UNIT - 2

2(i)

TWO PORT NETWORK PARAMETERS

Network :- Interconnection of elements of different characteristic

2. Circuit :- circuit is a Network with a closed path

Classification of Network [Based on the properties of elements]

1. Active / Passive Network
2. Linear / Non-linear Network
3. Lumped / distributed Network
4. Unilateral / Bilateral Network

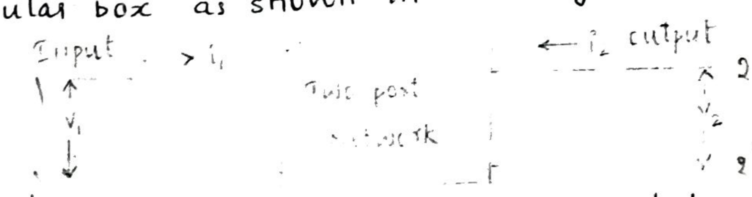
1. Active Network :- It is a network which contains atleast one active element. [It is a voltage source, current source, transistor]

2. Passive Network :- It is a network which does not contain active element

N/w → used is represented schematically by a rectangular box

Port :- It is a pair of terminals of the network from where the input is supplied (or) the output is extracted.

Two-port Network :- A network which contains two pairs of accessible terminals (one of i/p & other for o/p). Usually a two port network is represented using a rectangular box as shown in the diagram.



→ The value of current at port 2 will be takes as positive when it enters into the network.

Two port Network parameters :-

→ To describe the behaviour of an electrical network we will use the four variables of two port network (V_1, V_2, i_1, i_2) → No. of possible combination of four variables is $^4C_2 = 6$

→ Using the four variables we will construct mathematical equations from which the network behaviour is determined through certain constant values known as two-port network parameters.

→ There are 4 types of network parameters

1. Z-parameters

3. ABCD - parameters   4) Inverse ABCD (8) abcd parameters

4. h - parameters   6) g parameters.

1. <u>Z-parameters</u> :- Z-parameters determines the behaviour of the network in terms of its impedance values.

→ From ohm's Law, we know that

$$V = ZI$$

→ For a two port network, we have two voltages and two networks.

⇒ Writing into Matrix form

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [Z] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

, Where V_1 = port one's voltage , I_1 = port 1's current

V_2 = output voltage at port 2, I_2 = output current at port 2.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}_{2\times1} = \underbrace{\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}_{2\times2}} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}_{2\times1}$$

→ impedance parameters, These are calculated by setting each of the port current to zero.

$$\Rightarrow V_1 = Z_{11} I_1 + Z_{12} I_2 \longrightarrow ①$$

$$\Rightarrow V_2 = Z_{21} I_1 + Z_{22} I_2 \longrightarrow ②$$

Here I_1 and I_2 are independent variables and

V_1 and V_2 are dependent Variables.

<u>Determination of Z-parameters</u> :-

<u>case(i)</u> :- Let $I_1 = 0$ [i.e., When port 1 is open-circuited]

from eq ① and ②, we have

$$V_1 = Z_{12} I_2$$

$$V_2 = Z_{22} I_2$$

$$\therefore Z_{12} = \frac{V_1}{I_2}$$ [Where V_1 is open-circuited voltage at port 1]

$$Z_{22} = \frac{V_2}{I_2}$$ [Z_{22} is known as open-circuited output impedance].

→ Z_{12} is known as open-circuitted reverse transfer impedance.

Case(ii):- Let $I_2 = 0$ [When port 2 is open ci...]

from eq. ① & ②,

$$V_1 = Z_{11} I_1 \quad \text{and} \quad V_2 = Z_{21} I$$

$$Z_{11} = \frac{V_1}{I_1} \quad \text{and} \quad Z_{21} = \frac{V_2}{I_1}$$

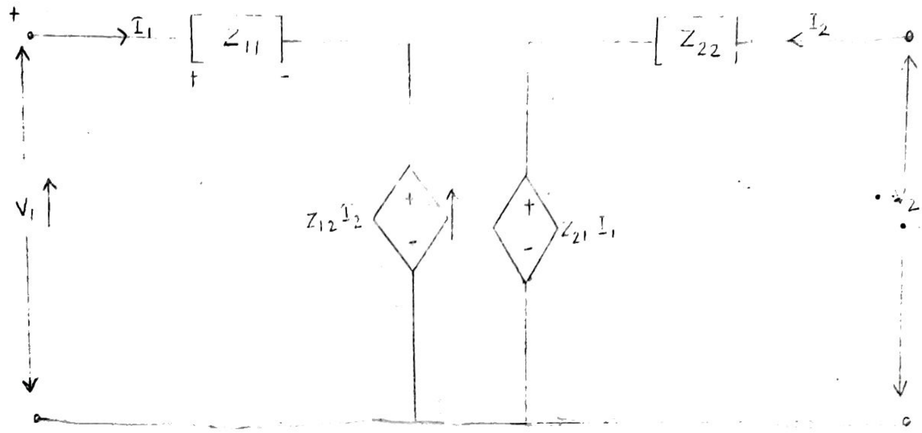
⇒ Where Z_{11} is open circuited input impedance and Z_{21} is open-circuited forward transfer impedance.

→ As Z-parameters are determined by open circuiting each port at a time, these parameters are known as open-circuited parameters.

Z-parametric Equivalent circuit :-

$V_1 = Z_{11}I_1 + Z_{12}I_2$ (KVL)

$V_2 = Z_{21}I_1 + Z_{22}I_2$



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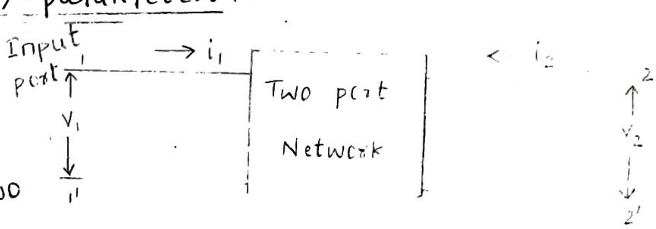
2. Admittance Parameters (or) Y-parameters :-

→ The admittance parameters gives the relationship of two port currents in terms of two port voltages.



→ From admittance (or) current form of ohm's Law,

$$I = \left(\frac{1}{Z}\right) V$$

$I = YV$ [Where Y is admittance, reciprocal of impedance].

→ For the given two-port network, the equation can be written as

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \underbrace{\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}}_{\text{admittance parameters}} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Where I_1, I_2 are dependent Variabl

Where V_1, V_2 are Independent parameters.

Calculated by setting each port voltage to zero

→ The Y-parameter equations are

$$\Rightarrow I_1 = Y_{11}V_1 + Y_{12}V_2 \longrightarrow ①$$

$$\Rightarrow I_2 = Y_{21}V_1 + Y_{22}V_2 \longrightarrow ②$$

...termination of ... meters :-

Case (i) - Let $V_1 = 0$ [short circuiting port 1]

from eq. ① & ②

$I_1 = Y_{12} V_2$ and $I_2 = Y_{22} V_2$

$Y_{12} = \frac{+(I_1)}{V_2} = \frac{+I_1}{V_2}$ [Y_{12} is known as short circuited ~~forward~~ reverse transfer admittance].

$Y_{22} = \frac{I_2}{V_2}$ [short circuited output admittance]

Case (ii) :- Let $V_2 = 0$ [short-circuiting port 1]

from eq. ① & ②, we have

$I_1 = Y_{11} V_1$

$\Rightarrow Y_{11} = \frac{I_1}{V_1}$ (Y_{11} is known as short-circuited input admittance)

$I_2 = Y_{21} V_1$ [Y_{21} is known as short-circuited ~~reverse~~ forward transfer admittance].

$Y_{21} = \frac{I_2}{V_1}$

$Y_{21} = \frac{+(I_2)}{V_1}$

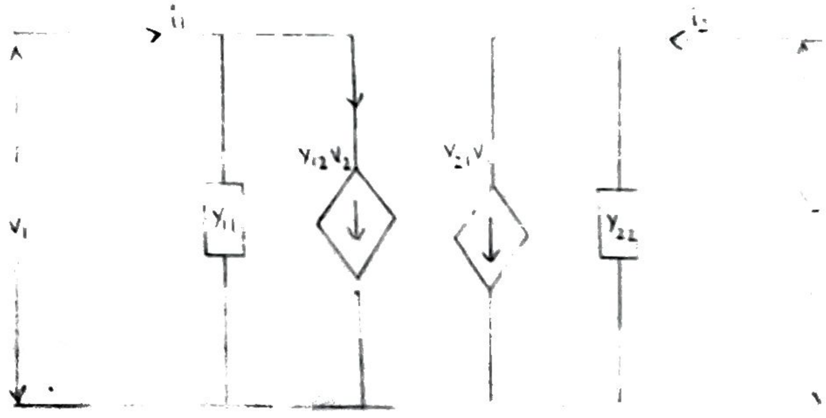
$Y_{21} = -I_2/V_1$

→ The negative sign is given because the direction of current in port 2 is opposite to the conventional two port direction when it gets short-circuited.

Y-parameteric equivalent circuit :-

→ From the Y-parametric equations we have,

$I_1 = Y_{11} V_1 + Y_{12} V_2$ and $I_2 = Y_{21} V_1 + Y_{22} V_2$

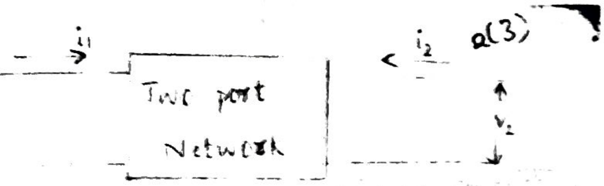


→ As admittance parameters are determined by short circuiting each port at a time. These parameters are also known as short circuiting parameters.

3. Transmission parameters (or) ABCD parameters :-

→ ABCD parameters gives the relationship between one end variables interms of the other end.

→ In general circuit Analysis, in a two-port network port 1 will be a sending end, where as port 2 will be receiving end.



→ The receiving end variables are considered as independent variables (V_2, I_2) and the sending end variables are dependent variables (V_1, I_1)

→ The ABCD parametric form is given as

$$V_1 = AV_2 - Bi_2 \qquad \Rightarrow \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$I_1 = CV_2 - Di_2$$

→ The negative sign is used because port 2 is receiving end, with the direction of current opposite to the conventional current.

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

<u>Determination of ABCD parameters :-</u>

<u>case(i)</u> :- [When output is open-circuit port-2]. Let $I_2 = 0$

$\Rightarrow V_1 = AV_2$

$A = \frac{V_1}{V_2}$ [Where A is open circuited **Reverse** voltage gain]

$\Rightarrow I_1 = CV_2$

$C = \frac{I_1}{V_2}$ [Where C is known as Open circuted ~~forward~~ reverse transfer admittance].

<u>Case(ii)</u> :- Let $V_2 = 0$ [short circuiting of port 2]

$V_1 = -Bi_2 \qquad\qquad I_1 = -DI_2$

$B = \frac{-V_1}{I_2} \qquad\qquad D = \frac{-I_1}{I_2}$ [Where D is known as Reverse current gain]

→ Where B is known as the short circuited ~~forward~~ reverse transfer impedance

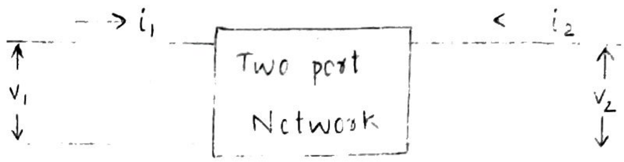
→ As these parameters are extensively used in general circuit Analysis and realisation of transmission networks, these are also known as General circuit parameters (or) Transmission parameters

Hybrid Parameters :-

→ Hybrid parameter gives the relationship between one port voltage and and other voltage current in terms of other two voltages.

→ The equation representation of hybrid parameters is gives as

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$



⇒ $V_1 = h_{11} I_1 + h_{12} V_2 \rightarrow ①$

$I_2 = h_{21} I_1 + h_{22} V_2 \rightarrow ②$

Determination of h-parameters :-

case(i) :- Let $V_2 = 0$ [short circuiting port 2]

from ① and ②, we have

$h_{ie} = h_{11} = \frac{V_1}{I_1}$ [sc Input impedance] $h_{fe} = \frac{I_2}{I_1} = h_{21}$ [short-circuited current gain]

→ h_{21} value will be always negative since when port 2 get short-circuited, direction of I_2 will be opposite to the conventional two port direction of current.

Case (ii) :- When $I_1 = 0$ [open circuited port 1]

$h_{re} = h_{12} = \frac{V_1}{V_2}$ [h_{12} known as Reverse voltage gain] $h_{oe} = h_{22} = \frac{I_2}{V_2}$ [open circuited output admittance]

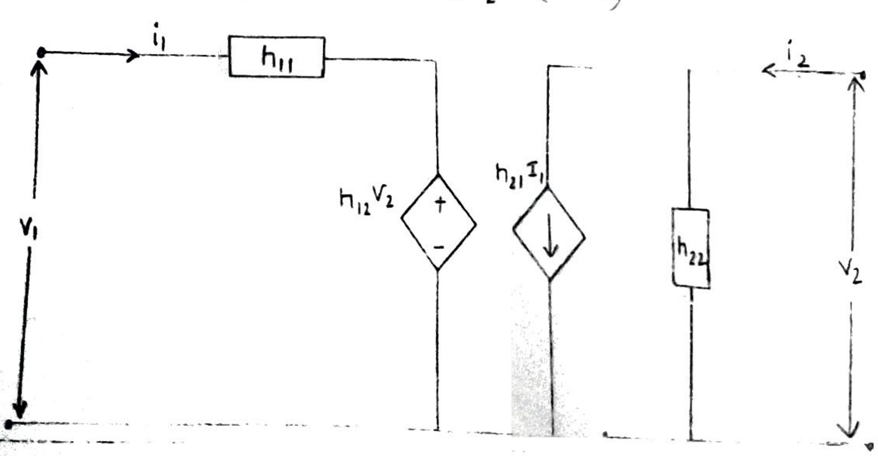
→ h-parameters are extensively used in electronic circuit Analysis.

→ There are also known as Mixed parameters.

h-parametric Equivalent circuit :-

$V_1 = h_{11} I_1 + h_{12} V_2$ (KVL)

$I_2 = h_{21} I_1 + h_{22} V_2$ (KCL)



* Conversion of one parameter into Another

1. From Z-parameters to Y-parameters:-

We know that from Z-parameters, we have $V_1 = Z_{11}I_1 + Z_{12}I_2 \rightarrow (1)$

$V_2 = Z_{21}I_1 + Z_{22}I_2 \rightarrow (2)$

$\Rightarrow$ from eq (2) $Z_{22}I_2 = V_2 - Z_{21}I_1$

$$I_2 = \frac{1}{Z_{22}}V_2 - \frac{Z_{21}}{Z_{22}}I_1 \rightarrow (3)$$

$\Rightarrow$ sub eq (3) in eq (1).

$$\Rightarrow V_1 = Z_{11}I_1 + Z_{12}\left[\frac{1}{Z_{22}}V_2 - \frac{Z_{21}}{Z_{22}}I_1\right] = \left[Z_{11} - \frac{Z_{12}Z_{21}}{Z_{22}}\right]I_1 + \frac{Z_{12}}{Z_{22}}V_2$$

$$\Rightarrow Z_{22}V_1 = \left[Z_{22}Z_{11} - Z_{12}Z_{21}\right]I_1 + Z_{12}V_2$$ Where $\left[\because \Delta Z = |Z| = Z_{22}Z_{11} - Z_{12}Z_{21}\right.$

$$\Rightarrow I_1 = \frac{Z_{22}V_1 - Z_{12}V_2}{Z_{22}Z_{11} - Z_{12}Z_{21}} = \frac{Z_{22}V_1 - Z_{12}V_2}{\Delta Z} = \frac{Z_{22}}{\Delta Z}V_1 - \frac{Z_{12}}{\Delta Z}V_2$$

$$\Rightarrow I_1 = \frac{Z_{22}}{\Delta Z}V_1 - \frac{Z_{12}}{\Delta Z}V_2 \rightarrow (4)$$

$\Rightarrow$ from eq. (2) $\Rightarrow$ substitute eq (4) in eq (2)

$$\Rightarrow V_2 = Z_{21}\left[\frac{Z_{22}}{\Delta Z}V_1 - \frac{Z_{12}}{\Delta Z}V_2\right] + Z_{22}I_2$$

$$\Rightarrow V_2 - \frac{Z_{21}Z_{22}}{\Delta Z}V_1 + \frac{Z_{21}Z_{12}}{\Delta Z}V_2 = Z_{22}I_2$$

$$\Rightarrow Z_{22}I_2 = \frac{Z_{22}Z_{11}}{\Delta Z}V_2 - \frac{Z_{21}Z_{22}}{\Delta Z}V_1$$

$$\Rightarrow I_2 = -\frac{Z_{21}}{\Delta Z}V_1 + \frac{Z_{11}}{\Delta Z}V_2 \rightarrow (5)$$

$\Rightarrow$ comparing eq (4) and eq (5) with Y-parametric equations

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \text{ and } I_2 = Y_{21}V_1 + Y_{22}V_2$$

We get, $Y_{11} = \frac{Z_{22}}{\Delta Z}$, $Y_{12} = -\frac{Z_{12}}{\Delta Z}$, $Y_{21} = \frac{-Z_{21}}{\Delta Z}$, $Y_{22} = \frac{Z_{11}}{\Delta Z}$ $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$\Rightarrow$ The Y-parametric Matrix is,

$$[Y] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{Z_{22}}{\Delta Z} & \frac{-Z_{12}}{\Delta Z} \\ \frac{-Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{bmatrix} = \frac{1}{\Delta Z}\begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix}$$

$; Y = \frac{}{|Z|} \cdot z]$

$\Rightarrow Y = Z^{-1}$

2. Z-parameters to ABCD parameters :-

$\Rightarrow$ We know that from z-parameters,

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \rightarrow ①$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \rightarrow ②$$

$\Rightarrow$ We know that from ABCD parameters

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

from ② $\Rightarrow Z_{21} I_1 = V_2 - Z_{22} I_2$

$$I_1 = \frac{V_2}{Z_{21}} - \frac{Z_{22}}{Z_{21}} I_2 \rightarrow ③$$

from eq. ①

$$\Rightarrow V_1 = Z_{11} I_1 + Z_{12} I_2$$

$\Rightarrow$ sub eq ③ in eq. ①

$$\Rightarrow V_1 = \frac{Z_{11}}{Z_{21}} V_2 - \frac{Z_{22} Z_{11}}{Z_{21}} I_2 + Z_{12} I_2$$

$$\Rightarrow V_1 = \frac{Z_{11}}{Z_{21}} V_2 - \left[\frac{Z_{22} Z_{11} - Z_{12} Z_{21}}{Z_{21}}\right] I_2 = \frac{Z_{11}}{Z_{21}} V_2 - \frac{\Delta Z}{Z_{21}} I_2$$

$$\Rightarrow V_1 = \frac{Z_{11}}{Z_{21}} V_2 - \frac{\Delta Z}{Z_{21}} I_2 \rightarrow ④$$

$\Rightarrow$ comparing eq. ③ & ④ with ABCD parameters,

$$\therefore A = \frac{Z_{11}}{Z_{21}}, \quad B = \frac{+\Delta Z}{Z_{21}}, \quad C = \frac{1}{Z_{21}}, \quad D = \frac{+Z_{22}}{Z_{21}}$$

3. Z-parameters to h-parameters :- [h-parameters in terms of Z-parameters]

$\Rightarrow$ We know from z-parameters,

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \ (\rightarrow ①) \quad \text{and} \quad V_2 = Z_{21} I_1 + Z_{22} I_2 \rightarrow ②$$

from eq. ②

$$\Rightarrow I_2 = \frac{V_2 - Z_{21} I_1}{Z_{22}} = \frac{1}{Z_{22}} V_2 - \frac{Z_{21}}{Z_{22}} I_1$$

$$\Rightarrow I_2 = \frac{1}{Z_{22}} V_2 - \frac{Z_{21}}{Z_{22}} I_1 \rightarrow ③ \Rightarrow I_2 = \frac{-Z_{21}}{Z_{22}} I_1 + \frac{1}{Z_{22}} V_2 \ (\rightarrow ⑤)$$

→ substitute eq ③ in eq ①.

$$\Rightarrow V_1 = Z_{11} I_1 + Z_{12}\left[\frac{1}{Z_{22}} V_2 - \frac{Z_{21}}{Z_{22}} I_1\right] = Z_{11} I_1 + \frac{Z_{12}}{Z_{22}} V_2 - \frac{Z_{21}}{Z_{22}} I$$

$$\Rightarrow V_1 = \frac{\Delta Z}{Z_{22}} I_1 + \frac{Z_{12}}{Z_{22}} V_2 \rightarrow ④$$

→ We know that h-parameters are $V_1 = h_{11} I_1 + h_{12} V_2$
$I_2 = h_{21} I_1 + h_{22} V_2$

→ comparing eq ④ & ⑤ with the above h-parametric equations, we get

$$h_{11} = \frac{\Delta Z}{Z_{22}}, \quad h_{12} = \frac{Z_{12}}{Z_{22}}, \quad h_{21} = \frac{-Z_{21}}{Z_{22}}, \quad h_{22} = \frac{1}{Z_{22}}$$

4. <u>Conversion of Y-parameters to Z-parameters</u>:-

→ We know from Y-parameters $I_1 = Y_{11} V_1 + Y_{12} V_2 \rightarrow ①$ and $I_2 = Y_{21} V_1 + Y_{22} V_2 \rightarrow ②$

→ from eq ② $V_2 = \dfrac{I_2 - Y_{21} V_1}{Y_{22}} = \dfrac{1}{Y_{22}} I_2 - \dfrac{Y_{21}}{Y_{22}} V_1 \rightarrow ③$

→ sub eq ③ in eq ①

$$\Rightarrow I_1 = Y_{11} V_1 + Y_{12}\left[\frac{1}{Y_{22}} I_2 - \frac{Y_{21}}{Y_{22}} V_1\right] = \frac{Y_{12}}{Y_{22}} I_2 + Y_{11} V_1 - \frac{Y_{12} Y_{21}}{Y_{22}} V_1$$

$$\Rightarrow I_1 = \frac{Y_{12}}{Y_{22}} I_2 + \frac{Y_{11} Y_{22} - Y_{12} Y_{21}}{Y_{22}} V_1 \qquad [\because \Delta Y = Y_{11} Y_{22} - Y_{12} Y_{21}]$$

$$\Rightarrow Y_{22} I_1 = Y_{12} I_2 + \Delta Z V_1 \Rightarrow \Delta y V_1 = Y_{22} I_1 - Y_{12} I_2$$

$$\Rightarrow V_1 = \frac{Y_{22}}{\Delta y} I_1 - \frac{Y_{12}}{\Delta y} I_2 \rightarrow ④$$

→ substitute V_1 in eq ②

$$\Rightarrow I_2 = \frac{Y_{21} Y_{22}}{\Delta Y} I_1 - \frac{Y_{21} Y_{12}}{\Delta Y} I_2 + Y_{22} V_2$$

$$\Rightarrow Y_{22} V_2 = I_2 + \frac{Y_{21} Y_{12}}{\Delta y} I_2 - \frac{Y_{21} Y_{22}}{\Delta Y} I_1$$

$$\Rightarrow Y_{22} V_2 = \frac{Y_{11} Y_{22}}{\Delta Y} I_2 - \frac{Y_{21} Y_{22}}{\Delta Y} I_1$$

$$\Rightarrow V_2 = \frac{-Y_{21}}{\Delta Y} I_1 + \frac{Y_{11}}{\Delta Y} I_2 \rightarrow ⑤$$

→ We know the Z-parameters $V_1 = Z_{11} I_1 + Z_{12} I_2$
$V_2 = Z_{21} I_1 + Z_{22} I_2$

...ing the ... ations with eq (4) & eq (5)

We get

$$\Rightarrow Z_{11} = \frac{Y_{22}}{\Delta Y}, \quad Z_{22} = \frac{Y_{11}}{\Delta Y}, \quad Z_{12} = \frac{-Y_{12}}{\Delta Y}, \quad Z_{21} = -\frac{Y_{21}}{\Delta Y}$$

5. Conversion of Y-parameters to ABCD-parameters:-

$\Rightarrow$ We have the Y-parameters $I_1 = Y_{11}V_1 + Y_{12}V_2$ (1) and $I_2 = Y_{21}V_1 + Y_{22}V_2$ (2)

$\Rightarrow$ We have the ABCD-parameters $V_1 = AV_2 - BI_2$ and $I_1 = CV_2 - DI_2$

from eq. (2) we have,

$$\Rightarrow Y_{21}V_1 = I_2 - Y_{22}V_2$$

$$\Rightarrow V_1 = \frac{1}{Y_{21}} I_2 - \frac{Y_{22}}{Y_{21}} V_2 \longrightarrow (3)$$

$$\Rightarrow V_1 = -\frac{Y_{22}}{Y_{21}} V_2 + \frac{1}{Y_{21}} I_2 \longrightarrow (3)$$

from eq (1) $\Rightarrow$ substitute eq (3) in eq (1).

$$\Rightarrow I_1 = Y_{11}\left[-\frac{Y_{22}}{Y_{21}} V_2 + \frac{I_2}{Y_{21}}\right] + Y_{12}V_2$$

$$\Rightarrow I_1 = \frac{Y_{11}}{Y_{21}} I_2 + V_2\left[\frac{-Y_{11}Y_{22} + Y_{12}Y_{21}}{Y_{21}}\right] = \frac{Y_{11}}{Y_{21}} I_2 - \frac{\Delta Y}{Y_{21}} V_2$$

$$\Rightarrow I_1 = -\frac{\Delta Y}{Y_{21}} V_2 + \frac{Y_{11}}{Y_{21}} I_2 \longrightarrow (4)$$

comparing eq. (3) and (4) with ABCD-parameters, we get

$$A = -\frac{Y_{22}}{Y_{21}}, \quad B = \frac{-1}{Y_{21}}, \quad C = \frac{-\Delta Y}{Y_{21}}, \quad D = \frac{-Y_{11}}{Y_{21}}$$

6. Conversion of Y-parameters to h-parameters:-

$\Rightarrow$ From Y-parameters $I_1 = Y_{11}V_1 + Y_{12}V_2 \rightarrow (1)$ & $I_2 = Y_{21}V_1 + Y_{22}V_2 \rightarrow (2)$

from eq. (2) $\Rightarrow$ $V_1 = \frac{1}{Y_{21}} I_2 - \frac{Y_{22}}{Y_{21}} V_2 \rightarrow (3)$

$\Rightarrow$ substitute the above eq (3) in eq. (1),

$$\Rightarrow I_1 = \frac{Y_{11}}{Y_{21}} I_2 - \frac{Y_{11}Y_{22}}{Y_{21}} V_2 + Y_{12}V_2 = \frac{Y_{11}}{Y_{21}} I_2 - \frac{\Delta Y}{Y_{21}} V_2$$

$$\Rightarrow Y_{21} I_1 = Y_{11} I_2 - \Delta Y V_2$$

$$\Rightarrow I_2 = \frac{Y_{21}I_1 + \Delta Y V_2}{Y_{11}} = \frac{Y_{21}}{Y_{11}} I_1 + \frac{\Delta Y}{Y_{11}} V_2$$

$$\therefore I_2 = \frac{Y_{21}}{Y_{11}} I_1 + \frac{\Delta Y}{Y_{11}} V_2 \rightarrow (4)$$

$\Rightarrow$ sub eq (4) in eq (3), we get

$$\Rightarrow V_1 = \frac{1}{Y_{11}} I_1 + \frac{\Delta Y}{Y_{21} Y_{11}} V_2 - \frac{Y_{22}}{Y_{21}} V_2$$

$$\Rightarrow V_1 = \frac{1}{Y_{11}} I_1 + \left[\frac{Y_{22}Y_{11} - Y_{12}Y_{21} - Y_{11}Y_{22}}{Y_{21}Y_{11}}\right] V_2$$

$$\Rightarrow V_1 = \frac{1}{Y_{11}} I_1 - \frac{Y_{12}Y_{21}}{Y_{21}Y_{11}} V_2 = \frac{1}{Y_{11}} I_1 - \frac{Y_{12}}{Y_{11}} V_2$$

$$\Rightarrow V_1 = \frac{1}{Y_{11}} I_1 - \frac{Y_{12}}{Y_{11}} V_2 \rightarrow (5)$$

$\Rightarrow$ We know the h-parameters are $V_1 = h_{11} I_1 + h_{12} V_2$
$I_2 = h_{21} I_1 + h_{22} V_2$

$\Rightarrow$ comparing the h-parameters with eq (4) & (5), we get

$$\Rightarrow h_{11} = \frac{1}{Y_{11}},\ h_{12} = \frac{-Y_{12}}{Y_{11}},\ h_{21} = \frac{Y_{21}}{Y_{11}},\ h_{22} = \frac{\Delta Y}{Y_{11}}$$

$\Rightarrow$ Conversion of ABCD - parameters to Z - parameters -

$\Rightarrow$ We know the ABCD - parameters are $V_1 = AV_2 - BI_2 \rightarrow (1)$
$I_1 = CV_2 - DI_2 \rightarrow (2)$

$\Rightarrow$ We know the Z - parameters are $V_1 = Z_{11} I_1 + Z_{12} I_2$
$V_2 = Z_{21} I_1 + Z_{22} I_2$

$\Rightarrow$ from eq (2) $I_1 = CV_2 - DI_2$

$$\Rightarrow CV_2 = I_1 + DI_2 \Rightarrow V_2 = \frac{1}{C} I_1 + \frac{D}{C} I_2 \rightarrow (3)$$

$\Rightarrow$ substitute eq (3) and eq (1)

$$\Rightarrow V_1 = \frac{A}{C} I_1 + \frac{AD}{C} I_2 - BI_2 = \frac{A}{C} I_1 + \left[\frac{AD - BC}{C}\right] I_2$$

$$\Rightarrow V_1 = \frac{A}{C} I_1 + \frac{(AD - BC)}{C} I_2 \longrightarrow (4)$$

$\Rightarrow$ comparing eq (3) and eq (4) with Z - parameters, we get

$$\Rightarrow Z_{11} = \frac{A}{C},\ Z_{12} = \frac{AD - BC}{C},\ Z_{21} = \frac{1}{C},\ Z_{22} = \frac{D}{C}$$

8 Conversion of ABCD parameters to Y-parameters :-

⇒ We get the ABCD-parameters are $V_1 = AV_2 - BI_2 \rightarrow (1)$

$I_1 = CV_2 - DI_2 \rightarrow (2)$

⇒ from eq.(1) $V_1 = AV_2 - BI_2$

$\Rightarrow I_2 = \frac{A}{B}V_2 - \frac{1}{B}V_1 \Rightarrow I_2 = -\frac{1}{B}V_1 + \frac{A}{B}V_2 \rightarrow (3)$

⇒ substitute eq (3) in eq (2)

$\Rightarrow I_1 = CV_2 - D\left[\frac{A}{B}V_2 - \frac{1}{B}V_1\right] = CV_2 - \frac{AD}{B}V_2 + \frac{D}{B}V_1$

$\Rightarrow I_1 = \frac{D}{B}V_1 + \frac{-(AD-BC)}{B}V_2 \longrightarrow (4)$

⇒ comparing eq (3) and eq (4) with Y-parameters $I_1 = Y_{11}V_1 + Y_{12}V_2$

$I_2 = Y_{21}V_1 + Y_{22}V_2$

We get,

$Y_{11} = \frac{D}{B}$, $Y_{12} = \frac{-(AD-BC)}{B}$, $Y_{21} = -\frac{1}{B}$, $Y_{22} = \frac{A}{B}$

9 Conversion of ABCD-parameters to h-parameters:-

⇒ we know the ABCD-parameters are $V_1 = AV_2 - BI_2 \rightarrow (1)$

$I_1 = CV_2 - DI_2 \rightarrow (2)$

⇒ from eq (2) $DI_2 = CV_2 - I_1$

$\Rightarrow I_2 = \frac{C}{D}V_2 - \frac{1}{D}I_1 \longrightarrow (3)$

⇒ substitute eq (3) in eq (1) $\Rightarrow V_1 = AV_2 - B\left[\frac{C}{D}V_2 - \frac{1}{D}I_1\right]$

$\Rightarrow V_1 = AV_2 - \frac{BC}{D}V_2 + \frac{B}{D}I_1$

$\Rightarrow V_1 = \frac{(AD-BC)V_2}{D} + \frac{B}{D}I_1 \longrightarrow (4)$

⇒ comparing eq (3) + (4) with h-parameters $V_1 = h_{11}I_1 + h_{12}V_2$

$I_2 = h_{21}I_1 + h_{22}V_2$

⇒ we get, $h_{12} = \frac{AD-BC}{D}$, $h_{11} = \frac{B}{D}$, $h_{22} = \frac{C}{D}$, $h_{21} = -\frac{1}{D}$

10. Conversion of h-parameters to Z-parameters:-

⇒ We know the h-parameters are $V_1 = h_{11}I_1 + h_{12}V_2 \rightarrow (1)$

$I_2 = h_{21}I_1 + h_{22}V_2 \rightarrow (2)$

⇒ from eq (2) $h_{22}V_2 = I_2 - h_{21}I_1$

$V_2 = \frac{1}{h_{22}}I_2 - \frac{h_{21}}{h_{22}}I_1$

$$\Rightarrow V_2 = \frac{-h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2 \longrightarrow ③$$

$\Rightarrow$ substitute eq. ③ in eq. ①

$$\Rightarrow V_1 = h_{11} I_1 + h_{12}\left[\frac{-h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_1\right] = \frac{(h_{11}h_{22} - h_{12}h_{21})}{h_{22}} I_1 + \frac{h_{12}}{h_{22}} I_2$$

$$\Rightarrow V_1 = \frac{\Delta h}{h_{22}} I_1 + \frac{h_{12}}{h_{22}} I_2 \longrightarrow ④$$

$\Rightarrow$ comparing eq. ③ & ④ with Z-parameters $V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

we get

$$\Rightarrow Z_{11} = \frac{\Delta h}{h_{22}}, \quad Z_{12} = \frac{h_{12}}{h_{22}}, \quad Z_{21} = \frac{-h_{21}}{h_{22}}, \quad Z_{22} = \frac{1}{h_{22}}$$

11 Conversion of h-parameters to Y-parameters :-

$\Rightarrow$ We know the h-parameters are $V_1 = h_{11} I_1 + h_{12} V_2 \longrightarrow ①$

$I_2 = h_{21} I_1 + h_{22} V_2 \longrightarrow ②$

$\Rightarrow$ from eq ① $h_{11} I_1 = V_1 - h_{12} V_2$

$$I_1 = \frac{1}{h_{11}} V_1 - \frac{h_{12} V_2}{h_{11}} \longrightarrow ③$$

$\Rightarrow$ substitute eq. ③ in eq. ②

$$\Rightarrow I_2 = h_{21}\left[\frac{1}{h_{11}} V_1 - \frac{h_{12} V_2}{h_{11}}\right] + h_{22} V_2 = \frac{h_{21}}{h_{11}} V_1 + h_{22} V_2 - \frac{h_{21} h_{12}}{h_{11}} V_2$$

$$\Rightarrow I_2 = \frac{h_{21}}{h_{11}} V_1 + \frac{\Delta h}{h_{11}} V_2 \longrightarrow ④$$

$\Rightarrow$ comparing eq. ③ and ④ with Y-parameters $I_1 = Y_{11} V_1 + Y_{12} V_2$

$I_2 = Y_{21} V_1 + Y_{22} V_2$

we get

$$\Rightarrow Y_{11} = \frac{1}{h_{11}}, \quad Y_{12} = \frac{-h_{12}}{h_{11}}, \quad Y_{21} = \frac{h_{21}}{h_{11}}, \quad Y_{22} = \frac{\Delta h}{h_{11}}$$

12. Conversion of h-parameters to ABCD-parameters :-

$\Rightarrow$ We know the h-parameters are $V_1 = h_{11} I_1 + h_{12} V_2 \rightarrow ①$

$I_2 = h_{21} I_1 + h_{22} V_2 \rightarrow ②$

$\Rightarrow$ from eq. ② $h_{21} I_1 = I_2 - h_{22} V_2$

$$I_1 = \frac{1}{h_{21}} I_2 - \frac{h_{22}}{h_{21}} V_2$$

$$I_1 = \frac{-h_{22}}{h_{21}} V_2 + \frac{1}{h_{21}} I_2 \longrightarrow ③$$

... the ABCD-parametric equations of network ...

$$\Rightarrow V_1 = h_{11}\left[\frac{I_2}{h_{21}} - \frac{h_{22}}{h_{21}}V_2\right] + h_{12}V_2 = \frac{h_{11}}{h_{21}}I_2 - \frac{h_{22}h_{11}V_2}{h_{21}} + h_{12}V_2$$

$$\Rightarrow V_1 = -\frac{\Delta h}{h_{21}}V_2 + \frac{h_{11}}{h_{21}}I_2 \rightarrow (4)$$

comparing eq. (3) & (4) with ABCD-parameters, $V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

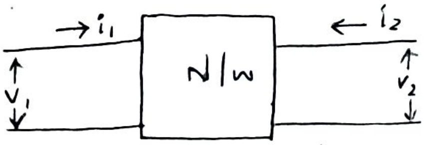
we get,

$$\Rightarrow A = \frac{-\Delta h}{h_{21}},\ B = \frac{-h_{11}}{h_{21}},\ C = \frac{-h_{22}}{h_{21}},\ D = \frac{-1}{h_{21}}$$

24/12/12

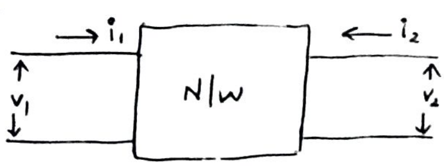
Conditions for symmetry and Reciprocity :-

Symmetry :- A network is said to be symmetrical if it offers same amount of impedance on either sides i.e., input impedance of the network at one port is equals to the impedance of the network at the other port.



→ To be symmetrical the given two-port network should satisfy the condition that $\frac{V_1}{i_1} = \frac{V_2}{i_2}$

Reciprocity :-



→ For the given network to be reciprocal it should satisfy the condition that $\frac{V_1}{i_2} = \frac{V_2}{i_1}$ (since reciprocity means ratio of excitation to response should be constant)

1. Conditions for Reciprocity and Symm... in terms of Z(Y)

Z- parameters :-

$V_1 = z_{11} I_1 + z_{12} I_2$ where $Z_{11} = \frac{V_1}{I_1}$, $Z_{12} = \frac{V_1}{I_2}$

$V_2 = Z_{21} I_1 + Z_{22} I_2$ $\quad Z_{21} = \frac{V_2}{I_1}$, $Z_{22} = \frac{V_2}{I_2}$.

⇒ From the condition of reciprocity $Z_{12} = Z_{21}$ (or) bilateral

condition of symmetry $Z_{11} = Z_{22}$

2. In terms of Y- parameters -

$I_1 = Y_{11} V_1 + Y_{12} V_2$ Where $Y_{11} = \frac{I_1}{V_1}$, $Y_{22} = \frac{I_2}{V_2}$

$I_2 = Y_{21} V_1 + Y_{22} V_2$ $\quad Y_{12} = \frac{I_1}{V_2}$, $Y_{21} = \frac{I_2}{V_1}$

⇒ condition for symmetry is $\frac{V_1}{I_1} = \frac{V_2}{I_2}$ → $\boxed{Y_{22} = Y_{11}}$ ⇒ $\frac{I_2}{V_2} = \frac{I_1}{V_1}$

condition for Reciprocity is $\frac{V_1}{I_2} = \frac{V_2}{I_1}$ → $\frac{i_1}{V_2} = \frac{i_2}{V_1}$ ⇒ $\boxed{Y_{12} = Y_{21}}$

3. In terms of ABCD parameters :-

$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

⇒ From conversion of ABCD to z-parameters, we have

$Z_{11} = \frac{A}{C}$, $Z_{12} = \frac{AD - BC}{C}$, $Z_{21} = \frac{1}{C}$, $Z_{22} = \frac{D}{C}$

⇒ condition of symmetery $Z_{11} = Z_{22}$ ⇒ $\frac{A}{C} = \frac{D}{C}$ → $\boxed{A = D}$

condition of Reciprocity $Z_{12} = Z_{21}$ ⇒ $\frac{AD - BC}{C} = \frac{1}{C}$ ⇒ $AD - BC =$

⇒ $\boxed{|ABCD| = 1}$

4 In terms of h- parameters -

$V_1 = h_{11} \cancel{V_2} + h_{12} \cancel{V_2} = h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$ where

⇒ From conversion of h to z-parameters, we have

$Z_{11} = \frac{\Delta h}{h_{22}}$, $Z_{12} = \frac{h_{12}}{h_{22}}$, $Z_{21} = \frac{-h_{21}}{h_{22}}$, $Z_{22} = \frac{1}{h_{22}}$

⇒ condition for symmetry $Z_{11} = Z_{22}$ ⇒ $\boxed{\Delta h = h_{12}}$ ⇒ $\Delta h = 1$

condition for Reciprocity $Z_{12} = Z_{21}$ ⇒ $\boxed{h_{12} = -h_{21}}$

	Z	Y	ABCD	h
1-Symmetry	$Z_{11} = Z_{22}$	$Y_{11} = Y_{22}$	$A = D$	$\Delta h = 1$
2-Reciprocity	$Z_{12} = Z_{21}$	$Y_{12} = Y_{21}$	$AD - BC = 1$	$h_{12} = -h_{21}$

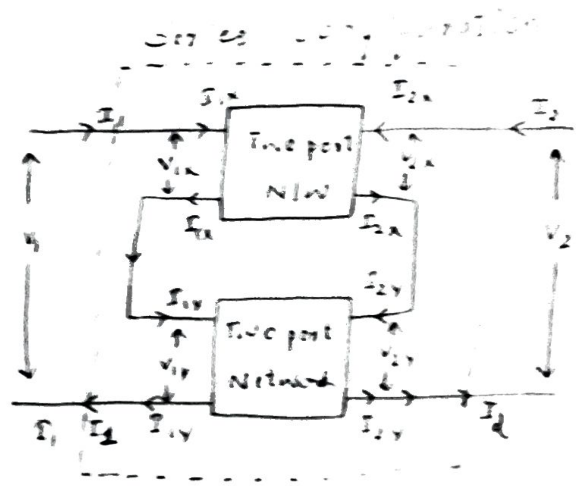
Interconnection of two-port Network :-

→ There are 3 possible ways that we interconnect two port network

1. Series configuration
2. Parallel configuration
3. Cascaded Network

1 Series configuration :-

→ Two two port networks are said to be in series, when current through each port of each network is same whereas the voltages are different



→ The series configuration of 2 two port networks is as shown in figure,

→ From the configuration,

V_1 = port 1 voltage of the series configuration

I_1 = port 1 current of the series configuration

V_2 = port 2 voltage of the series configuration

I_2 = port 2 current of the series configuration

V_{1x} = port 1 voltage of network x

V_{2x} = port 2 voltage of network x

→ I_{2x} = port 2 current of network x

I_{1x} = port 1 current of network x

V_{1y} = port 1 voltage of network y

V_{2y} = port 2 voltage of network y

I_{1y} = port 1 current of network y

I_{2y} = port 2 current of network y

→ From the basics of series configuration, we know that

$$V_1 = V_{1x} + V_{1y} \rightarrow (1), \quad I_1 = I_{1x} = I_{1y} \rightarrow (3)$$

$$\text{|||}^{ly} \; V_2 = V_{2x} + V_{2y} \rightarrow (2) \quad I_2 = I_{2x} = I_{2y} \rightarrow (4)$$

→ The series configuration of 2 port networks will be realize using z-parametric equations

→ From the given configuration, z-parameters of the entire configurations are

$$[V] = [Z][I]$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \rightarrow (5)$$

→ For a two port network x, $\begin{bmatrix} V_{1x} \\ V_{2x} \end{bmatrix} = \begin{bmatrix} Z_{11x} & Z_{12x} \\ Z_{21x} & Z_{22x} \end{bmatrix} \begin{bmatrix} I_{1x} \\ I_{2x} \end{bmatrix} \rightarrow 6$

→ For a two port network y, $\begin{bmatrix} V_{1y} \\ V_{2y} \end{bmatrix} = \begin{bmatrix} Z_{11y} & Z_{12y} \\ Z_{21y} & Z_{22y} \end{bmatrix} \begin{bmatrix} I_{1y} \\ I_{2y} \end{bmatrix} \rightarrow (7)$

→ sub eq. (1) & (2) in (5)

$$\Rightarrow \begin{bmatrix} V_{1x} + V_{1y} \\ V_{2x} + V_{2y} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} V_{1x} \\ V_{2x} \end{bmatrix} + \begin{bmatrix} V_{1y} \\ V_{2y} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \rightarrow (8)$$

→ sub eq. (6) & (7) in eq (8)

$$\Rightarrow \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Z_{11x} & Z_{12x} \\ Z_{21x} & Z_{22x} \end{bmatrix} \begin{bmatrix} I_{1x} \\ I_{2x} \end{bmatrix} + \begin{bmatrix} Z_{11y} & Z_{12y} \\ Z_{21y} & Z_{22y} \end{bmatrix} \begin{bmatrix} I_{1y} \\ I_{2y} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Z_{11x} & Z_{12x} \\ Z_{21x} & Z_{22x} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} + \begin{bmatrix} Z_{11y} & Z_{12y} \\ Z_{21y} & Z_{22y} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$= \left\{ \begin{bmatrix} Z_{11x} & Z_{12x} \\ Z_{21x} & Z_{22x} \end{bmatrix} + \begin{bmatrix} Z_{11y} & Z_{12y} \\ Z_{21y} & Z_{22y} \end{bmatrix} \right\} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Z_{11x}+Z_{11y} & Z_{12x}+Z_{12y} \\ Z_{21x}+Z_{21y} & Z_{22x}+Z_{22y} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Where

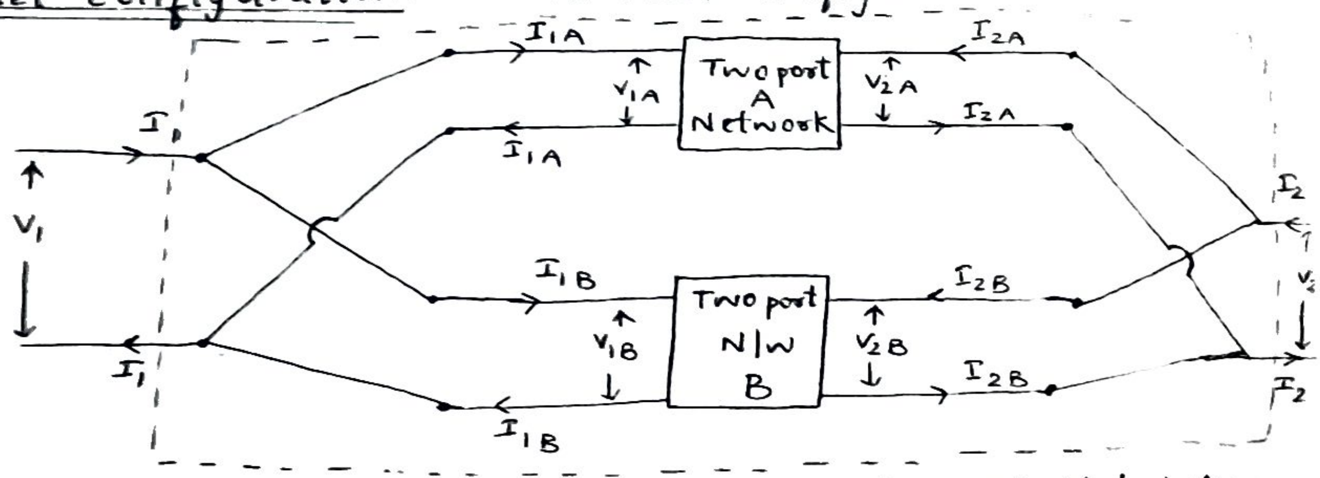
$\Rightarrow Z_{11} = Z_{11x}+Z_{11y}$, $Z_{12} = Z_{12x}+Z_{12y}$, $Z_{21} = Z_{21x}+Z_{21y}$, $Z_{22} = Z_{22x}+Z_{22y}$

$\Rightarrow$ If 'n' two port networks are connected in series configuration the Z-parameters of the total configuration is algebraic sum of individual Z-parameters i.e.,

$$\Rightarrow [Z_T] = [Z_1] + [Z_2] + \dots + [Z_n]$$

2. <u>Parallel Configuration:-</u>

Parallel configuration



→ Two two port networks are said to be in parallel when each port of each network have same voltage whereas current is different.

→ The parallel configuration of two two port networks is as shown in figure,

From the configuration,

V_1 = port 1 voltage of parallel configuration

V_2 = port 2 voltage of parallel configuration.

I_1 = port 1 current of parallel configuration.

I_2 = port 2 current of parallel configuration

V_{1A} = port 1 voltage of network A

V_{2A} = port 2 voltage of network B

→ The parallel configuration of 2 two port network is realized using Y-parametric equations

$\Rightarrow [I] = [Y][V]$

→ From the basics of parallel configuration, we have

$V_1 = V_{1A} = V_{1B} \rightarrow ①$; $I_1 = I_{1A} + I_{1B} \rightarrow ③$

$V_2 = V_{2B} = V_{2A} \rightarrow ②$; $I_2 = I_{2A} + I_{2B} \rightarrow ④$

$$\Rightarrow \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \rightarrow ⑤ \Rightarrow \begin{bmatrix} I_{1A}+I_{1B} \\ I_{2A}+I_{2B} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

→ For a two port network A, $\begin{bmatrix} I_{1A} \\ I_{2A} \end{bmatrix} = \begin{bmatrix} Y_{11A} & Y_{12A} \\ Y_{21A} & Y_{22A} \end{bmatrix} \begin{bmatrix} V_{1A} \\ V_{2A} \end{bmatrix} \rightarrow ⑥$

→ For a two port network B, $\begin{bmatrix} I_{1B} \\ I_{2B} \end{bmatrix} = \begin{bmatrix} Y_{11B} & Y_{12B} \\ Y_{21B} & Y_{22B} \end{bmatrix} \begin{bmatrix} V_{1B} \\ V_{2B} \end{bmatrix} \rightarrow ⑦$

→ sub eq ⑥ & ⑦ in ⑤

$$\Rightarrow \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Y_{11A}+Y_{11B} & Y_{12A}+Y_{12B} \\ Y_{21A}+Y_{21B} & Y_{22A}+Y_{22B} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

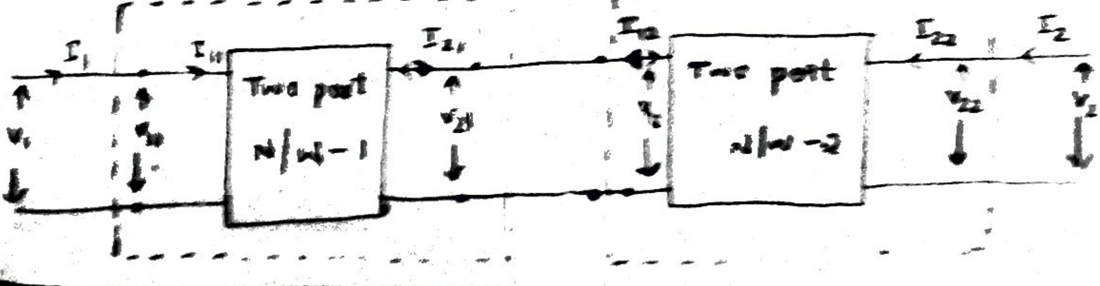
Where

$\Rightarrow Y_{11} = Y_{11A} + Y_{11B}$, $Y_{12} = Y_{12A} + Y_{12B}$, $Y_{21} = Y_{21A} + Y_{21B}$, $Y_{22} = Y_{22A} + Y_{22B}$

→ If 'n' two port networks are connected in parallel configuration, the Y-parameters of the total configuration is algebraic sum of individual Y-parameters, i.e.,

$\Rightarrow [Y] = [Y_1] + [Y_2] + \cdots\cdots + [Y_n]$

3. Cascade Connection (or) Ladder (or) Side by Side



⇒ Two 2 port networks are said to be in cascaded configuration when the output of 1 network is given as input to the other network. This type of connection is known as cascaded connection / ladder / side by side configurations.

→ Cascaded configuration of two 2 ports network is as shown in figure.

→ To realise the cascaded connection of 2 two port networks, we will use transmission parameters.

→ From the given configuration,

V_1 = port 1 voltage of entire configuration.

I_1 = port 1 current of entire configuration

V_2 = port 2 voltage of entire configuration

I_2 = port 2 current of entire configuration.

V_{11} = port 1 voltage of network 1

I_{11} = port 1 current of network 1

V_{21} = port 2 voltage of N/W 1

I_{21} = port 2 current of N/W 1

V_{12} = port 1 voltage of N/W 2

I_{12} = port 1 current of N/W 2

V_{22} = port 2 voltage of N/W 2

I_{22} = port 2 current of N/W 2

→ From the ABCD parametric equations, we know that

$$\Rightarrow \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \quad (1)$$

⇒ for Network 1 $\begin{bmatrix} V_{11} \\ I_{11} \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_{21} \\ -I_{21} \end{bmatrix}$ (2) ; for network 2, $\begin{bmatrix} V_{12} \\ I_{12} \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_{22} \\ -I_{22} \end{bmatrix}$ (3)

⇒ From the basics configuration,

$I_{11} = I_1$, $V_{11} = V_1$, $V_{12} = V_{21}$, $I_{12} = -I_{21}$, $V_{22} = V_2$, $I_{22} = I_2$

⇒ From the ABCD-parametric equations of network 1, 2 (ii)

$$\Rightarrow \begin{bmatrix} V_{11} \\ I_{11} \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_{21} \\ -I_{21} \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_{12} \\ +I_{12} \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V \\ - \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} V_{11} \\ I_{11} \end{bmatrix} = [A_1][A_2] \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = [A_1][A_2] \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \rightarrow (4)$$

⇒ comparing (1) & (4) $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = [A_1][A_2]$

⇒ If 'n' two port networks are connected in cascaded configuration, the ABCD parameters (transition) of final configuration is the product of ABCD-Matrices of individual network

i.e., $[A_T] = [A_1][A_2] \ldots\ldots [A_n]$

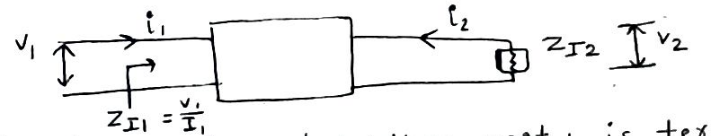
Conversion Table:-

to ↓ / from ←	Z		Y		ABCD		h	
Z	Z_{11}	Z_{12}	$\frac{Y_{22}}{\Delta Y}$	$\frac{-Y_{12}}{\Delta Y}$	$\frac{A}{C}$	$\frac{\Delta A}{C}$	$\frac{\Delta h}{h_{22}}$	$\frac{h_{12}}{h_{22}}$
	Z_{21}	Z_{22}	$\frac{-Y_{21}}{\Delta Y}$	$\frac{Y_{11}}{\Delta Y}$	$\frac{1}{C}$	$\frac{D}{C}$	$\frac{-h_{21}}{h_{22}}$	$\frac{1}{h_{22}}$
Y	$\frac{Z_{22}}{\Delta Z}$	$\frac{-Z_{12}}{\Delta Z}$	Y_{11}	Y_{12}	$\frac{D}{B}$	$\frac{-\Delta A}{B}$	$\frac{1}{h_{11}}$	$\frac{-h_{12}}{h_{11}}$
	$\frac{-Z_{21}}{\Delta Z}$	$\frac{Z_{11}}{\Delta Z}$	Y_{21}	Y_{22}	$\frac{-1}{B}$	$\frac{\Delta A}{B}$	$\frac{h_{21}}{h_{11}}$	$\frac{\Delta h}{h_{11}}$
ABCD	$\frac{Z_{11}}{Z_{21}}$	$\frac{\Delta Z}{Z_{21}}$	$\frac{-Y_{22}}{Y_{21}}$	$\frac{-1}{Y_{21}}$	A	B	$\frac{+\Delta h}{h_{21}}$	$\frac{+h_{11}}{h_{21}}$
	$\frac{1}{Z_{21}}$	$\frac{Z_{22}}{Z_{21}}$	$\frac{-\Delta Y}{Y_{21}}$	$\frac{-Y_{11}}{Y_{21}}$	C	D	$\frac{-h_{22}}{h_{21}}$	$\frac{-1}{h_{21}}$
h	$\frac{\Delta Z}{Z_{22}}$	$\frac{Z_{12}}{Z_{22}}$	$\frac{1}{Y_{11}}$	$\frac{-Y_{12}}{Y_{11}}$	$\frac{AD-BC}{D}$	$\frac{B}{D}$	h_{11}	h_{12}
	$\frac{-Z_{21}}{Z_{22}}$	$\frac{1}{Z_{22}}$	$\frac{Y_{21}}{Y_{11}}$	$\frac{\Delta Y}{Y_{11}}$	$\frac{C}{D}$	$\frac{-1}{D}$	h_{21}	h_{22}

26/12/16

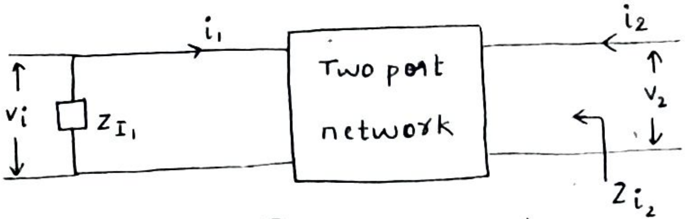
Image Parameters:- Image parameters are impedance parameters which describes the network behaviour. These are denoted by Z_{I_1} and Z_{I_2}. Image parameters are defined as impedances of each port when the other port is terminated with its image impedance i.e.,

Z_{I_1} = impedance at port 1 when port 2 is terminated with Z_{I_2}



Similarly,

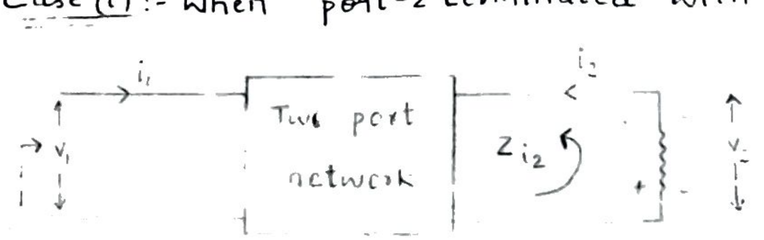
Zi_2 = Impedance at port 2 when port 1 is terminated with Zi_1



$$Z_{I_2} = \frac{V_2}{i_2}$$

Determination of Image parameters:- The values of Image parameters will be determined using the ~~transfer~~ transmission parameters of two port network i.e., Z_{I_1} & Z_{I_2} will be expressed in terms of ABCD parameters.

Case (i):- When port-2 terminated with its image ~~difference~~ (impedance)



$$Z_{I_1} = \frac{V_1}{I_1}$$

from ABCD parameters,

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

$$Z_{I_1} \Rightarrow \frac{V_1}{I_1} = \frac{AV_2 - BI_2}{CV_2 - DI_2} = \frac{I_2\left(A\frac{V_2}{I_2} - B\right)}{I_2\left(C\frac{V_2}{I_2} - D\right)}$$

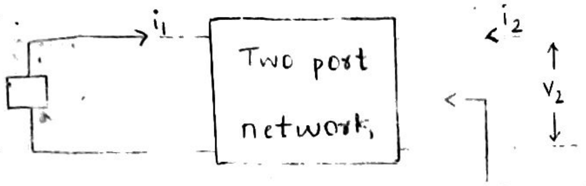
$\Rightarrow$ from diagram,

$V_2 = -I_2 Z_{I_2}$ (negative sign since current flow is opposite)

$$\Rightarrow \frac{V_2}{I_2} = -Z_{I_2}$$

$$\Rightarrow Z_{I_1} = \frac{-AZ_{I_2} - B}{-CZ_{I_2} - D} \Rightarrow Z_{I_1} = \frac{AZ_{I_2} + B}{CZ_{I_2} + D} \longrightarrow (1)$$

Case(ii):- When port 1 is terminated with Image parameters,



$$Z_{i_2} = \frac{V_2}{I_2}$$

from port 1,

$$V_1 = -I_1 Z_{I_1}$$

$$-Z_{I_1} = \frac{V_1}{I_1} = \frac{AV_2 - BI_2}{CV_2 - DI_2}$$

$$-Z_{I_1} = \frac{(AZ_{I_2} - B) I_2}{(CZ_{I_2} - D) I_2}$$

$$\Rightarrow Z_{I_1} = \frac{-(AZ_{I_2} - B)}{CZ_{I_2} - D} \longrightarrow (2)$$

$\Rightarrow$ from eq. (1) & (2), $\dfrac{AZ_{I_2} + B}{CZ_{I_2} + D} = \dfrac{-(AZ_{I_2} - B)}{CZ_{I_2} - D}$

$$\Rightarrow (AZ_{I_2} + B)(CZ_{I_2} - D) = -(AZ_{I_2} - B)(CZ_{I_2} + D)$$

$$\Rightarrow ACZ_{I_2}^2 - ADZ_{I_2} + BCZ_{I_2} - BD = -ACZ_{I_2}^2 - ADZ_{I_2} + BCZ_{I_2} + BD$$

$$\Rightarrow 2ACZ_{I_2}^2 = 2BD \Rightarrow Z_{I_2}^2 = \frac{BD}{AC}$$

$$\Rightarrow Z_{I_2} = \sqrt{\frac{BD}{AC}}$$

$\Rightarrow$ substituting Z_{I_2} eq. (1),

$$\Rightarrow Z_{I_1} = \frac{A\sqrt{\frac{BD}{AC}} + B}{C\sqrt{\frac{BD}{AC}} + D} = \frac{\frac{A}{\sqrt{A}}\sqrt{\frac{BD}{C}} + B}{\frac{C}{\sqrt{C}}\sqrt{\frac{BC}{A}} + D} = \frac{\frac{\sqrt{ABD}}{\sqrt{C}} + B}{\frac{\sqrt{CBD}}{\sqrt{A}} + D}$$

$$\Rightarrow Z_{I_1} = \frac{\sqrt{ABD} + B\sqrt{C}}{\sqrt{C}} \times \frac{\sqrt{A}}{\sqrt{CBD} + D\sqrt{A}} = \frac{A\sqrt{BD} + B\sqrt{AC}}{C\sqrt{BD} + D\sqrt{AC}} = \frac{\sqrt{AB}[\sqrt{AD} + \sqrt{B}}{\sqrt{DC}[\sqrt{BC} + \sqrt{A}}$$

$$\Rightarrow Z_{I_1} = \frac{\sqrt{AB}}{\sqrt{DC}}$$

$\Rightarrow$ If the network is symmetrical (or) bilateral,

$$\Rightarrow \frac{V_1}{I_1} = \frac{V_2}{I_2} \Rightarrow Z_{I_1} = Z_{I_2} \Rightarrow \frac{\sqrt{AB}}{\sqrt{DC}} = \frac{\sqrt{BD}}{\sqrt{AC}} \ (A = D)$$

$$\Rightarrow \sqrt{\frac{AB}{DC}} = \sqrt{\frac{BD}{AC}} \Rightarrow \sqrt{\frac{B}{C}} = \sqrt{\frac{B}{C}}$$

$$\Rightarrow Z_{I_1} = \sqrt{\frac{AB}{DC}} = \sqrt{\frac{B}{C}} \quad (\because A = D)$$

$$\Rightarrow Z_{I_2} = \sqrt{\frac{BD}{AC}} = \sqrt{\frac{B}{C}}$$

⇒ When network is symmetrical, the image parameters are equal in impedance,

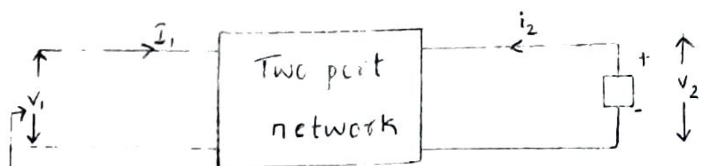
$$\Rightarrow Z_{I1} = Z_{I2} = \sqrt{\frac{B}{C}}$$

⇒ these value of impedance is known as characteristic impedance (or) iterative impedance & denoted by Z_i/Z_c.

$$Z_I = \sqrt{\frac{B}{C}} = Z_C$$

→ To determine the behaviour of network interms of reciprocity we will use one (or) more image parameters known as propagation constant and is denoted by γ.

Determination of propagation constant (γ):-



from 2 port network,

$$Z_{I1} = \frac{V_1}{I_1} \qquad Z_{I2} = -\frac{V_2}{I_2}$$

from ABCD parameters,

$$V_1 = AV_2 - BI_2$$

$$V_1 = V_2\left[A - \frac{B}{\left(\frac{V_2}{I_2}\right)}\right]$$

$$\Rightarrow \frac{V_1}{V_2} = A + \frac{B}{Z_{I2}} \longrightarrow ① \qquad \left(\because \frac{V_2}{I_2} = -Z_{I2}\right)$$

Similarly,

$$\Rightarrow I_1 = CV_2 - DI_2 = I_2\left[C\frac{V_2}{I_2} - D\right] = I_2\left[-CZ_{I2} - D\right]$$

$$\Rightarrow \frac{-I_1}{I_2} = CZ_{I2} + D \longrightarrow ②$$

But we know that $Z_{I2} = \sqrt{\frac{BD}{AC}}$

⇒ from eq. ① & ②

$$\Rightarrow \frac{V_1}{V_2} = A + B\sqrt{\frac{AC}{BD}} \qquad \Rightarrow \frac{-I_1}{I_2} = C\sqrt{\frac{BD}{AC}} + D$$

$$\Rightarrow \frac{V_1}{V_2} = A + \sqrt{\frac{ABC}{D}} \longrightarrow ③ \qquad \frac{-I_1}{I_2} = \sqrt{\frac{BCD}{A}} + D \longrightarrow ④$$

⇒ Multiplying eq. ③ and ④,

$$\Rightarrow \left(\frac{V_1}{V_2}\right) \times \left(\frac{-I_1}{I_2}\right) = \left(A + \sqrt{\frac{ABC}{D}}\right)\left(\frac{\sqrt{BCD}}{\sqrt{A}} + D\right)$$

$$\Rightarrow \left(\frac{V_1}{V_2}\right)\left(\frac{-I_1}{I_2}\right) = A\sqrt{\frac{BCD}{A}} + AD + BC + D\sqrt{\frac{ABC}{D}}$$

$$\Rightarrow \left(\frac{V_1}{V_2}\right)\left(\frac{-I_1}{I_2}\right) = AD + BC + 2\sqrt{ABCD}$$

$$= (\sqrt{AD})^2 + (\sqrt{BC})^2 + 2\sqrt{AD}\cdot\sqrt{BC}$$

$$= \left(\sqrt{AD} + \sqrt{BC}\right)^2$$

$$\Rightarrow \sqrt{AD} + \sqrt{BC} = \sqrt{\left(\frac{V_1}{V_2}\right)\left(\frac{-I_1}{I_2}\right)}$$

$\Rightarrow$ If the given network is reciprocal, then $AD - BC = 1$

$\Rightarrow AD - 1 = BC$

$$\Rightarrow \sqrt{AD} + \sqrt{AD-1} = \sqrt{\left(\frac{V_1}{V_2}\right)\left(\frac{-I_1}{I_2}\right)}$$

$\Rightarrow$ Let us assume $\cosh\phi = \sqrt{AD}$, then from hyperbolic fn,

$\Rightarrow \cosh^2\phi - \sinh^2\phi = 1$

$\Rightarrow \sinh^2\phi = \cosh^2\phi - 1 = AD$

$\Rightarrow \sinh\phi = \sqrt{AD-1}$

$$\Rightarrow \sqrt{AD} + \sqrt{AD-1} = \sqrt{\left(\frac{V_1}{V_2}\right)\left(\frac{-i_1}{i_2}\right)}$$

$$\Rightarrow \cosh\phi + \sinh\phi = \sqrt{\frac{Z_{I1}}{Z_{I2}}\,\frac{i_1}{i_2}\,\frac{i_1}{i_2}}$$

$$\Rightarrow e^{\phi} = \sqrt{\frac{Z_{I1}}{Z_{I2}}}\left(\frac{i_1}{i_2}\right)$$

$\Rightarrow$ If the network is symmetrical then $Z_{I1} = Z_{I2}$

$$e^{\phi} = \frac{i_1}{i_2}$$

$$\phi = \ln\left(\frac{i_1}{i_2}\right) = \gamma = \sigma + j\beta$$

where γ = propagation constant produced by network

$\Rightarrow \gamma = \sigma + j\beta$, where σ refers attenuation and β refers phase difference (or) phase shift.

Problem :-

19:- Determine the Z-parameters for the given two-port n/w and hence draw the equivalent circuit.

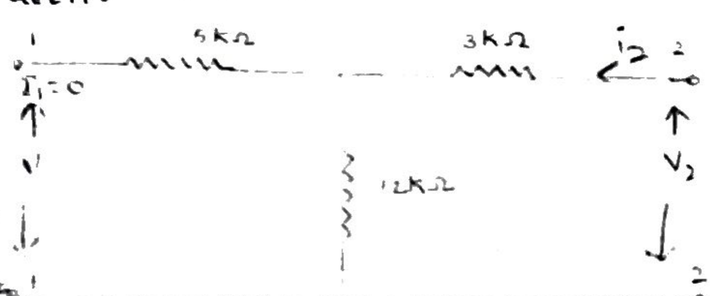
Sol:- We know that Z-parametric equations are,

$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

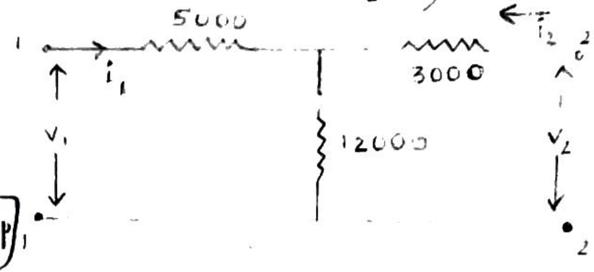
$Z_{12} = \frac{V_1}{I_2} = \frac{I_2 \cdot 12k}{I_1}$

$Z_{22} = \frac{V_2}{I_2}$



$\Rightarrow$ To determine Z-parameters open circuit each port at a time (i.e., $I_2 = 0$)

Case (i):- When port 2 is open circuited (i.e., $I_2 = 0$)



$\Rightarrow Z_{11} = \frac{V_1}{I_1} = \frac{V_1}{\left(\frac{V_1}{17k}\right)}$ $\left[I_1 = \frac{V_1}{Z_{11}} \right.$

$I_1 = \frac{V_1}{12k+5k}$

$\Rightarrow Z_{11} = 17k\Omega$ $\left. I_1 = \frac{V_1}{17k} \text{ Amp} \right]$

$\Rightarrow Z_{21} = \frac{V_2}{I_1}$ [Where V_2 is o.c voltage at port '2'

$V_2 = V_{12k}$ ($\because$ Both are parallel)

$\Rightarrow Z_{21} = \frac{V_2}{I_1}$ $V_2 = V_2 = (12k) I_1$]

$\Rightarrow Z_{21} = \frac{(12k) I_1}{I_1}$ $\Rightarrow Z_{21} = 12k\Omega$

Case (ii):- When port 1 is open circuited (i.e., $I_1 = 0$)

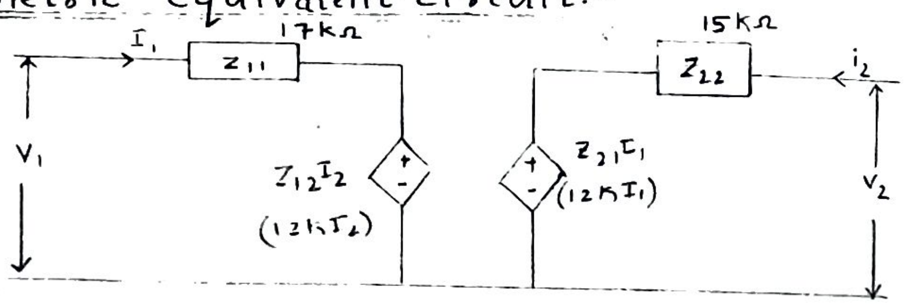
$\Rightarrow Z_{12} = \frac{V_1}{I_2} = \frac{(12k)I_2}{I_2}$ | $Z_{22} = \frac{V_2}{I_2} = \frac{V_2}{\left(\frac{V_2}{15k}\right)}$ $(V_1 = 12k)$ $\left[I_2 = \frac{V_2}{12+3} = \frac{V_2}{15k} \right]$

$\Rightarrow Z_{12} = 12k\Omega$ | $Z_{22} = 15k\Omega$

$\therefore Z_{11} = 17k\Omega, Z_{12} = 12k\Omega, Z_{21} = 12k\Omega, Z_{22} = 15k\Omega$

$$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 17k\Omega & 12k\Omega \\ 12k\Omega & 15k\Omega \end{bmatrix}$$

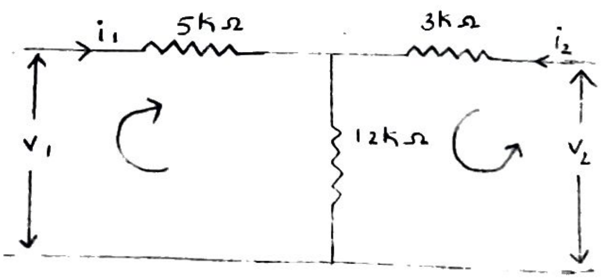
Z-parametric Equivalent circuit:-



Alternative Method:-

$\Rightarrow$ Applying KVL

In 1st loop,



$V_1 = 17i_1 + 12i_2$ | $V_1 = Z_{11}i_1 + Z_{12}i_2$

$V_2 = 15i_2 + 12i_1$ | $V_2 = Z_{21}i_1 + Z_{22}i_2$

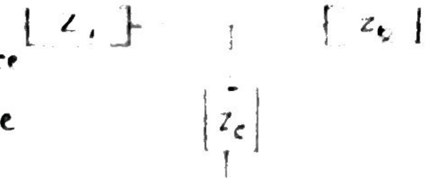
$V_1 = 5kI_1 + 12k(I_1 + I_2) = 17kI_1 + 12kI_2$

$V_2 = 3kI_2 + 12k(I_1 + I_2)$

$\therefore Z_{11} = 17k\Omega, Z_{12} = 12k\Omega,$

$Z_{21} = 12k\Omega, Z_{22} = 15k\Omega$

T-network :- A network is said to be T-network when its structure look like T-symbol.



→ Where Z_a, Z_b are Series Arm Impedance

Z_c - shunt Arm Impedance

→ The Z-parameters of the T-n/w is,

$Z_{11} = Z_a + Z_c$ (sum of all impedances connected to port 1)

$Z_{22} = Z_b + Z_c$ (sum of all impedances connected to port 2)

$Z_{21} = Z_c$ (mutual impedance between port 1 and port 2).

$Z_{12} = Z_c$ (mutual impedance between port 1 and 2)

→ From the above, we have $Z_{12} = Z_{21} = Z_c$, It means a T-network is always reciprocal in nature

→ From the condition of symmetry, if a network has to be symmetrical it should satisfy the condition that,

$\Rightarrow Z_{11} = Z_{22}$

$\Rightarrow Z_a + Z_c = Z_b + Z_c$

$\Rightarrow Z_a = Z_b$

$\therefore Z_a = Z_b$

→ A T-network is said to be symmetrical when its series Arm impedance are equal in magnitude

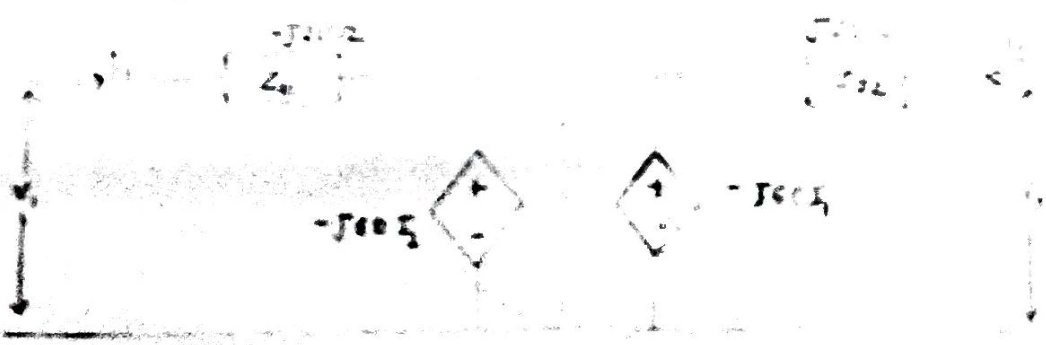
1Q - Determine the Z-parameters for the given 2-port n/w and hence draw the equivalent circuit

Sol - → $Z_{11} = J40 - J60 = -J20\,\Omega$

$Z_{22} = J80\,\Omega - J60\,\Omega = J20\,\Omega$

$Z_{12} = -J60\,\Omega$ and $Z_{21} = -J60\,\Omega$

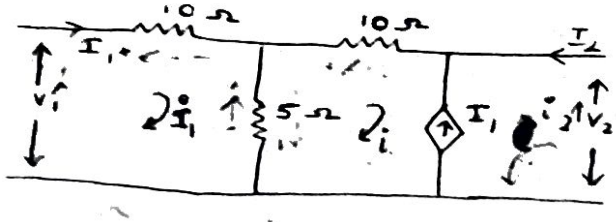
Apply KVL if question is given for max marks)



27/11/12

19 - Determine the z-parameters for given two port network and hence determine y-parameters?

Sol:-



⇒ Super Mesh Analysis:-

KVL for 1st Mesh:-

⇒ $V_1 - 10i_1 - 5(i_1 - i) = 0$

⇒ $V_1 - 15i_1 + 5i = 0$

⇒ $V_1 = 15i_1 - 5i \longrightarrow (1)$

⇒ combined loop equation ⇒ $5(i - i_1) + 10i + V_2 = 0$

⇒ $i + i_2 = -i_1$ ⇒ Super Mesh equation

⇒ $V_2 = -10i - 5(i - i_1)$ ⇒ $i = -(i_1 + i_2) \to (2)$

$= +10(i_1 + i_2) + 5(2i_1 + i_2) = 20i_1 + 15i_2 = Z_{21}i_1 + Z_{22}i_2 \to (3)$

⇒ from (1) & (2)

⇒ $V_1 = 15i_1 + 5(i_1 + i_2)$

⇒ $V_1 = 20i_1 + 5i_2 \longrightarrow (4)$

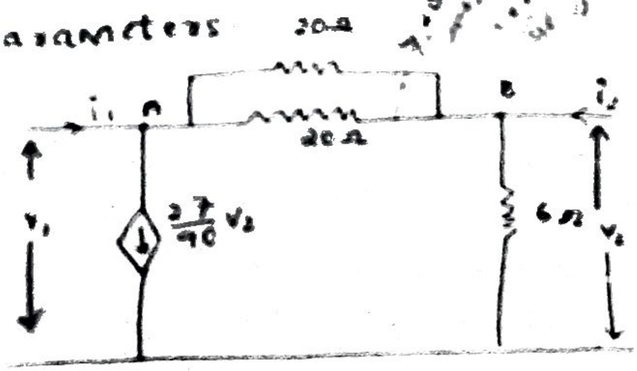
⇒ $V_1 = Z_{11}i_1 + Z_{12}i_2$

⇒ from (3) & (4) $Z = \begin{bmatrix} 20 & 5 \\ 20 & 15 \end{bmatrix}$ ⇒ $\frac{1}{20\times15-100}\begin{bmatrix} 15 & -5 \\ -20 & 20 \end{bmatrix} = Y$

⇒ $Y = Z^{-1} = \frac{1}{200}\begin{bmatrix} 15 & -5 \\ -20 & 20 \end{bmatrix}$

⇒ $Y = \begin{bmatrix} \frac{3}{40}℧ & -\frac{1}{40}℧ \\ -\frac{1}{10}℧ & \frac{1}{10}℧ \end{bmatrix}$

eg.- Determine the Y-parameters for the given network & hence determine the Z-parameters



Sol:- Y-parameters:-

$I_1 = Y_{11}V_1 + Y_{12}V_2$

$I_2 = Y_{21}V_1 + Y_{22}V_2$

→ Applying KCL at node A

$$\Rightarrow \frac{27}{90}V_2 + \frac{V_A - V_B}{10} = i_1$$

$$\Rightarrow i_1 = \frac{V_1 - V_2}{10} + \frac{27}{90}V_2$$

$$\Rightarrow i_1 = \frac{V_1}{10} + V_2\left[\frac{27}{90} - \frac{1}{10}\right]$$

$$\Rightarrow i_1 = \frac{V_1}{10} + V_2\left[\frac{27 - 9}{90}\right] = \frac{V_1}{10} + \frac{1}{5}V_2 = \frac{V_1}{10} + \frac{1}{5}V_2$$

→ At node B,

$$\Rightarrow i_2 = \frac{V_B}{6} + \frac{V_B - V_A}{10} = \frac{V_2}{6} + \frac{V_a - V_1}{10} = -\frac{V_1}{10} + V_2\left[\frac{1}{6} + \frac{1}{10}\right]$$

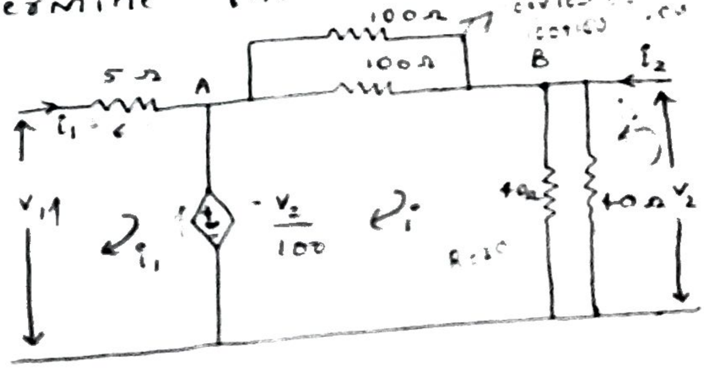
$$\Rightarrow i_2 = -\frac{V_1}{10} + V_2\left(\frac{1}{\frac{15}{4}}\right) = -\frac{V_1}{10} + \left(\frac{1}{\frac{15}{4}}\right)V_2 = -\frac{V_1}{10} + \frac{4}{15}V_2$$

$$\therefore Y = \begin{bmatrix} \frac{1}{10} & \frac{1}{5} \\ -\frac{1}{10} & \frac{4}{15} \end{bmatrix}$$

$$\Rightarrow Y = Z^{-1} \Rightarrow Y = \frac{1}{Z} \Rightarrow Z = \frac{1}{Y} = \frac{1}{\left(\frac{7}{150}\right)}\begin{bmatrix} \frac{4}{15} & -\frac{1}{5} \\ \frac{1}{10} & \frac{1}{10} \end{bmatrix}$$

$$\Rightarrow Z = \begin{bmatrix} 5.714 & -4.285 \\ 2.14 & 2.14 \end{bmatrix}$$

39.- Determine the open-circuited parameters (Z-parameters) for given two port network & hence determine the transmission parameters?



Sol:- → Nodal Analysis :-

KCL at node A :-

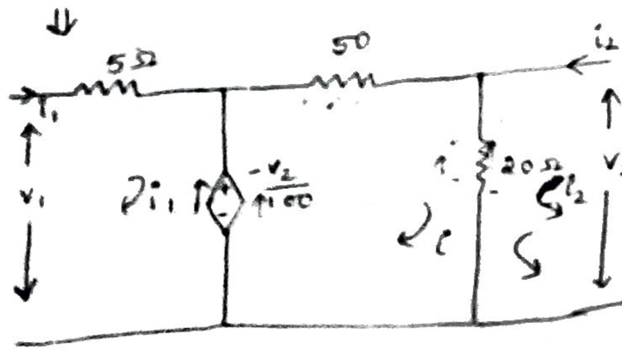
→ KVL at ① loop

$$\Rightarrow V_1 - 5i_1 - \frac{V_2}{100} = 0 \quad \rightarrow ①$$

$$\Rightarrow -20(i_2 + i) + V_2 = 0$$

$$\Rightarrow V_2 = 20(i + i_2)$$

⇓



$\Rightarrow V_1 = 5i_1 + \frac{V_2}{100} = 5i_1 + \frac{20(i+i_2)}{100} = 5i_1 + \frac{i+i_2}{5}$

KVL at ② loop,

$\Rightarrow \frac{V_2}{100} - 50i - 20(i+i_2) = 0$

$\Rightarrow \frac{V_2}{100} - 50i - 20i_2 - 20i = 0$ $\qquad V_2 = 20(i_2+i) \rightarrow ②$

$\Rightarrow \frac{i_2+i}{5} - 50i - 20i_2 - 20i = 0$

$\Rightarrow \frac{V_2}{100} = 70i + 20i_2 \Rightarrow 70i = \frac{V_2}{100} - 20i_2$

$\Rightarrow i = \frac{V_2}{7000} - \frac{20}{70} i_2 \rightarrow ①$

sub eq. ① in ②

$\Rightarrow V_2 = 20i_2 + 20\left[\frac{V_2}{7000} - \frac{20}{70} i_2\right]$

$\Rightarrow V_2 = 20i_2 + \frac{2V_2}{700} - \frac{40}{7} i_2$

$\Rightarrow V_2\left[\frac{700-2}{700}\right] = 20i_2 - \frac{40}{7} i_2$

$\Rightarrow V_2 \frac{698}{700} = 20i_2 - \frac{40}{7} i_2$

$\Rightarrow V_2 = \frac{700 \times 20}{698} i_2 + i_1(0) = 14.32 i_2 + i_1(0)$

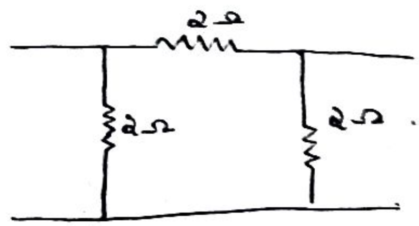
$V_1 = 5i_1 + \frac{V_2}{100}$ $\qquad V_2 = 0 i_1 + 14.32 i_2$

$V_1 = 5i_1 + \frac{14.32 i_2}{100}$ $\qquad V_1 = 5i_1 + 0.143 i_2$

31/12/12

19. Determine the transmission parameters for given two port network and hence deduce h-parameters.

sol:- ABCD - parameters

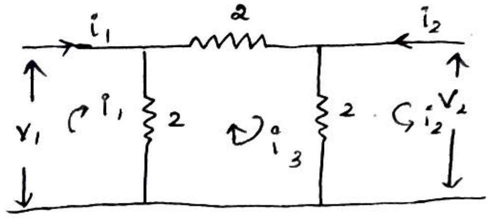


$\Rightarrow V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$.

$\Rightarrow V_1 = 2i_1 - 2i_3$

$\Rightarrow$ <u>case(i):-</u> $I_2 = 0$ (open circuiting)



$\Rightarrow V_1 - 2i_1 -$

$\Rightarrow 2(i_3 - i_1) + 2(i_3 + i_2) + 2i_3 = 0$

$\Rightarrow -2i_1 + 4i_3 + 4i_2 = 0$

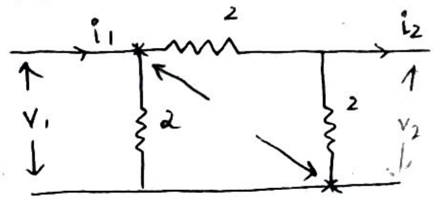
$6i_3 - 2i_1 + 2i_2 = 0$

$\Rightarrow 3i_3 - i_1 + i_2 = 0$

$V_2 = 2(i_3 + i_2)$

$V_1 = 2i_1$

$\Rightarrow V_2 =$ voltage at 2Ω. $I_2 = 0$



$V_2 = V_1 \cdot \frac{2}{2+2} = \frac{V_1}{2}$ $\quad A = \frac{V_1}{V_2} = \frac{V_1}{\left(\frac{V_1}{2}\right)}$

$A = 2$

$V_1 = AV_2 \Rightarrow A = \frac{V_1}{V_2}$

$I_1 = CV_2 \Rightarrow C = \frac{I_1}{V_2}$.

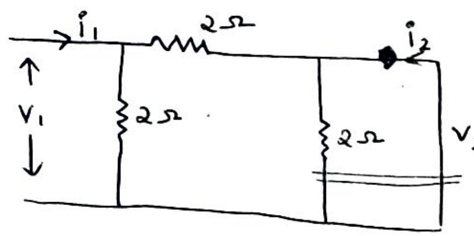
$i_1 = \frac{V_1}{R_T} \Rightarrow R_T = \frac{8}{6}$

$= \frac{V_1}{(8/6)} = \frac{6V_1}{8}$.

$C = \frac{I_1}{V_2} = \frac{I_1}{\left(\frac{V_1}{2}\right)} = \frac{2I_1}{V_1}$

$C = \frac{2 \times 6V_1}{8 \times V_1} = \frac{12}{8} = 1.5$

<u>case (ii):-</u> $V_2 = 0$ (short circuit)

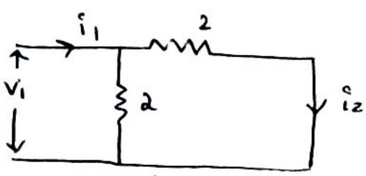


$\Rightarrow V_1 = -BI_2$ $\qquad I_1 = -DI_2$

$\Rightarrow B = -\frac{V_1}{I_2}$ $\qquad D = -\frac{I_1}{I_2}$

$\Rightarrow B = -\frac{V_1}{\left(\frac{V_1}{2}\right)} = -2\Omega$

$\Rightarrow D = -\frac{I_1}{I_2} = -\frac{I_1}{\left(\frac{I_1}{2}\right)} = -2$



$\frac{R_1 R_2}{R_1 + R_2} = \frac{2 \times 0}{2 + 0} =$

$2 \| 2 \Rightarrow i_2 = \frac{i_1}{2} = \frac{V_1}{2}$ amp

<u>h-parameters:-</u>

$V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$

$I_2 = \frac{CV_2 - I_1}{D} = \frac{C}{D} V_2 - \frac{1}{D} I_1$

$\boxed{I_2 = -\frac{1}{D} I_1 + \frac{C}{D} V_2}$

$\Rightarrow h_{11} = \frac{AD - BC}{D} =$

$V_1 = AV_2 - B\left[-\frac{1}{D} I_1 + \frac{C}{D} V_2\right]$.

$V_1 = AV_2 + \frac{B}{D} I_1 - \frac{BC}{D} V_2$

$V_1 = \left(\frac{AD - BC}{D}\right) V_2 + \frac{B}{D} I_1$

$$\Rightarrow \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 1.5 & -2 \end{bmatrix} \qquad V_1 = \frac{B}{D} I_1 + \left(\frac{AD-BC}{D}\right) V_2$$

$$\Rightarrow h_{11} = \frac{B}{D} = \frac{-2}{-2} = 1, \quad h_{12} = \frac{AD-BC}{D} = \frac{+1}{+2} = \frac{1}{2}.$$

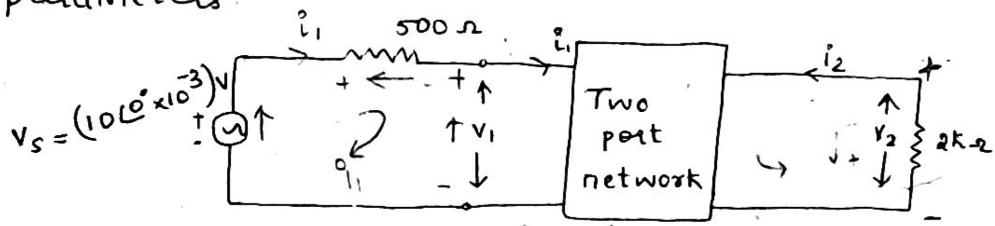
$$\Rightarrow h_{21} = -\frac{1}{D} = \frac{-1}{-2} = \frac{1}{2}, \quad h_{22} = \frac{C}{D} = \frac{1.5}{-2} = -0.75$$

2Q:- For the given Two port network, h-parameters are given as

$h_{11} = 1\,k\Omega$, $h_{12} = 0.003$, $h_{21} = 100$, $h_{22} = 50\,\mu℧$

Determine the value of V_2 and hence deduce the z-parameters.

Sol:-



⇒ from the diagram,

V_2 = voltage across $2\,k\Omega = -(i_2 \times 2000)$ volts.

$$\Rightarrow \quad V_1 = h_{11} i_1 + h_{12} V_2 \ \Rightarrow\ V_1 = 1000 i_1 + 0.003 V_2$$

$$i_2 = h_{21} i_1 + h_{22} V_2 \ \Rightarrow\ i_2 = 100 i_1 + 50 \times 10^{-6} V_2$$

⇒ sub V_2 value in V_1 & i_2

$$\Rightarrow \quad V_1 = 1000 i_1 + (0.003 \times 2000) - i_2$$

$$V_1 = 1000 i_1 - 6 i_2 \longrightarrow (1)$$

$$\Rightarrow \quad i_2 = 100 i_1 - i_2 (10^{-1}) \Rightarrow 1.1\, i_2 = 100 i_1$$

$$\Rightarrow i_2 = \frac{100}{1.1} i_1$$

⇒ sub i_2 in eq (1)

$$\Rightarrow V_1 = 1000 i_1 - 6 \times \frac{1000}{11} i_1 = \frac{5000}{11} i_1$$

$$\Rightarrow V_1 = \frac{5000}{11} i_1$$

⇒ Applying KVL on the source side,

$$\Rightarrow V_s - 500 i_1 - V_1 = 0$$

$$\Rightarrow V_1 = V_s - 500 i_1 \Rightarrow \frac{5000}{11} i_1 = V_s - 500 i_1$$

$$\Rightarrow \frac{10500}{11} i_1 = V_s = 10\angle 0^\circ \times 10^{-3}$$

$$\Rightarrow i_2 = \frac{1000}{11} i_1 \qquad \Rightarrow i_1 = \frac{110\angle 0^\circ \times 10^{-3}}{10500}$$

$$\Rightarrow i_2 = \frac{1000}{11} \times 1.047 \times 10^{-5} \qquad \Rightarrow i_1 = 1.047 \times 10^{-5} \text{ Amps.}$$

$$\Rightarrow i_2 = 9.518 \times 10^{-4} \text{ Amp}$$

$$\Rightarrow V_2 = -(i_2 \times 2000)\,V = -(9.518 \times 2000 \times 10^{-4}) = -1.904 \text{ volts.}$$

$h_{11} = \frac{V_1}{i_1} = 1k\Omega$

Z-parameters:-

$$\Rightarrow \begin{aligned} V_1 &= Z_{11} i_1 + Z_{12} i_2 \\ V_2 &= Z_{21} i_1 + Z_{22} i_2 \end{aligned} \quad \Bigg| \quad \begin{aligned} V_1 &= h_{11} i_1 + h_{12} V_2 \\ i_2 &= h_{21} i_1 + h_{22} V_2 \end{aligned}$$

$$\Rightarrow Z_{11} = \frac{\Delta h}{h_{22}}, \quad Z_{12} = \frac{h_{12}}{h_{22}} \qquad \Rightarrow V_2 = \frac{i_2}{h_{22}} - \frac{h_{21}}{h_{22}} i_1$$

$$\Rightarrow V_1 = h_{11} i_1 + \frac{h_{12}}{h_{22}} i_2 - \frac{h_{12} h_{21}}{h_{22}} i$$

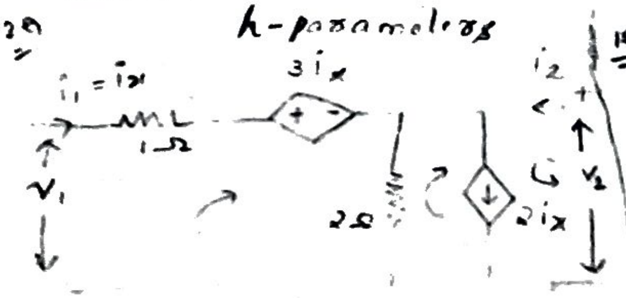
$$\Rightarrow Z_{21} = -\frac{h_{21}}{h_{22}}, \quad Z_{22} = \frac{+1}{h_{22}} \qquad \Rightarrow V_1 = \frac{\Delta h}{h_{22}} i_1 + \frac{h_{12}}{h_{22}} i_2.$$

$$\Rightarrow Z_{11} = -5000, \quad Z_{12} = 60$$

$$\Rightarrow Z_{21} = -2000000 = -2 \times 10^6, \quad Z_{22} = 2 \times 10^4.$$

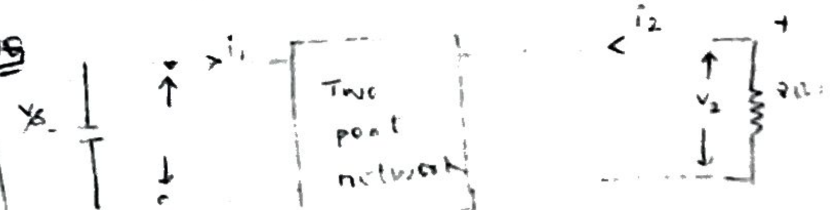
Homework

h-parameters



determine V_2 when $V_1 = 1V$

$R_L = 3\Omega$



$Z_{11} = Z_{22} = 10\Omega$

$Z_{12} = Z_{21} = 4\Omega$

determine V_2, I_1, I_2 when $V_s = 2V$

9.1 Introduction to Four Terminal Networks

- In the complex communication systems, the designer may need to use different networks such as filters, attenuators, equalizers etc. All these networks may be realized in the different forms of a four terminal network. It is essential to study different electrical properties of different types of the network. This chapter explains different electrical properties of symmetrical and asymmetrical networks and simple methods for deriving the expressions for the same.
- A four terminal network is a network having only one input port (or pair of terminals) and one output port.
- When the electrical properties of the network are unaffected even after interchanging input and output terminals, the network is called **symmetrical network**.
- When the electrical properties of the network get affected after interchanging input and output terminals, the network is called **asymmetrical network** or **dissymmetrical network**.
- Since there are two pairs of terminals, a four terminal network is also called **two port** network. Let us study the properties of the symmetrical and asymmetrical networks in detail.

9.2 Symmetrical Networks

- Any symmetrical network has two important electrical properties such as,
  1) Characteristic impedance (Z_0)
  2) Propagation constant (γ)
- The two networks having the same electrical properties i.e. characteristic impedance (Z_0) and propagation constant (γ) are called **equivalent networks** or **identical networks**.
- Let us study the properties of the symmetrical networks in detail.

9.2.1 Characteristic Impedance (Z_0)

- Consider that infinite number of identical symmetrical networks are connected in **cascade** or **tandem** as shown in the Fig. 9.2.1 (a). The input impedance measured at the input terminals of the first network in the chain of infinite networks will have some finite value which depends on the network composition. This impedance is the important property of a symmetrical network. Thus the characteristic impedance of a symmetrical network is the impedance measured at the input terminals of the first network in the chain of infinite networks in cascade and it is represented by Z_0.

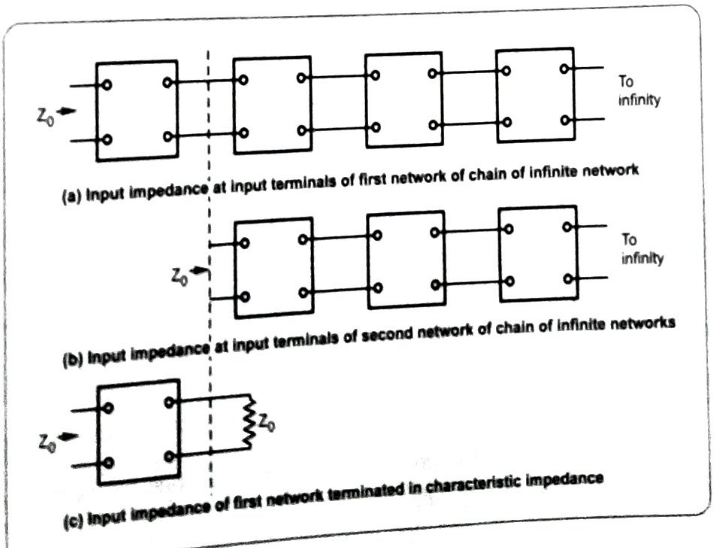


Fig. 9.2.1

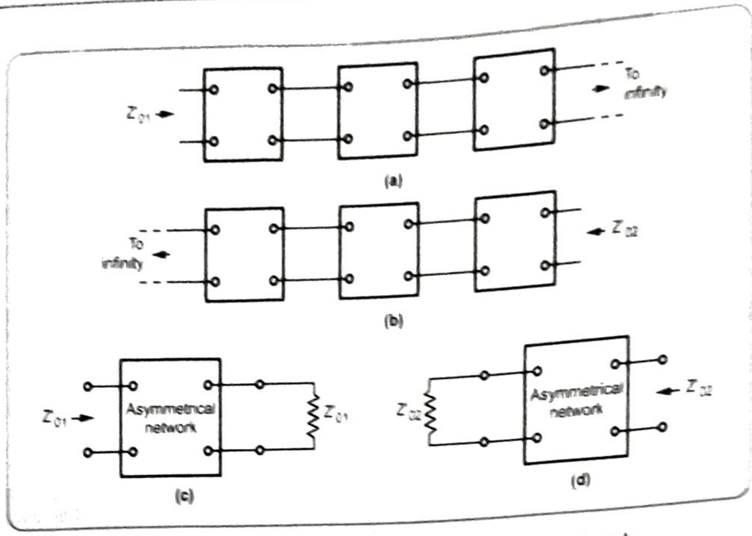


Fig. 9.3.1 Iterative impedances of asymmetrical network

terminals of the network when other pair of terminals is terminated in the impedance of the same value as shown in the Fig. 9.3.1 (c) and (d).

- The iterative impedances for any asymmetrical network are of different values when measured at different ports of the network. The iterative impedances are represented by Z'_{01} and Z'_{02} respectively at port 1 and port 2.

9.3.2 Image Impedances (Z_i)

- Similar to the iterative impedances, the image impedances are also of different values at different ports. Let the image impedances be denoted by Z_{i1} and Z_{i2}. Consider that the asymmetrical network is terminated with image impedance of port 2 i.e. Z_{i2} at it's output pair of terminals then the impedance measured at its input pair of terminals will be image impedance of port 1 i.e. Z_{i1}. Similarly if port one is terminated in the image impedance of port 1 i.e. Z_{i1} then the impedance measured at port two will be the image impedance of port 2 i.e. Z_{i2}. These conditions are illustrated by the Fig. 9.3.2 (a) and (b).

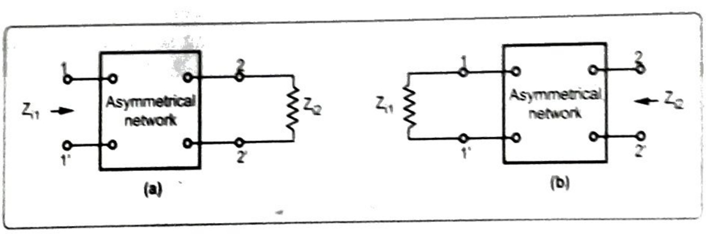


Fig. 9.3.2 Image impedances of asymmetrical network

- When an asymmetrical network is terminated in image impedances at both the ports the network is called **correctly terminated asymmetrical network** as shown in the Fig. 9.3.3.

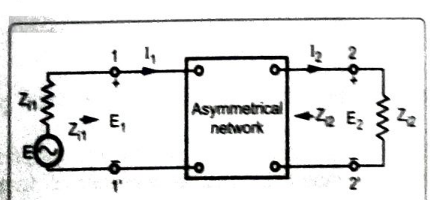


Fig. 9.3.3 Properly or correctly terminated asymmetrical network

9.3.3 Image Transfer Constant (e^{θ})

- When an asymmetrical network is terminated in its image impedances at both the ports as shown in the Fig. 9.3.3, then the ratio of currents $\frac{I_1}{I_2}$ will be different from the ratio of voltages $\frac{E_1}{E_2}$. Hence image transfer constant θ is defined as

$$\therefore \quad \boxed{e^{\theta} = \sqrt{\frac{E_1 I_1}{E_2 I_2}}}$$

The real part of image transfer constant is called **image attenuation constant**; while the imaginary part is called **image phase constant.**

After detailed discussion of various properties of symmetrical and asymmetrical networks let us study the properties of some of the important symmetrical and asymmetrical networks including symmetrical T, π, lattice networks and asymmetrical networks such as half sections, L sections.

Questions

Fig. 7.5.5

Fig. 7.5.6

7.6 Network Functions for Two Port Network

JNTU : May-04, 06, 12, 15, Dec.-03, 07, 12

- For the two port network, there are two ports. The variables measured at port 1 are $v_1(t)$ and $i_1(t)$ while the variables measured at port 2 are $v_2(t)$ and $i_2(t)$. This is shown in the Fig. 7.6.1.

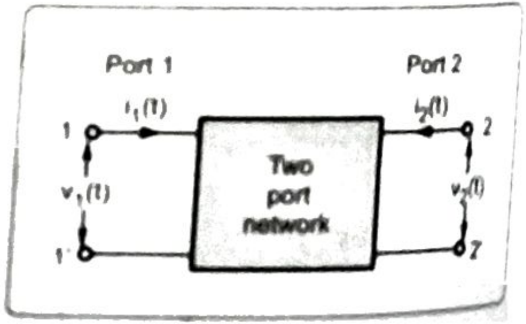


Fig. 7.6.1 Variables of two port network

- The ratio of the variables measured at the same port either port 1 or port 2 defines driving point function. Hence for two port network, we can define,

1. Driving point impedance functions :

This is the ratio of $V_1(s)$ and $I_1(s)$ at port 1, denoted as $Z_{11}(s)$ or the ratio of $V_2(s)$ to $I_2(s)$ at port 2, denoted as $Z_{22}(s)$. Both are driving point impedance functions.

$$Z_{11}(s) = \frac{V_1(s)}{I_1(s)} \quad \text{and} \quad Z_{22}(s) = \frac{V_2(s)}{I_2(s)}$$

2. Driving point admittance functions :

The reciprocals of the driving point impedance functions at the two ports give the driving point admittance functions at the respective ports. These are denoted as $Y_{11}(s)$ and $Y_{22}(s)$ at the two ports respectively.

$$\therefore Y_{11}(s) = \frac{I_1(s)}{V_1(s)} \quad \text{and} \quad Y_{22}(s) = \frac{I_2(s)}{V_2(s)}$$

7.6.1 Transfer Functions for Two Port Network

- The function defined as the ratio of Laplace of two variables such that one variable is defined at one port while the other variable is defined at second port. Accordingly four transfer functions can be defined for two port network as,

1. Voltage ratio transfer function :

It is the ratio of Laplace transforms of voltage at one port to voltage at other port. It is denoted as G(s).

$$\therefore \quad G(s) = \frac{V_2(s)}{V_1(s)} \text{ or } V_1(s)$$

2. Current ratio transfer function :

It is the ratio of Laplace transforms of current at one port to current at other port. It is denoted as $\alpha(s)$.

$$\therefore \quad \alpha(s) = \frac{I_2(s)}{I_1(s)} \text{ or } \frac{I_1(s)}{I_2(s)}$$

3. Transfer impedance function :

It is the ratio of Laplace transformers of voltage at one port to current at other port. It is denoted as $Z_{12}(s)$ or $Z_{21}(s)$.

$$Z_{21}(s) = \frac{V_2(s)}{I_1(s)} \quad \text{and} \quad Z_{12}(s) = \frac{V_1(s)}{I_2(s)}$$

4. Transfer admittance function :

It is the ratio of Laplace transforms of current at one port to voltage at other port. It is denoted as $Y_{12}(s)$ or $Y_{21}(s)$.

$$Y_{21}(s) = \frac{I_2(s)}{V_1(s)} \quad \text{and} \quad Y_{12}(s) = \frac{I_1(s)}{V_2(s)}$$

Example 7.6.1 ***For given two port network find driving point impedance function and voltage ratio transfer function.***

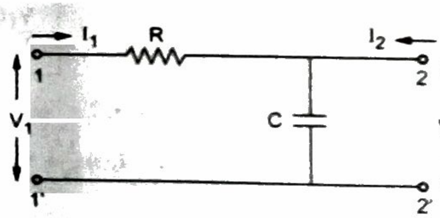


Fig. 7.6.2

Solution : Transforming given network in s domain, the network will be as shown in the Fig. 7.6.2 (a).

To find voltage ratio transfer function, we will first find V_2 using potential divider rule as follows,

7.7 Poles and Zeros of Network Function

JNTU : Dec.-12, May-12

- As the network function is the ratio of Laplace transforms of one variable to other, it can be expressed as the ratio of two polynomials in 's' as,

$$H(s) = \frac{P(s)}{Q(s)}$$

- The P(s) is the numerator polynomial in 's' having say degree 'm' while Q(s) is the denominator polynomial in 's' having say degree 'n'. So H(s) can be further expressed as,

$$H(s) = \frac{a_0 s^m + a_1 s^{m-1} + a_2 s^{m-2} + ... + a_m}{b_0 s^n + b_1 s^{n-1} + b_2 s^{n-2} + ... + b_n}$$

- There are 'm' factors of the numerator and 'n' factors of the denominator. So $H(s)$ can be expressed in the factorised form as,

$$H(s) = \frac{K(s-z_1)(s-z_2)....(s-z_m)}{(s-p_1)(s-p_2)....(s-p_n)}$$

- where $z_1, z_2, ... z_m$ are the roots of the polynomial $P(s) = 0$ while $p_1, p_2, ... p_n$ are the roots of the polynomial $Q(s) = 0$.
- The constant K is called **system gain factor** or **scale factor**.
- The network function is also called a **system function** or **transfer function** when it is a ratio of Laplace transform of output variable to the Laplace transform of input variable of that system.

7.7.1 Poles

- The values of 's' i.e. complex frequencies, which make the system function infinite when substituted in the denominator of a system function are called **poles** of the system function.
- So values $s = p_1, p_2, ... p_n$ are the poles of H(s).
- If such poles are real and non-repeated, these are called **simple poles**. If a particular pole has same value twice or more than that, it is called **repeated pole**. A pair of poles with complex conjugate values is called a pair of **complex conjugate poles**.
- The poles are the roots of the equation obtained by equating denominator polynomial of a system function to zero. Such an equation is called **characteristic equation** of a system.

7.7.2 Zeros

- The values of 's' i.e. complex frequency values, which make the system function zero when substituted in the numerator of a system function are called **zeros** of the system function.
- So values $s = z_1, z_2, ... z_m$ are the zeros of H(s).
- Similar to the poles, zeros also can be simple zeros, repeated zeros or complex conjugate zeros.
- The zeros are the roots of the equation obtained by equating numerator polynomial of a system function to zero.

Let

$$H(s) = \frac{10(s+4)}{s(s^2+2s+2)}$$

So $P(s) = 0$ gives $(s+4) = 0$. Hence $s = -4$ is the zero of H(s).

While $Q(s) = 0$ gives $s\left(s^2 + 2s + 2\right) = 0$. This gives,

$s = 0, \ -1 + j, \ -1 - j$

- So there are 3 poles. While 10 is the scale factor.
- Poles and zeros are the **critical frequencies**. At the poles, the network function becomes infinite while at the zeros the network function becomes zero. At all the other frequencies, the network function has a finite non-zero value.
- For a network function, if poles and zeros at zero and infinity are taken into account in addition to the finite poles and zeros then the total number of zeros is equal to the total number of poles.

7.7.3 Singularities

- A complex system function H(s) is said to be **analytic** in a region if H(s) and all its **derivatives** exist in that region. The points in s-plane at which the function H(s) is analytic are called **ordinary points**.
- There are some points in s-plane at which the function H(s) is not analytic i.e. H(s) and its **derivatives** are nonexisting. Such points are called **singular points** or **singularities** of the function H(s).
- At all the poles, H(s) approaches infinity and is non-existing at the poles. Thus poles of H(s) are the singularities of the function H(s).

7.7.4 D.C. Gain

- The d.c. has a zero frequency. The H(s) is complex frequency function. The value of H(s) at zero frequency i.e. at s = 0 is called **d.c. gain** of the system function. When s = 0 is substituted in H(s), the resulting value is a constant, indicating gain at zero frequency.

For example, $$H(s) = \frac{20\,(s+2)}{\left(s^2 + 2s + 5\right)}$$

Then at s = 0, $$H(s)_{s=0} = \frac{20 \times 2}{5} = 8$$

- Thus 8 is the d.c. gain of the system function H(s).
- As $s = j\omega$ in frequency domain, the d.c. gain is also indicated as H(j0), Z(j0) etc. This is because $\omega = 0$ for d.c. hence s = j0 indicates d.c. condition.

7.7.5 Pole-Zero Plot

Consider a complex plane with x-axis indicating real axis denoted as σ axis while y-axis indicating imaginary axis denoted as $j\omega$ axis. Such a plane is used to indicate values of the variable 's' and hence called **s-plane**. All the poles and zeros which are nothing but the values of 's' can be easily indicated in such s-plane.

The plot obtained by locating all the poles and zeros of the system function in s-plane is called **pole-zero** plot of the system function H(s). The s-plane is shown in the Fig. 7.7.1.

> The poles are indicated by 'cross' (X) in s-plane.
> The zeros are indicated by 'zero' mark (O) in s-plane.

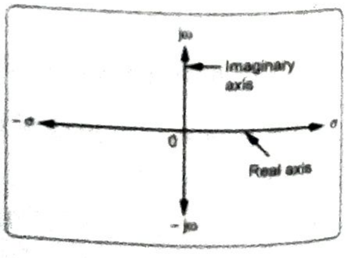


Fig. 7.7.1

For repeated poles or zeros, the marks equal to number of repeated poles or zeros must be shown.

For example, $H(s) = \dfrac{20(s+1)(s+5)}{s\left(s^2+2s+2\right)(s+7)}$

There are two zeros located at $s = -1$ and $s = -5$ while there are 4 poles located at $s = 0$, $s = -1 \pm j$ and $s = -7$

The corresponding pole-zero plot is shown in the Fig. 7.7.2.

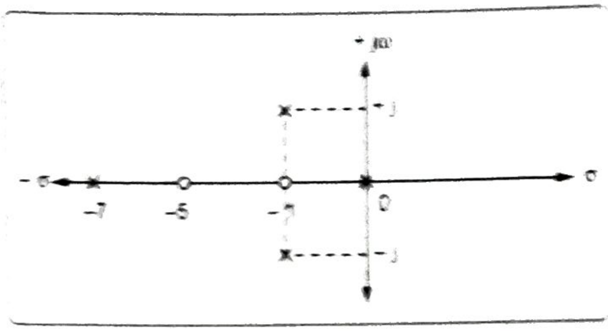


Fig. 7.7.2

Example 7.7.1 ***For the network shown in the Fig. 7.7.3, find driving point function Z(s). Plot the poles and zeros of Z(s) on s-plane.***

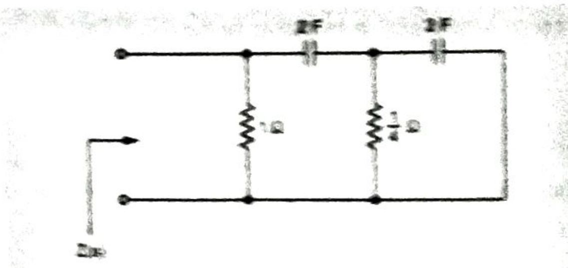


Fig. 7.7.3

Solution : Transforming given network in s-domain as shown in the Fig. 7.7.3 (a). In transformed network, an inductor L transforms to impedance sL, while a capacitor C transforms to impedance $\dfrac{1}{sC}$.

Now, impedances $\dfrac{1}{2s}$ and $\dfrac{1}{4}$ in parallel,

$$Z_1(s) = \frac{1}{2s} \parallel \frac{1}{4} = \frac{\frac{1}{2s} \cdot \frac{1}{4}}{\frac{1}{2s} + \frac{1}{4}}$$

$$= \frac{1}{4+2s} = \frac{1}{2(s+2)}$$

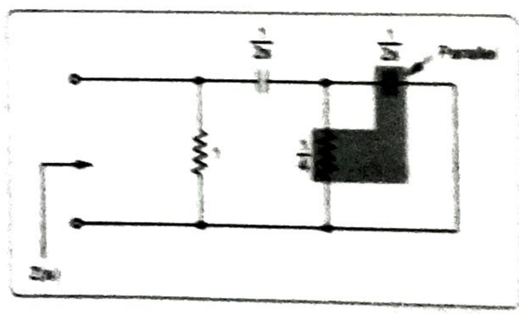


Fig. 7.7.3 (a)

Now impedance $Z_1(s)$ appears in series with $\left(\dfrac{1}{2s}\right)$.

$$Z_2(s) = \frac{1}{2s} + Z_1(s) = \frac{1}{2s} + \frac{1}{2(s+2)} = \frac{s+2+s}{2s(s+2)} = \frac{2(s+1)}{2s(s+2)}$$

$$Z_2(s) = \frac{(s+1)}{s(s+2)}$$

Now finally the impedance $Z_2(s)$ appears in parallel with 1 Ω.

$$Z(s) = (1) \parallel Z_2(s) = \frac{(1)\left[\frac{(s+1)}{s(s+2)}\right]}{1+\frac{s+1}{s(s+2)}} = \frac{s+1}{s(s+2)+s+1}$$

$$\boxed{Z(s) = \frac{N(s)}{D(s)} = \frac{s+1}{s^2+3s+1}} \quad \text{... (1)}$$

Pole-zero plot of Z(s) : To find poles and zeros of Z(s) equating polynomials in **numerator and denominator to zero** respectively.

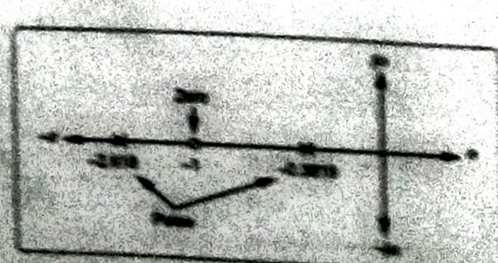


Fig. 7.7.3 (b) Pole-zero plot of Z(s)

Zeros :

$$N(s) = 0$$

$$s + 1 = 0$$

i.e.

$$s = -1$$

Poles :

$$D(s) = 0$$

$$\therefore \quad s^2 + 3s + 1 = 0$$

$$\therefore \quad s_{1,2} = \frac{-3 \pm \sqrt{9-4}}{2} = \frac{-3 \pm \sqrt{5}}{2}$$

$$\therefore \quad s_1 = -0.3819$$

or

$$s_2 = -2.618$$

Hence the zero is located at $s = -1$ while the poles are located at $s = -0.3819$ and $s = -2.618$ as shown in the Fig. 7.7.3 (b).

Example 7.7.2 ***Determine the system function if the d.c. gain of the system is 10 and pole-zero plot is as shown in the Fig. 7.7.4.***

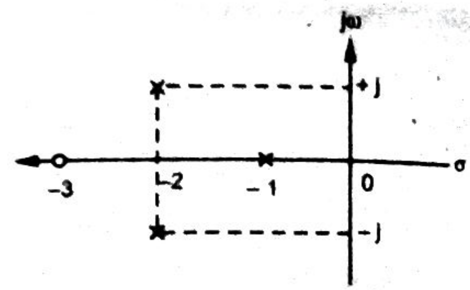


Fig. 7.7.4

Solution : From the pole-zero plot, system function has 3 poles at $s = -1, -2+j, -2-j$ and one zero at $s = -3$.

$$\therefore \quad H(s) = \frac{K(s+3)}{(s+1)(s+2-j)(s+2+j)} = \frac{K(s+3)}{(s+1)\left(s^2+4s+5\right)}$$

Now d.c. gain is value of H(s) at s = 0.

$$\therefore \quad H(s)\big|_{s=0} = \frac{K \times 3}{1 \times 5} = 10$$

$$\therefore \quad K = \frac{50}{3} = 16.667$$

Hence the system function is,

$$H(s) = \frac{16.667\,(s+3)}{(s+1)\left(s^2+4s+5\right)}$$

7.9 Necessary Conditions for Driving Point Functions

JNTU : May-03, 12, 13, 15, Dec.-05, 14

- The driving point impedance function is nothing but the input impedance of the network as viewed through the input terminals. The driving point admittance function is reciprocal of impedance function. Hence driving point functions are called **immitance functions.** While defining these functions, all initial conditions are assumed zero. The immitance functions is generally a ratio of two separate polynomials in 's'

$$Z(s) \text{ or } Y(s) = \frac{P(s)}{Q(s)}$$

- After cancelling the common factors in the numerator polynomial $P(s)$ and denominator polynomial $Q(s)$, the necessary conditions for the driving point functions are as follows :

1. The coefficients of the numerator polynomial P(s) and the denominator polynomial Q(s) must be real and positive.
2. If poles and zeros are imaginary then such poles and zeros must be conjugate.
3. The real part of all the poles and zeros must be negative or zero and if the real part is zero, then the pole or zero must be simple.
4. There should not be any missing term between the highest and lowest degrees in the polynomials P(s) and Q(s) unless all the even or all the odd terms are missing.
5. The degree of the polynomial in numerator and denominator should differ by either zero or one.
6. The terms of lowest degree in P(s) and Q(s) may differ in degree at the most by one.

Review Questions

1. *Explain the necessary conditions for driving point function.*
JNTU : May-03, 12, Dec.-05, May-13,15,Dec.-14, Marks 7

2. *Write short note on driving point function.*
JNTU : May-12, Marks 5

3. *State whether the following function is suitable for driving point function analysis :*

$$Z(s) = \frac{5\left(4s^3 + 2s^2 + s + 2\right)}{s^4 + 3s^3 + 4s^2}$$

[Ans. : Not suitable]

4. *State whether the following function is suitable for driving point function analysis :*

$$Z(s) = \frac{5(3s^3 + 4s^2 + s + 4)}{s^5 + 3s^4 + 2s^3 + s^2}$$

[Ans. : Not suitable]

7.10 Necessary Conditions for Transfer Functions

JNTU : May-12, Dec.-06, 12

The transfer functions are also the ratios of two separate polynomials in 's'.

$$\text{Transfer function} = \frac{P(s)}{Q(s)}$$

After cancelling the common factors in the polynomials P(s) and Q(s), the necessary conditions for the transfer functions are as follows :

1. The coefficients of P(s) and Q(s) must be real and positive.
2. The complex and imaginary poles and zeros must be conjugate.
3. The real part of the poles must be negative or zero and if it is zero, the pole must be simple, including origin.
4. There should not be any missing term between the highest and lowest degree of Q(s), unless all the even or odd terms are missing.
5. The polynomial P(s) may have negative terms or even some missing terms between the highest and lowest degree.
6. The degree of polynomial P(s) may be as small as zero independent of the degree of the polynomial Q(s).

Review Question

1. *Explain the necessary conditions for transfer functions.*
JNTU : May-12, Dec.-06, 12, Marks 7

7.11 Time Domain Behaviour from Pole-Zero Plot

JNTU : May-12, 13

- The behaviour of a system can be predicted only by looking at the pole-zero plot. The poles are the complex frequencies explaining the time responses. Also zeros are useful in finding partial fraction expansion.
- We will discuss graphical methods for obtaining the residues by partial-fraction expansion directly from pole-zero plot.
- Consider the network function as follows,

$$F(s) = \frac{A_0\,(s - z_0)\,(s - z_1)}{(s - p_0)\,(s - p_1)\,(s - p_2)}$$

Then we can expand F(s) as,

$$F(s) = \frac{K_0}{s-p_0} + \frac{K_1}{s-p_1} + \frac{K_2}{s-p_2}$$

- The poles and zeros are plotted on the complex plane as shown in Fig. 7.11.1.

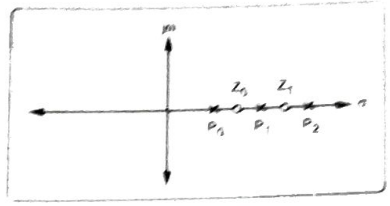


Fig. 7.11.1 Pole-zero plot

- "The residue K at any pole p is equal to the ratio of product of the vectors from the zeros to that pole to the product of the vectors from the other poles to that pole."

Key Point **Note that the residues of the conjugate poles are also conjugate in nature.**

- With the help of ruler and protractor we can determine the length and angles of the vectors so that residues can be determined. Consider following function,

$$F(s) = \frac{3\,s}{s^3 + 4\,s^2 + 6\,s + 4}$$

- For finding partial expansion, factorize the denominator by general method, we get

$$s^3 + 4\,s^2 + 6\,s + 4 = (s+2)(s^2 + 2\,s + 2)$$

$$\therefore \qquad F(s) = \frac{3\,s}{(s+2)(s^2 + 2\,s + 2)}$$

Finding roots of $s^2 + 2\,s + 2 = 0$ by the formula,

$$s = \frac{-b \pm \sqrt{b^2 - 4\,a\,c}}{2\,a} = -1 \pm j\,1$$

$$\therefore \qquad F(s) = \frac{3\,s}{(s+2)(s+1+j1)(s+1-j1)}$$

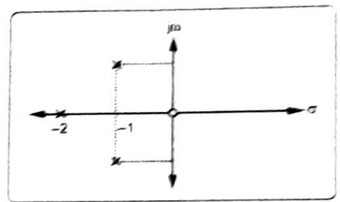


Fig. 7.11.2 Pole-zero plot

- The pole-zero plot is as follows, shown in the Fig. 7.11.2.

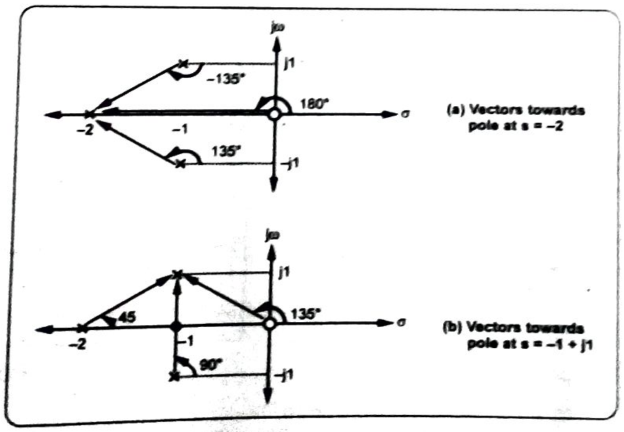


Fig. 7.11.3 Evaluation of residues

The partial fractions can be written as,

$$\frac{3\,s}{s^3 + 4\,s^2 + 6\,s + 4} = \frac{A}{s+2} + \frac{B}{(s+1)+j1} + \frac{C}{(s+1)-j1}$$

- To find A, draw vectors from other poles and zeros. Find magnitude and angle with +ve real axis of all vectors.