

$$N(\cancel{\sqrt{R_2}+\sqrt{R_1}})(\sqrt{R_2}-\sqrt{R_1}) = (\sqrt{R_1}+\sqrt{R_2})^{\cancel{2}}$$

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$$N(\sqrt{R_2}-\sqrt{R_1}) = \sqrt{R_1}+\sqrt{R_2}$$

sub $\sqrt{R_2} = \frac{R_0}{\sqrt{R_1}}$

$\frac{R_0}{\sqrt{R_2}}$

$$N\left(\frac{R_0}{\sqrt{R_1}} - \sqrt{R_1}\right) = \sqrt{R_1}+\frac{R_0}{\sqrt{R_1}}$$

$$N\left(\frac{R_0-R_1}{\cancel{\sqrt{R_1}}}\right) = \frac{R_1+R_0}{\cancel{\sqrt{R_1}}}$$

$$\boxed{R_1 = R_0\left[\frac{N-1}{N+1}\right]}$$

sub $\sqrt{R_1} = \frac{R_0}{\sqrt{R_2}}$

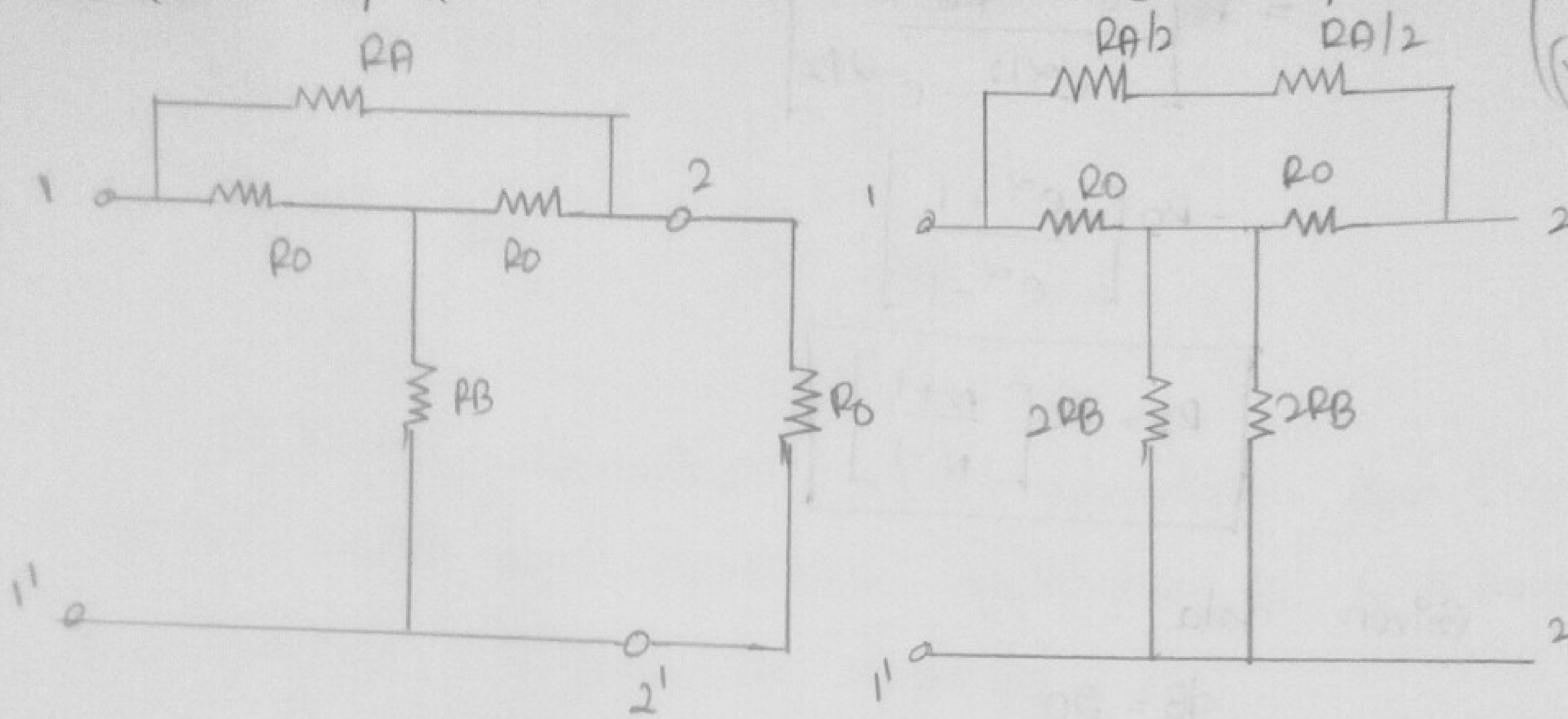
$$N\left(\sqrt{R_2} - \frac{R_0}{\sqrt{R_2}}\right) = \frac{R_0}{\sqrt{R_2}} + \sqrt{R_2}$$

$$N\left(\frac{R_2-R_0}{\cancel{\sqrt{R_2}}}\right) = \frac{R_0+R_2}{\cancel{\sqrt{R_2}}}$$

$$NR_2 - NR_0 = R_2+R_0$$

$$\boxed{R_2 = R_0\left[\frac{N+1}{N-1}\right]}$$

4) Explain bridged T-attenuator and also design it whith an attenuation of 20 dB and terminated in a load of 500 Ω



It is similar to the T-attenuator with extra bridge over it. This attenuator is said to be symmetrical when its series arm resistance is equal. The design of the bridged T-symmetrical attenuator will be carried through bisection theorem

Bisection theorem states that the n/w with mirror image symmetry can be converted into equivalent lattice structure were the series arm impedance of the equivalent lattice structure is equal to short circuited i/p impedance of the bisected n/w were as diagonal arm impedance is equal open circuited impedance of the bisected n/w.

The above can be redrawn as shown in figure from the bisection theorem $R_1 = R_{SC}$.

$$(R_A + 2R_0)\left[\frac{N-1}{N+1}\right] = R_A$$

$$R_A\left[\frac{N-1}{N+1}\right] + 2R_0\left[\frac{N-1}{N+1}\right] = R_A$$

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$$2R_0\left[\frac{N-1}{N+1}\right] = R_A\left[1 - \frac{N-1}{N+1}\right]$$

$$2R_0\left[\frac{N-1}{N+1}\right] = R_0\left[\frac{\cancel{N}+1-\cancel{N}+1}{N+1}\right]$$

$$\frac{2R_0\left[\frac{N-1}{\cancel{N+1}}\right]}{\frac{2}{\cancel{N+1}}} = R_A$$

$$\boxed{R_A = R_0[N-1]}$$

$$R_2 = R_0\left[\frac{N+1}{N-1}\right] \quad -④$$

equating ③ and ④

$$R_0 + 2R_B = R_0\left[\frac{N+1}{N-1}\right]$$

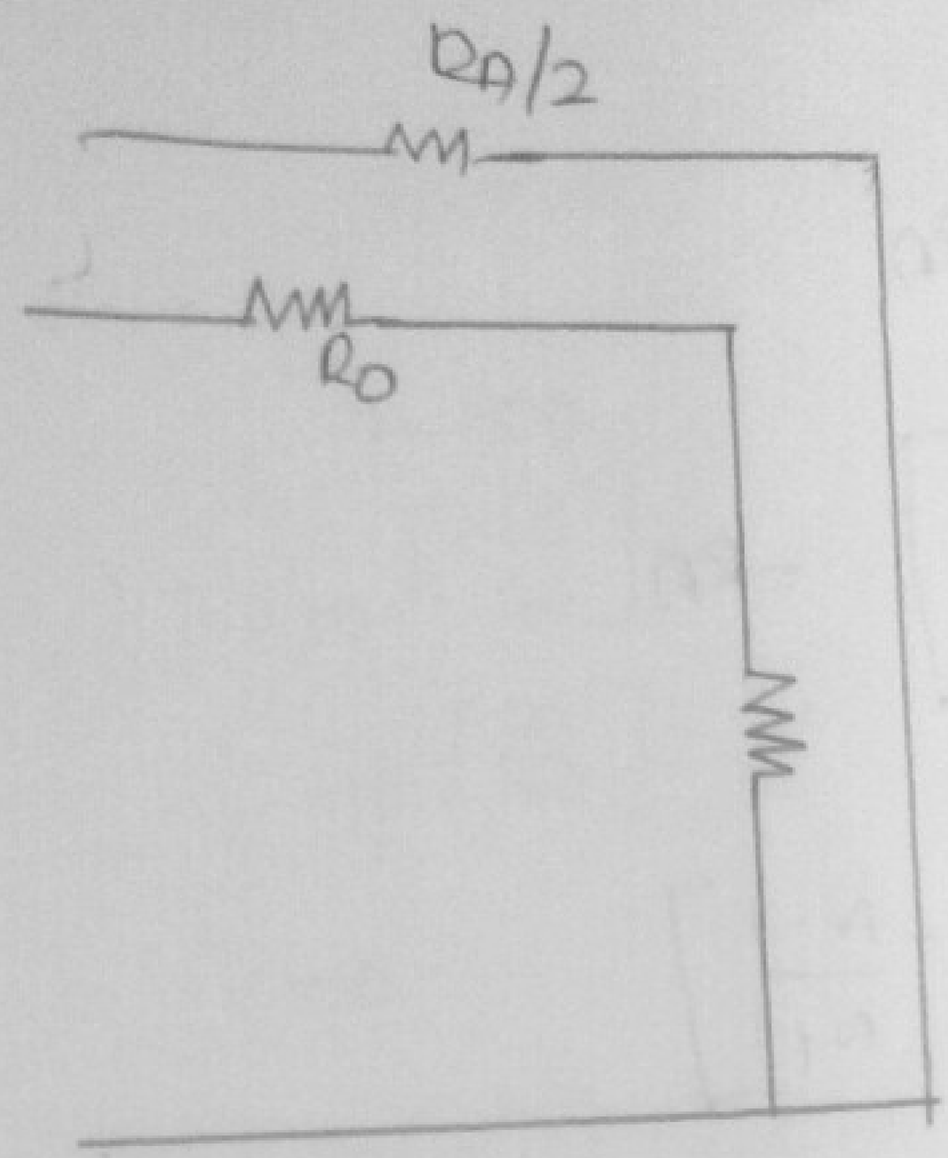
$$2R_B = R_0\left[\frac{N+1}{N-1} - 1\right]$$

$$2R_B = R_0\left[\frac{\cancel{N}+1-\cancel{N}+1}{N-1}\right]$$

$$\boxed{R_B = \frac{R_0}{N-1}}$$

The bridged T-n/w is said to be symmetrical when the bridge arm and shunt arm satisfies

$$R_A = R_B = R_0^2$$

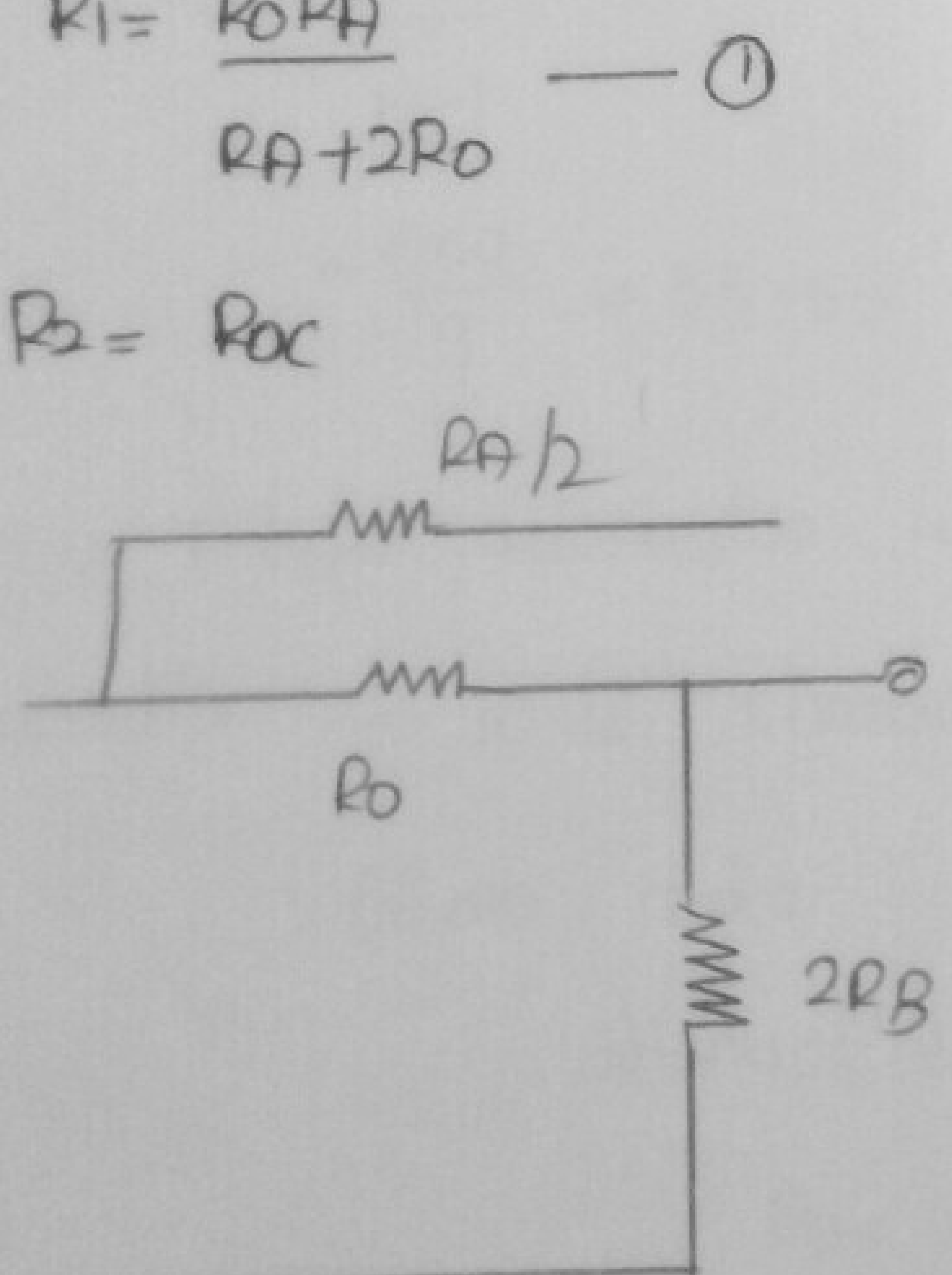


$R_{SC} = R_A/2 \parallel R_O$

$$R_{SC} = \frac{\frac{R_A}{2} \cdot R_O}{\frac{R_A}{2} + R_O} = \frac{R_O R_A}{R_A + 2R_O} \qquad (\because R_{SC} = R_1)$$

$$R_1 = \frac{R_O R_A}{R_A + 2R_O} \quad \text{— (1)}$$

$R_2 = R_{OC}$



$$R_{OC} = R_O + 2R_B \quad \text{— (2)}$$

From the lattice attenuator we know that

$$R_1 = R_O \left[\frac{N-1}{N+1}\right] \quad \text{— (3)}$$

equating eq (1) and (3)

$$\cancel{R_O}\left[\frac{N-1}{N+1}\right] = \frac{\cancel{R_O} R_A}{R_A + 2R_O}$$

Given $dB = 20$

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$R_o = 500$

$N = 10^{\left[\frac{dB}{20}\right]}$

$= 10^{\left[\frac{20}{20}\right]}$

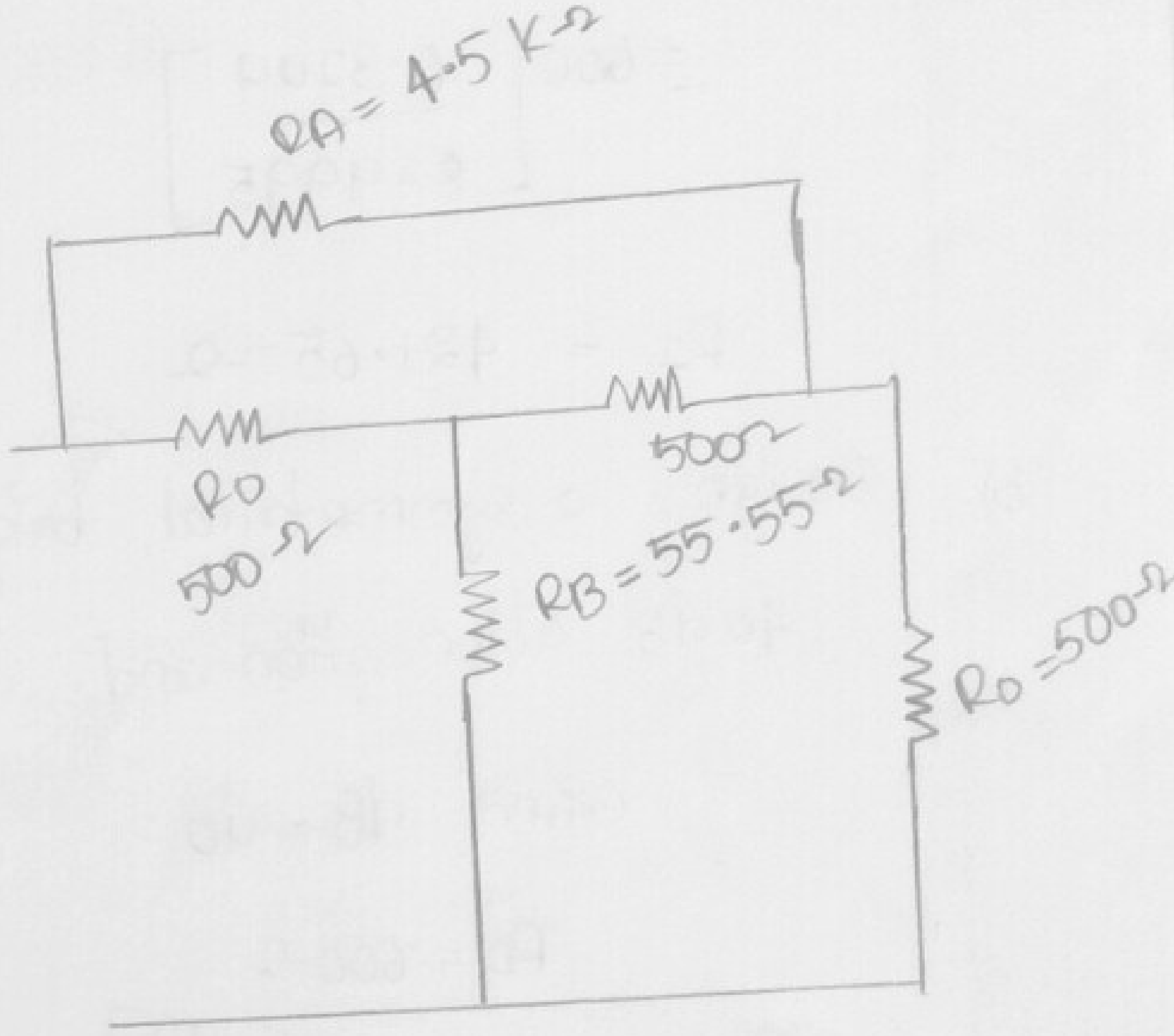
$N = 10$

$R_A = R_o[N-1]$

$= 500[10-1]$

$= 4500$

$= 4.5\,K\Omega$



$R_B = \frac{R_o}{N-1}$

$= \frac{500}{9} = 55.55\ \Omega$

5 a) design a T-type attenuator with the following specification attenuation = 10dB, characteristic impedance = 600 Ω

Given data

attenuation = 10dB

$R_o = 600\ \Omega$

$N = 10^{\left[\frac{dB}{20}\right]}$

$= 10^{\left[\frac{10}{20}\right]} = 3.1622$

$R_1 = R_o\left[\frac{N-1}{N+1}\right]$

$= 600\left[\frac{3.1622-1}{3.1622+1}\right] = 600\left[\frac{2.1622}{4.1622}\right]$

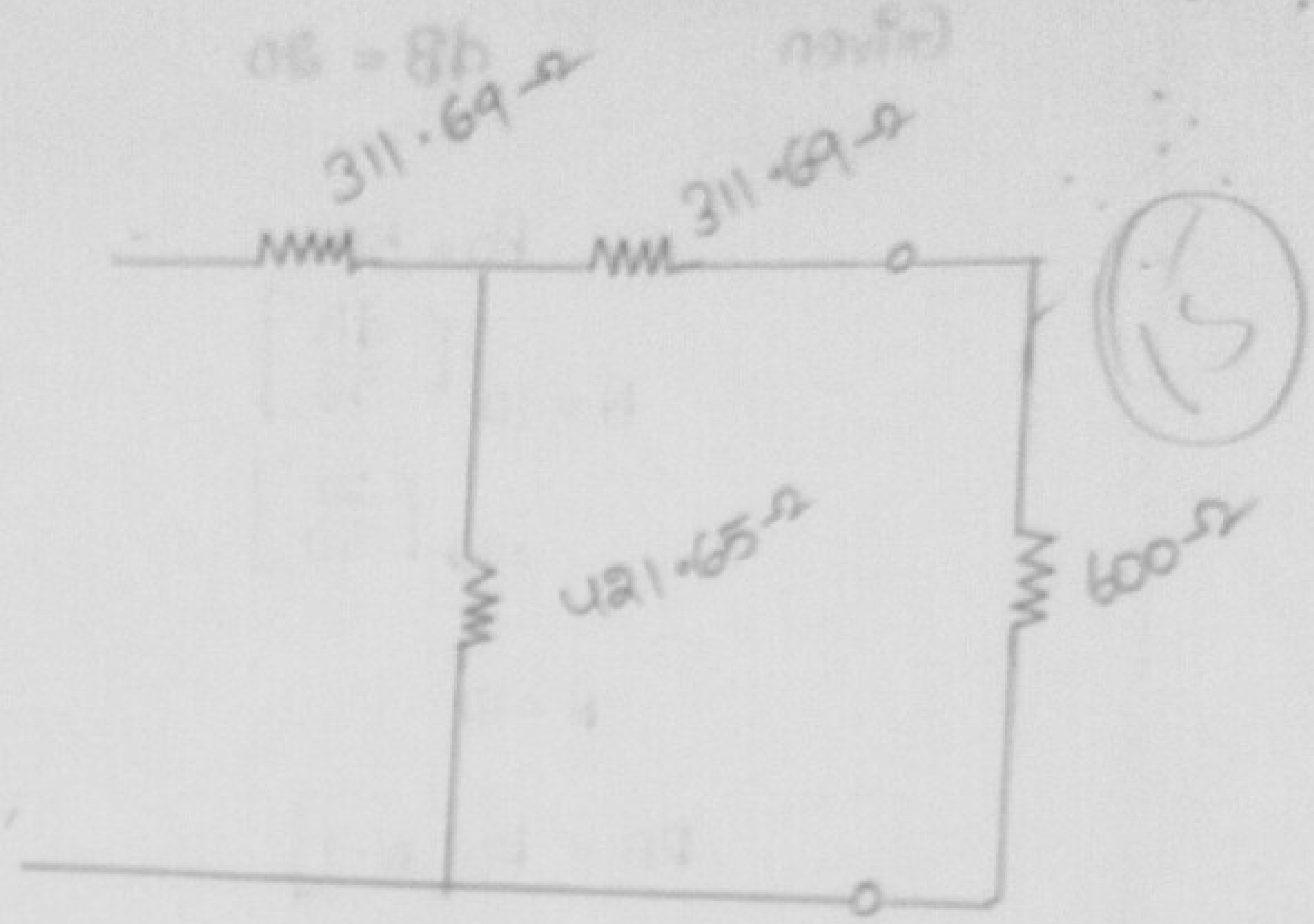
$= 311.69\ \Omega$

$$R_2 = R_0\left[\frac{2N}{N^2-1}\right]$$

$$= 600\left[\frac{2[3.1622]}{(3.1622)^2-1}\right]$$

$$= 600\left[\frac{6.3244}{8.9995}\right]$$

$$R_2 = 421.65\ \Omega$$



b) Design a symmetrical bridged T-type attenuator with 40 dB attenuation and design impedance of 600 Ω

Given dB = 40

$R_0 = 600\ \Omega$

$$N = 10^{\left[\frac{40}{20}\right]}$$

$N = 100$

$$R_A = R_0[N-1]$$

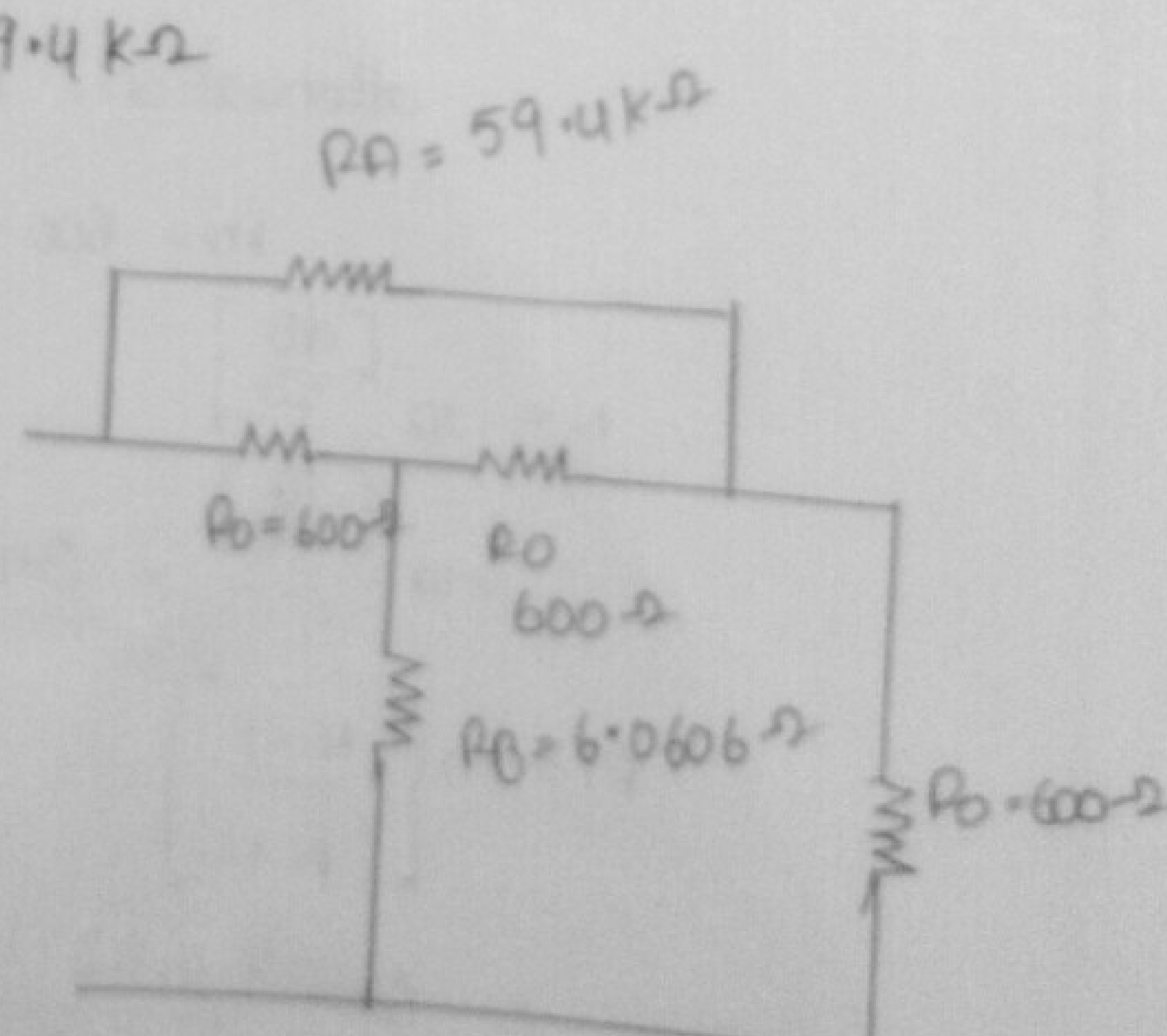
$$= 600[100-1]$$

$$= 600[99] = 59.4\ k\Omega$$

$$R_B = \frac{R_0}{N-1}$$

$$= \frac{600}{99}$$

$$R_B = 6.0606\ \Omega$$



3 a) Design a symmetrical lattice attenuator with 20dB attenuation and characteristic impedance of 600Ω.

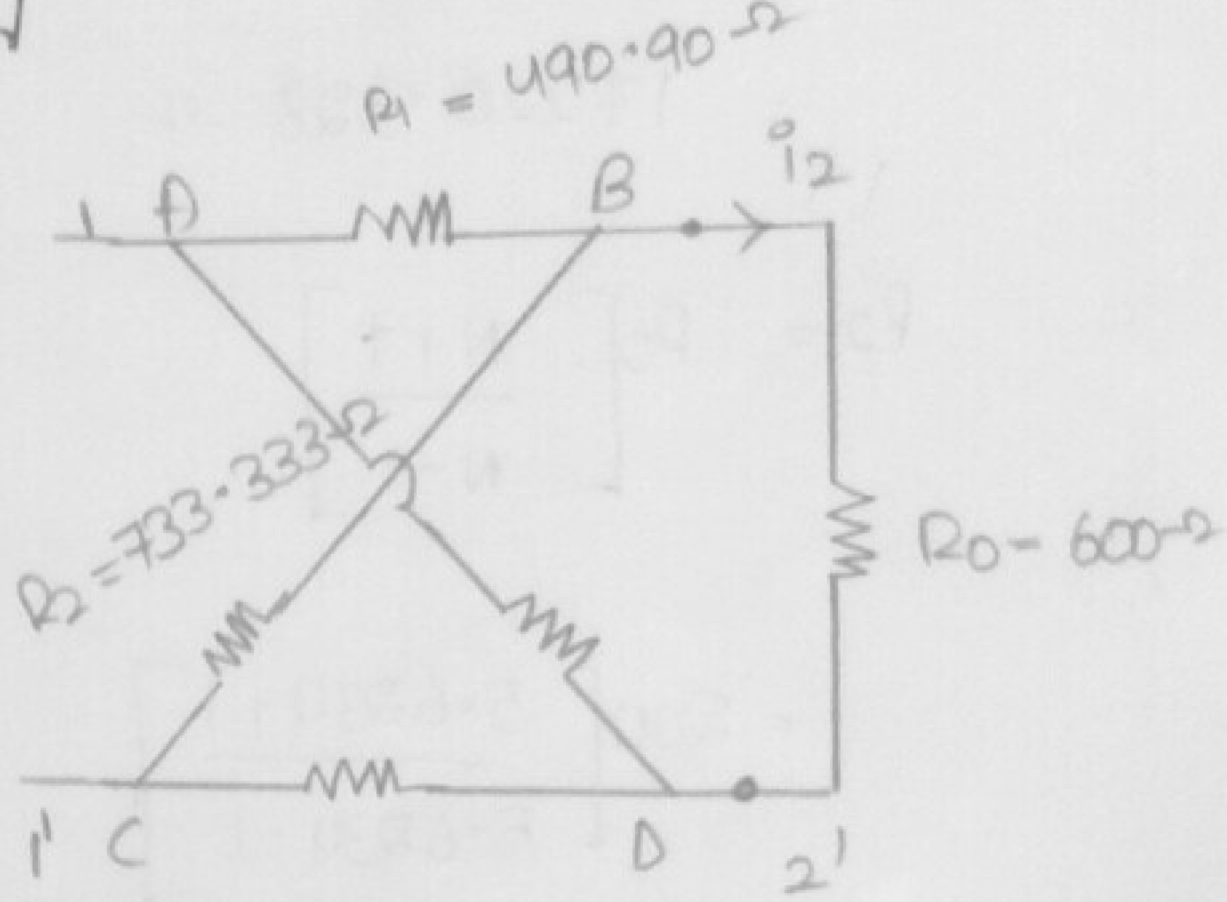
Given data

$dB = 20$

$R_0 = 600$

$N = 10^{\left[\frac{dB}{20}\right]} = 10^{\left[\frac{20}{20}\right]} = 10$

$N = 10$



$R_1 = R_0\left[\frac{N-1}{N+1}\right]$

$= 600\left[\frac{10-1}{10+1}\right]$

$= 600\left[\frac{9}{11}\right] = 490.90\,\Omega$

$R_2 = R_0\left[\frac{N+1}{N-1}\right]$

$= 600\left[\frac{11}{9}\right] = 733.333\,\Omega$

b) Design a π type attenuator to provide an attenuation of 15dB. characteristic impedance $= 200\,\Omega$

Given data

$dB = 15$

$R_0 = 200\,\Omega$

$N = 10^{\left[\frac{dB}{20}\right]}$

$= 10^{\left[\frac{15}{20}\right]}$

$N = 5.6234$

$$R_1 = R_0\left[\frac{N^2-1}{2N}\right]$$

$$= 200\left[\frac{(5\cdot6234)^2-1}{2(5\cdot6234)}\right]$$

$$= 200\left[86\cdot1016\right]$$

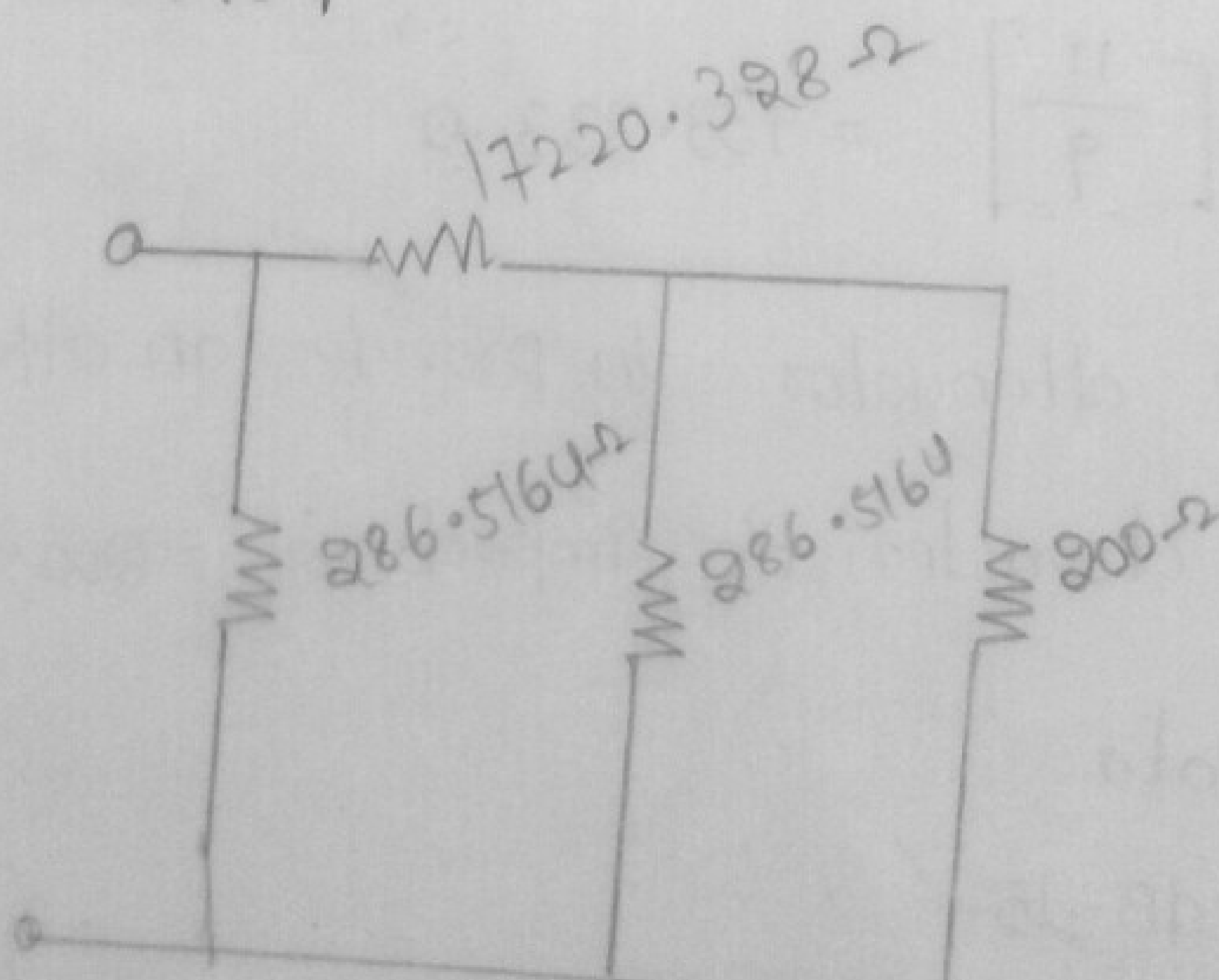
$$= 17220\cdot328\ \Omega$$

$$R_2 = R_0\left[\frac{N+1}{N-1}\right]$$

$$= 200\left[\frac{5\cdot6234+1}{5\cdot6234-1}\right]$$

$$= 200\left[\frac{6\cdot6234}{4\cdot6234}\right]$$

$$= 286\cdot5164\ \Omega$$



$\Rightarrow z_1 e^{\gamma} = z_2 (e^{\gamma} - 2 + e^{-\gamma}) e^{\gamma}$

$\Rightarrow z_1 = z_2 (e^{\gamma} - 2 + e^{-\gamma})$

$\Rightarrow z_1 = z_2 (e^{\gamma} + e^{-\gamma}) - 2 z_2$

$\Rightarrow \dfrac{z_1 + 2z_2}{z_2} = e^{\gamma} + e^{-\gamma}$

$\Rightarrow \dfrac{z_1 + 2z_2}{2z_2} = \dfrac{e^{\gamma} + e^{-\gamma}}{2}$

$\Rightarrow 1 + \dfrac{z_1}{2z_2} = \cosh\gamma$

$\Rightarrow$ From the hyperbolic relations, we know that

$z_2 = \dfrac{R_A R_B}{R_A + R_B}$

$\Rightarrow \cosh 2\gamma = 1 + 2\sinh^2\gamma$

$\Rightarrow \cosh\gamma = 1 + \dfrac{z_1}{2z_2}$

$\Rightarrow \cosh 2 \cdot \dfrac{\gamma}{2} = 1 + 2\sinh^2\dfrac{\gamma}{2} = 1 + \dfrac{z_1}{2z_2}$

$\Rightarrow 2\sinh^2\dfrac{\gamma}{2} = \dfrac{z_1}{2z_2}$

$\Rightarrow \sinh\dfrac{\gamma}{2} = \sqrt{\dfrac{z_1}{4z_2}}$

$\Rightarrow$ we know that the propogation constant $\gamma = \alpha + J\beta$

where α is attenuation &

β is phase shift.

$\Rightarrow \sinh\dfrac{\gamma}{2} = \sinh\left(\dfrac{\alpha + J\beta}{2}\right) = \sqrt{\dfrac{z_1}{4z_2}}$

$\Rightarrow \sinh\left(\dfrac{\alpha}{2} + J\dfrac{\beta}{2}\right) = \sqrt{\dfrac{z_1}{4z_2}}$

$\Rightarrow \sinh\dfrac{\alpha}{2}\cos\dfrac{\beta}{2} + J\cosh\dfrac{\alpha}{2}\sin\dfrac{\beta}{2} = \sqrt{z_1/4z_2}$

Characteristic Impedance and classification of pass and stop band:-

$\Rightarrow$ 1. characteristic impedance of symmetrical T-n/w:

$$Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}$$

$$\Rightarrow Z_{0\pi} = \sqrt{\frac{Z_1 Z_2}{1 + \frac{Z_1}{4Z_2}}}$$

$$\Rightarrow \sinh\frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}}$$

$$\Rightarrow \sinh\frac{\alpha}{2}\cos\frac{\beta}{2} + j\cosh\frac{\alpha}{2}\sin\beta/2 = \sqrt{\frac{Z_1}{4Z_2}}$$

Case(i):- When Z_1 and Z_2 are of same reactances, then

$$\Rightarrow \left|\frac{Z_1}{4Z_2}\right| > 0 \Rightarrow \left|\frac{Z_1}{4Z_2}\right| > 0 \quad \text{i.e.,} \sqrt{\frac{Z_1}{4Z_2}} = x + j\cdot 0 \text{ [real value]}$$

$\Rightarrow$ But, we know that

$$\Rightarrow \sinh\gamma/2 = x + j(0)$$

$$\Rightarrow \sinh\frac{\alpha}{2}\cdot\cos\beta/2 + j\cosh\frac{\alpha}{2}\sin\beta/2 = x + j(0)$$

$\Rightarrow$ comparing real and imaginary parts on both sides

$$\Rightarrow \sinh\frac{\alpha}{2}\cos\beta/2 = x \longrightarrow (1)$$

$$\cosh\frac{\alpha}{2}\sin\beta/2 = 0 \longrightarrow (2)$$

$\Rightarrow$ eq. (1) & (2) will be satisfy when $\beta = 0$, Hence from eq (1)

$$\Rightarrow \sinh\alpha/2 = x$$

$$\Rightarrow \alpha = 2\sinh^{-1}\left(\sqrt{\frac{Z_1}{4Z_2}}\right) \Rightarrow \alpha = 2\sinh^{-1}\left(\sqrt{\frac{Z_1}{4Z_2}}\right) \neq 0.$$

$\Rightarrow$ When both the reactances are of same ~~reactance~~ type there exist attenuation band (or) stop band (since $\alpha \neq 0$ stop band)

Case(ii):- When Z_1 and Z_2 are of opposite reactances,

$$\Rightarrow \frac{Z_1}{4Z_2} < 0 \Rightarrow \frac{Z_1}{4Z_2} < 0$$

i.e.,

$$\Rightarrow \sqrt{\frac{Z_1}{4Z_2}} = 0 + j\cdot x \text{ (imaginary value)}$$

$\Rightarrow \sinh\frac{\alpha}{2}\cos\frac{\beta}{2} + j\cosh\frac{\alpha}{2}\sin\frac{\beta}{2} = 0 + jx$

$\Rightarrow \sinh\frac{\alpha}{2}\cos\frac{\beta}{2} = 0 \longrightarrow ①$

$\cosh\frac{\alpha}{2}\sin\frac{\beta}{2} = x \longrightarrow ②$

$\Rightarrow$ eq ① will be satisfied when $\alpha = 0$ (pass band) and

$\beta = \pi, 3\pi, \ldots n\pi$

$\beta = (2n+1)\pi$ where n is integer

$\Rightarrow$ <u>case(A)</u> :- When $\alpha = 0$ there exists a pass band since the attenuation is zero.

$\Rightarrow$ <u>case(B)</u> :- When $\beta = \pi$,

from eq. ② $\Rightarrow \cosh\frac{\alpha}{2} = x$

$\alpha = 2\cosh^{-1}(x) \neq 0$

$\Rightarrow$ Hence there exists a stop band.

<u>[illegible]</u>

Condition for pass band and stop band with the value of characteristic impedance:-

We know that from propagation constant,

$$\sinh\frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}}$$

$$\Rightarrow \sinh\frac{\alpha}{2}\cos\frac{\beta}{2} + j\cosh\frac{\alpha}{2}\sin\frac{\beta}{2} = \sqrt{\frac{Z_1}{4Z_2}}$$

<u>case(i)</u>:-

$\rightarrow$ When both impedances are of same type, the value of $\frac{Z_1}{4Z_2} > 0$ (which is a real value)

$\rightarrow$ Under this condition, the value of $\alpha = 2\sinh^{-1}\sqrt{\frac{Z_1}{4Z_2}} \neq 0$

Hence there exists a stop band i.e., the condition for stop band is that $\frac{Z_1}{4Z_2} > 0$

Case 2:-

When Z_1 and Z_2 are of opposite reactances $\sqrt{\frac{Z_1}{4Z_2}}$ is an imaginary value, which gives two sets of equations 3(8)

$$\sinh\frac{\alpha}{2}\cos\frac{\beta}{2} = 0 \longrightarrow ①$$

$$\cosh\frac{\alpha}{2}\sin\frac{\beta}{2} = x \longrightarrow ②$$

→ When $\beta = \pi$, eq.① $= 0$

from eq② $\cosh\frac{\alpha}{2} = x$

$$\alpha = 2\cosh^{-1}(x) = 2\cosh^{-1}\sqrt{\frac{Z_1}{4Z_2}} \neq 0$$

→ Again there exists a stop band.

→ And the condition for stop band here is when $\frac{Z_1}{4Z_2} < -1$

→ When $\alpha = 0$,

eq.① $= 0$ and there is (exists) a pass band

from eq.② $\sin\frac{\beta}{2} = x = \sqrt{\frac{Z_1}{4Z_2}}$

$$\beta = 2\sin^{-1}\sqrt{\frac{Z_1}{4Z_2}}$$

→ As we know the limits of sin exists between -1 to 1 (-1 to 1), Hence the value of $\frac{Z_1}{4Z_2}$ will also be lies between -1 to 1.

** → But we know that when $\frac{Z_1}{4Z_2} > 0$, it will enter into the attenuation band. Hence to have pass band, the value of $\frac{Z_1}{4Z_2}$ should be between -1 to 0.

$$-1 < \frac{Z_1}{4Z_2} < 0$$

→ The value of frequency at which the filter changes its state from stop band to pass band and vice versa is termed as cutoff frequency (or) nominal frequency.

→ From the pass band condition, we have

$$-1 \le \frac{Z_1}{4Z_2} \le 0$$

from the lower limit

$$\frac{Z_1}{4Z_2} = -1 \Rightarrow Z_1 + 4Z_2 = 0$$

→ from the upper limit

$$\Rightarrow \frac{Z_1}{4Z_2} = 0 \Rightarrow Z_1 = 0.$$

** → From symmetrical T network, we know the characteristic impedance is,

$$Z_{OT} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}$$

(opposite type of reactance)

→ Under stop band condition i.e., when $\frac{Z_1}{4Z_2} > 0$ (or) $\frac{Z_1}{4Z_2} < -1$

(same type of reactance)

→ The value of characteristic impedance

$$Z_{OT} = \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)}$$

case(i) :-

Let $Z_1 = Jx_1$, $Z_2 = Jx_2$ and $\frac{Z_1}{4Z_2} > 0$

$$\Rightarrow Z_{OT} = \sqrt{J^2 x_1 x_2 \left(1 + \frac{x_1}{4x_2}\right)}$$

$$\Rightarrow Z_{OT} = \sqrt{-1}\,(K) = JK \text{ (imaginary)}$$

→ Under stop band condition, is always imaginary.

→ Hence the characteristic impedance under attenuation band is always imaginary

case(ii) :- Under pass band

$$-1 < \frac{Z_1}{4Z_2} < 0 \text{ (opposite reactances)}$$

$$Z_1 = J x_1$$

$$Z_2 = -J x_2$$

$$\Rightarrow Z_{OT} = \sqrt{-J^2 x_1 x_2 \left(1 + \frac{Jx_1}{-4Jx_2}\right)} = \sqrt{x_1 x_2 \left(1 - \frac{x_1}{4x_2}\right)} = K$$

$1 - \frac{x_1}{4x_2} > 0$ (since from pass band condition)

⇒ Hence under pass band condition, the characteristic impedance will be always real.

Characteristic of filters:- 3(9)17

1. Under pass band condition, the characteristic impedance of the filter is real always
2. Under stop band condition, the characteristic network of the filter is imaginary always

Relation between Z_{OT} and $Z_{O\pi}$:-

$\Rightarrow$ $Z_{OT} = \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)}$, $Z_{O\pi} = \sqrt{\frac{Z_1 Z_2}{\frac{Z_1}{4Z_2} + 1}}$

$\Rightarrow$ $Z_{OT} = \sqrt{Z_1 Z_2 \sqrt{Z_{O\pi}}}$

$\Rightarrow$ $Z_{OT}^2 = Z_1^2 Z_2^2 Z_{O\pi}^2$ $\Rightarrow$ $Z_{O\pi} = \frac{Z_1 Z_2}{Z_{OT}}$

$\Rightarrow$ Under stop band $Z_{O\pi} = \frac{K}{JK} = \frac{Jx_1 Jx_2}{JK} = j\frac{x_1 x_2}{K}$

$Z_{O\pi} = jA$ (imaginary)

$\Rightarrow$ Under pass band $Z_{O\pi} = \frac{(Jx_1)(-Jx_2)}{K'} = -\frac{x_1 x_2}{K} = B$

$\Rightarrow$ So impedance is real.

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Type of filters:-

The filters are primarily classified into 2 types basically

1. Active filters (constructed with operational amplifier)
2. Passive filters (constructed with L, C, elements)

→ The passive filters are again classified into two types

1. constant k (or) prototype filters
2. m- derived filters.

1. Constant-K Filters :-

→ A filter is said to be constant-K when its elements satisfy the relationship that

$$Z_1 Z_2 = K^2$$

Where Z_1 = series arm impedance

Z_2 = shunt arm impedance

K = Nominal (or) design impedance

→ constant K-filters are classified in 2 types.

1. constant-K low pass filter.
2. constant-K High pass filter.

1. constant-K Low pass filter :-

→ When the series arm impedance is inductance & the shunt arm impedance is capacitance, then that filter exhibits the characteristics of a LPF.

⇒ Let $Z_1 = JWL$ and $Z_2 = \frac{1}{JWC} = -JX_C$

then $Z_1 Z_2 = JWL \cdot \frac{1}{JWC} = \frac{L}{C} = K^2$

$$\frac{L}{C} = K^2$$

$$\therefore K^2 = \frac{L}{C}$$

∴ Hence it is a constant K-filter (since it is satisfying the filter $Z_1 Z_2 = K^2$)

⇒ From the pass band condition,

$$\Rightarrow -1 < \frac{Z_1}{4Z_2} < 0$$

$$\Rightarrow -1 < \frac{JWL}{4 \cdot \frac{1}{JWC}} < 0$$

$$\Rightarrow -1 < \frac{J^2 W^2 LC}{4} < 0$$

$$\Rightarrow -1 < -\frac{(2\pi f)^2 LC}{4} < 0$$

$$\Rightarrow -1 < -\frac{4\pi^2 f^2 LC}{4} < 0$$

$$\Rightarrow -1 < -\pi^2 f^2 LC < 0$$

$$\Rightarrow -1 < -\frac{f^2}{\left(\frac{1}{\pi^2 LC}\right)} < 0 \longrightarrow ①$$

→ from the condition of cut-off frequencies

upper,

$$\Rightarrow \frac{Z_1}{4Z_2} = 0 \quad \text{and} \quad Z_1 = -4Z_2$$

$$\Rightarrow Z_1 = 0 \qquad JWL = \frac{-4}{JWC}$$

$$\Rightarrow JWL = 0 \qquad J^2 W^2 = \frac{-4}{LC}$$

$$\Rightarrow f = 0 \qquad W^2 = \frac{4}{LC} \Rightarrow f_c^2 = \frac{1}{\pi^2 LC}$$

$$f_c = \frac{1}{\pi\sqrt{LC}}$$

$\frac{1}{jwC} = -4jwL$

$\frac{1}{4} = w^2 LC$

$\rightarrow 4\pi^2 f_c^2 LC = \frac{1}{4}$

$\Rightarrow f_c^2 = \frac{1}{16\pi^2 LC}$

→ Where f_c is known as cut-off frequency (or) nominal (Which separates stop band & transmission band)

→ substituting f_c in eq. ①

$$\Rightarrow -1 < \frac{-f^2}{f_c^2} < 0$$

$$\Rightarrow -1 < -\left(\frac{f}{f_c}\right)^2 < 0$$

→ from Lower Limit

$$\Rightarrow -1 < -\left(\frac{f}{f_c}\right)^2$$

$$\Rightarrow -\left(\frac{f}{f_c}\right)^2 > -1$$

$$\Rightarrow \left(\frac{f}{f_c}\right)^2 < 1$$

$$\Rightarrow f < f_c$$

Upper Limit

$$-\left(\frac{f}{f_c}\right)^2 < 0$$

$$\left(\frac{f}{f_c}\right)^2 > 0$$

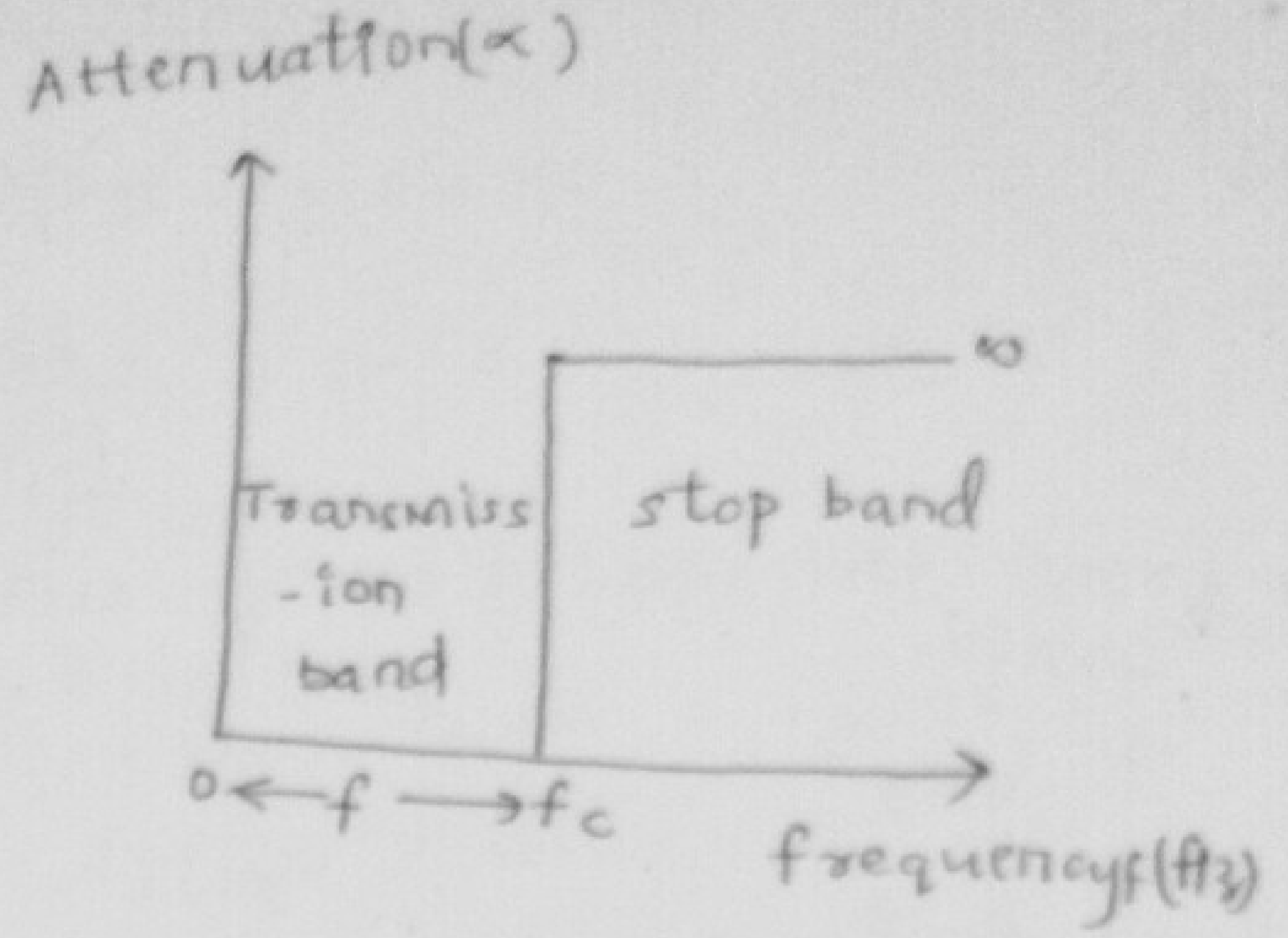
$$f > (f_c \times 0)$$

$$f > 0$$

→ The range of frequencies that the filter can transmit is $0 < f < f_c$.

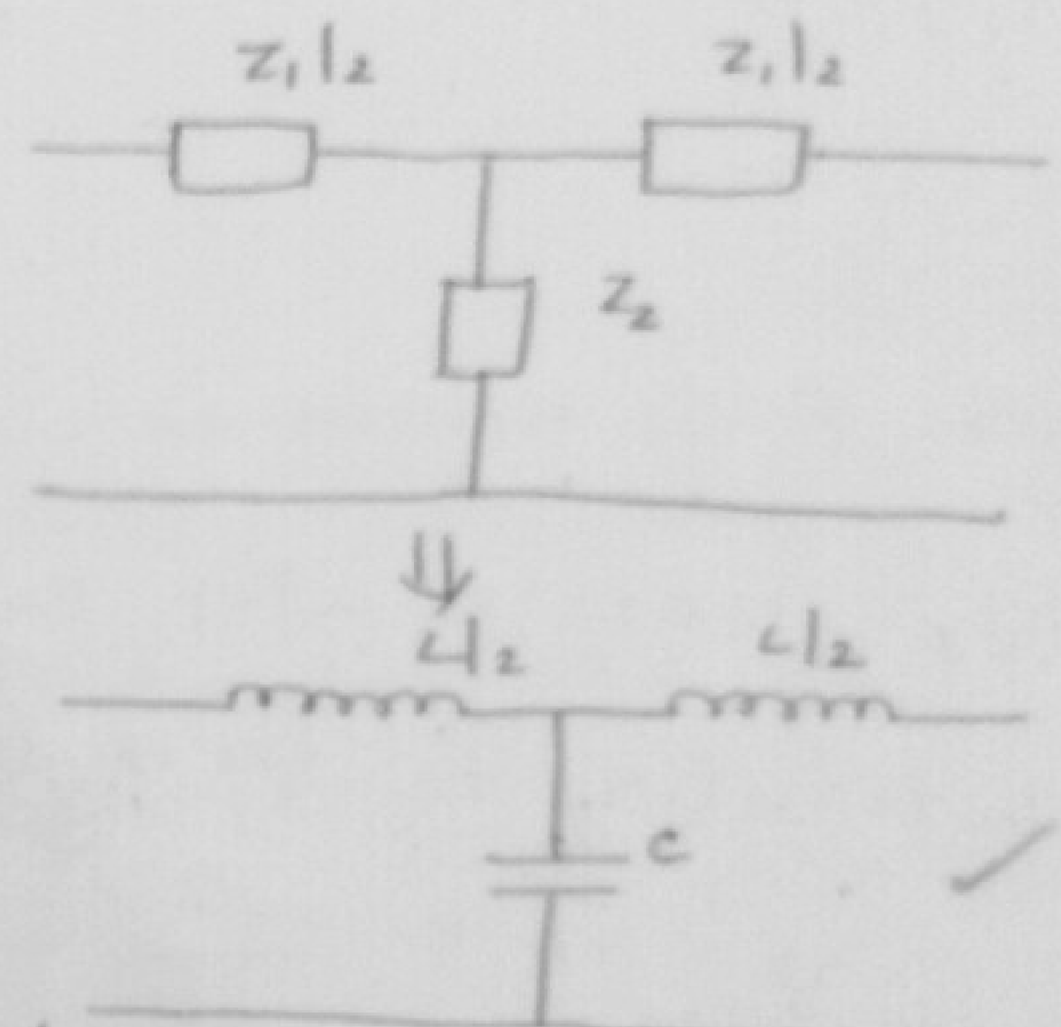
→ As the filter allows the frequencies below a cut off frequencies it is a Low pass filter.

$0 < f < f_c$



Symmetrical T-filter:-

→ We know that the configuration of symmetrical T-filter is,



here,

$Z_1 = JWL$

$Z_2 = -JX_c$

i.e, $\frac{Z_1}{2Z_2} = \frac{JWL}{2} = JW\left(\frac{L}{2}\right)$

$Z_2 = \frac{1}{JWC}$

⇒ The characteristic impedance of symmetrical T-network, is

$$Z_{OT} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2} = \sqrt{Z_1 Z_2\left(1 + \frac{Z_1}{4Z_2}\right)}$$

$$= \sqrt{K^2\left(1 - \left(\frac{f}{f_c}\right)^2\right)}$$

$$\Rightarrow \frac{Z_1}{4Z_2} = -\left(\frac{f}{f_c}\right)^2$$

$$Z_1 Z_2 = K^2$$

1. When $f = 0 \Rightarrow Z_{OT} = K =$ real

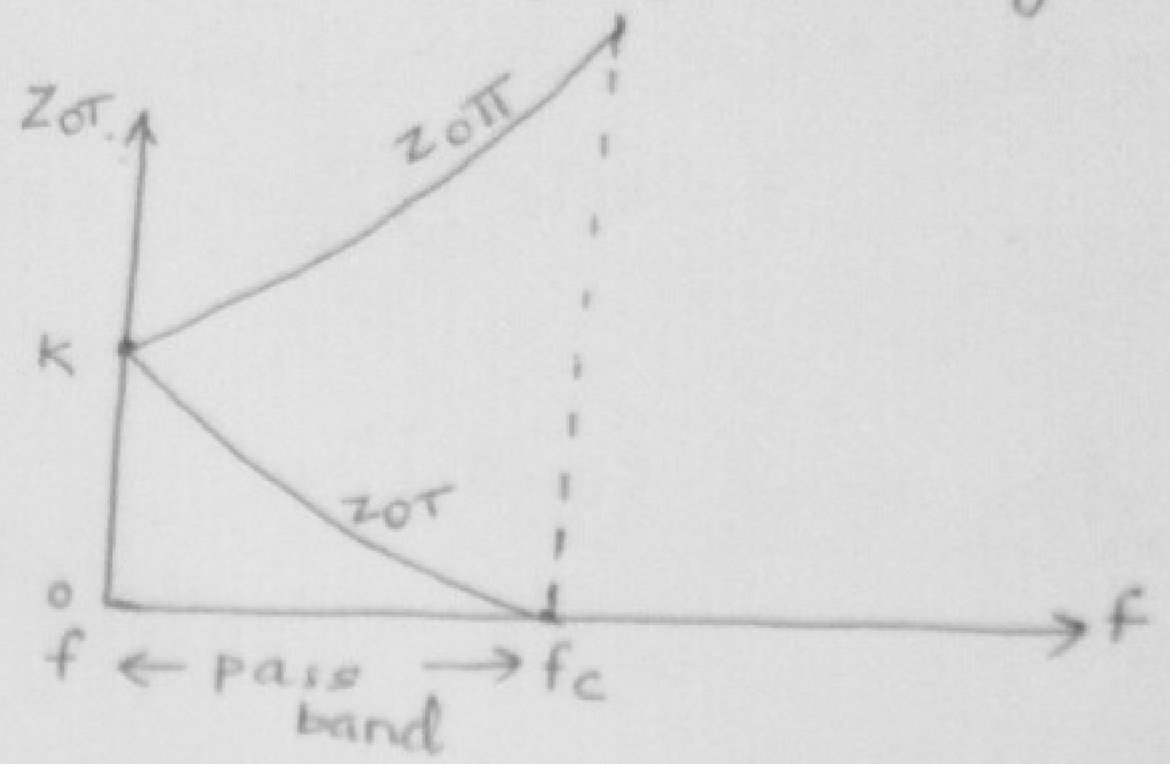
2. When $f < f_c \Rightarrow \frac{f}{f_c} < 1 \Rightarrow \left(\frac{f}{f_c}\right)^2 < 1 \Rightarrow 1 - \left(\frac{f}{f_c}\right)^2 > 0$

$\Rightarrow Z_{OT} > 0$ (real value)

3. When $f = f_c \Rightarrow Z_{OT} = 0$

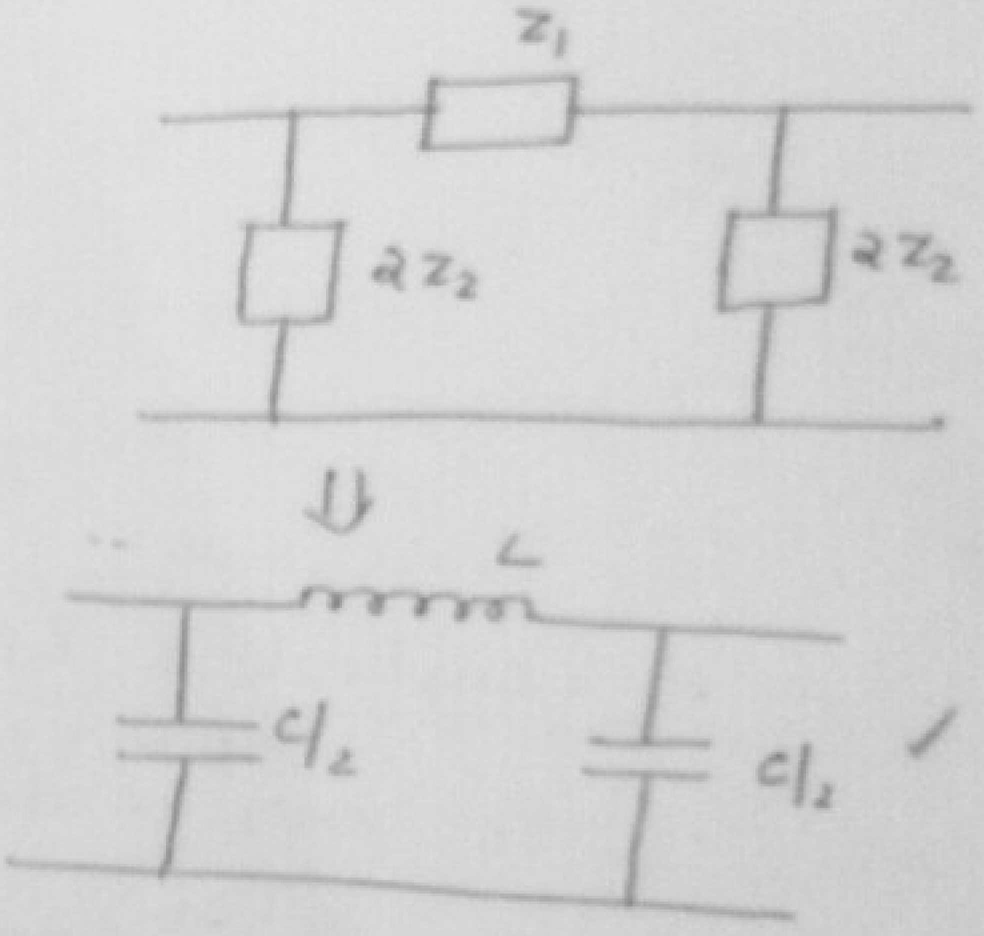
4. When $f > f_c \Rightarrow \left(\frac{f}{f_c}\right) > 1 \Rightarrow \left(\frac{f}{f_c}\right)^2 > 1 \Rightarrow 1 - \left(\frac{f}{f_c}\right)^2 < 0$ 3(11)

$\Rightarrow Z_{0T} = \text{Imaginary}$



Symmetrical π-filter:-

⇒ we know that symmetrical π-configuration is,



$\Rightarrow Z_1 = JWL$

$Z_2 = \frac{1}{JWC}$

$\Rightarrow 2Z_2 = \frac{2}{JWC} = \frac{1}{JW\left(\frac{C}{2}\right)}$

⇒ The characteristic impedance of symmetrical π-network is,

$$\Rightarrow Z_{0\pi} = \sqrt{\frac{Z_1 Z_2}{1 + \frac{Z_1}{4Z_2}}}$$

$$\Rightarrow Z_{0\pi} = \sqrt{\frac{K^2}{1 + -\left(\frac{f}{f_c}\right)^2}} = \sqrt{\frac{K^2}{1 - \left(\frac{f}{f_c}\right)^2}}$$

$$\Rightarrow Z_{0\pi} = \frac{K}{\sqrt{1 - \left(\frac{f}{f_c}\right)^2}}$$

1. When $f = 0 \Rightarrow Z_{0\pi} = K$ (real)
2. When $f < f_c \Rightarrow \frac{f}{f_c} < 1 \Rightarrow 1 - \left(\frac{f}{f_c}\right)^2 > 0 \Rightarrow Z_{0\pi} = \text{real}(\uparrow)$
   $Z_{0\pi}(\uparrow)$
3. When $f > f_c \Rightarrow Z_{0\pi} = \text{imaginary}$.
4. When $f = f_c \Rightarrow Z_{0\pi} = \infty$

Variation of α, β with frequency :-

Under

⇒ Pass Band condition:- $\alpha = 0$

$\cosh^{-1}\left(\frac{5}{4}\right) = 0.69$

$\cosh^{-1}\left(\frac{8}{4}\right) = 1.31$

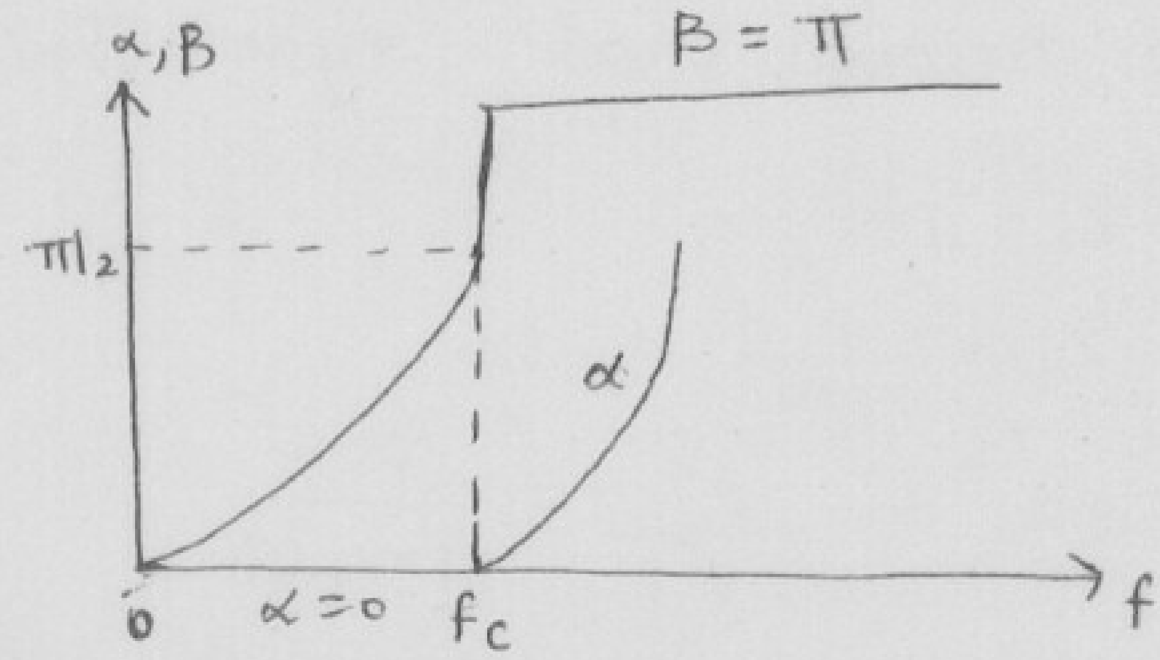
$\cosh^{-1}\left(\frac{10}{4}\right) = 1.56$

$$\beta = 2\sin^{-1}\left(\sqrt{\frac{Z_1}{4Z_2}}\right) = 2\sin^{-1}\left(j\frac{f}{f_c}\right)$$

⇒ Under stop band condition;

$$\alpha = 2\cosh^{-1}\left(\sqrt{\frac{Z_1}{4Z_2}}\right) = 2\cosh^{-1}\left(j\frac{f}{f_c}\right)$$

$$\beta = \pi$$



Design of constant-k LPF :-

⇒ Any filter can be designed for a specified values of cut-off frequency and nominal impedance i.e., always a designer will be provided with k and f_c value.

⇒ from the basics of constant -k Low pass filter, we know that

$$\Rightarrow k^2 = \frac{L}{C} \Rightarrow L = k^2 C \longrightarrow ①$$

$$\Rightarrow k = \sqrt{\frac{L}{C}}$$

$$L = k^2 C$$

$$L = k^2 \frac{1}{\pi k f_c}$$

$$L = \frac{k}{\pi f_c} \text{ Henrys.}$$

$$f_c = \frac{1}{\pi\sqrt{LC}}$$

$$\Rightarrow f_c^2 = \frac{1}{\pi^2 LC}$$

$$\Rightarrow f_c^2 \pi^2 LC = 1$$

from eq. ①

$$\Rightarrow f_c^2 \pi^2 k^2 \cdot C \cdot C = 1$$

$$\Rightarrow C^2 = \frac{1}{f_c^2 \pi^2 k^2}$$

$$\Rightarrow C = \frac{1}{\pi k f_c} \text{ farads}$$

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2. Constant-k High pass filter :- A

→ Where the series arm impedance is capacitance and the shunt arm impedance is inductance, then that filter exhibits the characteristics of a HPF,

⇒ Let $Z_1 = \frac{1}{JWC}$ and $Z_2 = JWL$

⇒ then $Z_1 Z_2 = \frac{1}{JWC} \cdot JWL = \frac{L}{C} = K^2$ ⇒ $K^2 = \frac{L}{C}$

⇒ From the pass-band condition,

$$\Rightarrow -1 < \frac{Z_1}{4Z_2} < 0 \Rightarrow -1 < \frac{1}{4J^2W^2LC} < 0.$$

$$\Rightarrow -1 < \frac{-1}{4(4\pi^2 f^2)LC} < 0 \Rightarrow -1 < \frac{-1}{16\pi^2 f^2 LC} < 0.$$

$$\Rightarrow -1 < \frac{-\left(\frac{1}{16\pi^2 LC}\right)}{f^2} < 0 \longrightarrow (1).$$

⇒ from the condition of cut-off frequencies,

Upper boundary

$$\Rightarrow \frac{Z_1}{4Z_2} = 0$$

$$\Rightarrow Z_1 = 0 \Rightarrow \frac{1}{JWC} = 0 \Rightarrow \frac{1}{f} = 0$$

$$\Rightarrow f = \infty$$

Lower Limit

$$Z_1 = -4Z_2$$

$$\Rightarrow \frac{1}{JWC} = -4JWL \Rightarrow W^2 LC = \frac{1}{4}$$

$$\Rightarrow 4\pi^2 f_c^2 LC = \frac{1}{4}$$

$$\Rightarrow f_c^2 = \frac{1}{16\pi^2 LC}$$

$$\Rightarrow \therefore f_c = \frac{1}{4\pi\sqrt{LC}}$$ is known as cut-off frequency (or) nominal frequency.

⇒ substituting f_c in eq. (1)

$$\Rightarrow -1 < -\left(\frac{f_c^2}{f^2}\right) < 0 \Rightarrow -1 < -\left(\frac{f_c}{f}\right)^2 < 0.$$

⇒ Lower Limit

$$\Rightarrow -1 < -\left(\frac{f_c}{f}\right)^2$$

$$\Rightarrow -\left(\frac{f_c}{f}\right)^2 > -1$$

$$\Rightarrow \left(\frac{f_c}{f}\right)^2 < 1$$

$$\Rightarrow f_c < f$$

$$\Rightarrow \boxed{f > f_c}$$

Upper Limit

$$\Rightarrow -\left(\frac{f_c}{f}\right)^2 < 0$$

$$\Rightarrow \left(\frac{f_c}{f}\right)^2 > 0 \Rightarrow \left(\frac{f}{f_c}\right)^2 < \frac{1}{0}$$

$$\Rightarrow f_c > 0 \quad \Rightarrow \boxed{f < \infty}$$

→ The range of frequencies that the filter can transmit is for $f > f_c$ and $f < \infty$. → $f_c < f < \infty$

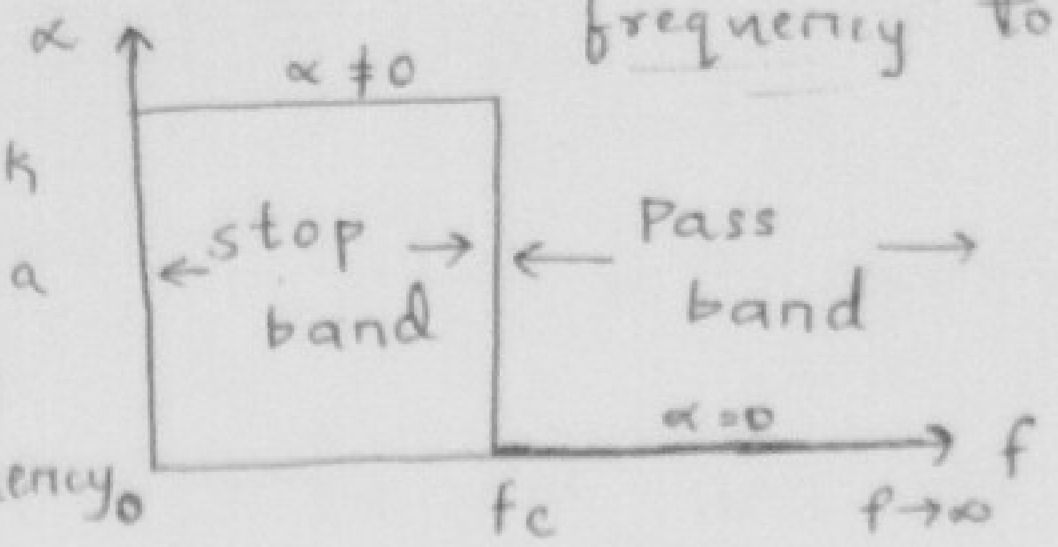
stop band → $0 < f < f_c$, $\alpha \neq 0$

pass band → $f_c < f < \infty$, $\alpha = 0$

→ A HPF is one which allows the frequencies above the cut-off frequencies it is a High pass filter.

$f > f_c$

frequency to attenuation characteristic

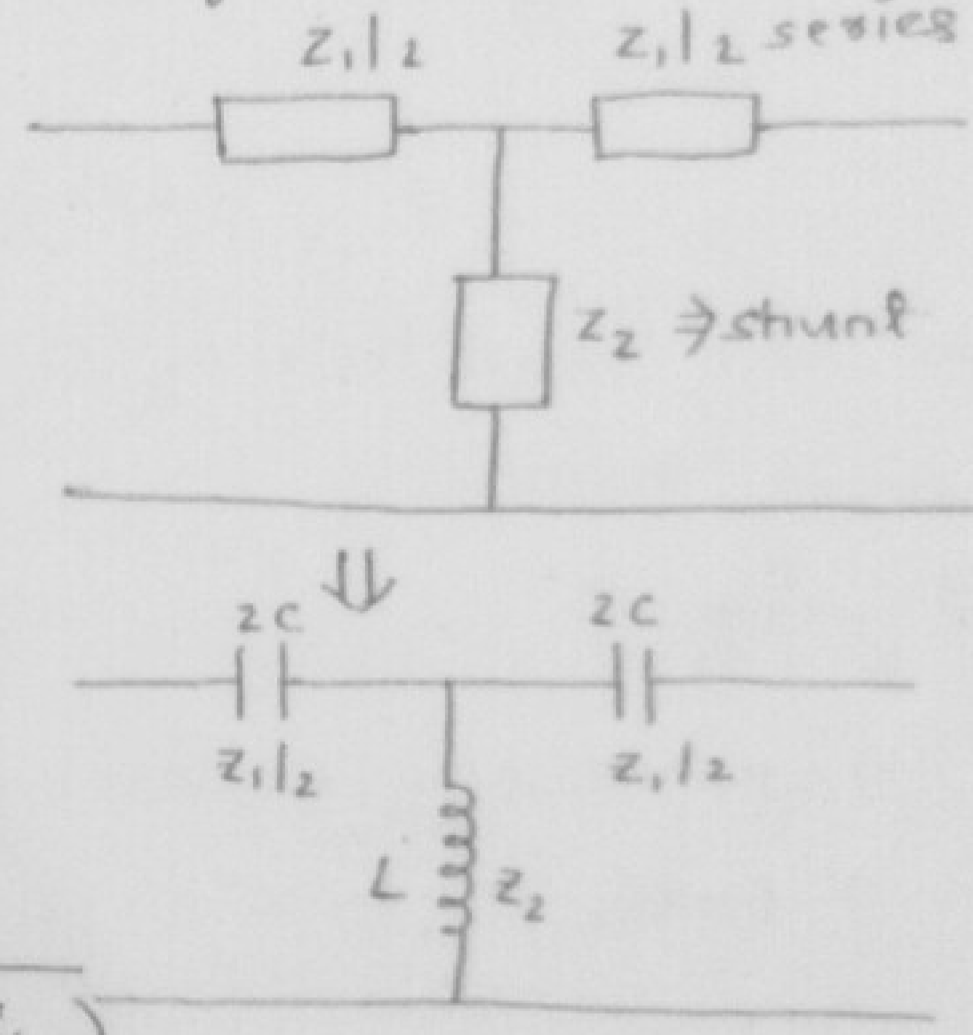


Note:- The constructed constant-k filter has a pass band above a cut off frequency and stop below the cut-off frequency. Hence it is a HPF

Symmetrical T-filter:-

→ We know that the configuration of symmetrical T-filter is, $Z_1 = \frac{1}{j\omega C}$ and $Z_2 = j\omega L$

$Z_1/2$ series → $\frac{Z_1}{2}, \frac{Z_1}{2}$



→ i.e., $\frac{Z_1}{2} = \frac{1}{j\omega\left(\frac{C}{2}\right)} = \frac{\left(\frac{1}{j\omega C}\right)}{2} = \frac{1}{j\omega(2C)}$

$\frac{Z_1}{2} = 2C$

→ The characteristic impedance of symmetrical T-network is

$$\Rightarrow Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2} = \sqrt{Z_1 Z_2\left(1 + \frac{Z_1}{4Z_2}\right)}$$

$$Z_{0T} = \sqrt{k^2\left(1 - \left(\frac{f_c}{f}\right)^2\right)} \quad \left(\text{since } \frac{Z_1}{4Z_2} = -\left(\frac{f_c}{f}\right)^2 \text{ and } Z_1 Z_2 = k^2\right)$$

1. When $f = 0 \Rightarrow Z_{0T} = (-\infty) = (-\text{ve value})$ imaginary
2. When $f < f_c \Rightarrow \frac{f}{f_c} < 1 \Rightarrow f_c > f \Rightarrow \frac{f_c}{f} > 1 \Rightarrow \left(\frac{f_c}{f}\right)^2 > 1 \Rightarrow 1 - \left(\frac{f_c}{f}\right)^2 < 0$ (imaginary)
3. When $f = f_c \Rightarrow Z_{0T} = 0$.
4. When $f > f_c \Rightarrow f_c < f \Rightarrow \left(\frac{f_c}{f}\right)^2 < 1 \Rightarrow 1 - \left(\frac{f_c}{f}\right)^2 > 0 \Rightarrow Z_{0T} > 0$ (real value)

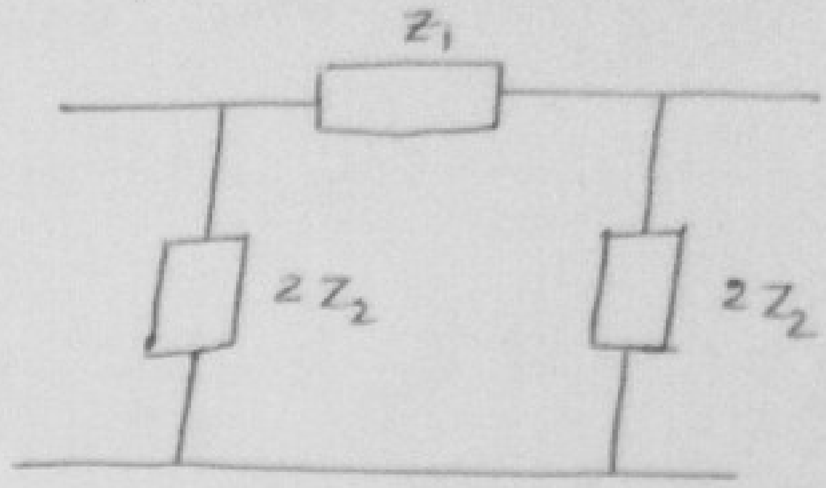
$f < f_c \Rightarrow f_c > f$

$\Rightarrow \left(\frac{f_c}{f}\right)^2 > 1$

$\Rightarrow 1 - \left(\frac{f_c}{f}\right)^2 < 0$

Symmetrical π - filter:-

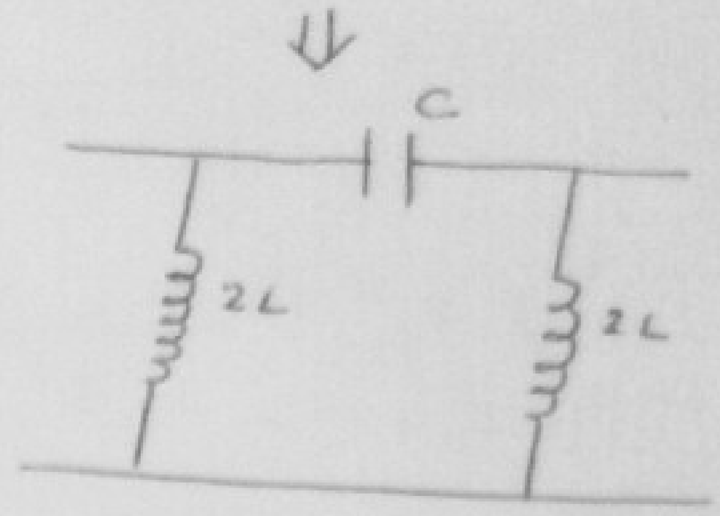
⇒ We know that the configuration of symmetrical π - filte



⇒ $Z_1 = \frac{1}{JWC}$ and $Z_2 = JWL$

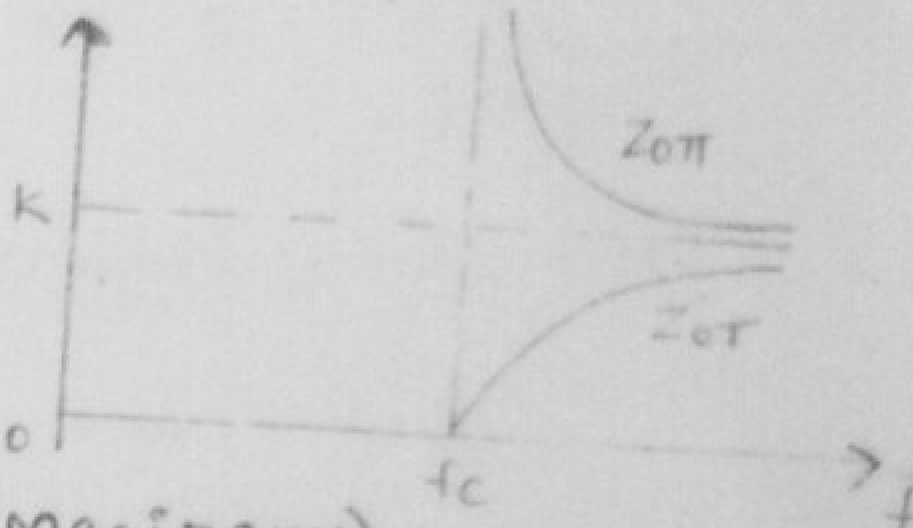
⇒ i.e., $2Z_2 = \frac{2}{1/JWL}$ ⇒ $2JWL$ ⇒ $JW(2L)$

⇒ The characteristic impedance of symmetrical π - network is,



$$\Rightarrow Z_{0\pi} = \sqrt{\frac{Z_1 Z_2}{1+\frac{Z_1}{4Z_2}}} = \sqrt{\frac{K^2}{1-\left(\frac{f_c}{f}\right)^2}}$$

$$\Rightarrow Z_{0\pi} = \frac{K}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$



1. When $f=0$ ⇒ $Z_{0\pi} = \frac{\bullet}{1-\infty} = 0.$
2. When $f < f_c$ ⇒ $f_c > f$ ⇒ $1-\left(\frac{f_c}{f}\right)^2 < 0$ (imaginary)
3. When $f = f_c$ ⇒ $f_c = f$ ⇒ $Z_{0\pi} = \infty$
4. When $f > f_c$ ⇒ $f_c < f$ ⇒ $\left(\frac{f_c}{f}\right)^2 < 1$ ⇒ $1-\left(\frac{f_c}{f}\right)^2 > 0$ (real value)

Variation of α, β with frequency:-

1. Under pass band condition:- $\alpha = 0$

$$\beta = 2\sin^{-1}\left(\sqrt{\frac{Z_1}{4Z_2}}\right) = 2\sin^{-1}\left(\sqrt{\frac{Z_1}{4Z_2}}\right)$$

$$\beta = 2\sin^{-1} J\left(\frac{f_c}{f}\right) = 2\sin^{-1} J\left(\frac{f_c}{f}\right)$$

2. Under stop band condition:-

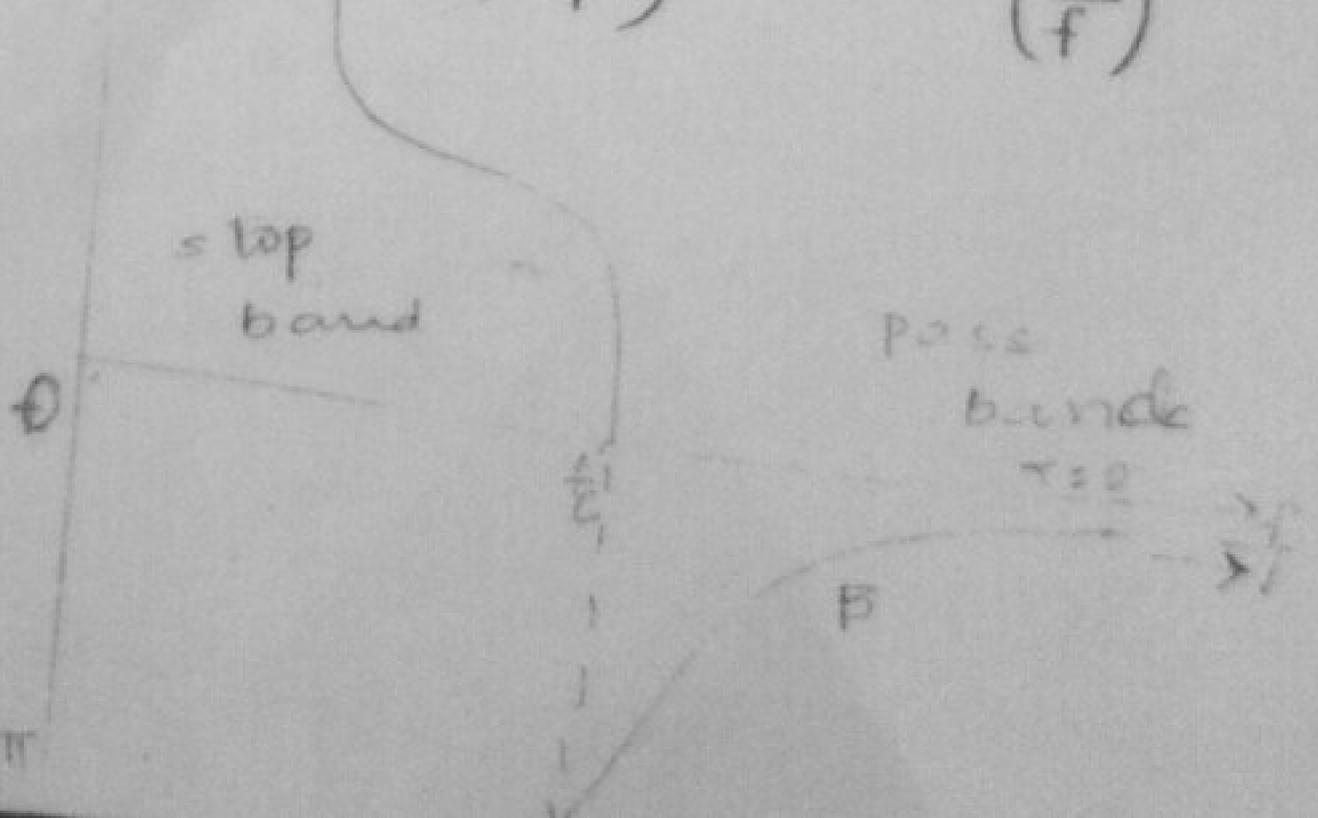
$$\alpha = 2\cosh^{-1}\sqrt{\frac{Z_1}{4Z_2}} = 2\cosh^{-1}\left(J\frac{f_c}{f}\right) = 2\cos^{-1}\left(\frac{f_c}{f}\right)$$

$$\beta = -\pi$$

⇒ $2\cos^{-1}\left(\frac{4}{5}\right) = 1.28$

⇒ $2\cos^{-1}\left(\frac{4}{8}\right) = 2.09$

⇒ $2\cos^{-1}\left(\frac{9}{10}\right) = 2.31$



Design of constant k-High pass filter:-

→ from the basics of constant-k HPF, we have

$$\Rightarrow k^2 = \frac{L}{C} \Rightarrow L = k^2 C \longrightarrow ①$$

$$\Rightarrow f_c = \frac{1}{4\pi\sqrt{LC}}$$

$$L = k^2 C$$

$$L = \frac{k}{4\pi f_c} \text{ Henry}$$

$$\Rightarrow f_c^2 = \frac{1}{16\pi^2 LC} \quad \text{from eq. ①}$$

$$\Rightarrow f_c^2 = \frac{1}{16\pi^2 k^2 c^2} \Rightarrow c^2 = \frac{1}{16\pi^2 k^2 f_c^2}$$

$$\Rightarrow \therefore c = \frac{1}{4\pi k f_c} \text{ farads}$$

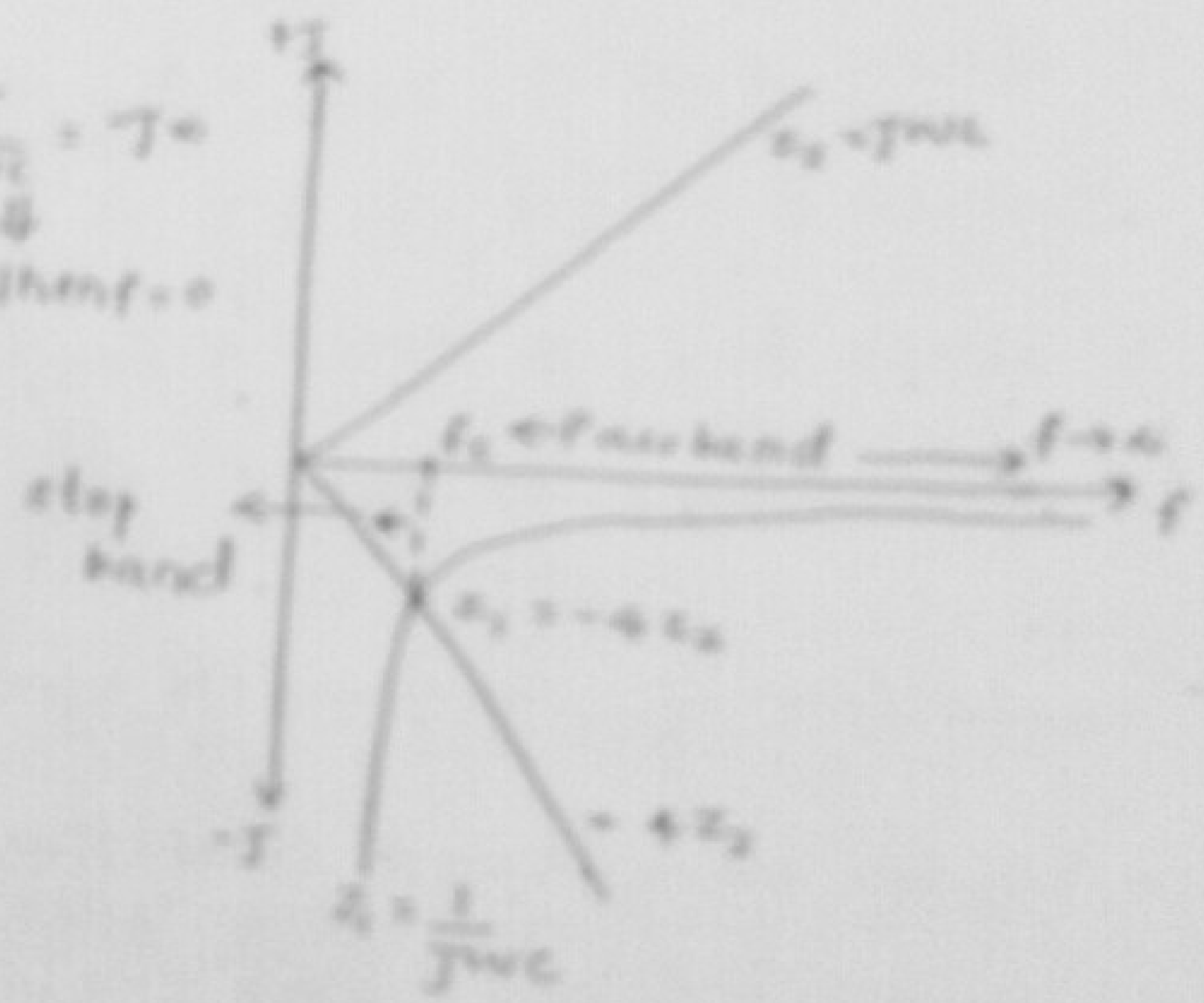
Impedance diagram:-

$$Z_1 = -jX_C = \frac{1}{j\omega C} \downarrow = \frac{-j}{\omega c} = j\infty$$

↓ when $f = 0$

$$Z_2 = j\omega L \uparrow$$

$$\Rightarrow Z_1 =$$



Symmetrical T-filter:-

$$Z_{0T} = k\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

1. when $f = 0 \Rightarrow Z_{0T}$ = imaginary
2. when $f \uparrow \Rightarrow \frac{f_c}{f} \downarrow \Rightarrow 1-\left(\frac{f_c}{f}\right)^2 \uparrow \Rightarrow Z_{0T} \uparrow$
3. when $f = f_c \Rightarrow Z_{0T} = 0$
4. when $f > f_c \Rightarrow Z_{0T}$ = real
5. when $f = \infty \Rightarrow Z_{0T} = k$

Symmetrical π-filter:-

$$Z_{0\pi} = \frac{k}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

1. When $f = \infty \Rightarrow Z_{0\pi} = k$ (and it is real always)

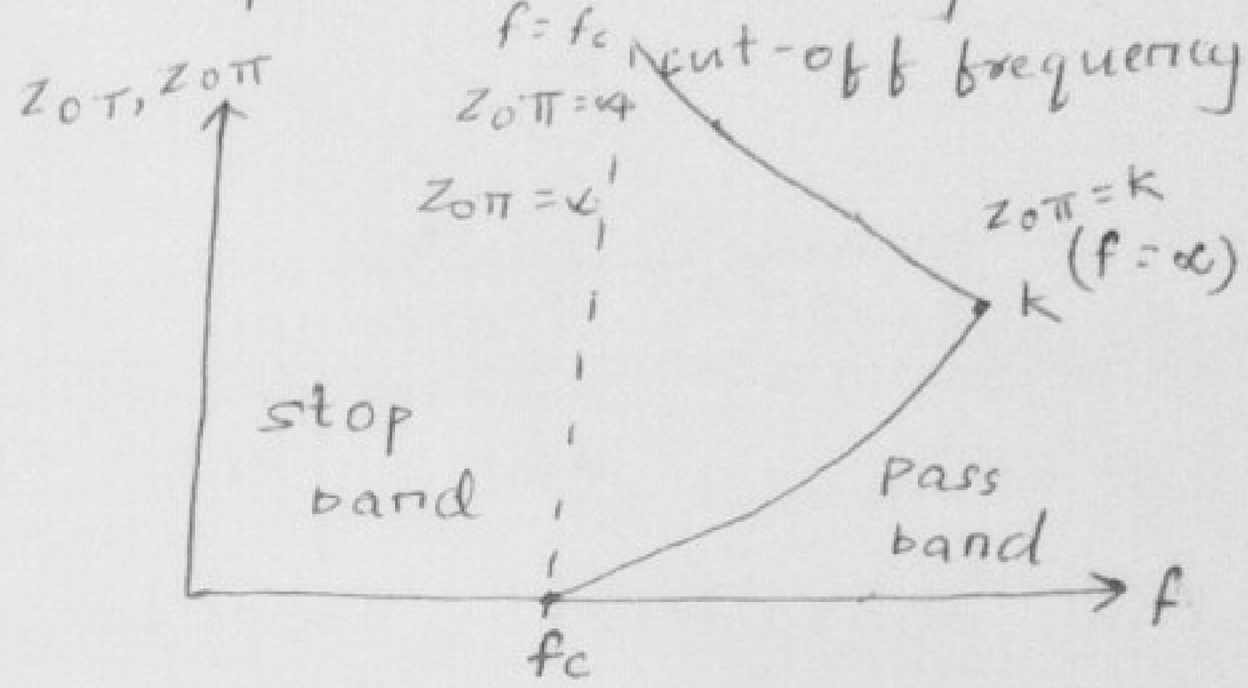
2. When $f = f_c \Rightarrow Z_{0\pi} = \infty$

Variation in characteristic impedance with respect to frequency:-

Z_{0T}

$\Rightarrow f > f_c \Rightarrow \frac{f_c}{f} < 1 \Rightarrow \left(\frac{f_c}{f}\right)^2 << 1$

$\Rightarrow \left(1 - \frac{(f_c)^2}{f^2}\right) > 0 \uparrow\uparrow$



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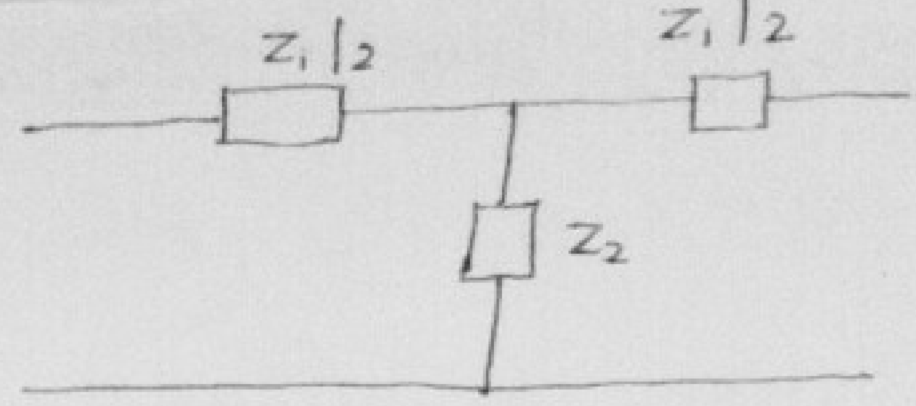
M-derived filters :- As the attenuation in the stop band for a prototype filter is not infinite (or) is not sharp there is a need for us to go for a new filter which will produce sharp attenuation in the stop band. This can be achelved in 2 ways,

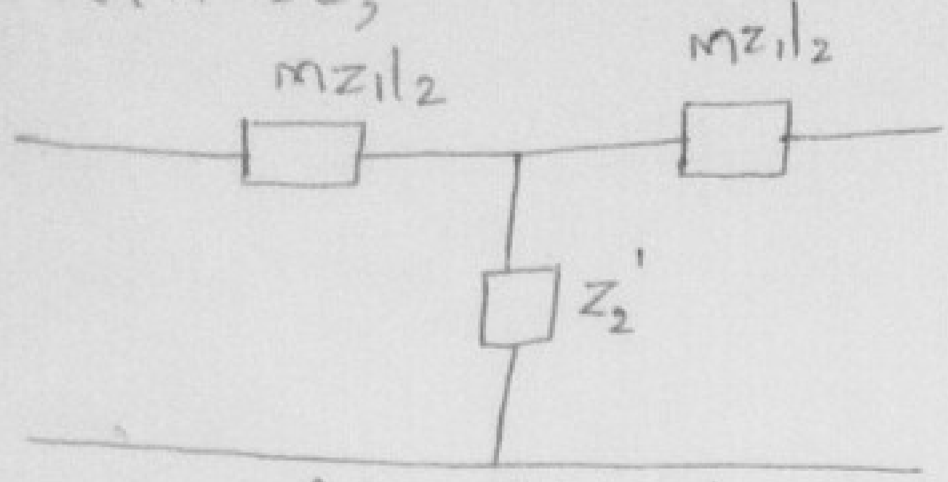
1. By cascading no. of prototype filters which has same characteristic impedance, same pass band & same stop band. However this is disadvantageous since the elements are not pure. (practically L, C will have some internal resistance,
2. Sharp attenuation will be acheived by deriving a new filter from the prototype which are usually known as M-derived filter. The condition for deriving a filter is the characteristic impedance should be same i.e., $Z_0 = Z_0'$ where Z_0 is characteristic impedance of prototype filter. Z_0' is characteristic impedance of derived filter

1. M-derived T-Section filter:-

→ We know that the basic T-section filter is,



$\Rightarrow$ and m-derived T-section will be,

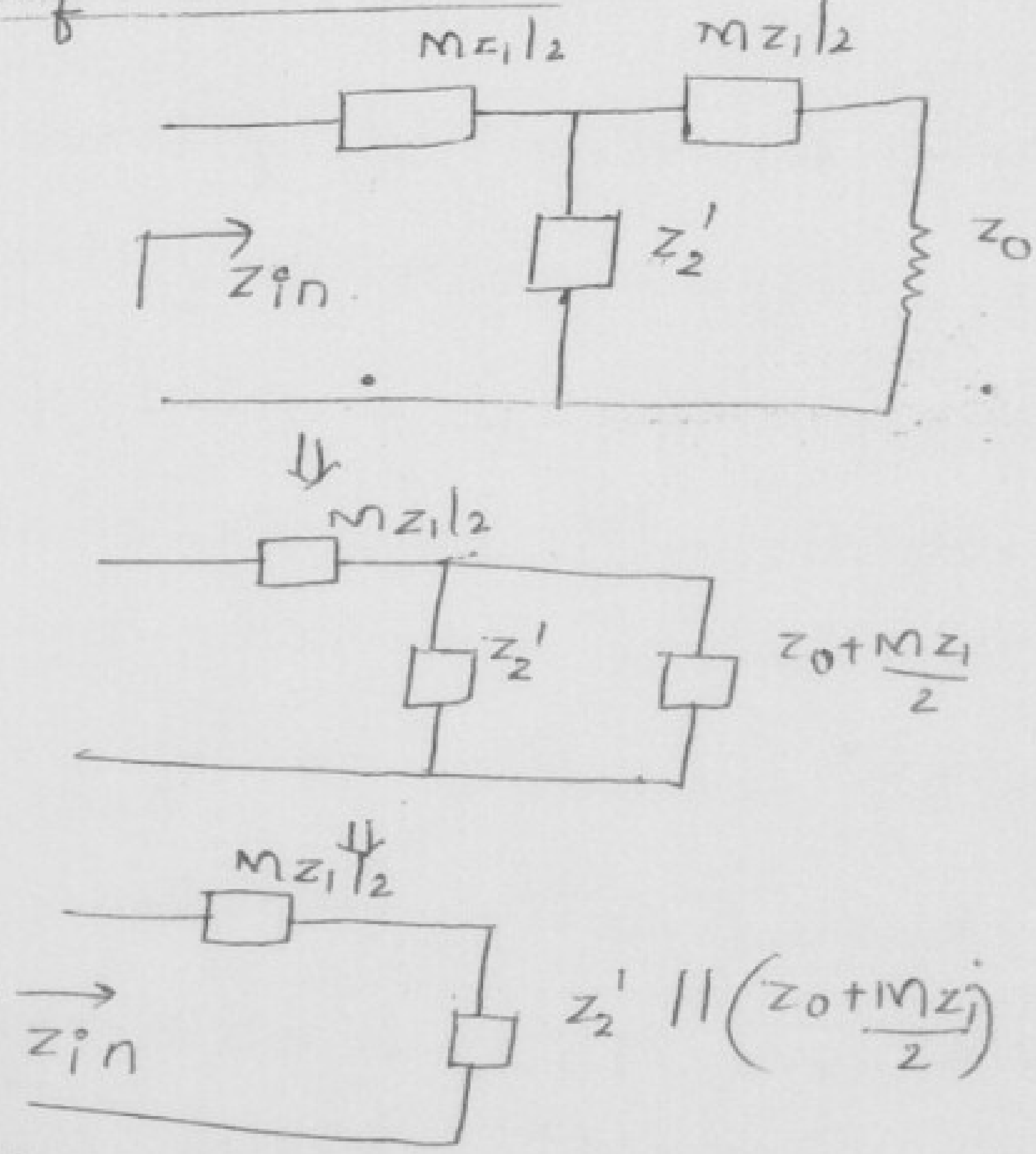


$\Rightarrow$ <u>Characteristic Impedance of prototype T-filter is</u> :-

$$Z_{OT} = \sqrt{Z_1 Z_2 + \frac{Z_1^2}{4}}$$

$\Rightarrow$ <u>The characteristic Impedance of m-derived</u> T-section filter, is

Z_{OT}'



$$\Rightarrow \quad Z_{in} = \left(\left(Z_0 + \frac{mZ_1}{2}\right) || Z_2'\right) + \frac{mZ_1}{2}$$

$$\Rightarrow Z_{in} = \frac{mZ_1}{2} + \left(Z_2' \,||\, Z_0 + \frac{mZ_1}{2}\right)$$

$$\Rightarrow Z_{in} = \frac{mZ_1}{2} + \frac{Z_2' \cdot \left(Z_0 + \frac{mZ_1}{2}\right)}{Z_2' + Z_0 + \frac{mZ_1}{2}}$$

Symmetrical $Z_{in} = Z_0$

$$\Rightarrow \quad Z_0 = \frac{\frac{mZ_1}{2}\left(Z_2' + Z_0 + \frac{mZ_1}{2}\right) + Z_2' Z_0 + Z_2' \frac{mZ_1}{2}}{Z_2' + Z_0 + \frac{mZ_1}{2}}$$

$$\Rightarrow \quad \cancel{Z_2' Z_0} + Z_0^2 + \cancel{\frac{mZ_1}{2} Z_0} = Z_2' \frac{mZ_1}{2} + \cancel{\frac{mZ_1}{2} Z_0} + \frac{m^2 Z_1^2}{4} + \cancel{Z_2' Z_0} + \frac{mZ_1 Z_2'}{2}$$

$$\Rightarrow \quad Z_0^2 = mZ_1 \cdot Z_2' + m^2 \frac{Z_1^2}{4}$$

$$\Rightarrow \quad Z_0 = \sqrt{m^2 \frac{Z_1^2}{4} + mZ_1 Z_2'} = Z_{OT}'$$

$\Rightarrow$ From, the <u>condition of</u> derivation, we have

$$\Rightarrow \boxed{Z_{OT} = Z_{OT}'}$$

$$\Rightarrow \quad Z_1 Z_2 + \frac{Z_1^2}{4} = m^2 \frac{Z_1^2}{4} + mZ_1 Z_2'$$

$$\Rightarrow \quad Z_1 Z_2 - mZ_1 Z_2' = (m^2 - 1)\frac{Z_1^2}{4}$$

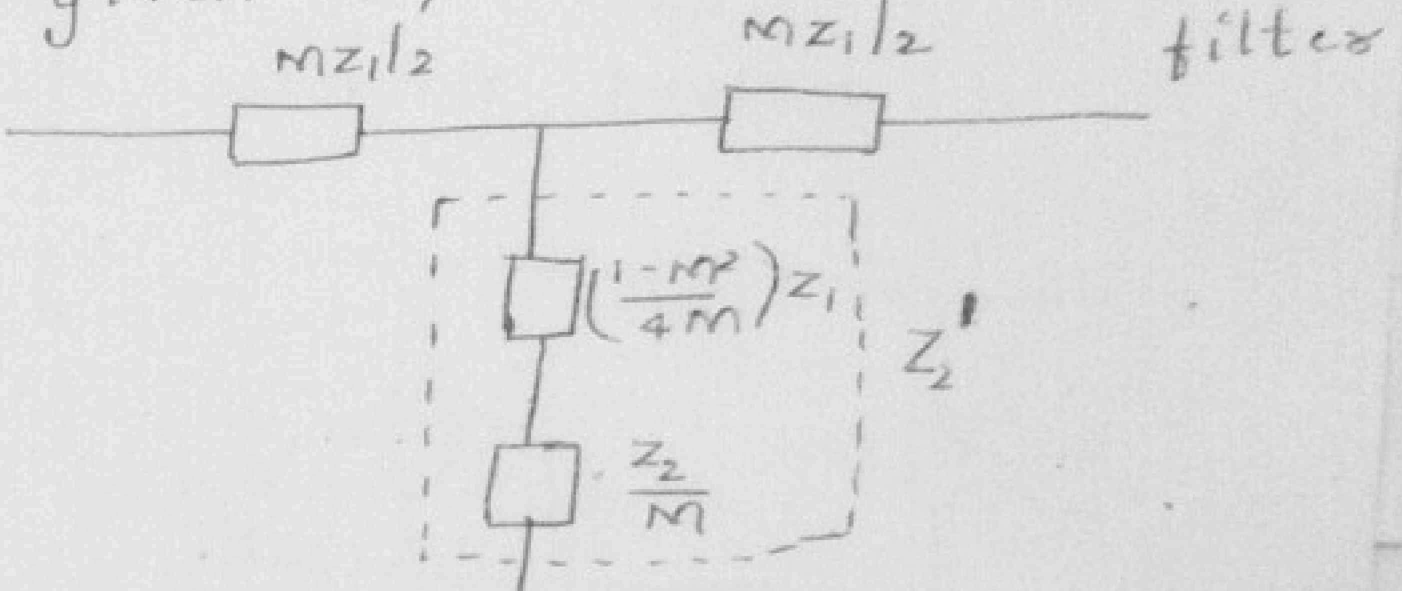
$$\Rightarrow mZ_1Z_2' = (1-m^2)\frac{Z_1^2}{4} + Z_1Z_2$$

$$\Rightarrow Z_2' = (1-m^2)\frac{Z_1}{4m} + \frac{Z_2}{m}$$

$$\Rightarrow Z_2' = \left(\frac{1-m^2}{4m}\right)Z_1 + \frac{Z_2}{m}$$

⇒ Hence for an m-derived T-section filter, the shunt impedance is not a single impedance it is the series combination of two impedances. And the symmetrical m-derived T-section is given as,

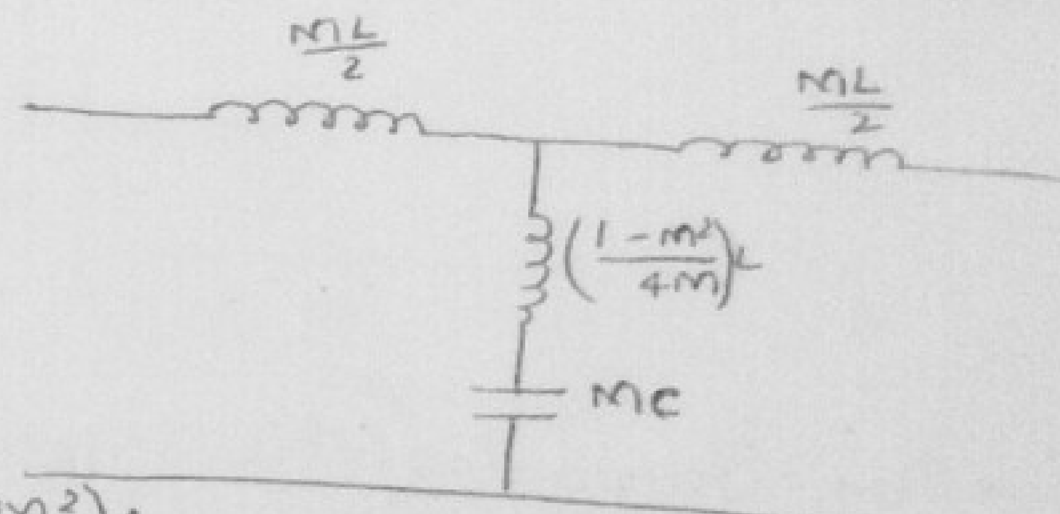
m-derived T-section filter



m-derived Low pass filter of T-section:-

⇒ series ⇒ $\frac{1}{JWC} = Z_2$

shunt ⇒ $JWL = Z_1$



$$\Rightarrow \frac{mZ_1}{2} = \frac{m(JWL)}{2} = JW\left(\frac{mL}{2}\right)$$

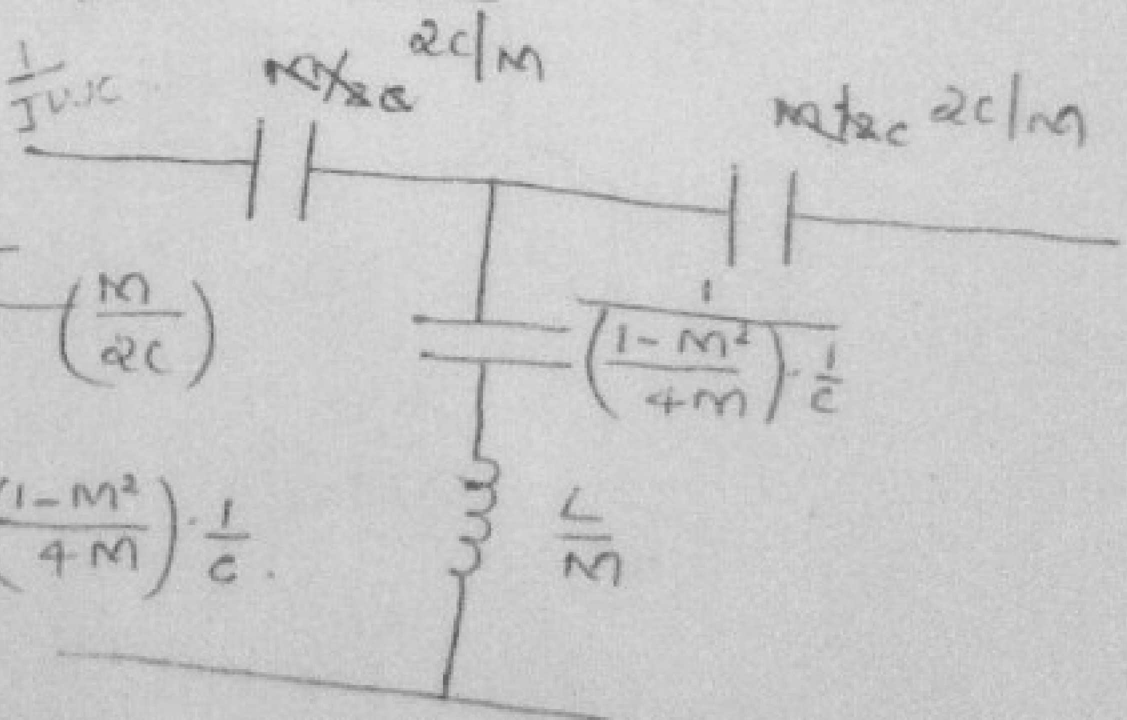
$$\Rightarrow \left(\frac{1-m^2}{4m}\right)Z_1 = \left(\frac{1-m^2}{4m}\right)(JWL) = JW\left(\frac{1-m^2}{4m}\right)L$$

$$\Rightarrow \frac{Z_2}{m} = \frac{1}{JW(mC)} = \frac{-J}{W(mC)}.$$

m-derived High pass filter of T-section:-

$Z_1 = \frac{1}{JWC}$, $Z_2 = JWL$

$\frac{1}{JW\left(\frac{2C}{m}\right)} \Rightarrow \frac{1}{JWC}$



$$\Rightarrow \frac{mZ_1}{2} = \frac{m}{2JWC} = \frac{1}{JW}\left(\frac{m}{2C}\right) = \frac{-J}{W}\left(\frac{m}{2C}\right)$$

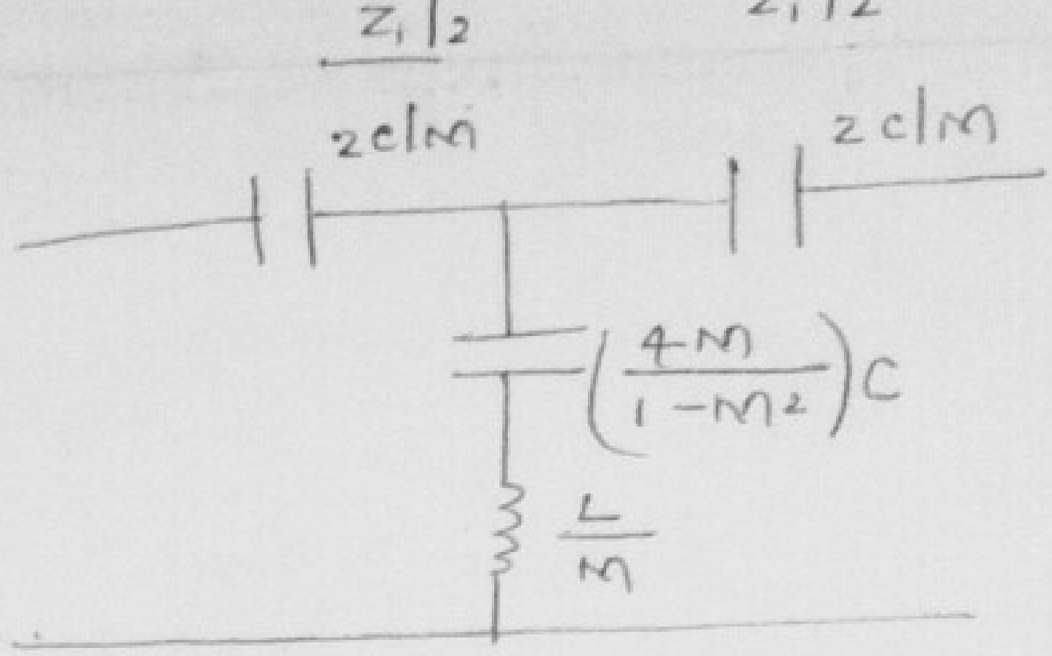
$$\Rightarrow \left(\frac{1-m^2}{4m}\right)Z_1 = \frac{1}{JW}\left(\frac{1-m^2}{4m}\right)\frac{1}{C} = \frac{-J}{W}\left(\frac{1-m^2}{4m}\right)\cdot\frac{1}{C}.$$

$$\Rightarrow \frac{Z_2}{m} = \frac{JWL}{m} = JW\left(\frac{L}{m}\right)$$

$$\Rightarrow \left(\frac{1-m^2}{4m}\right)Z_1 = \left(\frac{1-m^2}{4m}\right)\left(\frac{1}{j\omega C}\right)$$

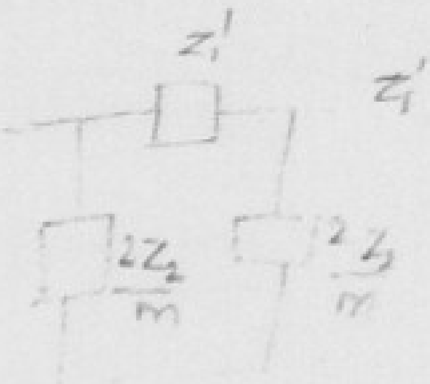
$$= \frac{1}{j\omega}\,\frac{(1-m^2)}{(4mC)}$$

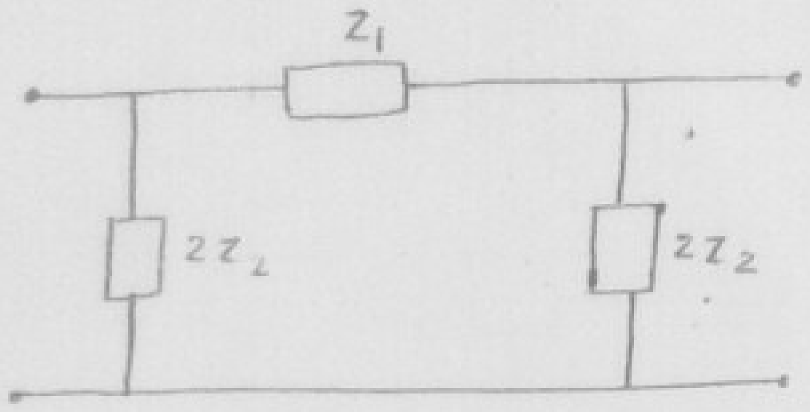
$$= \frac{1}{j\omega\left(\frac{4mC}{1-m^2}\right)} = \frac{1}{j\omega C}$$

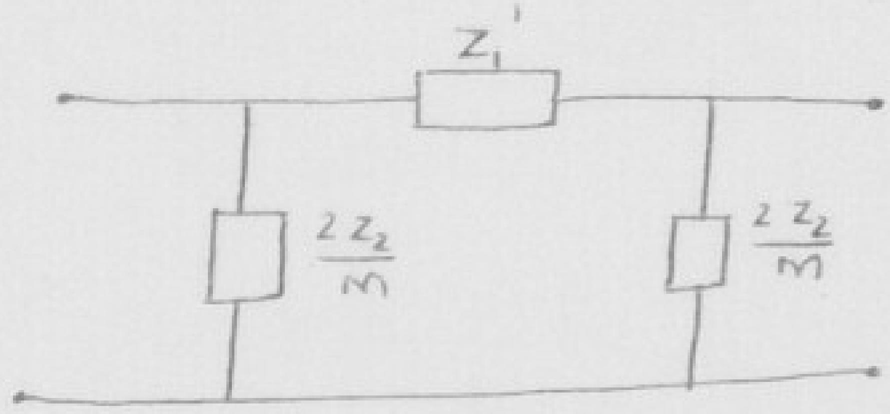


2. m-derived π-section filter :-

→ we know the basic π-section filter and the m-derived π-section filter,





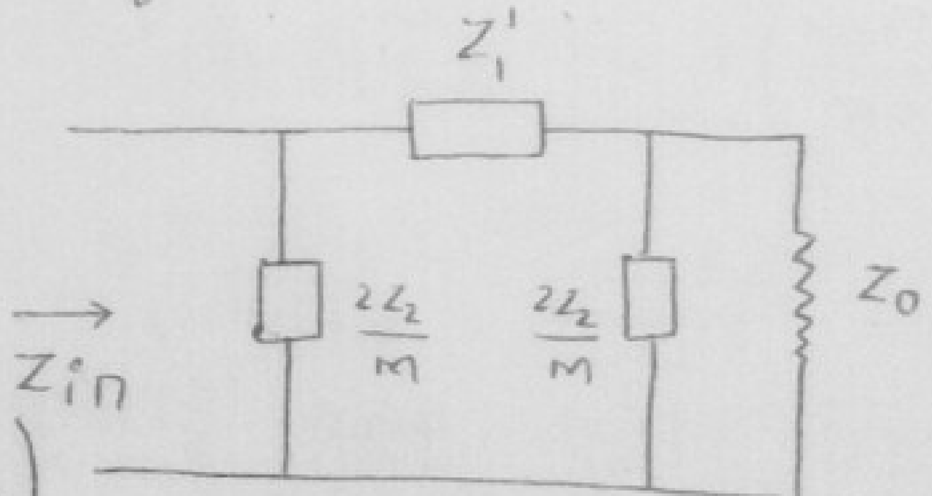


→ characteristic impedance of prototype π-filter is

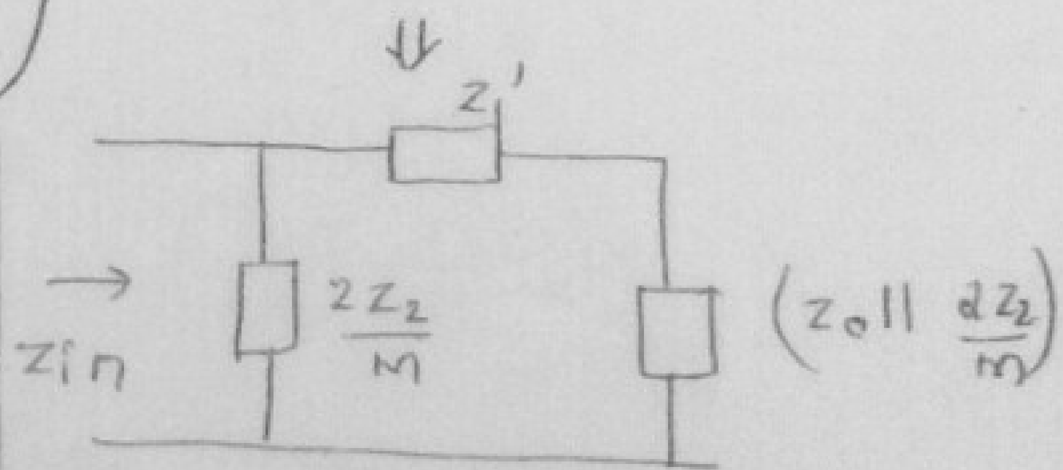
$$Z_{0\pi} = \sqrt{\frac{Z_1 Z_2}{1 + \frac{Z_1}{4Z_2}}}$$

→ The characteristic impedance of m-derived π-section filter is,

$$\Rightarrow Z_{in} = \frac{2Z_2}{m} \;\|^{el}\; \left(Z_1' + \frac{Z_0\,\frac{2Z_2}{m}}{Z_0 + \frac{2Z_2}{m}}\right)$$

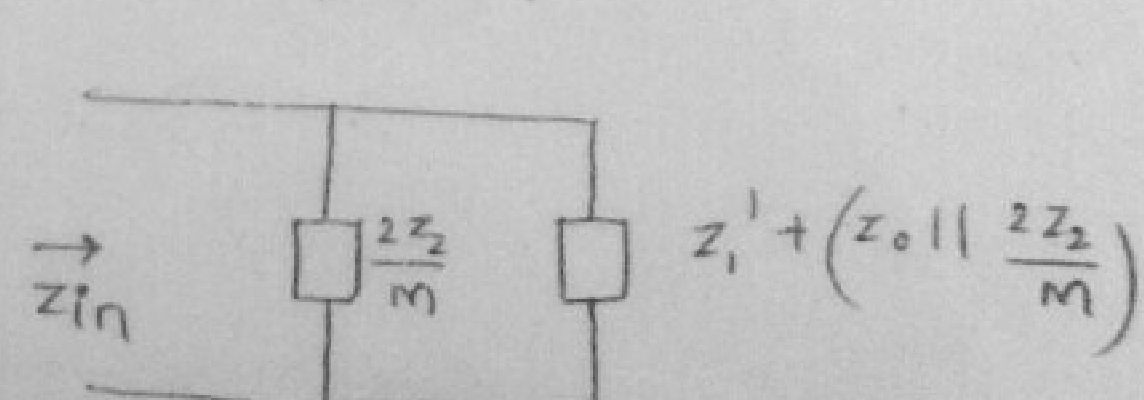


$$\Rightarrow Z_{in} = \frac{2Z_2}{m} \;\|^{el}\; \left(\frac{Z_1'\left(Z_0 + \frac{2Z_2}{m}\right) + Z_0\frac{2Z_2}{m}}{Z_0 + \frac{2Z_2}{m}}\right)$$



$$\Rightarrow Z_{in} = \frac{\frac{2Z_2}{m}\left(Z_1' Z_0 + \frac{2Z_2}{m} Z_1' + Z_0\frac{2Z_2}{m}\right)}{\frac{2Z_2}{m} + Z_0 + \frac{2Z_2}{m}}$$

$$\Rightarrow Z_{in} = \frac{\frac{2Z_2}{m} Z_1' Z_0 + \frac{4Z_2^2}{m^2} Z_1' + \frac{4Z_2^2}{m^2} Z_0}{\frac{4Z_2}{m} + Z_0}$$



condition:-

$\Rightarrow$ symmetrical $Z_{in} = Z_0$.

$$\Rightarrow \quad Z_0 = \frac{\frac{2Z_2}{m} Z_1' Z_0 + \frac{4Z_2^2}{m^2} Z_1' + \frac{4Z_2^2}{m^2} Z_0}{\frac{4Z_2}{m} + Z_0}$$

$$\Rightarrow Z_0^2 + \frac{4Z_2}{m} Z_0 = \frac{2Z_2}{m} Z_1' Z_0 + \frac{4Z_2^2}{m^2} Z_1' + \frac{4Z_2^2}{m^2} Z_0$$

$$\Rightarrow Z_{in} = \frac{\frac{2Z_2}{m}\left(\frac{Z_1' Z_0 + \frac{2Z_2}{m} Z_1' + Z_0 \frac{2Z_2}{m}}{Z_0 + \frac{2Z_2}{m}}\right)}{\frac{2Z_2}{m} + \frac{\frac{2Z_2}{m} Z_1' + \frac{2Z_2}{m} Z_0 + Z_1' Z_0}{Z_0 + \frac{2Z_2}{m}}}$$

$$\Rightarrow Z_{in} = \frac{\frac{2Z_2}{m}\left(Z_1' Z_0 + \frac{2Z_2}{m} Z_1' + \frac{2Z_2}{m} Z_0\right)}{\frac{2Z_2}{m} Z_0 + \left(\frac{2Z_2}{m}\right)^2 + \frac{2Z_2}{m} Z_1' + Z_1' Z_0 + \frac{2Z_2}{m} Z_0}.$$

$\Rightarrow$ symmetrical $Z_{in} = Z_0$

$$\Rightarrow Z_0 = \frac{\cancel{\frac{2Z_2}{m}}\left(Z_1' Z_0 + \frac{2Z_2}{m} Z_1' + \frac{2Z_2}{m} Z_0\right)}{\cancel{\frac{2Z_2}{m}}\left(Z_0 + \frac{2Z_2}{m} + Z_1' + Z_1' Z_0 \frac{m}{2Z_2} + Z_0\right)}$$

$$\Rightarrow Z_0^2 + \frac{2Z_2}{m} Z_0 + Z_0 Z_1' + Z_1' Z_0^2 \frac{m}{2Z_2} + Z_0^2 = Z_1' Z_0 + \frac{2Z_2}{m} Z_1' + \frac{2Z_2}{m} Z_0$$

$$\Rightarrow \quad 2Z_0^2 = \frac{2Z_2}{m} Z_1' - Z_1' Z_0^2 \frac{m}{2Z_2} \Rightarrow 2Z_0^2 + Z_0^2 Z_1' \frac{m}{2Z_2} = \frac{2Z_2}{m} Z_1'$$

$$\Rightarrow Z_0^2\left(2 + Z_1' \frac{m}{2Z_2}\right) = \frac{2Z_2}{m} Z_1'$$

$$\Rightarrow \quad Z_0^2 = \frac{\frac{2Z_2}{m} Z_1'}{2 + Z_1'\left(\frac{m}{2Z_2}\right)} = \frac{\frac{Z_2}{m} Z_1'}{1 + Z_1'\left(\frac{m}{4Z_2}\right)}$$

$$\Rightarrow Z_0^2 = \sqrt{\frac{\frac{Z_2}{m} \cdot Z_1'}{1 + Z_1' \frac{1}{4\left(\frac{Z_2}{m}\right)}}} = Z_{0\pi}'$$

⇒ From, the condition of derivation, we have

$$\Rightarrow Z_{0\pi} = Z_{0\pi}'$$

$$\Rightarrow \sqrt{\frac{Z_1 Z_2}{1 + \frac{Z_1}{4Z_2}}} = \sqrt{\frac{Z_1' \frac{Z_2}{m}}{1 + \frac{Z_1'}{4\left(\frac{Z_2}{m}\right)}}}$$

$$\Rightarrow \frac{Z_1 Z_2}{1 + \frac{Z_1}{4Z_2}} = \frac{\frac{Z_2}{m} Z_1'}{1 + \frac{Z_1' m}{4Z_2}}$$

$$\Rightarrow Z_1 Z_2 + \frac{Z_1' Z_1 m}{4} = Z_1' \frac{Z_2}{m} + \frac{Z_1 Z_1'}{4m}$$

$$\Rightarrow Z_1' \left(\frac{Z_1 m}{4} + \frac{-Z_2}{m} - \frac{Z_1}{4m} \right) = -Z_1 Z_2$$

$$\Rightarrow Z_1' = \frac{Z_1 Z_2}{\frac{Z_1}{4m} + \frac{Z_2}{m} - \frac{mZ_1}{4}} = \frac{Z_1 Z_2}{\frac{Z_2}{m} + \frac{Z_1}{4m}(1 - m^2)}$$

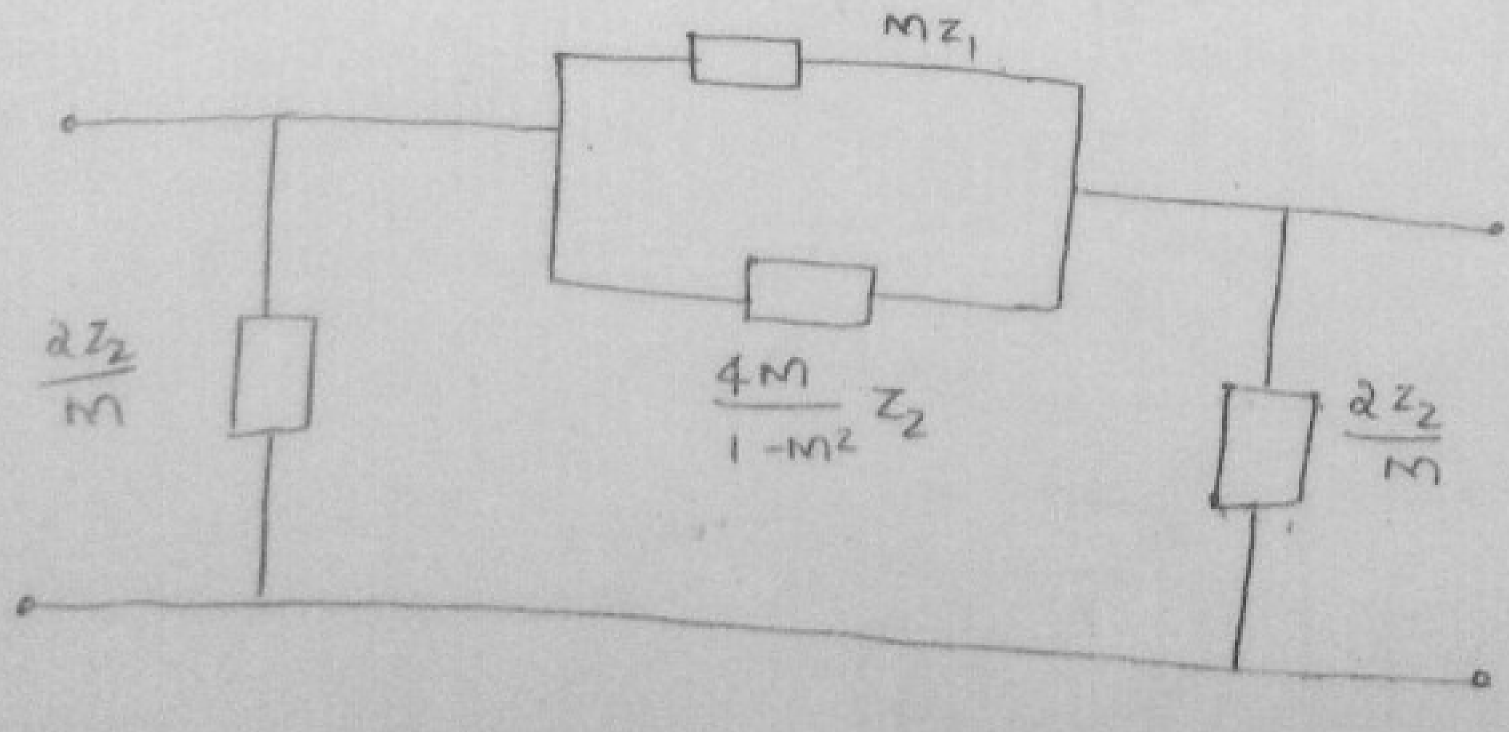
⇒ Multiplying and dividing by $\left(\frac{4m^2}{1-m^2}\right)$

$$\Rightarrow Z_1' = \frac{Z_1 Z_2 \times \left(\frac{4m^2}{1-m^2}\right)}{\left(\frac{Z_2}{m} + \frac{Z_1}{4m}(1-m^2)\right)\left(\frac{4m^2}{1-m^2}\right)} = \frac{Z_1 Z_2 \left(\frac{4m^2}{1-m^2}\right)}{\frac{Z_2 4m}{1-m^2} + \frac{Z_1 m^2}{m}}$$

$$\Rightarrow Z_1' = \frac{(mZ_1)\left(\frac{4m}{1-m^2} Z_2\right)}{mZ_1 + \frac{4m}{1-m^2} Z_2}$$

∴ Hence for a m-derived π-section filter, the series arm impedance is a parallel combination of mZ_1 and $\frac{4m}{1-m^2} Z_2$.

And the symmetrical m-derived π-section is given as,



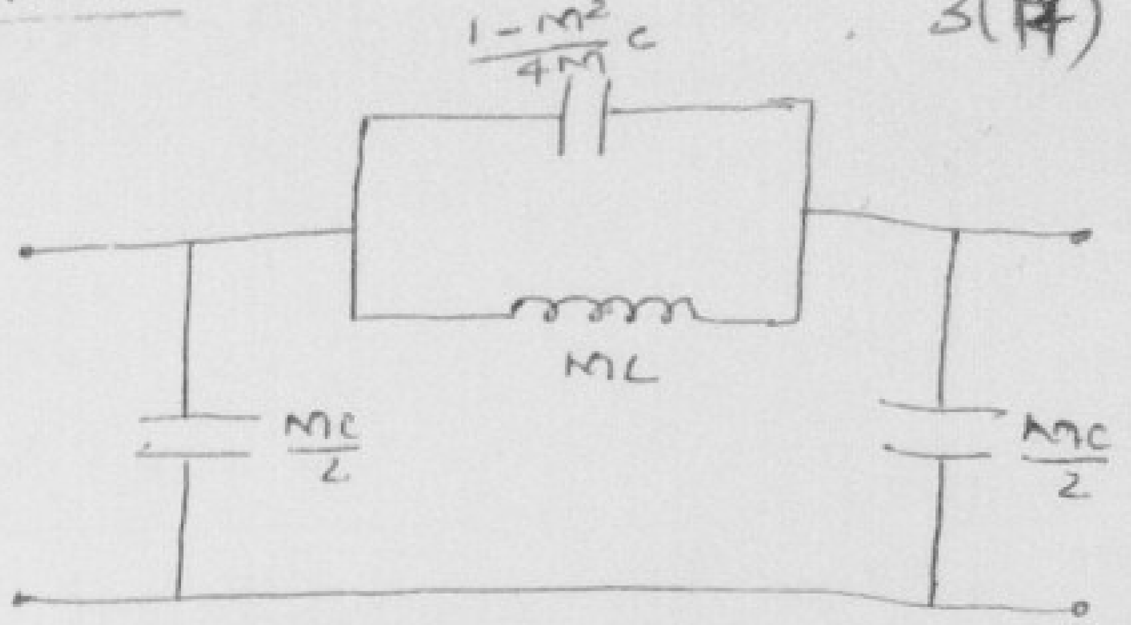
$Z_1 = \frac{1}{j\omega c}$

$Z_2 = j\omega L$

$mZ_1 = \frac{1}{j\omega \left(\frac{C}{m}\right)}$

7

m-derived LPF of π-section filter:-



$\Rightarrow Z_1 = j\omega L$ and $Z_2 = \frac{1}{j\omega C}$

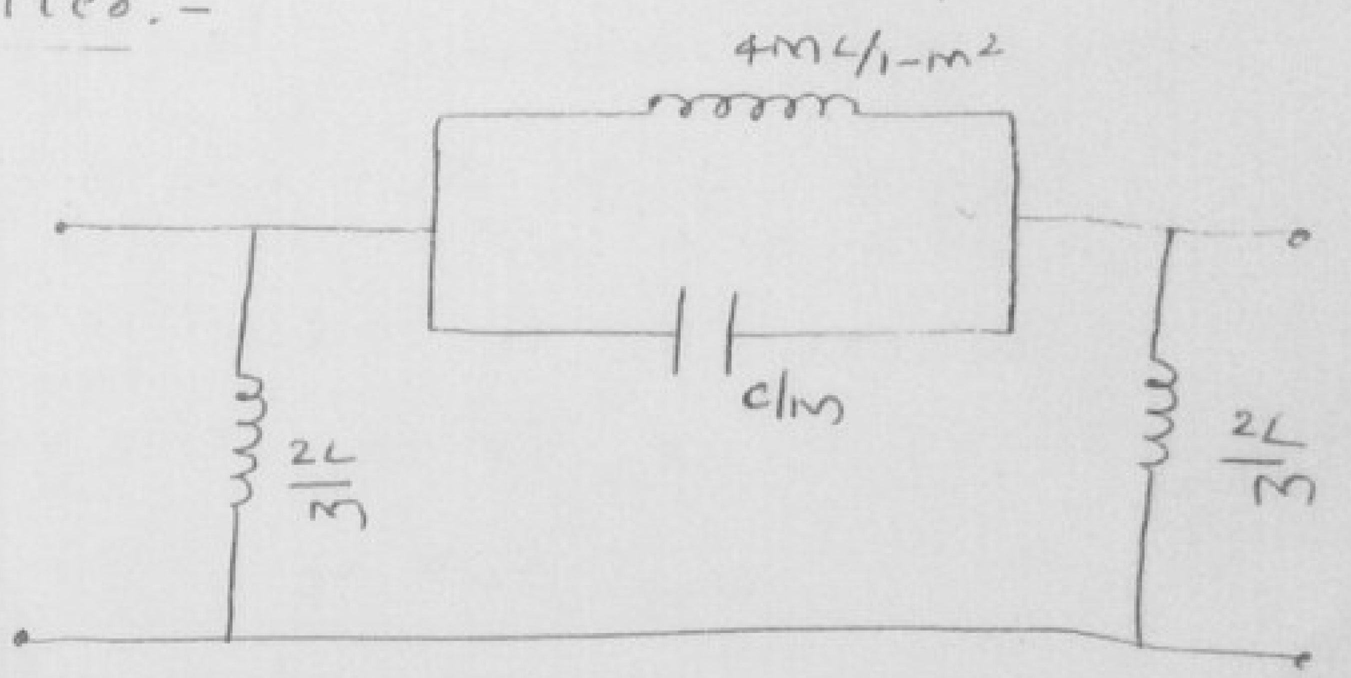
$\Rightarrow mZ_1 = j\omega(mL)$

$$\Rightarrow \frac{4m}{1-m^2}(Z_2) = \frac{4m}{(1-m^2)(j\omega C)} = \frac{1}{C\,j\omega\left(1/\frac{4m}{1-m^2}\right)}$$

$$= \frac{1}{j\omega\left(\frac{1-m^2}{4m}\right)C}$$

$$\Rightarrow \frac{2Z_2}{m} = \frac{2}{j\omega(mC)} = \frac{1}{j\omega\left(\frac{mC}{2}\right)}$$

m-derived HPF of π-filter:-



$\Rightarrow Z_1 = \frac{1}{j\omega C}$ & $Z_2 = j\omega L$

$$\Rightarrow mZ_1 = \frac{m}{j\omega C} = \frac{1}{j\omega\left(\frac{C}{m}\right)}$$

$$\Rightarrow \frac{4m}{1-m^2}Z_2 = j\omega\left(\frac{4m}{1-m^2}\right)L$$

$$\Rightarrow \frac{2Z_2}{m} = j\omega\left(\frac{2L}{m}\right)$$

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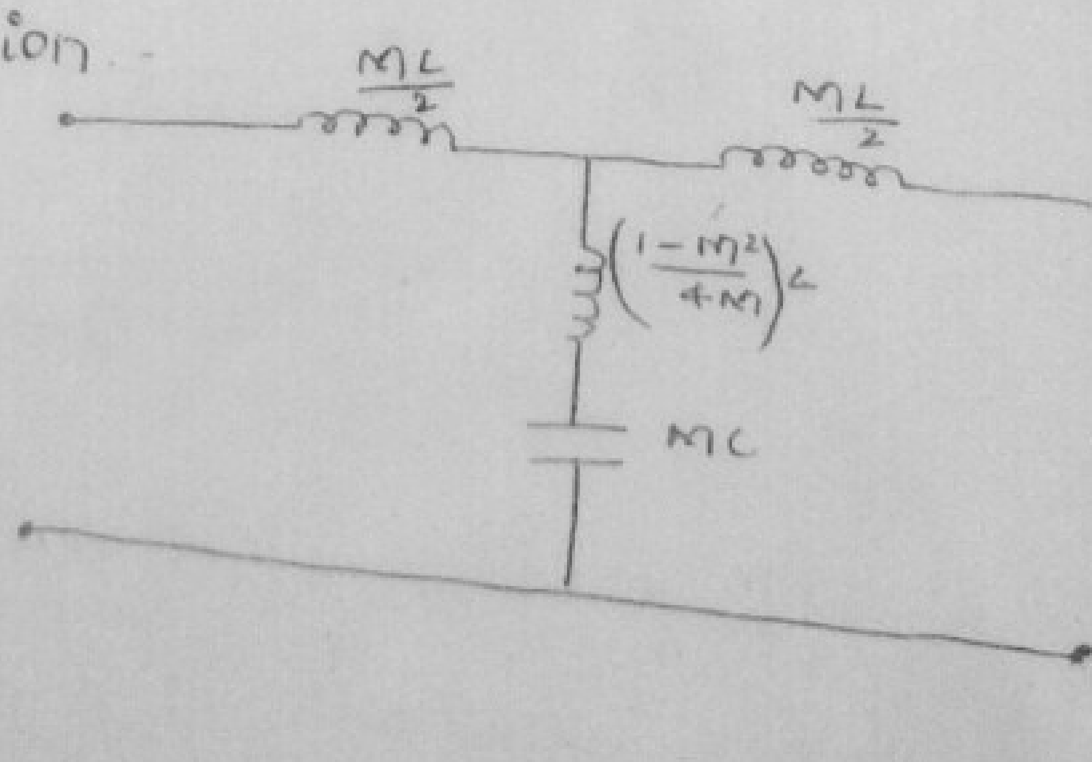
1. m-derived Low pass Filter:-

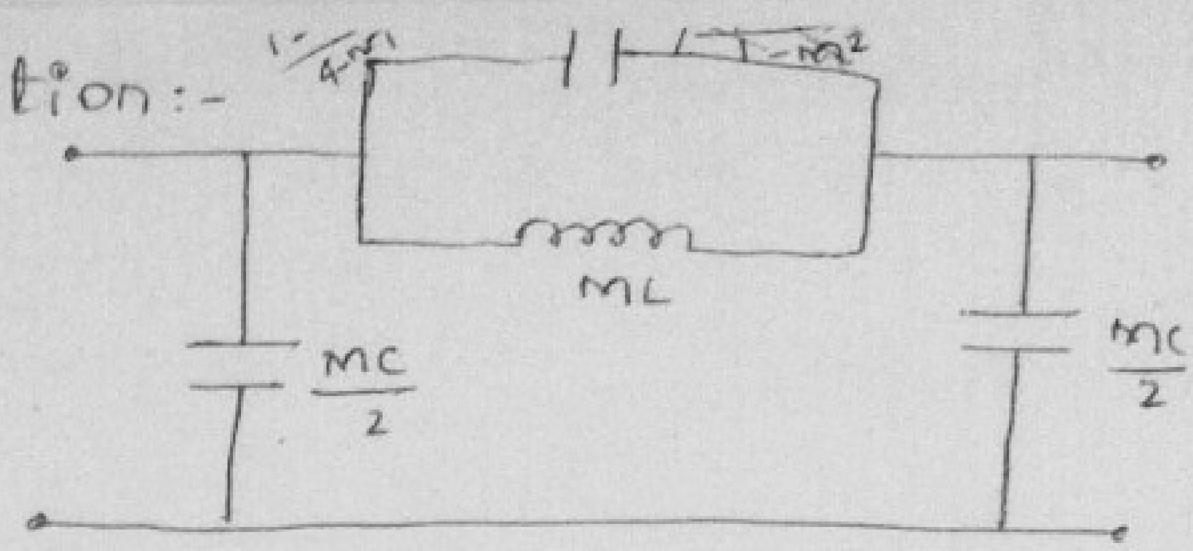
→ It is the filter which is derived from basic LPF which will produce infinite attenuation in the stop band at a desired frequency.

→ From the basic symmetrical T and π section, we can derive the LPF as

m-derived T-section.



M-derived π-section:-



(i)

→ From the symmetrical T-section filter, infinite attenuation will be acheived at a frequency when the shunt arm resonates. The shunt arm of symmetrical T- filter consists of two elements in series.

→ From the basics of resonance, we know that the condition for series resonance is net reactance of circuit = 0.

i.e., $X = 0 \Rightarrow X_L - X_C = 0 \Rightarrow X_L = X_C$ at $f = f_r$

Where X_L is inductive reactance of the coil (Ω)

X_C is capactive reactance

$\Rightarrow X_L = X_C$

$\Rightarrow W_r L = \frac{1}{W_r C}$

$\Rightarrow W_r^2 = \frac{1}{L_1 C_1}$ ⇒ but from diagram $L_1 = \left(\frac{1-m^2}{4m}\right)L$

$C_1 = mC$

$$\Rightarrow W_r^2 = \frac{1}{\left(\frac{1-m^2}{4m}\right)L(mC)}$$

W_r = radians/sec

$$\Rightarrow W_r^2 = \frac{4}{(1-m^2)LC} \qquad \because W_r = 2\pi f_r$$

$$\Rightarrow 4\pi^2 f_r^2 = \frac{4}{(1-m^2)LC}$$

$$\Rightarrow f_r^2 = \frac{4}{4\pi^2(1-m^2)LC} = \frac{1}{\pi^2(1-m^2)LC}$$

$$\Rightarrow f_r = \frac{1}{\pi\sqrt{(1-m^2)LC}}$$

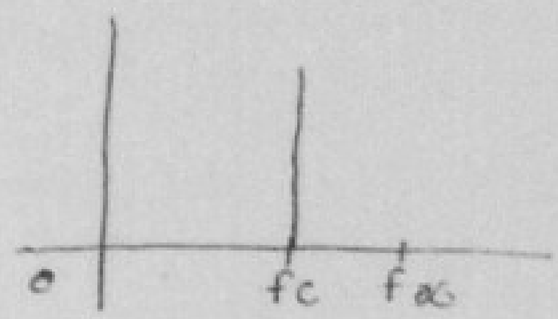
f_r is the frequency at which infinite attenuation occurs.

→ But from LPF configuration, we know that the cut off frequency $f_c = \frac{1}{\pi\sqrt{LC}}$

$$\Rightarrow f_\infty = \frac{f_c}{\sqrt{1-M^2}} \Rightarrow \sqrt{1-M^2} = \frac{f_c}{f_\infty}$$

$$\Rightarrow 1-M^2 = \left(\frac{f_c}{f_\infty}\right)^2 \Rightarrow 1-\left(\frac{f_c}{f_\infty}\right)^2 = M^2$$

$$\Rightarrow M = \sqrt{1-\left(\frac{f_c}{f_\infty}\right)^2}$$



$\Rightarrow$ As f_∞ will be always greater than cut off frequency, the value of M will be less than 1.

1. $f_\infty > f_c \Rightarrow \frac{f_c}{f_\infty} < 1 \Rightarrow 1-\left(\frac{f_c}{f_\infty}\right) \Rightarrow M > 0$.

2. Minimum value (M) at $f_\infty = f_c \Rightarrow$ then $M = 0$.

3. Maximum value of M f_∞ is infinite $\Rightarrow M = 1$

$\Rightarrow$ The value of M is between 0 & 1.

$$0 \le M < 1$$

(ii) From the symmetrical π-configuration infinite attenuat will be acheived when the series arm resonates. The series arm consists of L and C elements in parallel whose resonant condition is given by net susceptance is equal to zero i.e., $B = 0$.

$$Y = JB \quad (Y = \overset{0}{G} + JB)$$

$\Rightarrow$ When $B = 0$ then $Y = 0$

$$Z = \frac{1}{0} = \infty$$

for the series branch,

$$I = 0$$

$$\Rightarrow Y_T = Y_1 + Y_2 = \frac{1}{Z_1} + \frac{1}{Z_2}$$

(since Impedance is in $\|^{le}$ admittance is in serie

$$= \frac{1}{JWML} + \frac{+1}{\left(\frac{1}{JW\left(\frac{1-M^2}{4M}\right)C}\right)}$$

$$= \frac{1}{JWML} + J\frac{(1-M^2)WC}{4M}$$

$$= \frac{-J}{WML} + J\frac{(1-M^2)WC}{4M} = 0 + J\left[\left(\frac{1-M^2}{4M}\right)WC - \frac{1}{WML}\right]$$

$$Y_T = G + JB$$

→ At resonant frequency $\omega = \omega_r$ then $B = 0$

$$\Rightarrow \frac{1-m^2}{4m}\omega_r C = \frac{1}{\omega_r m L}$$

$$\Rightarrow \omega_r^2 = \frac{4}{(1-m^2)LC}$$

$$\Rightarrow (2\pi f_r)^2 = \frac{4}{(1-M^2)LC}$$

$$\Rightarrow f_r^2 = \frac{1}{\pi^2 LC(1-M^2)}$$

$$\Rightarrow f_r = \frac{1}{\pi\sqrt{LC}\sqrt{1-M^2}}$$

$$f_c = \frac{1}{\pi\sqrt{LC}} \quad \Rightarrow f_\infty = \frac{1}{f_c\sqrt{1-m^2}}$$

$$\Rightarrow m = \sqrt{1-\left(\frac{f_c}{f_\infty}\right)^2}$$

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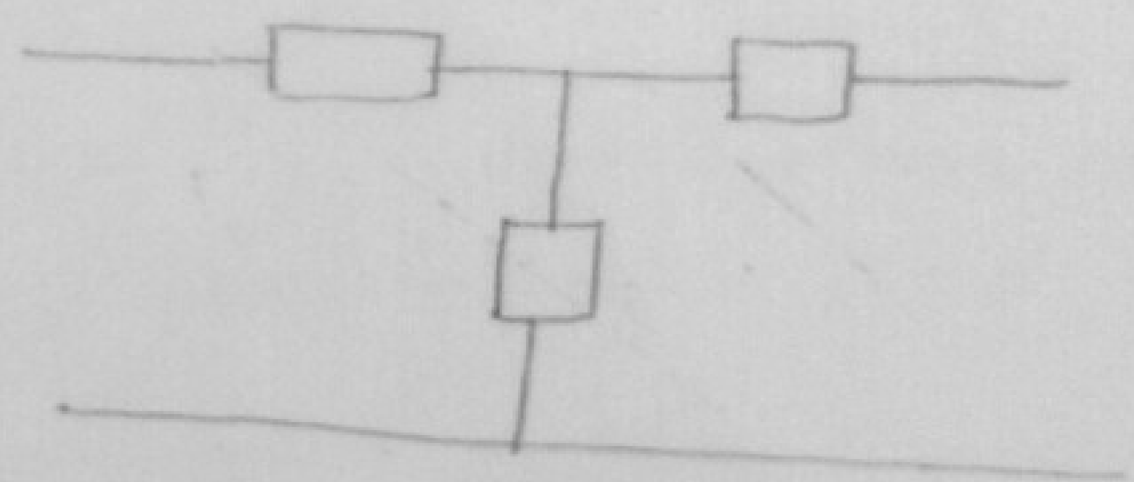
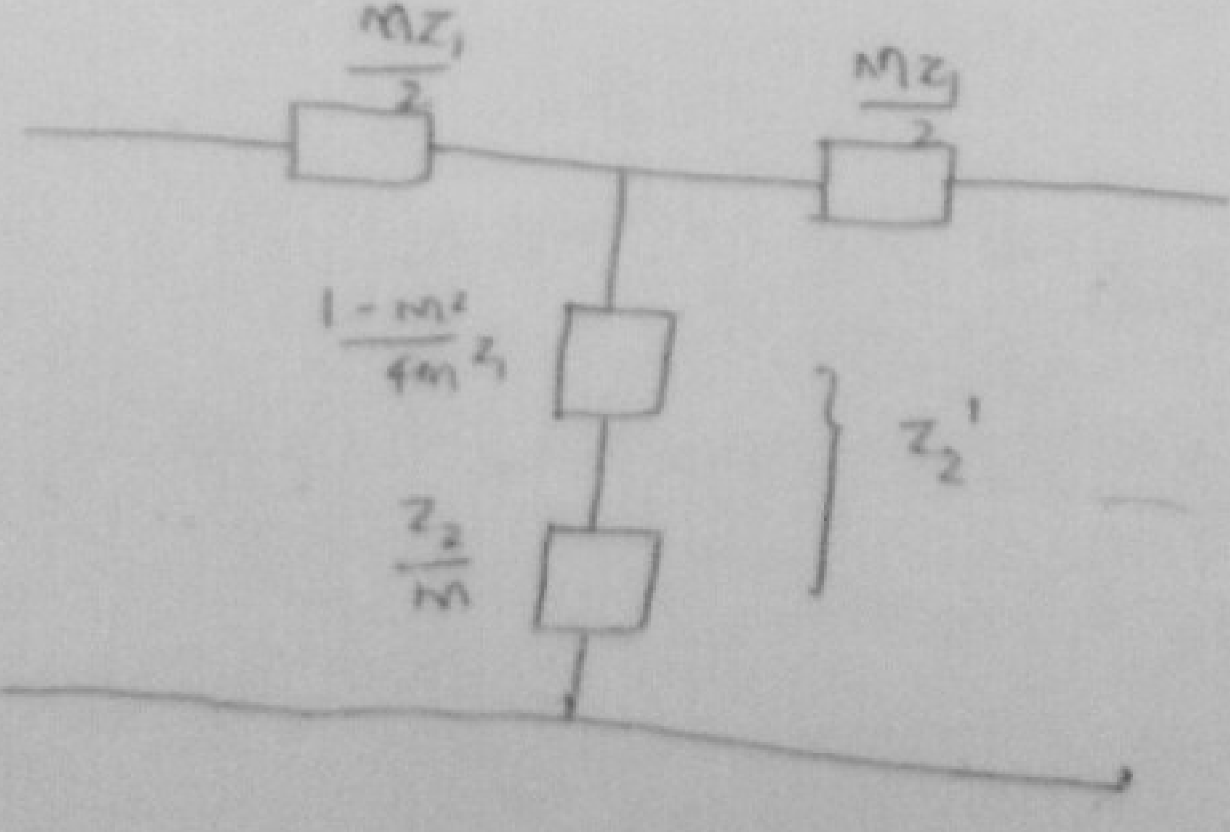
2 M-derived High pass filter:-

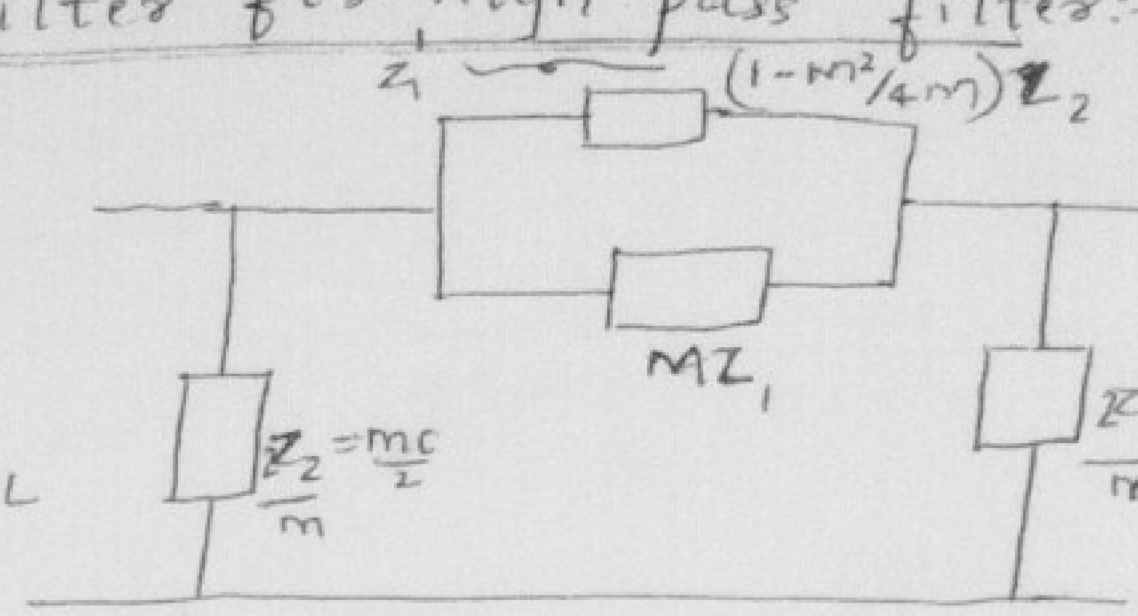
A m-derived HPF is the one in which we can acheive infinite attenuation at a desired frequency below cut-off frequency.

i) m-derived T-section HPF :-





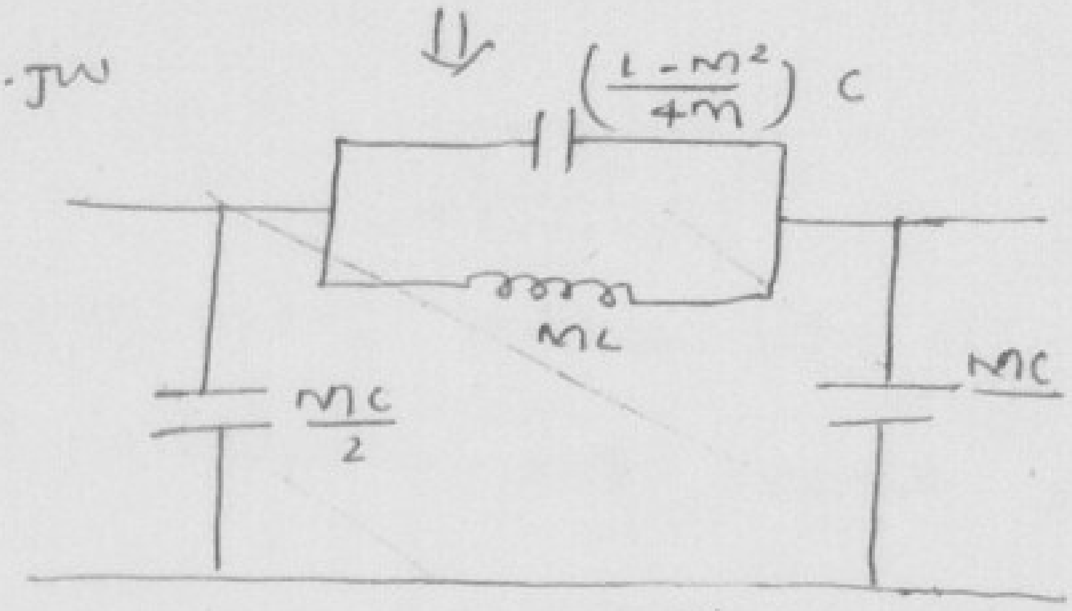
(ii) Symmetrical π- filter for High pass filter:-



Where $\cancel{Z_1 = \frac{1}{JWC}}$, $\cancel{Z_2 = \frac{1}{JWC}}$

$Z_1 = \frac{1}{JWC}$, $Z_2 = JWL$

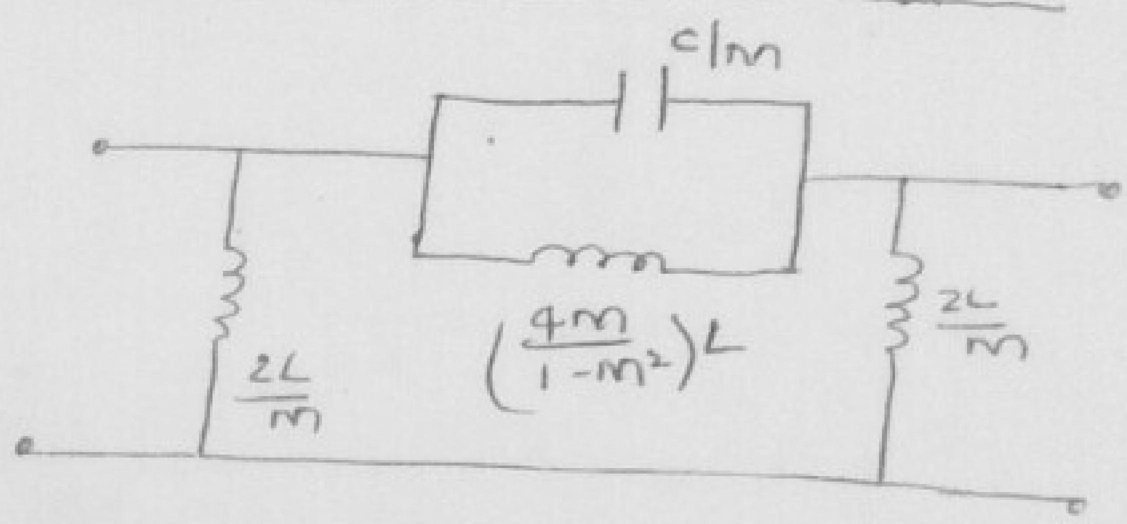
$\Rightarrow \left(\frac{1-m^2}{4m}\right)Z_2 = \left(\frac{1-m^2}{4m}\right)L \cdot JW$



$\Rightarrow \cancel{\frac{2Z_2}{m} = \frac{Z_2}{JW\left(\frac{mC}{2}\right)}}$

$\Rightarrow mZ_1 = \frac{1}{JW\left(\frac{C}{m}\right)}$

$\Rightarrow \frac{2Z_2}{m} = JW\left(\frac{2L}{m}\right)$



$\Rightarrow Y = G + JB$

$B = 0$

$$\Rightarrow Y_T = \frac{1}{Z_x} + \frac{1}{Z_y} = \frac{1}{JW\left(\frac{C}{m}\right)} + \frac{1}{JW\frac{4mL}{1-m^2}}$$

$$= 0 + \left(JW\frac{C}{m} + \frac{1-m^2}{J4WmL}\right)$$

$$= 0 + J\left(\frac{WC}{m} - \frac{1-m^2}{4WmL}\right)$$

$$Y_T = G + JB$$

$\Rightarrow$ At $f_r \Rightarrow B = 0 \Rightarrow (W = W_r \text{ at } f_r)$

$$\Rightarrow \frac{W_r C}{m} = \frac{1-m^2}{4W_r mL}$$

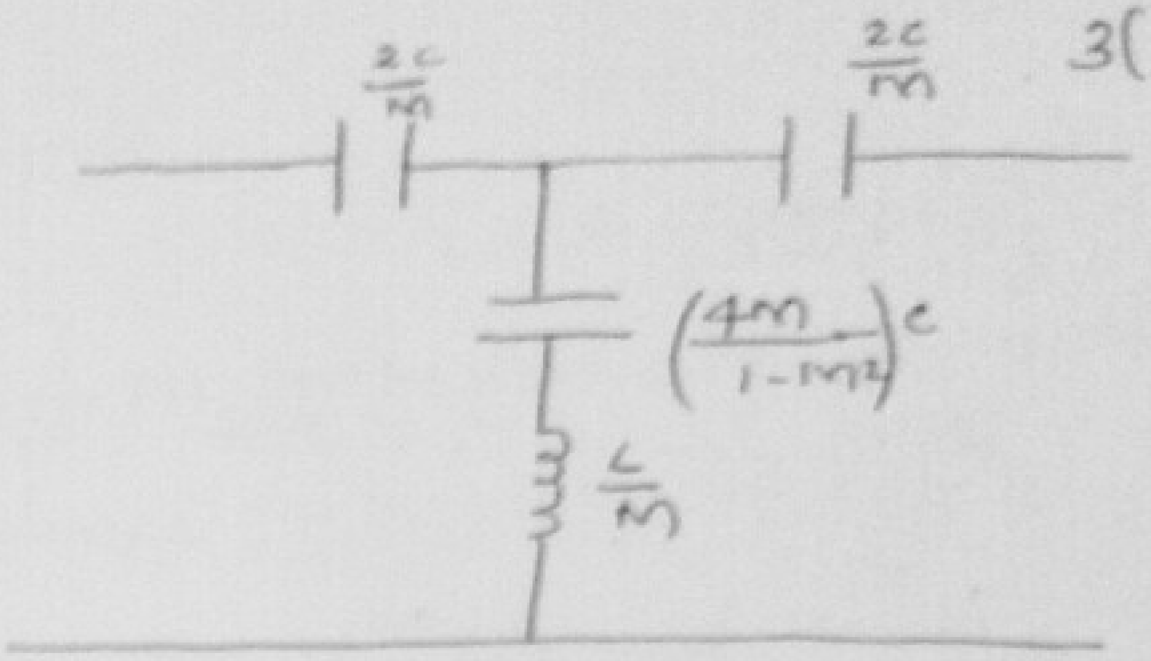
$$\Rightarrow W_r C = \frac{1-m^2}{4W_r L}$$

$$\Rightarrow W_r^2 = \frac{1-m^2}{4LC} = \frac{1-m^2}{4LC}$$

$$\Rightarrow 4\pi^2 f_r^2 = \frac{1-m^2}{4LC}$$

$$\Rightarrow f_r = \frac{\sqrt{1-m^2}}{4\pi\sqrt{LC}}$$

→ The above is said to be High pass when $Z_1 = \frac{1}{j\omega C}$, $Z_2 = j\omega L$



3(19)

$$\Rightarrow Z_1 = \frac{1}{j\omega C}$$

$$\Rightarrow \frac{mZ_1}{2} = \frac{m}{j\omega 2C} = \frac{1}{j\omega\left(\frac{2C}{m}\right)}$$

$$\Rightarrow \frac{Z_2}{m} = j\omega\left(\frac{L}{m}\right)$$

$$\Rightarrow \frac{1-m^2}{4m} Z_1 = \frac{1}{j\omega\left(\frac{4m}{1-m^2}\right)C}$$

→ At infinite attenuation frequency, the shunt branch will resonate where the condition is $X_L = X_C$.

$$\Rightarrow \omega_r L = \frac{1}{\omega_r C}$$

$$\Rightarrow \omega_r^2 = \frac{1}{LC}$$

$$\Rightarrow 4\pi^2 f_r^2 = \frac{1}{LC}$$

$$\Rightarrow \pi^2 4 f_r^2 = \frac{1}{\frac{L}{m}\left(\frac{4m}{1-m^2}\right)C} \Rightarrow f_r^2 = \frac{1-m^2}{16\pi^2 LC}$$

$$\Rightarrow f_r = \frac{\sqrt{1-m^2}}{4\pi\sqrt{LC}} = f_\infty$$

where f_∞ indicates the frequency of infinite attenuation.

To find m :-

→ From the characteristics of HPF, we know that the value of cut-off frequency,

$$f_c = \frac{1}{4\pi\sqrt{LC}}$$

$$\Rightarrow f_\infty = \frac{\sqrt{1-m^2}}{1/f_c}$$

$$\Rightarrow f_\infty = f_c\sqrt{1-m^2}$$

$$\Rightarrow \frac{f_\infty}{f_c} = \sqrt{1-m^2} \Rightarrow 1-m^2 = \left(\frac{f_\infty}{f_c}\right)^2$$

$$\Rightarrow m = \sqrt{1-\left(\frac{f_\infty}{f_c}\right)^2}$$

1. When $f_\infty = f_c \Rightarrow m = 0$

1. When $f_\infty = 0 \Rightarrow m = 1$

$\Rightarrow$ m lies between 0 and 1 $\Rightarrow 0 < m < 1$

$\Rightarrow f_\alpha = f_c\sqrt{1-m^2}$

$\Rightarrow m = \sqrt{1-\left(\frac{f_\alpha}{f_c}\right)^2}$

19:- Design an m-derived LPF with a cut-off frequency of 10KHz (f_c) and a characteristic impedance of 400Ω (K). The frequency of infinite attenuation is given as 20KHz (f_∞).

$C = \frac{1}{\pi K f_c}$

$L = \frac{K}{\pi f_c}$

sol:- Given data,

→ given that it is m-derived LPF

→ cut-off frequency $f_c = 10KHz$

→ characteristic impedance (or) Design $K = 400Ω$ impedance.

→ $f_\alpha = 20KHz$

For a LPF, $Z_1 = j\omega L$, $Z_2 = \frac{1}{j\omega C}$ Where $L = \frac{K}{\pi f_c}$

$\Rightarrow L = \frac{400}{\pi \times 1000 \times 10}$ and $C = \frac{1}{\pi K f_c}$ $C = \frac{1}{\pi K f_c}$

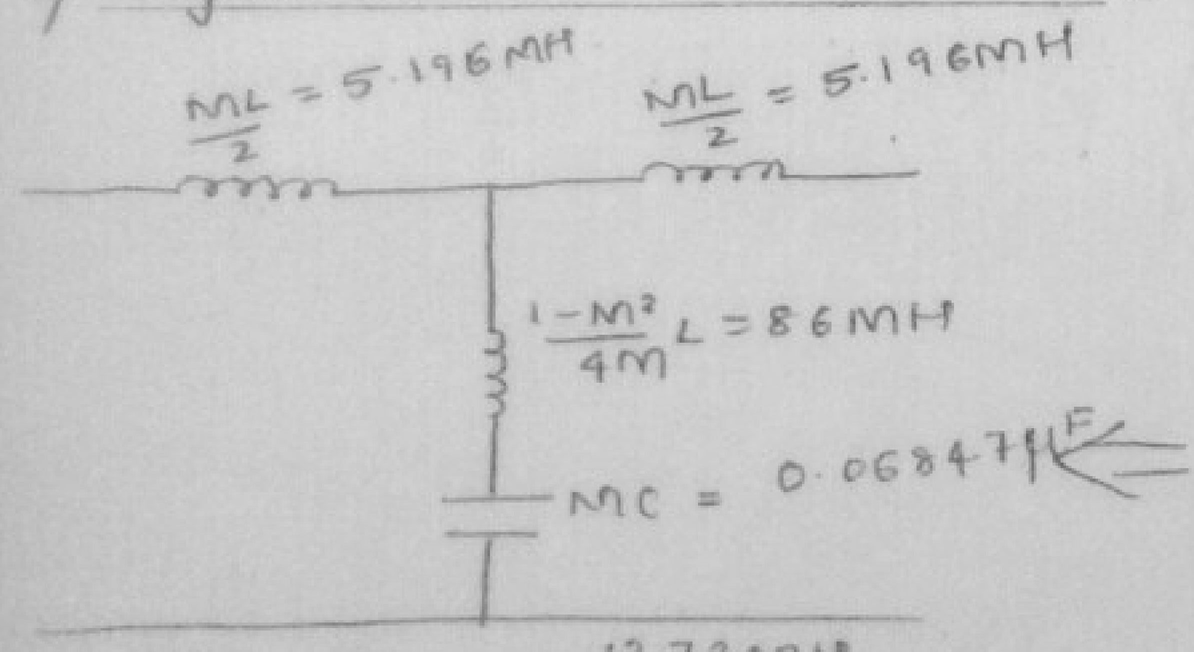
$\Rightarrow L = 0.012$ Henry $C = 0.0795\mu F$

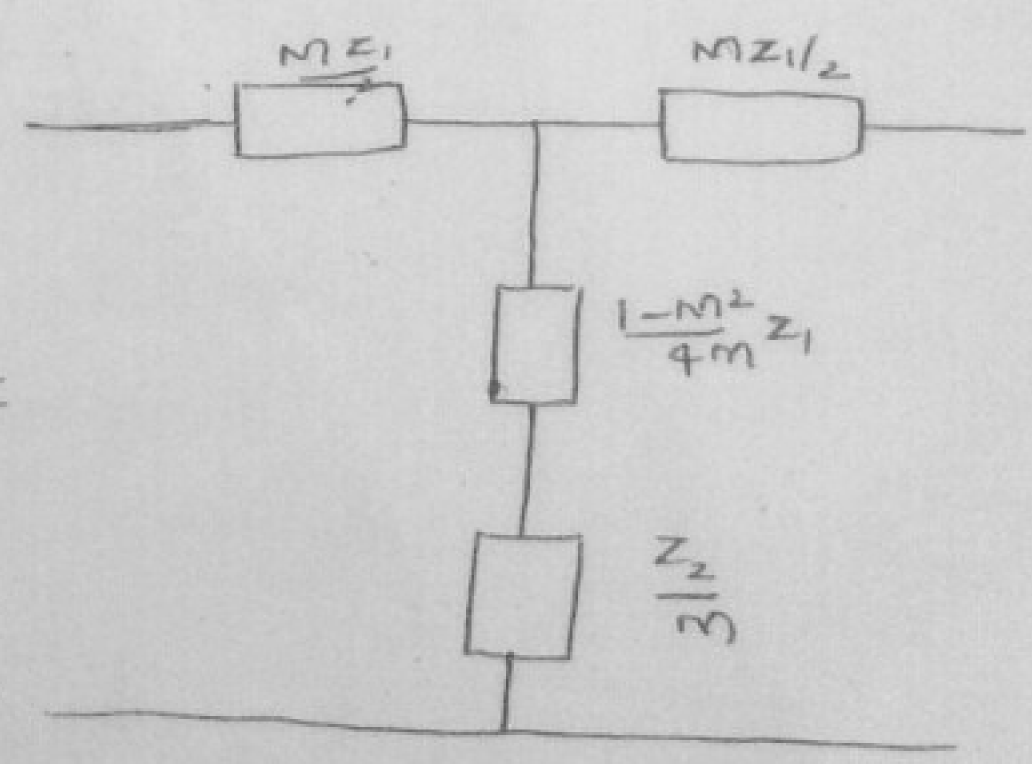
$\Rightarrow L = 12mH$

The value of m is,

$$m = \sqrt{1-\left(\frac{f_c}{f_\infty}\right)^2} = \frac{\sqrt{3}}{2} = 0.866$$

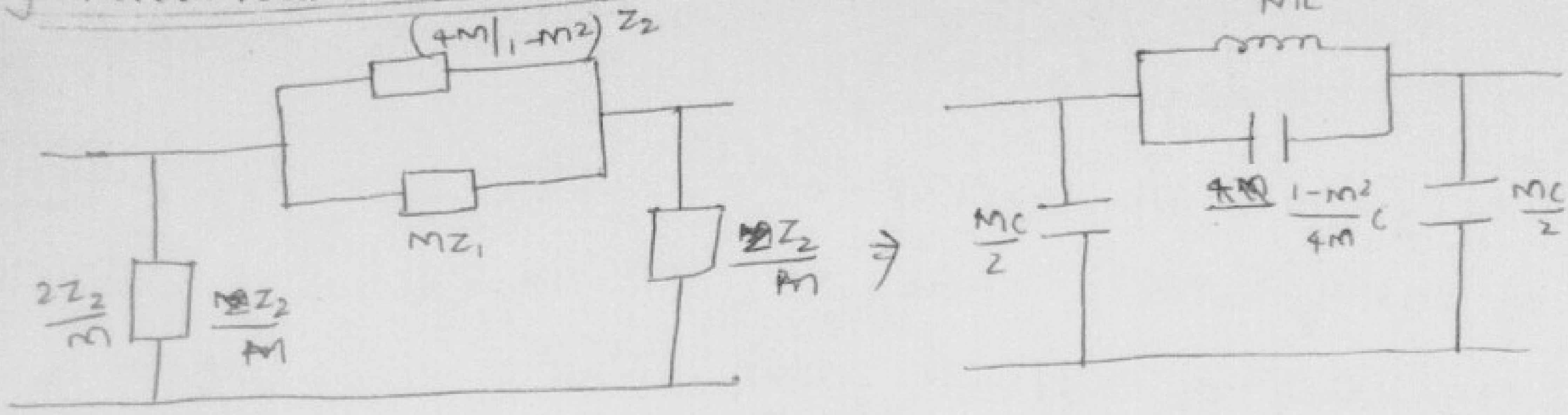
→ symmetrical T-section filter, (LowPF)





$\Rightarrow \frac{mL}{2} = \frac{(0.866)\times 12mH}{2} = 5.196mH$

Symmetrical π-section LPF :-



Q:- Design an m-derived HPF for a nominal impedance of 500Ω (k) with a cut-off frequency of 25kHz (f_c) to attain infinite attenuation at a frequency of 12.5kHz (f_∞). →

Sol:- Given, $k = 500\Omega$

$$f_c = 25\,kHz$$

$$f_\infty = 12.5\,kHz$$

→ For a HPF $Z_1 = \frac{1}{j\omega C}$ and $Z_2 = j\omega L$

where $L = \frac{k}{4\pi f_c}$ & $C = \frac{1}{4\pi k f_c}$

$\Rightarrow L = \frac{500}{4\pi \times 25\times10^3} = 1.591\,mHenry$ $\quad C = \frac{1}{4\pi\times 500\times 25}\,mF = 6.366\times10^{-9}F$

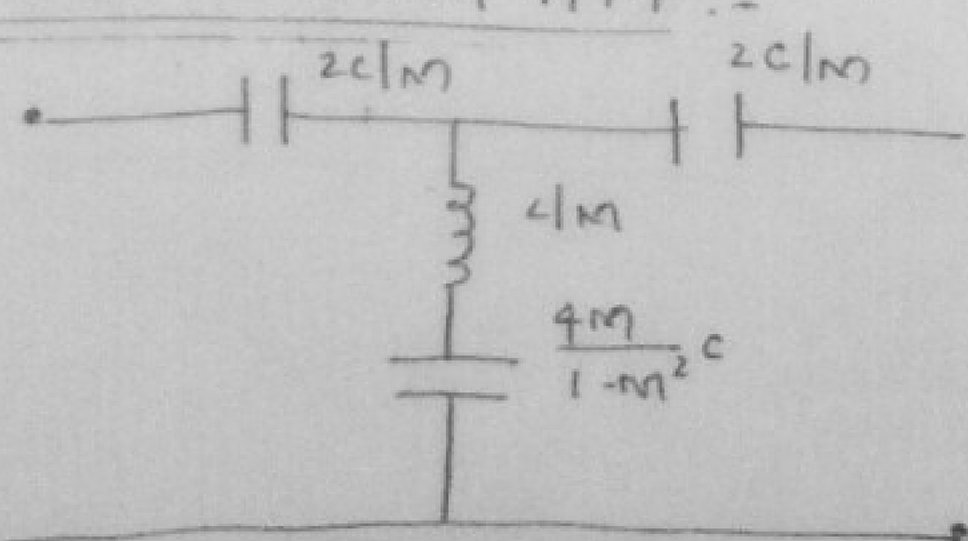
$C = 0.00636\,\mu F$

∴ The value of m is $m = \sqrt{1-\left(\frac{f_\infty}{f_c}\right)^2} = \sqrt{1-\left(\frac{12.5}{25}\right)^2} = \frac{\sqrt{3}}{2}$

$m = 0.86602$

$m = 0.866$

Symmetrical T-section HPF:-

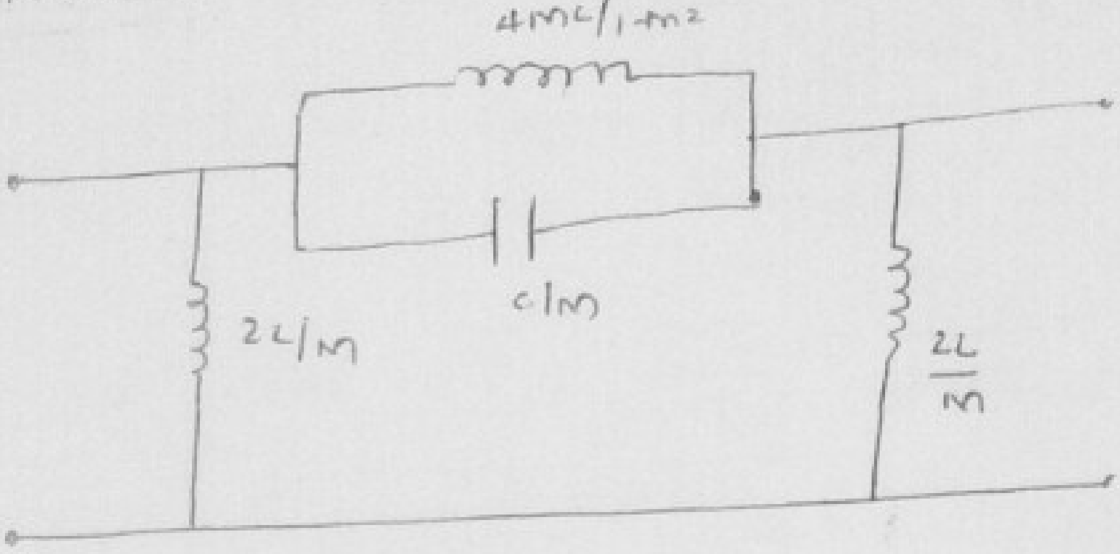


$\frac{2C}{m} = 0.0147\,\mu F$

$\frac{L}{m} = 1.8371\,mH$

$\frac{4m}{1-m^2}C = 0.8819\,\mu F$

Symmetrial π-section HPF:-



$$\frac{4mL}{1-m^2} = 22.04\,mH$$

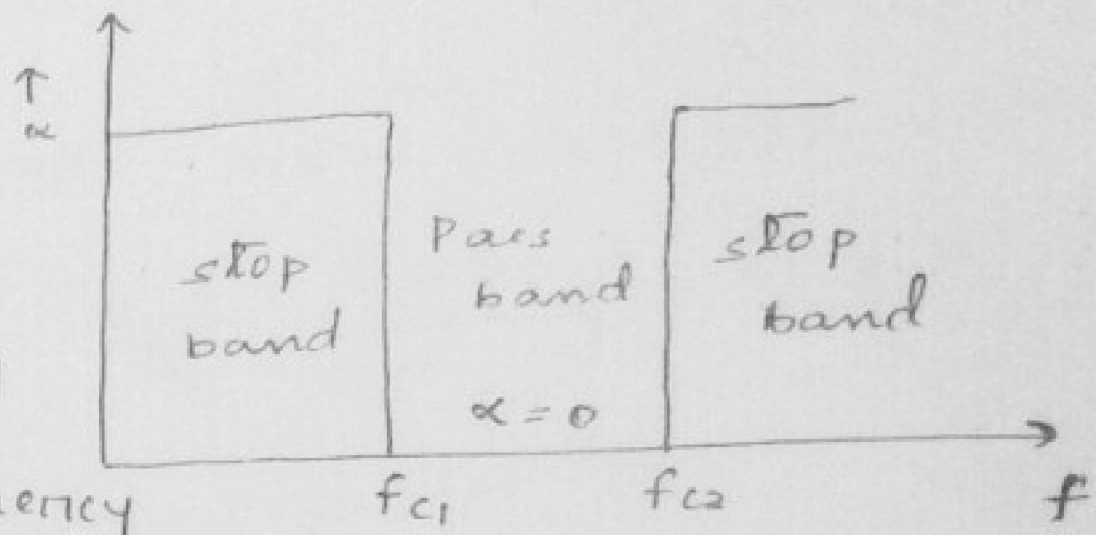
$$\frac{C}{m} = 7.351 \times 10^{-3}\,\mu F$$

$$\frac{2L}{m} = 3.674\,mH$$

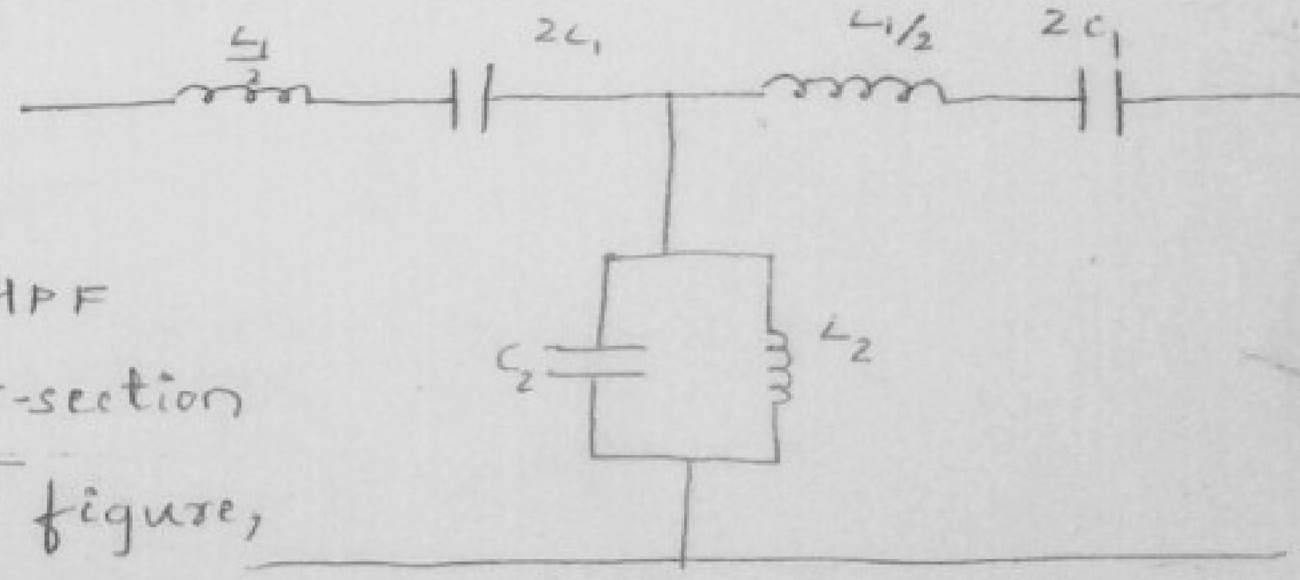
21/1/13

Band pass filter:- A BPF is a one which transmits a certai[n] band of frequencies and eliminates the remaining.

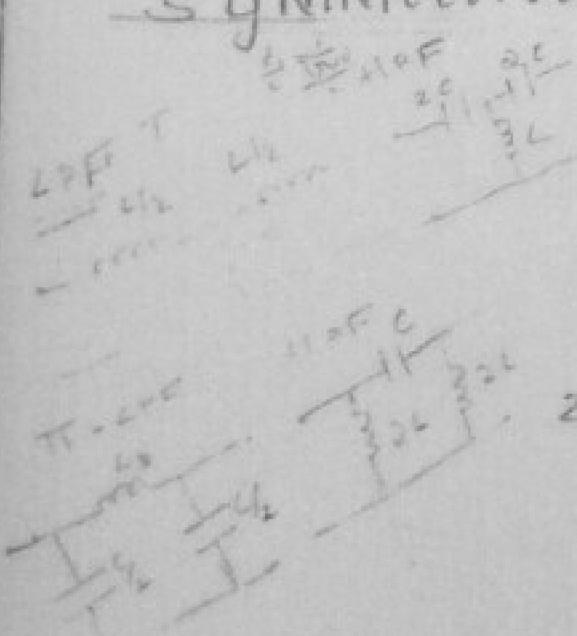
→ The Band pass Filter can be constructed by joining a HPF with a LPF where the LPF cut off frequency will be the upper cut off frequency & the cut-off frequency of HPF will be the lower cut off frequency. But it is an uneconomical to join two prototype filters of different characteristics

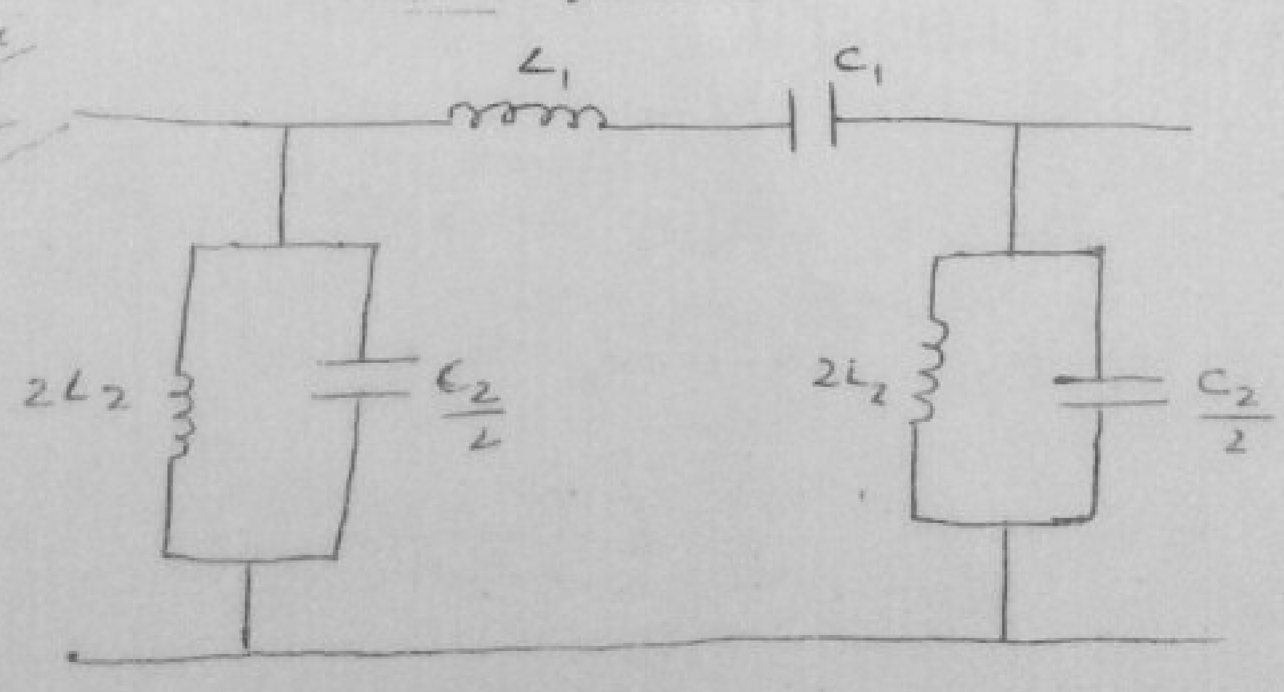


→ Hence the BPF will be constructed by joining the elements of LPF and HPF together. A symmetrical T-section BPF will be as shown in figure,



Symmetrical π-section filter:-





→ The series Arm impedance is,

$$Z_1 = JWL_1 + \frac{1}{JWC_1} = J\left(\frac{W^2L_1C_1-1}{WC_1}\right) \longrightarrow ①$$

$$Z_2 = L_2 \| C_2 = JWL_2 \| \frac{1}{JWC_2} = \frac{\frac{JWL_2}{JWC_2}}{JWL_2 + \frac{1}{JWC_2}} = \frac{L_2(JWC_2)}{C_2(1-W^2L_2C_2)}$$

$$Z_2 = \frac{JWL_2}{1-W^2L_2C_2} \longrightarrow ②$$

The condition to have transmission band for a BPF is the series arm and shunt arm should resonate at the same frequency.

→ For the series arm, the resonant condition is net reactance = 0

i.e., $0 + J\left(\frac{W_r^2L_1C_1-1}{WC_1}\right) = 0.$

$$\Rightarrow W_r^2L_1C_1 - 1 = (WC_1)0$$

$$\Rightarrow W_0^2L_1C_1 = (\cancel{WC_1})^{0} + 1$$

$$\Rightarrow W_r^2 = \frac{1}{L_1C_1} \longrightarrow ③$$

→ For the shunt arm, net susceptance = 0 (∵ $\|^{el}$ LC Resonant ckt)

$$\Rightarrow Y = \frac{1}{Z_2} = \frac{1-W^2L_2C_2}{JWL_2} = \frac{-J(1-W^2L_2C_2)}{WL_2} = G + JB$$

$$\Rightarrow B = 0 \Rightarrow \frac{1-W^2L_2C_2}{WL_2} = 0$$

$$\Rightarrow 1 - W_r^2L_2C_2 = 0$$

$$\Rightarrow W_r^2 = \frac{1}{L_2C_2} \longrightarrow ④$$

From eq.s ③ & ④, $\frac{1}{L_1C_1} = \frac{1}{L_2C_2}$

$$\Rightarrow L_1C_1 = L_2C_2 \longrightarrow ⑤$$

→ For a constant k-type filter, $Z_1Z_2 = k^2$, for the given BPF

$$\Rightarrow Z_1Z_2 = J^2\frac{WL_2}{WC_1}\left(\frac{W^2L_1C_1-1}{1-W^2L_2C_2}\right)$$

$$\Rightarrow Z_1Z_2 = \frac{L_2}{C_1}\left(\frac{1-W^2L_1C_1}{1-W^2L_2C_2}\right)$$

from eq. ⑤ $\Rightarrow Z_1 Z_2 = \frac{L_2}{C_1}\left(\frac{1-w^2L_1C_2}{1-w^2L_2C_2}\right) = \frac{L_2}{C_1}$

$L_1C_1 = L_2C_2$

$\Rightarrow Z_1 Z_2 = \frac{L_2}{C_1} = k^2$ which is a constant-K filter.

* Relation between Resonating frequencies & cut-off frequencies:

→ From the condition of pass band, we have $-1 < \frac{Z_1}{4Z_2} < 0$

$\Rightarrow \frac{Z_1}{4Z_2} = -1$

$\Rightarrow Z_1 = -4Z_2$ Multiply Z_1 on both sides

$\Rightarrow Z_1^2 = -4Z_1Z_2 \qquad \because Z_1Z_2 = k^2$

$\Rightarrow Z_1^2 = -4k^2$

$\Rightarrow Z_1 = \pm 2Jk$

→ At low cut off frequency $Z_1 = -2Jk \ (\because X_{C1} > X_{L1})$ and at upper cut off frequency $Z_1 = +2Jk \ (X_{L1} > X_{C1})$.

Z_1 at lower cut of frequency at $w = w_1$

Z_1 at $w = w_1 = Jw_1L_1 + \frac{1}{Jw_1C_1} = -2Jk$

Z_1 at $w = w_2 = Jw_2L_1 + \frac{1}{Jw_2C_1} = +2Jk$

$\Rightarrow Jw_1L_1 + \frac{1}{Jw_1C_1} = -\left(Jw_2L_1 + \frac{1}{Jw_2C_1}\right)$

$\Rightarrow Jw_1L_1 + \frac{1}{Jw_1C_1} = -Jw_2L_1 - \frac{1}{Jw_2C_1}$

$\Rightarrow \frac{J^2w_1^2L_1C_1 + 1}{Jw_1C_1} = \frac{-J^2w_2^2L_1C_1 - 1}{Jw_2C_1}$

$\Rightarrow (-w_1^2L_1C_1 + 1)w_2 = -(-w_2^2L_1C_1 + 1)w_1$

$\Rightarrow -w_1^2L_1C_1w_2 + w_2 = -(-w_1w_2^2L_1C_1 + w_1)$

$\Rightarrow -L_1C_1w_1w_2(w_1 + w_2) = -(w_1 + w_2)$

$\Rightarrow w_1w_2 = \frac{+1}{L_1C_1} \qquad \left(\text{since } \frac{1}{L_1C_1} = w_r^2\right)$

$\Rightarrow \frac{1}{w_r^2} = \frac{1}{w_1w_2} \Rightarrow w_r^2 = w_1w_2 \Rightarrow w_r = \sqrt{w_1w_2}$

$\Rightarrow \frac{1}{w_r} = \frac{1}{\sqrt{w_1w_2}} \Rightarrow \frac{1}{2\pi f_r} = \frac{1}{\sqrt{2\pi f_1 \times 2\pi f_2}} \Rightarrow f_r = \sqrt{f_1}$

Design of a Band pass Filter:-

Z_1 at $\omega = \omega_1 \Rightarrow -2jk = j\omega_1 L_1 + \frac{1}{j\omega_1 C_1}$

$$\Rightarrow 1 + j^2\omega_1^2 L_1 C_1 = -2j^2 k \omega_1 C_1 = 2k\omega_1 C_1$$

$$\Rightarrow \cancel{\omega_1 L_1 = -2k} + 1 - \omega_1^2 L_1 C_1 = 2\omega_1 k C_1$$

(since $L_1 C_1 = \frac{1}{\omega_r^2}$)

$$\Rightarrow \cancel{L_1 = \frac{-2k}{\omega_1}}$$

$$\Rightarrow 2jk = j\omega_2 L_1 + \frac{1}{j\omega_2 C_1}$$

$$\Rightarrow 2j^2 k\omega_2 C_1 = j^2\omega_2^2 L_1 C_1 + 1$$

$$\Rightarrow -2k\omega_2 C_1 = 1 - \omega_2^2 L_1 C_1$$

$$\Rightarrow \cancel{L_1 C_1 = \frac{1}{k^2}} \Rightarrow \cancel{-2k\omega_2 C_1 = \omega_2^2 k}$$

$$\because L_1 C_1 = \frac{1}{\omega_r^2} \Rightarrow -2k\omega_2 C_1 = 1 - \frac{\omega_2^2}{\omega_r^2}$$

$$\Rightarrow -2k\omega_2 C_1 = 1 - \frac{f_2^2}{f_r^2}$$

$$\Rightarrow -2k\omega_2 C_1 = 1 - \frac{f_2}{f_1}$$

$$\Rightarrow -2k(2\pi f_2 f_1) C_1 = f_1 - f_2$$

$$\Rightarrow -4\pi k C_1 f_1 f_2 = f_1 - f_2$$

$$\Rightarrow \boxed{C_1 = \frac{f_2 - f_1}{4\pi k f_1 f_2}}$$

→ It is a constant k-filter, where values of shunt arm,

$$\Rightarrow Z_1 Z_2 = k^2 = \frac{L_2}{C_1} = \frac{L_1}{C_2} = k^2$$

$$\Rightarrow L_1 C_1 = L_2 C_2$$

$$\Rightarrow \boxed{L_2 = C_1 k^2 = \frac{k(f_2 - f_1)}{4\pi f_1 f_2} \text{ Henrys}}$$

$$\Rightarrow \boxed{C_2 = \frac{L_1}{k^2} = \frac{1}{\pi k (f_2 - f_1)} \text{ Farads}}$$

$$\Rightarrow 1 - \omega_1^2 L_1 C_1 = 2\omega_1 k C_1$$

$$\Rightarrow 1 - \frac{\omega_1^2}{\omega_r^2} = 2\omega_1 k C_1$$

$$\Rightarrow 1 - \left(\frac{f_1}{f_r}\right)^2 = 4\pi k f_1 C_1$$

since $\omega_r = \sqrt{\omega_1 \omega_2}$

$$f_r = \sqrt{f_1 f_2}$$

$$f_r^2 = f_1 f_2$$

$$\omega_1^2 = 4\pi^2 f_1^2 \text{ and } \omega_r^2 = 4\pi^2 f_r^2$$

$$\Rightarrow 1 - \frac{f_1^2}{f_1 f_2} = 4\pi k f_1 C_1$$

$$\Rightarrow 1 - \frac{f_1}{f_2} = 4\pi k f_1 C_1$$

$$\Rightarrow f_2 - f_1 = 4\pi k f_1 f_2 C_1$$

$$\Rightarrow C_1 = \frac{f_2 - f_1}{4\pi k f_1 f_2} \text{ Farads}$$

and $L_1 C_1 = \frac{1}{\omega_r^2}$

$$L_1 = \frac{1}{\omega_r^2 C_1}$$

$$\Rightarrow L_1 = \frac{4\pi k f_1 f_2}{\omega_r^2 (f_2 - f_1)}$$

$$\Rightarrow L_1 = \frac{4\pi k f_1 f_2}{4\pi^2 f_r^2 (f_2 - f_1)}$$

$$\Rightarrow L_1 = \frac{k f_1 f_2}{\pi f_1 f_2 (f_2 - f_1)}$$

$$\Rightarrow L_1 = \frac{k}{\pi (f_2 - f_1)} \text{ Henr}$$

19:- Design a Band pass filter whose cut off frequencies are $f_1 = 5KHz$ and $f_2 = 10KHz$ for a nominal impedance of $k = 600\Omega$ all. 3(23)

Determine the value of resonant frequency also. ($f_0 = ?$)

Sol:- Given data,

$k = 600\Omega$,

Lower-cut of frequency $f_1 = 5KHz$.

Upper-cut of frequency $f_2 = 10KHz$

We know that from the design of Band pass filter,

$$\boxed{L_1 = \frac{k}{\pi(f_2 - f_1)} \text{ Henry} \quad \& \quad L_2 = \frac{k(f_2 - f_1)}{4\pi f_1 f_2} \qquad C_1 = \frac{f_2 - f_1}{4\pi k f_1 f_2} \quad C_2 = \frac{1}{\pi k(f_2 - f_1)}}$$

$f_r = \sqrt{f_1 f_2}$

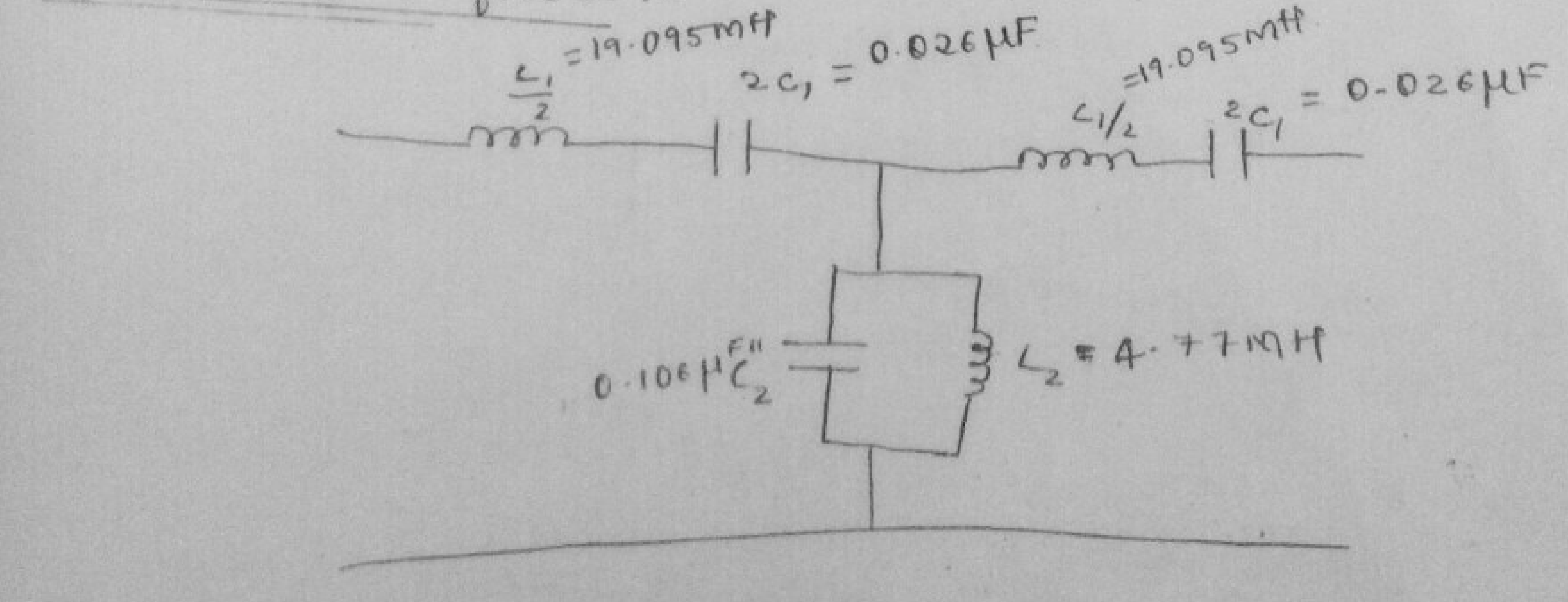
$$\Rightarrow L_1 = \frac{k}{\pi(f_2 - f_1)} = \frac{600}{\pi(5k)} = 38.19mH$$

$$\Rightarrow C_1 = \frac{f_2 - f_1}{4\pi k f_1 f_2} = \frac{5k}{50k^2 \times 600 \times 4\pi} = 1.32 \times 10^{-8} = 0.013\mu F$$

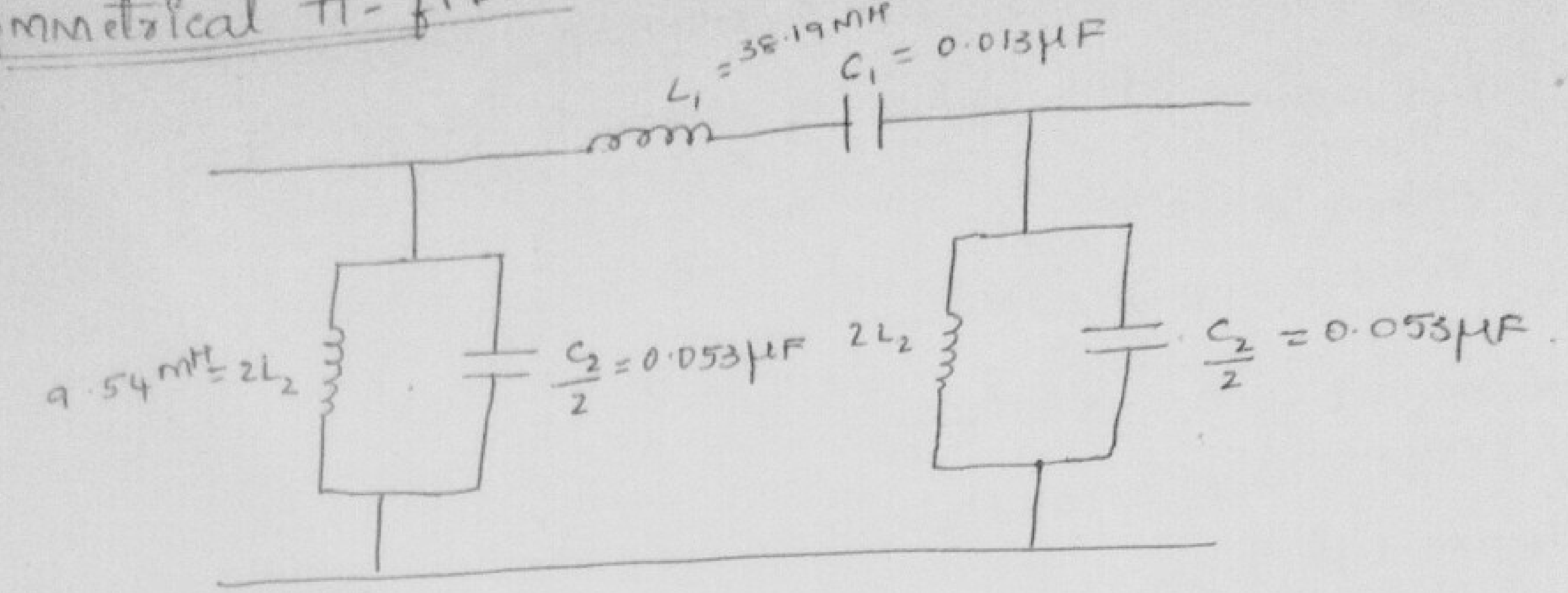
$$\Rightarrow L_2 = \frac{600(5k)}{4\pi \times 50 \times (10^3)^2} = 4.77 \; \cancel{4.77}mH$$

$$\Rightarrow C_2 = \frac{1}{\pi \times 600 \times (5k)} = 0.106\mu F$$

Symmetrical T-filter :-



Symmetrical π- filter:- (BPF)



We know that,

$\Rightarrow f_r = \sqrt{f_1 f_2}$

$\Rightarrow f_r = \sqrt{5 \times 10k^2} = \sqrt{50}\ kHz = 7.071 kHz$

29:- Design a Band pass filter whose cut of frequencies are 1kHz and 10kHz and the nominal impedance is 500Ω. Draw the symmetrical T and π configuration.

sol:- Given data,

Nominal impedance $k = 500\Omega$

Lower cut frequency $f_1 = 1kHz$

$f_2 = 10kHz$.

We know that,

$L_1 = \frac{k}{\pi(f_2 - f_1)}$ H & $C_1 = \frac{f_2 - f_1}{4\pi k f_1 f_2}$ F

$L_2 = \frac{k(f_2 - f_1)}{4\pi f_1 f_2}$ & $C_2 = \frac{1}{\pi k(f_2 - f_1)}$

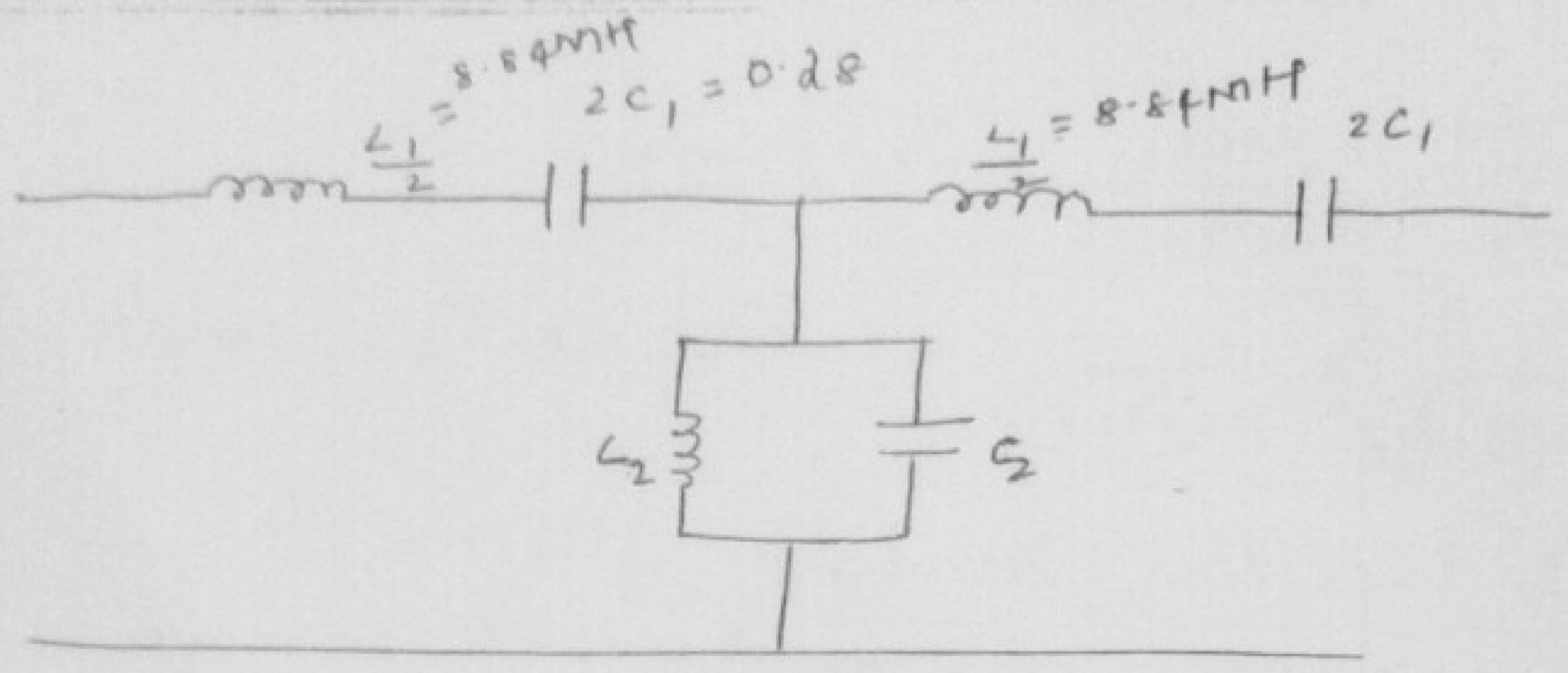
$\Rightarrow L_1 = \frac{500}{\pi \times 9k} = 17.68mH$

$\Rightarrow C_1 = \frac{9k}{4\pi(500)\,10k^2} = 1.432 \times 10^{-7} = 0.14\mu F$

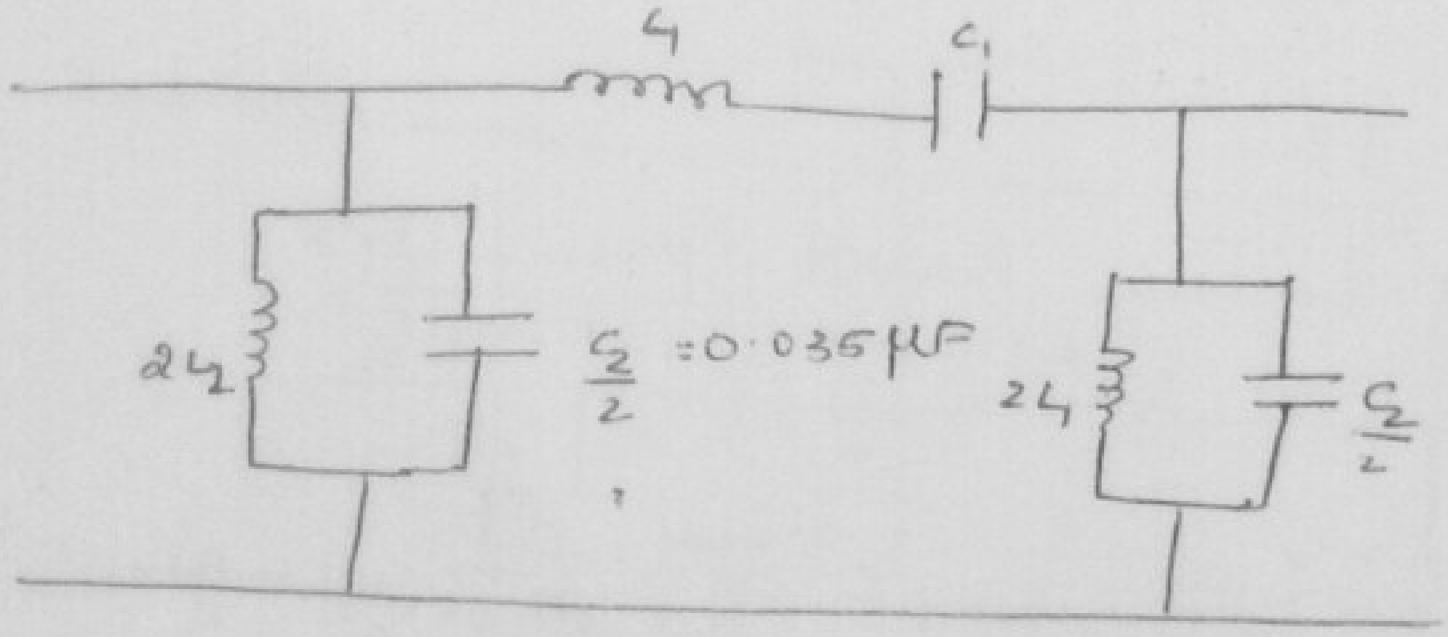
$\Rightarrow C_2 = \frac{1}{\pi(500)(9k)} = 7.07 \times 10^{-7} = 0.07\mu F$

$\Rightarrow L_2 = \dfrac{500 \times 9k}{4\pi \times 10k^2} = 35.80 mH$.

Symmetrical T-section BPF :-



Symmetrial π- section BPF :-



39:- Design a symmetrical BPF of T & π sections whose cut off frequencies are 6 KHz and 3 KHz, the nominal impedance is given as 600Ω.

Sol:- Given,

Lower cut of frequency $= f_1 = 3 KHz$

Upper cut of frequency $= f_2 = 6 KHz$

Nominal Impedance $k = 600\Omega$

We know that,

$L_1 = \dfrac{k}{\pi (f_2 - f_1)}$ Henry & $C_1 = \dfrac{f_2 - f_1}{4\pi k f_1 f_2}$ F

$L_2 = \dfrac{k(f_2 - f_1)}{4\pi f_1 f_2}$ & $C_2 = \dfrac{1}{\pi k (f_2 - f_1)}$

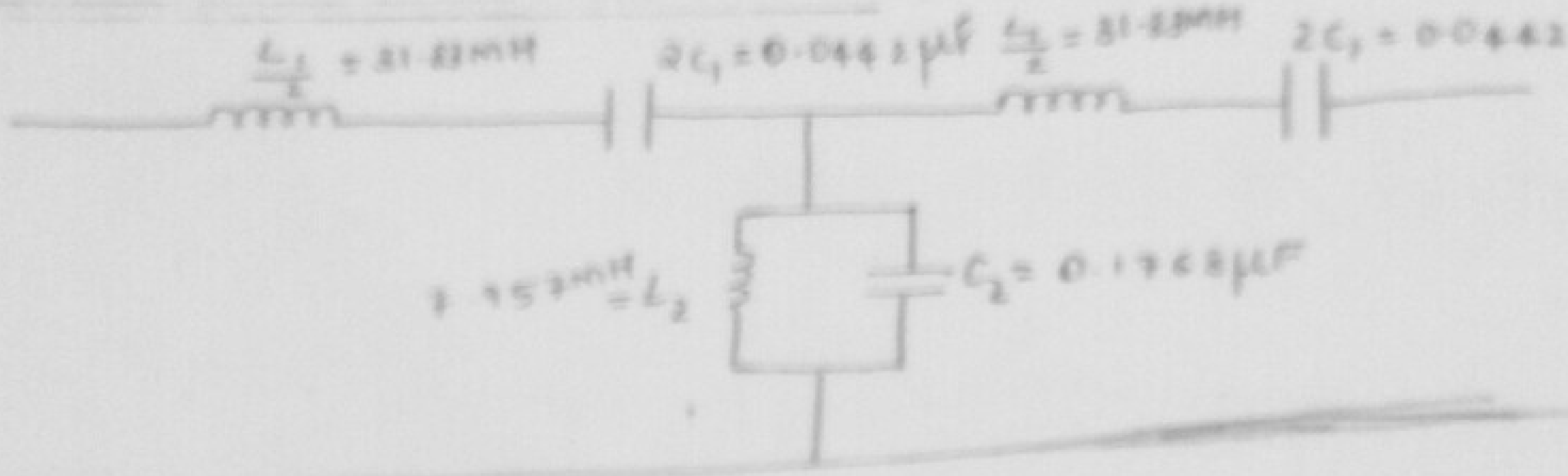
$$L_1 = \frac{k}{\pi(f_2 - f_1)} = \frac{600}{\pi \times 3k} = 63.66\,mH$$

$$C_1 = \frac{f_2 - f_1}{4\pi k f_1 f_2} = \frac{3}{4\pi \times 600 \times 18k} = 2.2104\times10^{-8} = 0.0221\,\mu F$$

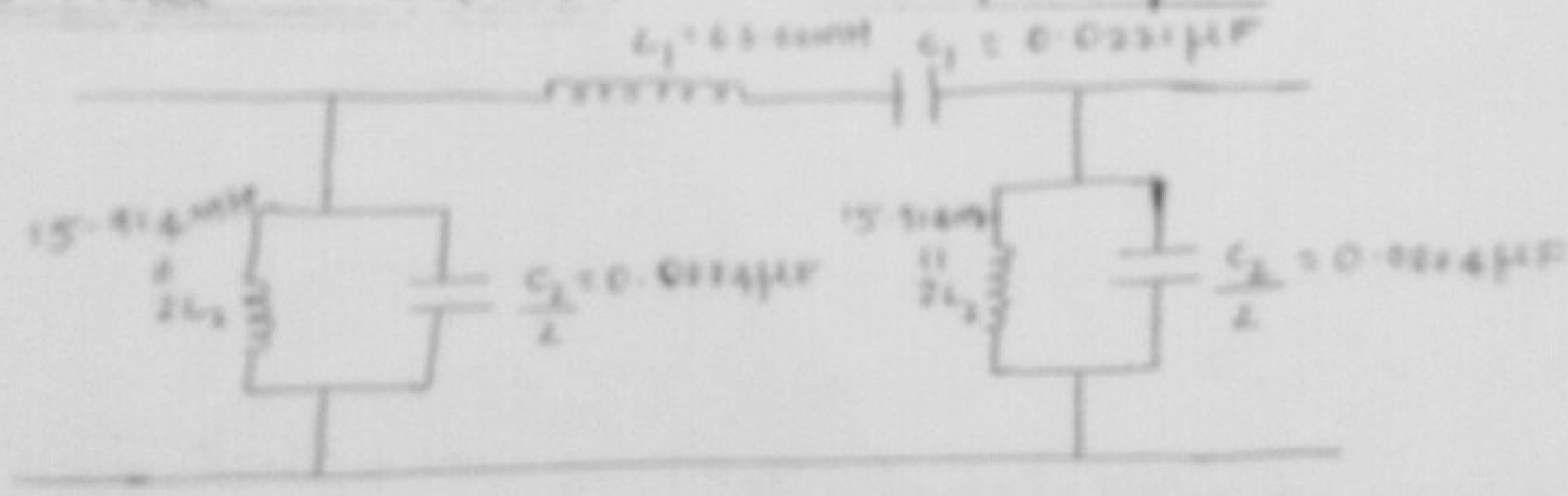
$$L_2 = \frac{k(f_2 - f_1)}{4\pi f_1 f_2} = \frac{600\times3}{4\pi\times18k} = 7.957\,mH$$

$$C_2 = \frac{1}{\pi k(f_2 - f_1)} = \frac{1}{\pi\times600\times3k} = 1.7683\times10^{-7} = 0.17683\,\mu F$$

Symmetrical T-section Band pass filter:-



Symmetrical π-section Band pass filter



Band Elimination Filter:-

→ A BEF is a one which attenuates a particular band of frequencies and allows the remaining.

→ The frequency to attenuation characteristics are shown as below,

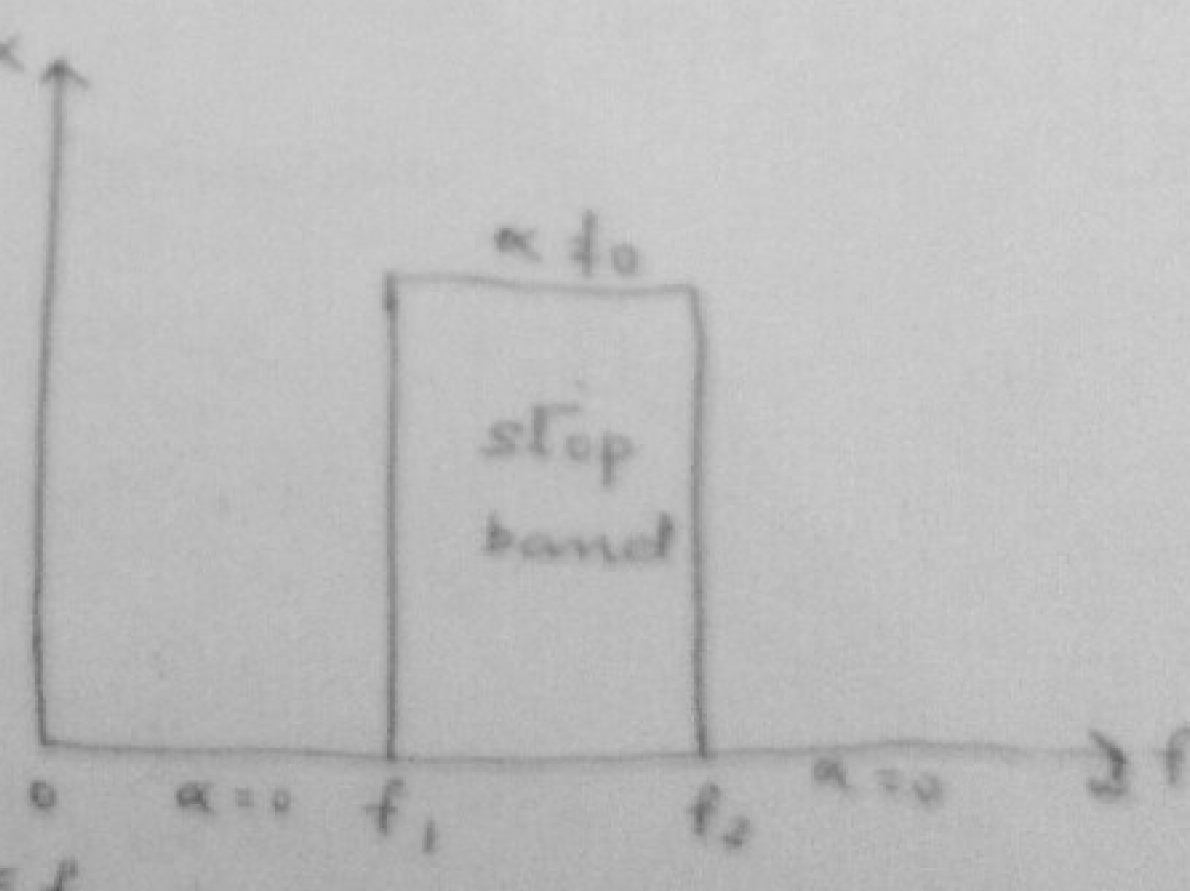
pass band → $0 \to f_1$, f

$f_1 \to \infty$

$(f \le f_1)$

$f < f_1$ & $f > f_2$

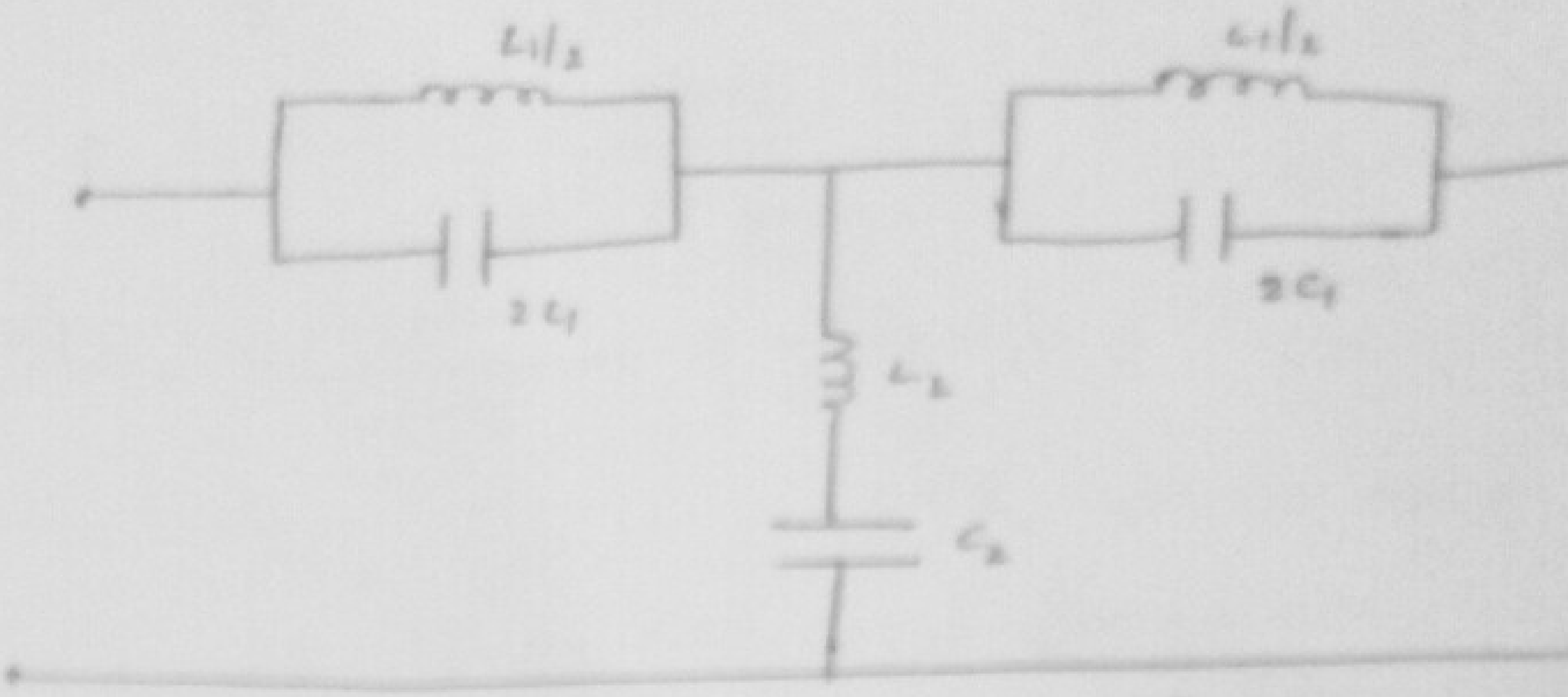
$\boxed{f_2 < f < f_1}$



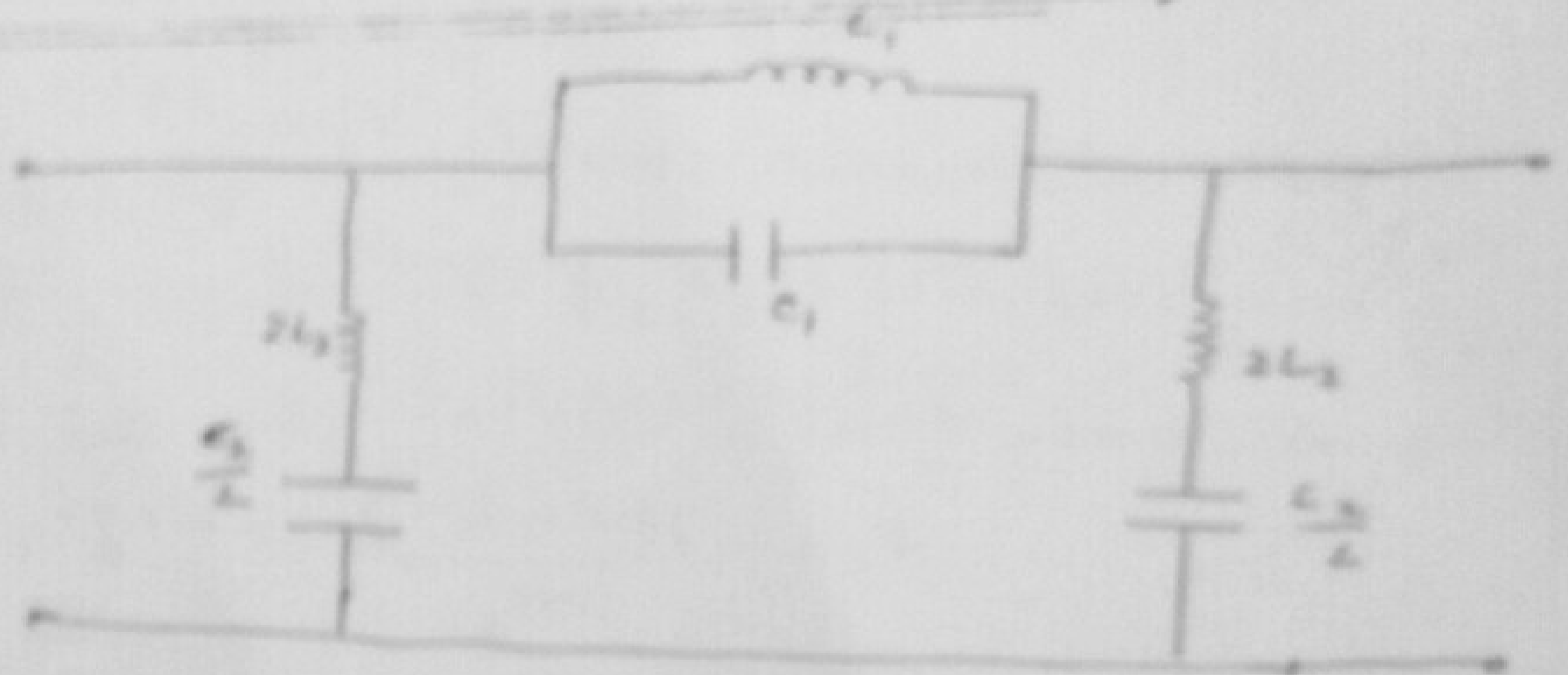
stop band $f_1 \to f_2 \Rightarrow f_1 < f < f_2$

→ BEF will be constructed by joining a Low pass filter with a High pass filter in parallel. 3(25)

Symmetrical T-section Band Elimination Filter:-



Symmetrical π-section Band elimination filter:-



→ from symmetrical T-section,

$$\Rightarrow Z_1 = \left\{\left(j\omega \frac{L_1}{2}\right) \| \left(\frac{1}{j\omega 2C_1}\right)\right\} + \left\{\left(j\omega \frac{L_1}{2}\right) \| \left(\frac{1}{j\omega 2C_1}\right)\right\}$$

$$\Rightarrow Z_1 = 2 \times \frac{j\omega \frac{L_1}{2} \times \frac{1}{j\omega 2C_1}}{\frac{j\omega L_1}{2} + 1/j\omega 2C_1} = \frac{2 \times \frac{L_1}{4C_1}}{\frac{j\omega L_1 + 4j\omega C_1}{2}} \Rightarrow \frac{2 \cdot \frac{L_1}{4C_1}}{\frac{-\omega^2 L_1 C_1 + 1}{2j\omega C_1}} = \frac{\frac{L_1}{C_1}}{j\omega(L_1 + 4C_1)}$$

$$\Rightarrow Z_1 = \frac{L_1 \cdot j\omega}{1 - \omega^2 L_1 C_1}$$

$$\Rightarrow Z_1 = \frac{j\omega L_1}{1 - \omega^2 L_1 C_1}$$

$$\Rightarrow Z_2 = j\omega L_2 + \frac{1}{j\omega C_2} = \frac{-\omega^2 L_2 C_2 + 1}{j\omega C_2} = \frac{j(\omega^2 L_2 C_2 - 1)}{\omega C_2}$$

→ To obtain Band elimination characteristics the series arm and shunt arm should resonate at same frequency. The series arm consists of parallel resonant ckt whose resonant condition will be obtained at its net susceptance zero

→ We know that impedance of the series branch,

$$\Rightarrow Z_1 = \frac{JWL_1}{1 - W^2 L_1 C_1}$$

$$\Rightarrow Y_1 = \frac{1}{Z_1} = \frac{1 - W^2 L_1 C_1}{JWL_1} = \frac{J(W^2 L_1 C_1 - 1)}{WL_1} = 0 + JB$$

⇒ At $W = W_r$, $B = 0$

$$W_r^2 L_1 C_1 = 1$$

$$W_r = \frac{1}{\sqrt{L_1 C_1}} \longrightarrow ①$$

→ The shunt branch consists of a series resonant ckt which will resonate at a frequency where its net reactance = 0

→ we know that,

$$Z_2 = \frac{J(W^2 L_2 C_2 - 1)}{WC_2} = 0 + JX$$

→ At $W = W_r$, $X = 0$

$$\Rightarrow W_r^2 L_2 C_2 = 1$$

$$\Rightarrow W_r = \frac{1}{\sqrt{L_2 C_2}} \longrightarrow ②$$

∴ from eq ① & ②

$$L_1 C_1 = L_2 C_2 \Rightarrow \frac{1}{L_1 C_1} = \frac{1}{L_2 C_2}$$

Taking the product of $Z_1 Z_2$,

$$\Rightarrow Z_1 Z_2 = \frac{JWL_1}{(1 - W^2 L_1 C_1)} \times \frac{J(1 + W^2 L_2 C_2)}{WC_2} = \left(\frac{1 - W^2 L_2 C_2}{1 - W^2 L_1 C_1}\right)\frac{L_1}{C_2}$$

$$\Rightarrow Z_1 Z_2 = \frac{L_1}{C_2} = K^2$$

(from resonant condⁿ $L_1 C_1 = L_2 C_2$)

⇒ As the filter satisfy the relationship $Z_1 Z_2 = K^2$, its a constant - K filter.

→ from the condition of pass band, we have

$$-1 < \frac{Z_1}{4 Z_2} < 0$$

Lower limit,

$\Rightarrow \frac{Z_1}{4Z_2} = -1$

$\Rightarrow Z_1 = -4Z_2$ (Multiply on both sides with Z_2)

$\Rightarrow Z_1 Z_2 = -4Z_2^2$

$\Rightarrow k^2 = -4Z_2^2$ $\Rightarrow Z_2 = \pm \frac{Jk}{2}$ Where the value of Z_2 at lower cut-off frequency is opposite to the value of Z_2 at upper cut-off frequency.

$\Rightarrow$ We have $Z_2 = J\omega L_2 + \frac{1}{J\omega C_2}$

(i) $\Rightarrow$ At $\omega = \omega_1$, i.e., at lower cut-off frequency capacitive reactance dominates inductive reactance and hence Z_2 is negative and is given by

$$Z_2 = \frac{-Jk}{2} \quad (X_C > X_L)$$

(ii) At $\omega = \omega_2$, i.e., at upper cut-off frequency, inductive reactance dominates capacitive reactance $(X_L > X_C)$ and Z_2 is +ve and is given by

$$Z_2 = \frac{+Jk}{2}$$

$\Rightarrow \therefore Z_2$ at $\omega = \omega_1 = -(Z_2$ at $\omega = \omega_2)$ i.e., $Z_2 = \left(J\omega_1 L_2 + \frac{1}{J\omega_1 C_2}\right) = -\left(J\omega_2 L_2 + \frac{1}{J\omega_2 C_2}\right)$

$$\Rightarrow J\omega_1 L_2 + \frac{1}{J\omega_1 C_2} = -J\omega_2 L_2 - \frac{1}{J\omega_2 C_2}$$

$$\Rightarrow JL_2(\omega_1 + \omega_2) = -\frac{1}{J\omega_2 C_2} - \frac{1}{J\omega_1 C_2}$$

$$\Rightarrow JL_2(\omega_1 + \omega_2) = J\left(\frac{1}{\omega_2 C_2} + \frac{1}{\omega_1 C_2}\right)$$

$$\Rightarrow L_2(\omega_1 + \omega_2) = \frac{1}{C_2}\left(\frac{\omega_1 + \omega_2}{\omega_1 \omega_2}\right)$$

$$\Rightarrow L_2 C_2 = \frac{1}{\omega_1 \omega_2}$$

$$\Rightarrow \omega_1 \omega_2 = \frac{1}{L_2 C_2} = \frac{1}{1/\omega_r^2} = \omega_r^2$$

$$\Rightarrow \omega_r = \sqrt{\omega_1 \omega_2}$$

Design of Band Elimination filter:-

→ The value of Z_2 at $w = w_1 = \frac{-Jk}{2}$

$$\Rightarrow Jw_1L_2 + \frac{1}{Jw_1C_2} = \frac{-Jk}{2}$$

$$\Rightarrow J\left(w_1L_2 - \frac{1}{w_1C_2}\right) = \frac{-Jk}{2}$$

$$\Rightarrow -w_1L_2 + \frac{1}{w_1C_2} = \frac{k}{2}$$

$$\Rightarrow \frac{1 - w_1^2L_2C_2}{w_1C_2} = \frac{k}{2} \Rightarrow \frac{1 - \frac{w_1^2}{w_r^2}}{w_1C_2} = \frac{k}{2}$$

$$\Rightarrow 1 - \left(\frac{w_1}{w_r}\right)^2 = w_1C_2\frac{k}{2}$$

$$\Rightarrow 1 - \frac{w_1^2}{w_1w_2} = w_1C_2\frac{k}{2} \Rightarrow 1 - \frac{w_1}{w_2} = w_1C_2\frac{k}{2} \Rightarrow \frac{w_2 - w_1}{w_2} = w_1C_2\frac{k}{2}$$

$$\Rightarrow \frac{2\pi(f_2 - f_1)}{2\pi f_2} = 2\pi f_1C_2\frac{k}{2} \Rightarrow \frac{f_2 - f_1}{f_2} = \pi f_1C_2k$$

$$\Rightarrow C_2 = \frac{f_2 - f_1}{k\pi f_1f_2} \text{ farads.}$$

→ we know that,

$$\Rightarrow w_r^2 = \frac{1}{L_2C_2} \Rightarrow L_2 = \frac{1}{w_r^2C_2} = \frac{k\pi f_1f_2}{w_1w_2(f_2 - f_1)}$$

$$\Rightarrow L_2 = \frac{k\pi f_1f_2}{4\pi^2f_1f_2(f_2 - f_1)} = \frac{k}{4\pi(f_2 - f_1)}$$

$$L_2 = \frac{k}{4\pi(f_2 - f_1)} \text{ Henrys}$$

we know that

$$\Rightarrow Z_1Z_2 = \frac{L_1}{C_2} = k^2 \Rightarrow \frac{L_2}{C_1} = k^2 = \frac{L_1}{C_2}$$

$$\Rightarrow L_1 = k^2C_2 = \frac{k(f_2 - f_1)}{\pi f_1f_2} \text{ Henrys.}$$

$$\Rightarrow C_1 = \frac{L_2}{k^2} = \frac{1}{4\pi k(f_2 - f_1)} \text{ Farads.}$$

$jw_1L_2 + jw_1C_2$

$jw_2 + jw_1L_2$

$j_2w_1^2L_2C_2 + 1 = \frac{-jk}{2} jw_1C_2$

$1 - \frac{w_1^2}{w_r^2} = -j^2k\pi f_1C_2$

$1 - \frac{f_1}{f_2} = k\pi f_1C_2$

$C_2 = \frac{f_2 - f_1}{k\pi f_1f_2}$

3(27) as

29/11/18

Q:- Design a LPF of constant k-type whose cut-off frequency f_c is 20kHz and for a nominal impedance of k 400Ω and also determine the values of α, β, Z_0 at a frequency of a) 10kHz
b) 30kHz

Sol:- Given data,

It is a Low pass filter of constant k-type

cut-off frequency f_c = 20kHz

Nominal Impedance $k = 400\Omega$

Design:-

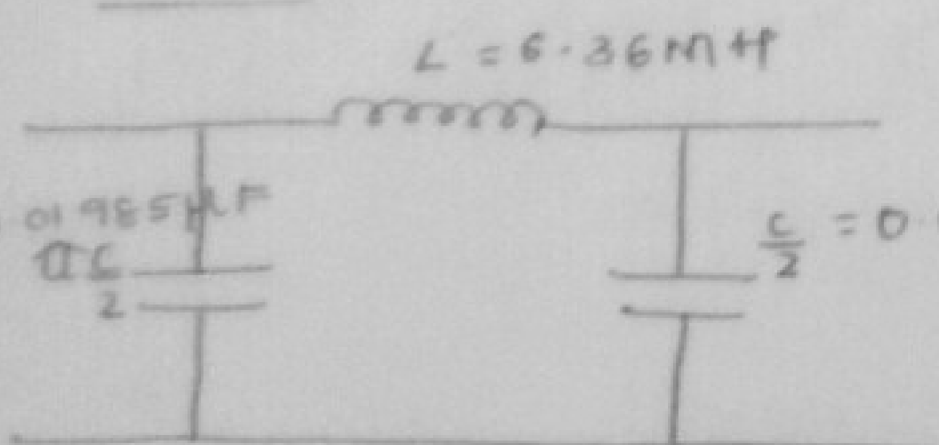
→ We know that, the value of L and C from LPF is

$L = \frac{k}{\pi f_c}$ & $C = \frac{1}{k\pi f_c}$

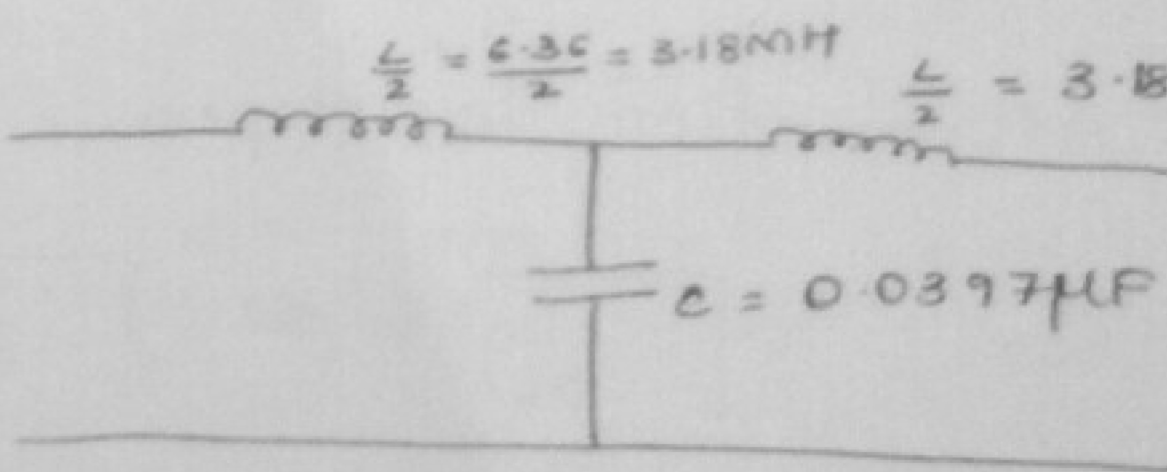
$L = \frac{400}{\pi(20k)}$ $C = \frac{1}{400\pi \times 20k} = 0.0397\mu F$

$L = 6.36mH$

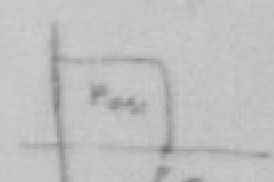
Π-section :-



T-section



Determination of α, β, Z_0:-



a) 10kHz, Given that $f = 10kHz$

$f = 10kHz < f_c = 20kHz$

Hence it lies in transmission band.

→ According to condn of transmission band, $\left(\frac{Z_1}{4Z_2} = \frac{f}{f_c}\right)$

$\alpha = 0$, $\beta = 2\sin^{-1}\sqrt{\left|\frac{Z_1}{4Z_2}\right|}$

$\beta = 2\sin^{-1}\left(\frac{f}{f_c}\right) = 2\sin^{-1}\left(\frac{10}{20}\right)$ $\because \frac{Z_1}{4Z_2} = -\left(\frac{f}{f_c}\right)^2$

$\beta = 2\sin^{-1}(0.5) = 2\times 30° = 60°$

$$Z_{0\pi} = \frac{k}{\sqrt{1-\left(\frac{f}{f_c}\right)^2}} = \frac{400\times 2}{\sqrt{3}} = 461.880\Omega$$

b) At a frequency of $f = 30kHz$.

$\Rightarrow f = 30kHz > f_c = 20kHz \Rightarrow f > f_c$

It lies in stop band

In the stop band,

$$\beta = \pi, \quad \alpha = 2\cosh^{-1}\sqrt{\left|\left(\frac{Z_1}{4Z_2}\right)\right|} = 2\cosh^{-1}\left(\frac{f}{f_c}\right)$$

$$\alpha = 2\cosh^{-1}\left(\frac{30}{20}\right) = 2\cosh^{-1}(1.5) = 2\times 0.962$$

$$\alpha = (1.92)\ \text{Nepers}$$

Note:-

→ Attenuation is expressed in terms of decibels (or) Nepers

→ Attenuation in dB is given by $\alpha = 20\log\left(\frac{V_1}{V_2}\right) = 20\log\left(\frac{i_1}{i_2}\right)$

$$= 10\log\left(\frac{P_1}{P_2}\right) dB.$$

→ Attenuation in Nepers is given by $\alpha = \ln\left(\frac{V_1}{V_2}\right) = \ln\left(\frac{i_1}{i_2}\right)$

$$Z_{0T} = k\sqrt{1-\left(\frac{f}{f_c}\right)^2} = 400\sqrt{1-\frac{9}{4}} = J\,447.213\Omega$$

$$Z_{0\pi} = \frac{400}{\sqrt{1-\frac{9}{4}}} = -J\,\cancel{447.213}\ 357.7\Omega$$

29- Design a constant k-HPF whose cut-off frequency is 25kHz for a nominal impedance of 600Ω and also determine the values of α, β, Z_0 at frequencies of 50kHz and 12.5kHz

Sol:- Given data,

It is High pass filter.

$f_c = 25kHz$

$K = 600\Omega$ 3(28) 46

Design :-

We know that the values of L and C from HPF is

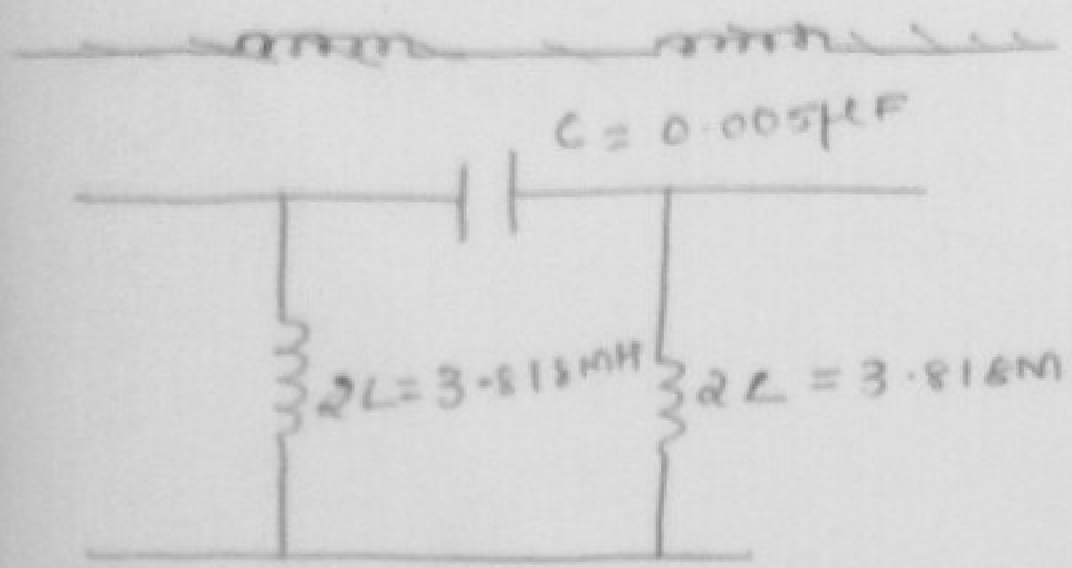
$$L = \frac{K}{4\pi f_c} \quad \& \quad C = \frac{1}{4K\pi f_c}$$

$$L = \frac{600}{4\pi \times 25k} \qquad C = \frac{1}{4\pi \times 600 \times 25k}$$

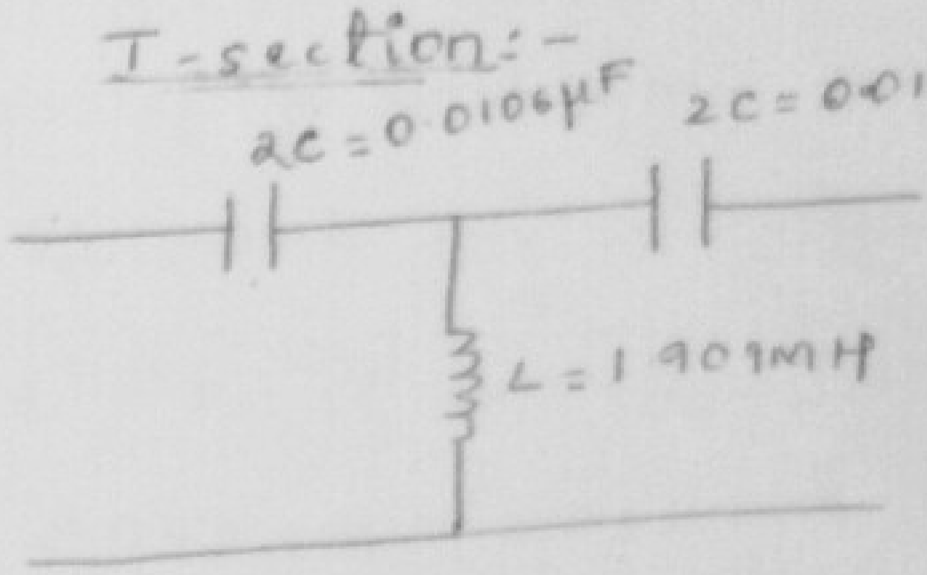
$$L = 1.909 MH \qquad C = 5.305 \times 10^{-9} = 0.005305 \mu F$$

$Z_1 = \frac{1}{j\omega \frac{C}{2}}$

Π-section :-



T-section :-



Determination of α, β, Z_0 :-

a) At a frequency of 50kHz

$f = 50kHz, f_c = 25kHz$

$\Rightarrow f > f_c \Rightarrow$ It lies in pass band

$\Rightarrow$ Acc. to pass band,

$$\alpha = 0, \ \beta = 2\sin^{-1}\sqrt{\left|\frac{Z_1}{4Z_2}\right|} = 2\sin^{-1}\left(\frac{f_c}{f}\right) = 2\sin^{-1}\left(\frac{25}{50}\right) = 2\sin^{-1}\left(\frac{1}{2}\right)$$

$$\beta = 2 \times 30^\circ$$

$$\beta = 60^\circ$$

$$\Rightarrow Z_{0T} = K\sqrt{1-\left(\frac{f_c}{f}\right)^2} = K\sqrt{1-\frac{1}{4}} = 519.615\Omega$$

$$\Rightarrow Z_{0\pi} = \frac{K}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{K}{\sqrt{1-\frac{1}{4}}} = 692.82\Omega.$$

b) At a $f = 12.5\,kHz$

$f < f_c \Rightarrow$ It lies in stop band.

Where $\beta = \pi$, $\alpha = 2\cosh^{-1}\left(\frac{f_c}{f}\right) = 2\cosh^{-1}\left(\frac{25}{12.5}\right) = (2.633)$ Nepers

$Z_{0T} = k\sqrt{1-\left(\frac{f_c}{f}\right)^2} = j1039.23\,\Omega$

$Z_{0\pi} = -j\,346.410\,\Omega$

39:- Design a Band elimination filter, to eliminate the frequencies between 2kHz and 5kHz. The characteristic impedance of filter should be 500Ω.

sol:- Given data, It is a Band elimination filter.

$f_1 = 2\,kHz$ & $f_2 = 5\,kHz$.

$k = 500\Omega$

soln:-

$L_1 = \frac{k(f_2 - f_1)}{\pi f_1 f_2}$ & $C_1 = \frac{1}{4\pi k(f_2 - f_1)}$

$L_1 = 47.746\,mH$ $C_1 = 0.053\,\mu F = 0.053\,\mu F$

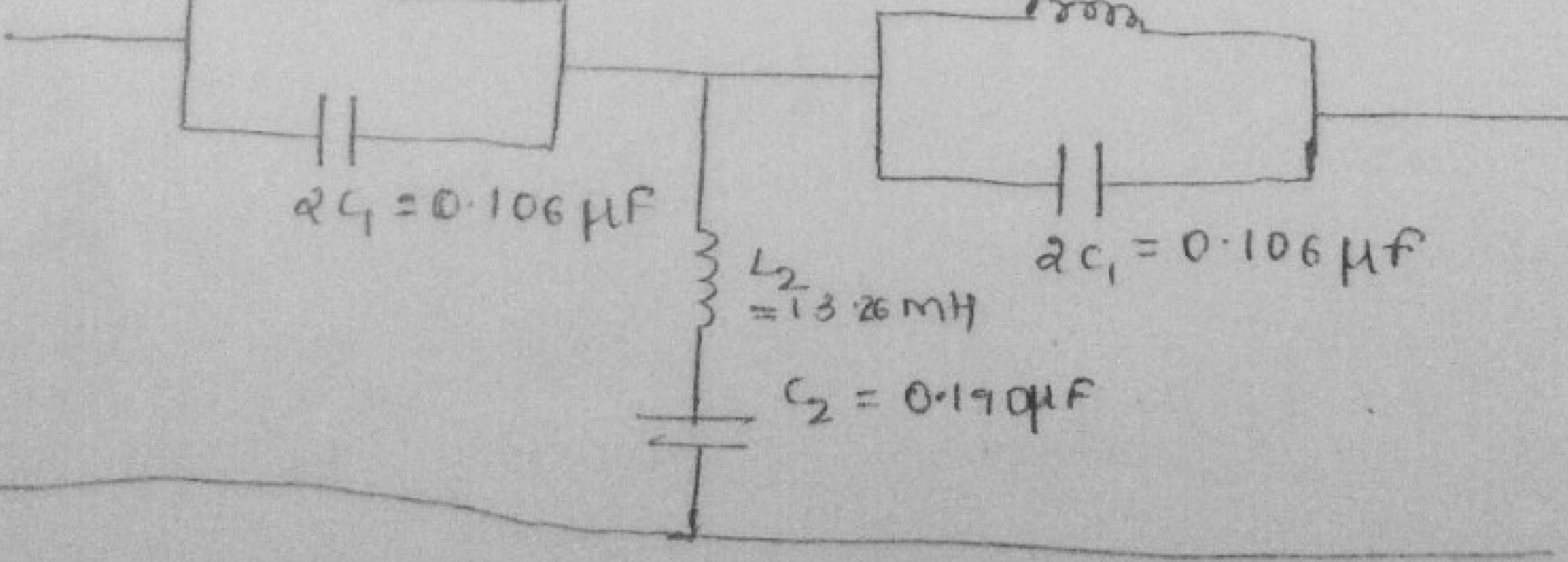
$L_2 = \frac{k}{4\pi(f_2 - f_1)}$ $C_2 = \frac{f_2 - f_1}{\pi k f_1 f_2}$

$L_2 = 13.26\,mH$

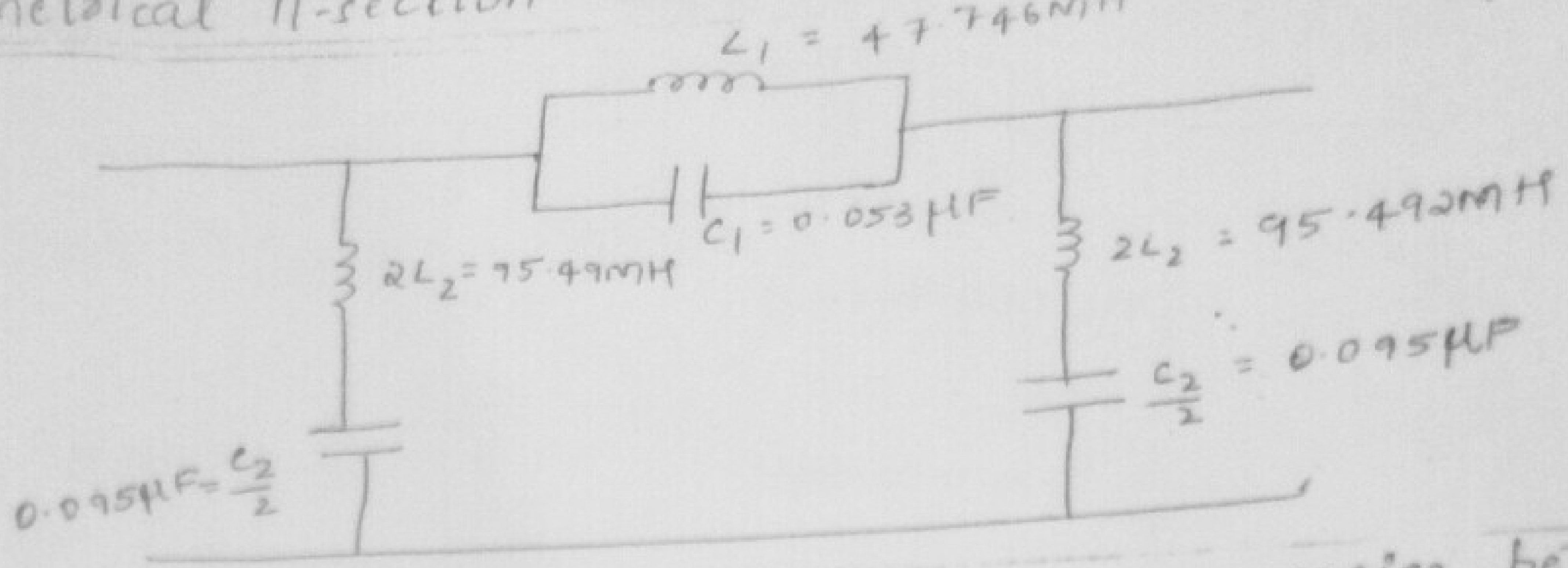
$C_2 = 1.909 \times 10^{-4} = 0.190\,\mu F$

$= 0.190\,\mu F$

Symmetrical T-section BEF:-



Symmetrical π-section BEF :-



49:- Design a BEF, to eliminate the frequencies between 2kHz and 4kHz for a nominal impedance of 600Ω

It is a Band elimination filter

Given data,
$f_1 = 2kHz$ & $f_2 = 4kHz$
$K = 600\Omega$

soln
$$L_1 = \frac{k(f_2 - f_1)}{\pi f_1 f_2} = 47.746mH$$

$$C_1 = \frac{1}{4\pi k(f_2-f_1)} = \frac{1}{4\pi \times 600 \times 2k}$$

$$C_1 = 6.631 \times 10^{-8}$$

$$C_1 = 0.063\mu F$$

$$L_2 = \frac{k}{4\pi(f_2 - f_1)} = \frac{600}{4\pi(2k)}$$

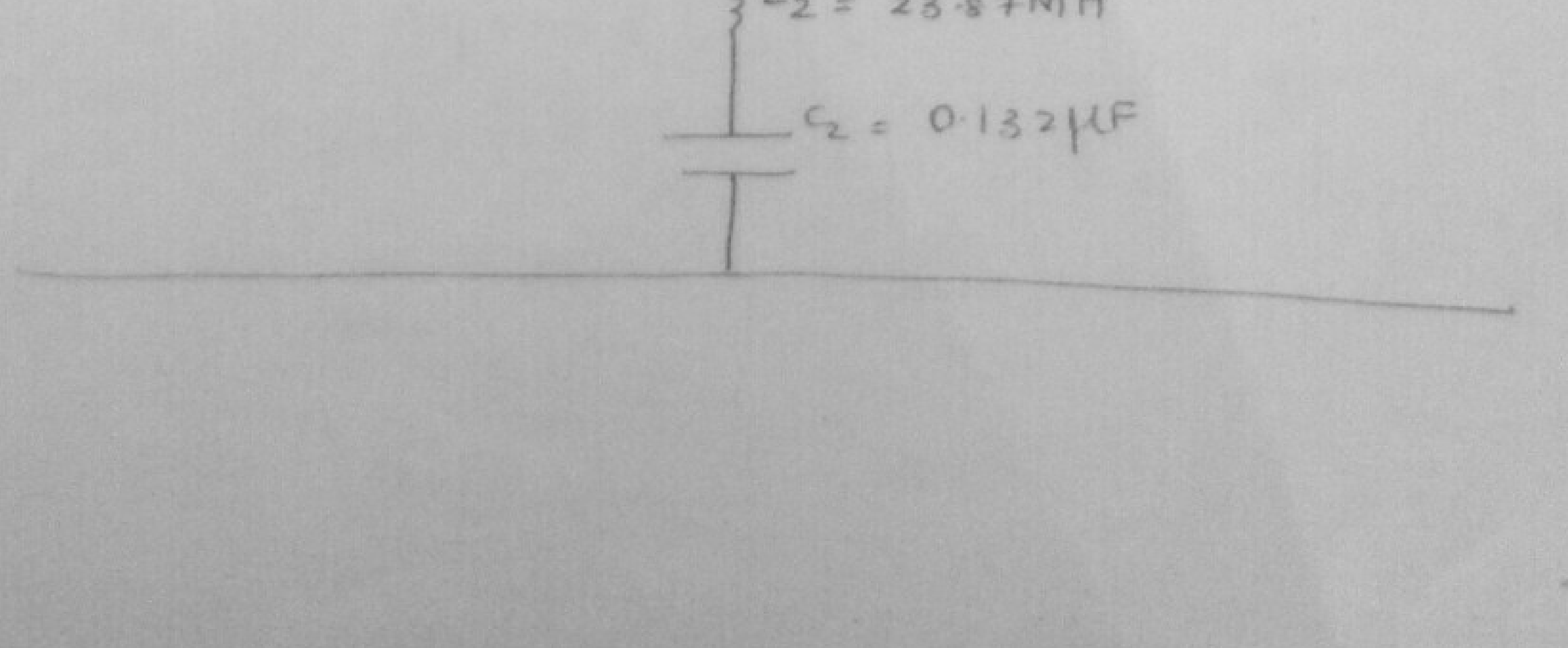
$$L_2 = 23.87\ mH$$

$$C_2 = \frac{f_2 - f_1}{\pi k f_1 f_2} = \frac{2k}{\pi \times 600 \times 8k^2}$$

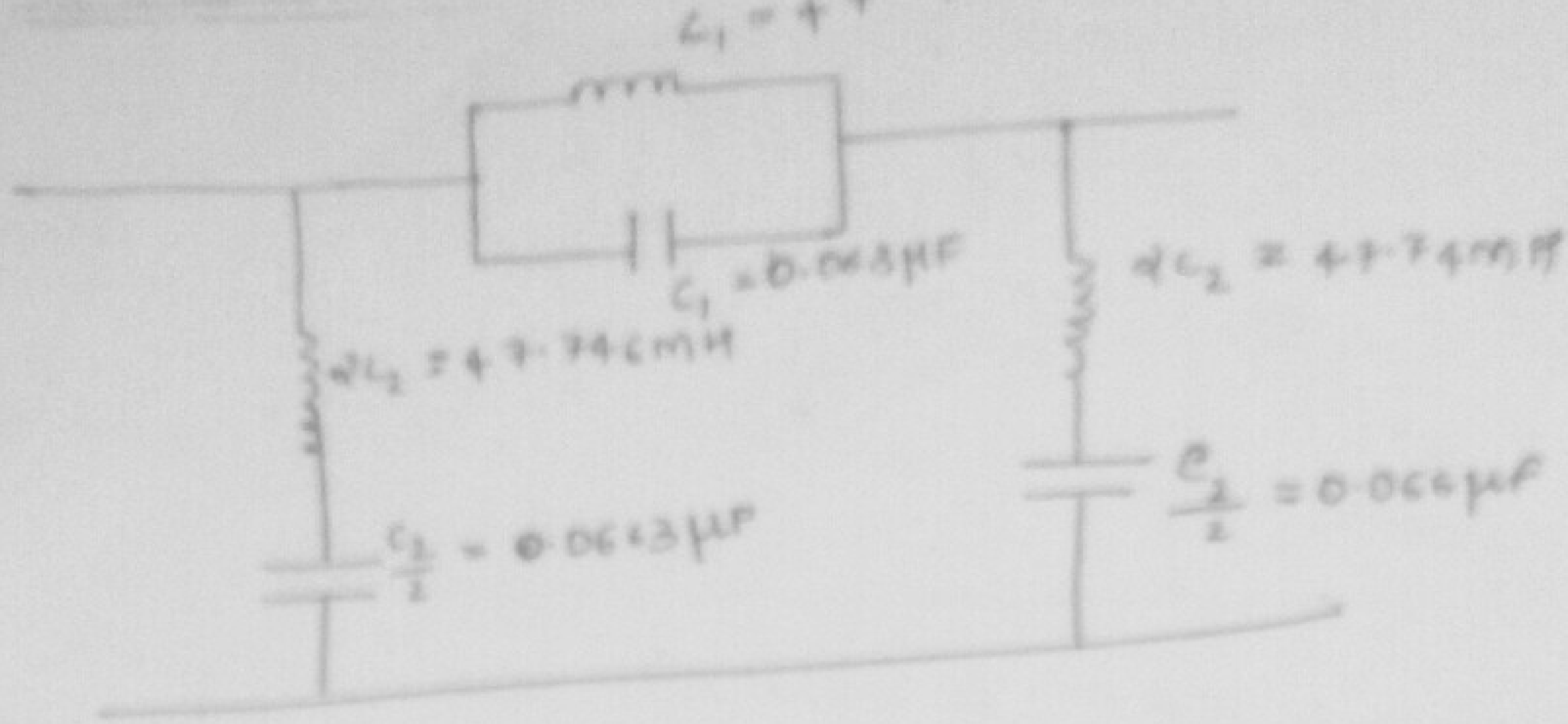
$$C_2 = 1.326 \times 10^{-4} \times 10^{-3}$$

$$C_2 = 1.326 \times 10^{-7} = 0.1326\mu F$$

Symmetrical T-section BEF :-



Symmetrical π-section BEF:-



Q. Design a BEF to eliminate the frequencies between 1kHz and 5kHz for a nominal impedance of 400Ω.

Q. Design a m-derived LPF whose cut-off frequency is 20kHz to attain infinite attenuation at a frequency of 24kHz for a nominal impedance of 400Ω.

Given data, It is a m-derived Low pass filter

Sol: cut-off frequency $f_c = 20kHz$

$k = 400\Omega$

$f_\infty = f_r = 24kHz$

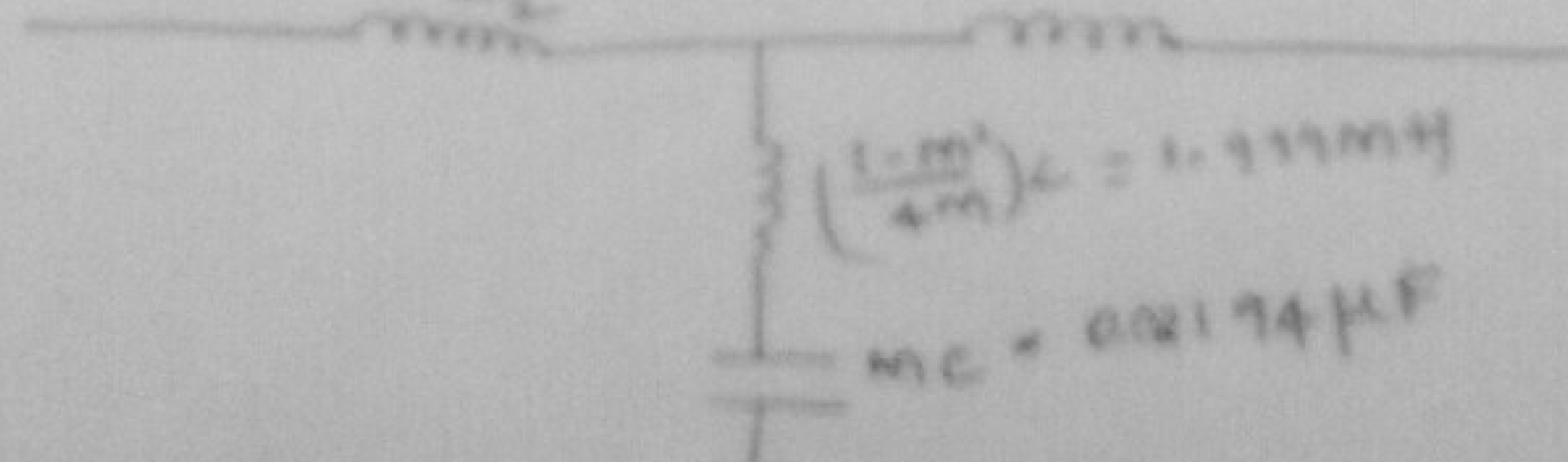
$$m = \sqrt{1-\left(\frac{f_c}{f_\infty}\right)^2} = \sqrt{1-\left(\frac{20}{24}\right)^2} = 0.5527$$

for a LPF, we know that,

$$L = \frac{k}{\pi f_c} \qquad C = \frac{1}{k\pi f_c}$$

$$L = 6.366mH \qquad C = 0.0397\mu F$$

m-derived T-section of LPF:-



Design a low pass filter (both π and T sections) having a cut-off frequency of 2KHz to operate with a Terminated load resistance of 500Ω 3(30)

Given data :-

$f_c = 2\,KHz = 2000\,Hz$

$K = 500\,\Omega$

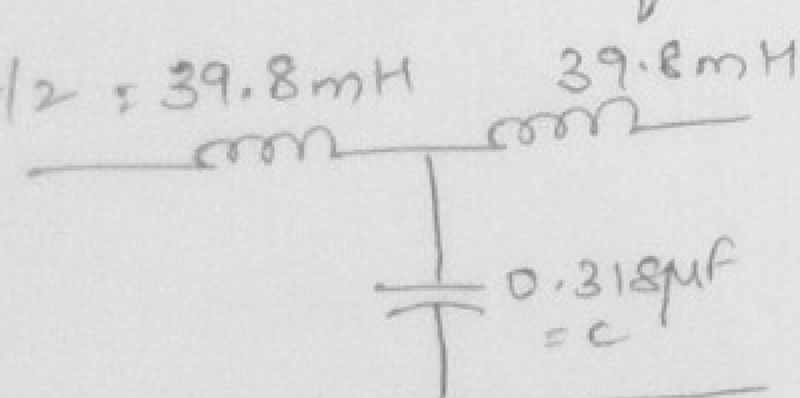
we know that

$$K = \sqrt{\frac{L}{C}} = 500\,\Omega$$

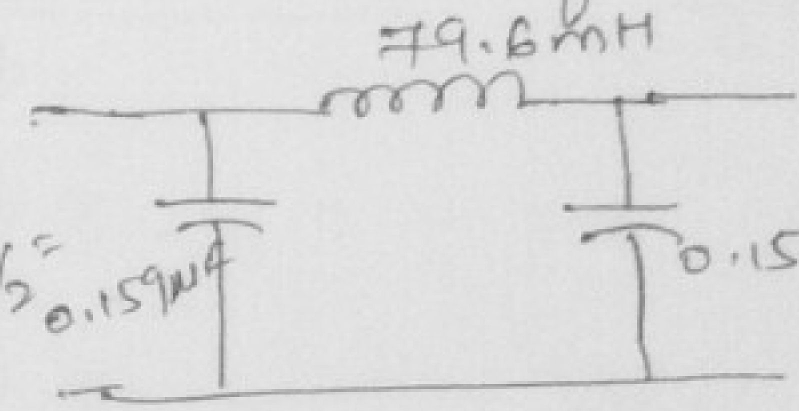
$$L = \frac{K}{\pi f_c} = \frac{500}{\pi \times 2000} = 0.0796\,H \Rightarrow 79.6\,mH$$

$$C = \frac{1}{\pi f_c K} = \frac{1}{3.14 \times 2000 \times 500} = 3.184 \times 10^{-7} \Rightarrow 0.318\,\mu F$$

The T section of the filter is



The π section of the filter:



Design a high pass filter having a cut off frequency of 1KHz with a load resistance of 600Ω.

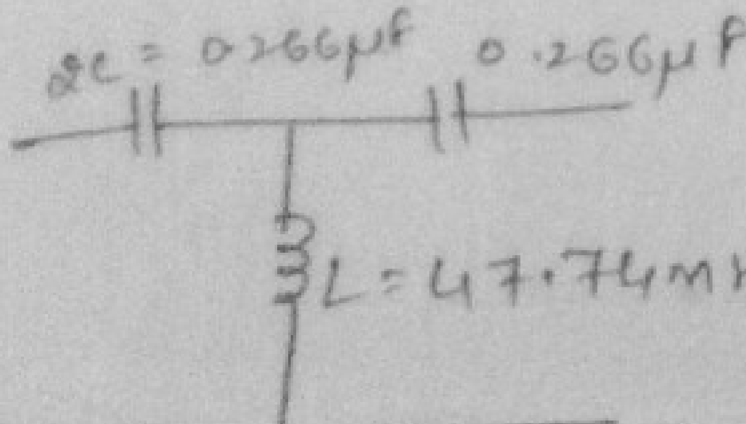
Given data :-

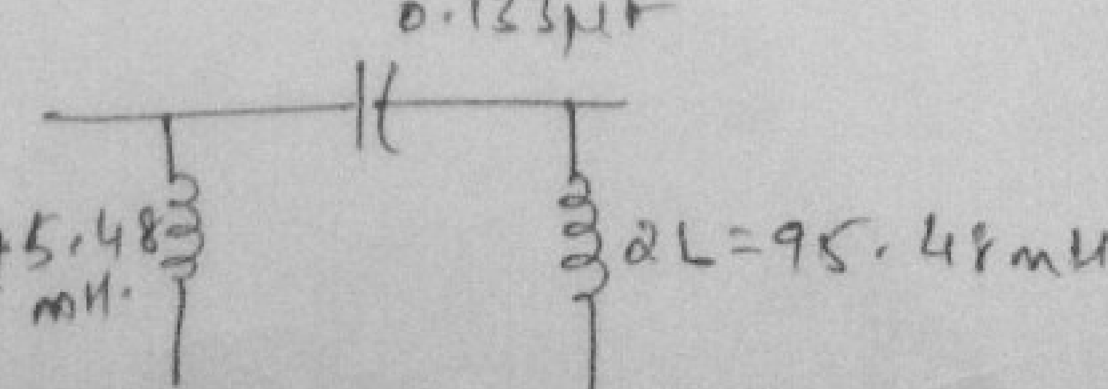
$f_c = 1KHz = 1000\,Hz$

$K = 600\,\Omega$

$$L = \frac{K}{4\pi f_c} = \frac{600}{4 \times \pi \times 1000} = 47.74\,mH$$

$$C = \frac{1}{4\pi K f_c} = \frac{1}{4\pi \times 600 \times 1000} = 0.133\,\mu F$$





3) Design a m-derived low pass filter having cut-off freq of 1KHz, design the impedance of 400Ω and the resonant freq of 1100 Hz.

Sol $K = 400\Omega$, $f_c = 1KHz$, $f_\infty = 1100\,Hz$.

$$m = \sqrt{1-\left(\frac{f_c}{f_\infty}\right)^2} = \sqrt{1-\left(\frac{1000}{1100}\right)^2} = 0.416$$

From const. k type filter the values of L and C are

$$L = \frac{K}{\pi f_c} = \frac{400}{\pi \times 1000} = 127.32\,mH$$

$$C = \frac{1}{K\pi f_c} = \frac{1}{\pi \times 400 \times 1000} = 0.795\,\mu F$$

The modified values of L and C for m-derived filter are

$$\frac{mL}{2} = \frac{0.416 \times 127.32 \times 10^{-3}}{2} = 26.48\,mH$$

$$mC = 0.416 \times 0.795 \times 10^{-6} = 0.33\,\mu F$$

$$\frac{1-m^2}{4m} L = \frac{1-(0.416)^2}{4(0.416)} \times 127.32 \times 10^{-3} = 63.27\,mH$$

π section elements :-

$$\frac{mC}{2} = \frac{0.416 \times 0.795 \times 10^{-6}}{2} = 0.165\,\mu F$$

$$\frac{(1-m^2)}{4m} \times C = \frac{1-(0.416)^2}{4 \times 0.416} \times 0.795 \times 10^{-6} = 0.395\,\mu F$$

$$mL = 0.416 \times 127.32 \times 10^{-3} = 52.965\,mH$$

m-derived low pass filter are