Laplace Transform

Laplace is very useful tool in circuit analysis. It transforms integro-diff eq's into linear polyminal eq's.

Laplace is a mathematical tool which transforms a function in time domain in to a function in frequency domain.

If $f(t)$ is function defined over the limits $0<t<\alpha$.

The laplace transform is given by.

For any function to apply L.T, it must satisfy condition $\int_0^{\alpha} f(t) e^{-st} dt < \alpha$

$$f(t) \xrightarrow{L.T} \int_0^{\alpha} f(t) e^{-st} dt$$

A laplace transform is said to be bilateral when it takes the limits between $-\alpha$ to $+\alpha$. It is said to be unilateral when it takes the limits from 0 to ∞

1) Functional Laplace :- These are laplace transforms of the special functions

2) Operational Laplace :- defines mathematical property of L.T

i) Unit step function :- $f(t) = u(t)$

$$u(t) = \begin{cases} 1, & t>0 \\ 0, & t<0 \end{cases}$$

$$L[f(t)] = \int_0^{\alpha} f(t) e^{-st} dt = \int_0^{\alpha} 1 \cdot e^{-st} dt = -\frac{1}{s}\left[e^{-st}\right]_0^{\alpha}$$

$$= -\frac{1}{s}\left[e^{\alpha} - e^{0}\right] = \frac{1}{s}$$

$f(t) \rightarrow \boxed{u(t) \xrightarrow{L.T} \frac{1}{s}}$

ii) Unit impulse function :-

$$\delta(t) = 1, \quad t=0$$
$$= 0, \quad t>0$$

$L\{\delta(t)\} = 1 \cdot e^{-s.0} dt = 1.$ $\quad \boxed{\delta(t) \xrightarrow{L.T} 1.}$

Unit Ramp function :-

$\int uv = u\int v - \int \frac{du}{dt}\int v$

$$r(t) = at, \quad t > 0$$
$$= 0, \quad t < 0$$

if $a = 1 \Rightarrow r(t) = t, \; t > 0$

$$L[r(t)] = \int_0^\infty r(t)\, e^{-st}\, dt = \int_0^\infty t\, e^{-st}\, dt$$

$$= \left.\frac{te^{-st}}{-s}\right|_0^\infty - \int 1 \cdot \int e^{-st}\, dt\, dt = \left[\frac{te^{-st}}{-s}\right]_0^\infty - \int_0^\infty \frac{e^{-st}}{-s}\, dt$$

$$= \left[\frac{\infty e^{-\infty}}{-s} - \frac{0 \cdot e^{-0}}{s}\right] + \frac{1}{s}\left[\frac{e^{-st}}{-s}\right]_0^\infty = 0 - \frac{1}{s^2}\left[e^{-\infty} - e^{-0}\right] = \frac{1}{s^2}$$

$$\boxed{t \xrightarrow{L.T} \frac{1}{s^2}}$$

Exponential function :- $f(t) = e^{-at}$

$$L[e^{-at}] = \int_0^\infty e^{-at} \cdot e^{-st}\, dt = \int_0^\infty e^{-(s+a)t}\, dt$$

$$= \frac{-1}{s+a}\left[e^{-(s+a)t}\right]_0^\infty = \frac{-1}{s+a}\left[e^{-\infty} - e^{0}\right] = \frac{1}{s+a}$$

$$\boxed{e^{-at} \xrightarrow{L.T} \frac{1}{s+a}}$$

Cosine function :- $f(t) = \cos\omega t$

$$L[\cos\omega t] = \int_0^\infty \cos\omega t\, e^{-st}\, dt = \int_0^\infty \left[\frac{e^{j\omega t} + e^{-j\omega t}}{2}\right] e^{-st}\, dt$$

$$= \frac{1}{2}\left\{\int_0^\infty e^{-(s-j\omega)t}\, dt + \int_0^\infty e^{-(s+j\omega)t}\, dt\right\}$$

$$= \frac{1}{2}\left\{\frac{-1}{s-j\omega} \cdot \left[e^{-(s-j\omega)t}\right]_0^\infty + \frac{-1}{s+j\omega}\left[e^{-(s+j\omega)t}\right]_0^\infty\right\}$$

$$= \frac{1}{2}\left\{ \frac{-1}{s-j\omega}\left[e^{\alpha}-e^{0}\right] - \frac{1}{s+j\omega}\left[e^{\alpha}-e^{0}\right]\right\}$$

$$= \frac{1}{2}\left\{ \frac{1}{s-j\omega} - \frac{1}{s+j\omega}\right\} = \frac{1}{2}\left[\frac{s+j\omega}{s^2+\omega^2} - \frac{s-j\omega}{s^2+\omega^2}\right]$$

$$= \frac{1}{\cancel{2}}\left[\frac{\cancel{2}s}{s^2+\omega^2}\right] = \frac{s}{s^2+\omega^2}$$

$$\Rightarrow \boxed{\cos\omega t \xrightarrow{L.T} \frac{s}{s^2+\omega^2}}$$

sine function :- $f(t) = \sin\omega t$

$$L[\sin\omega t] = \int_0^{\alpha} f(t) e^{-st} dt$$

$$= \int_0^{\alpha} \sin\omega t\, e^{-st} dt = \int_0^{\alpha} \left(\frac{e^{j\omega t} - e^{-j\omega t}}{2j}\right) e^{-st} dt$$

$$= \frac{1}{2j}\left\{ \int_0^{\alpha} e^{-(s-j\omega)t} dt - \int_0^{\alpha} e^{-(s+j\omega)t} dt \right\}$$

$$= \frac{1}{2j}\left[\frac{e^{-(s+j\omega)t}}{s+j\omega} - \frac{e^{-(s-j\omega)t}}{s-j\omega}\right]_0^{\alpha}$$

$$= \frac{1}{2j}\left[\frac{(-1)}{s+j\omega} - \frac{(-1)}{s-j\omega}\right] = \frac{1}{2}\left[\frac{-s+j\omega}{s^2+\omega^2} + \frac{s+j\omega}{s^2+\omega^2}\right]$$

$$= \frac{1}{2j}\left[\frac{-\cancel{s}+j\omega+\cancel{s}+j\omega}{s^2+\omega^2}\right] = \frac{\cancel{2j}\omega}{\cancel{2j}(s^2+\omega^2)} = \frac{\omega}{s^2+\omega^2}$$

$$\boxed{\sin\omega t \xrightarrow{L.T} \frac{\omega}{s^2+\omega^2}}$$

Type	f(t)	F(s)
Impulse	$\delta(t)$	1
step	$u(t)$	$1/s$
Ramp	t	$1/s^2$
Exponential	e^{-at}	$1/s+a$
sine	$\sin\omega t$	$\omega / s^2+\omega^2$
cosine	$\cos\omega t$	$s / s^2+\omega^2$
first Differentive	$\frac{df(t)}{dt}$	$sF(s)-F(0)$
Integral	$\int_0^t f(t)dt$	$\frac{F(s)}{s}$
	$e^{-at}f(t)$	$F(s+a)$

$\int uv = u\int v - \int u' \int$

Diff

$$\int_0^\infty \frac{d}{dt}f(t)e^{-st}dt = \int \underbrace{\frac{df(t)}{dt}}_{v}\underbrace{e^{-st}}_{u} dt$$

$$= \left[f(t)e^{-st}\right]_0^\infty - \int_0^\infty (-s)e^{-st}\left[\int_0^\infty \frac{df(t)}{dt}dt\right]dt$$

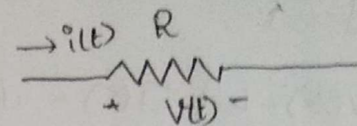
$$= \left[0 - f(0)\right] + s\int_0^\infty e^{-st}f(t)\,dt = -F(0) + sF(s) = sF(s) - f(0)$$

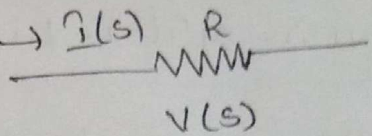
$$L\left\{\int_0^t f(t)dt\right\} = \int_0^\infty \underbrace{e^{-st}}_{v}\Big(\underbrace{\int_0^t f(t)dt}_{u}\Big)dt = \underbrace{\left[\int_0^t f(t)dt \cdot \frac{e^{-st}}{-s}\right]_0^\infty}+\int_0^\infty f(t)\frac{e^{-st}}{-s}$$

$$= 0 + \frac{1}{s}\int_0^\infty f(t)e^{-st}dt = \frac{1}{s}F(s)$$

Resistor in the frequency domain :-

Consider the resistor connected to voltage source $v(t)$. Ohm's law,





$$v(t) = R\,i(t)$$

Taking laplace transform on both sides

$$V(s) = R\,I(s)$$

The ratio of frequency domain representation of voltage to the frequency domain repsesentation of current is transform impedence of resistor,

$$\boxed{Z(s) = \frac{V(s)}{I(s)} = R} \; ; \text{ units } - \text{'}\Omega\text{'}$$

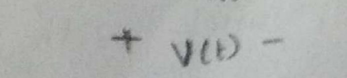
The impedence of a resistor does not change while going from time domain to the frequency domain, as it is independent of frequency.

The admittance $Y(s)$ of a resistor is debined as the ratio of $I(s)$ to $V(s)$

$$\boxed{Y(s) = \frac{I(s)}{V(s)} = \frac{1}{R} = G} \rightarrow \text{units - siemen 's'}$$

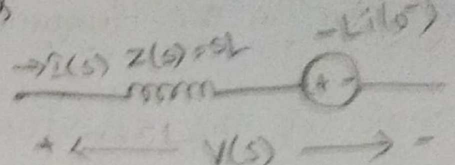
Inductor in frequency domain :-

Consider an inductor connected to time varying voltage source $v(t)$,



$$v(t) = L\,\frac{di(t)}{dt}$$

Taking laplace transform on both sides

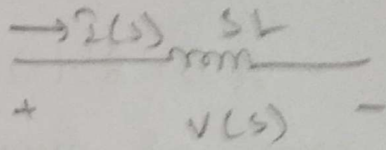


$$V(s) = L\left[sI(s) - i(0^-)\right]$$

$$\Rightarrow \boxed{V(s) = sLI(s) - Li(0^-)} \longrightarrow (1) \rightarrow$$

If the energy stored in an inductor is initially zero i.e., $i(0^-)=0$, then

$$V(s) = sLI(s)$$

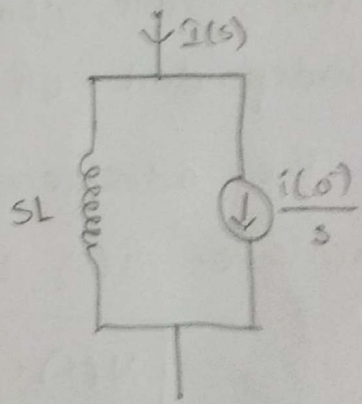


$\Rightarrow \boxed{Z(s) = \frac{V(s)}{I(s)} = sL}$, the transformed impedence of inductor.

If $s=j\omega$ $\Rightarrow \boxed{Z(j\omega) = j\omega L}$

This equation represent's that an impedence of sL ohms is connected in series with an independent voltage source of $Li(0^-)$

from Eq (1), we can write as



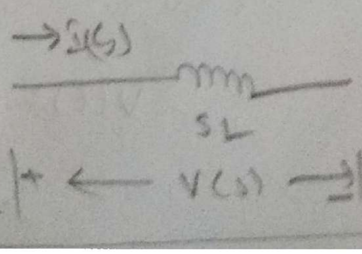
$$sLI(s) = V(s) + Li(0^-)$$

$$\Rightarrow I(s) = \frac{V(s)}{sL} + \frac{\cancel{L}i(0^-)}{s\cancel{L}}$$

$$\Rightarrow \boxed{I(s) = \frac{V(s)}{sL} + \frac{i(0^-)}{s}} \longrightarrow (2)$$

It represents that circuit contains an impedence 'sL' in parallel with the current source $\frac{i(0^-)}{s}$.

If $i(0^-)=0$, then $\boxed{I(s) = \frac{V(s)}{sL}}$



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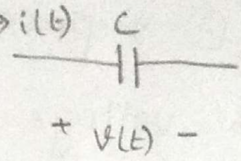
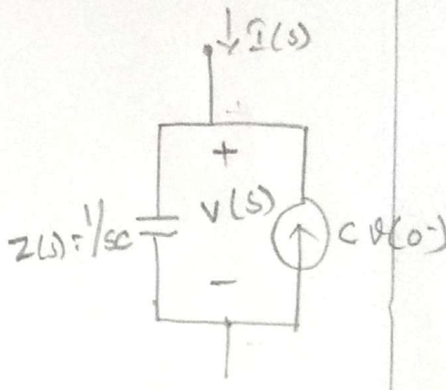
Capacitor in s domain :-

Consider an initially charged capacitor & the voltage-current relation is given as



$$i = C\frac{dv}{dt}$$

Taking laplace transform on both sides



$$I(s) = C\left[sV(s) - v(0^-)\right]$$

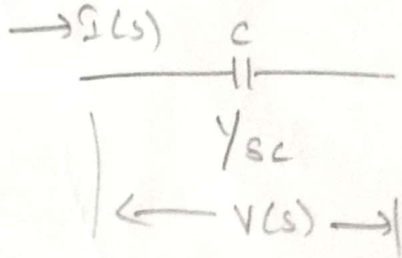
$$\Rightarrow \boxed{I(s) = sCV(s) - Cv(0^-)} \rightarrow (1) \quad I = V/Z \; (8)(VY)$$

Eq (1) represents that the ckt contains an admittance of capacitor sC in parallel with current source $Cv(0^-)$.

* If capacitor is initially stored no energy i.e., $v(0^-) = 0$

$$\Rightarrow I(s) = sCV(s)$$

$$\Rightarrow \boxed{Z(s) = \frac{V(s)}{I(s)} = \frac{1}{sC}}$$

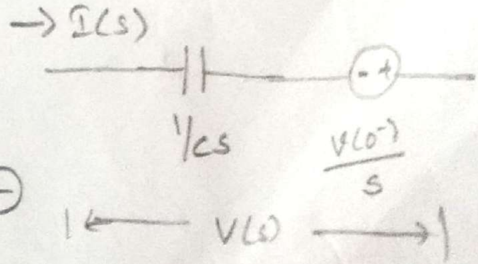


from Eq (1), we can also write

$$CsV(s) = I(s) + Cv(0^-)$$

$$\Rightarrow V(s) = \frac{I(s) + Cv(0^-)}{Cs}$$

$$\Rightarrow \boxed{V(s) = \frac{I(s)}{Cs} + \frac{v(0^-)}{s}} \rightarrow (2)$$



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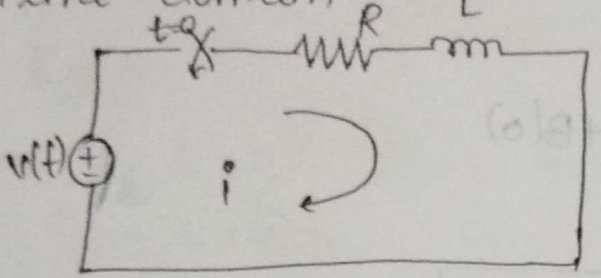
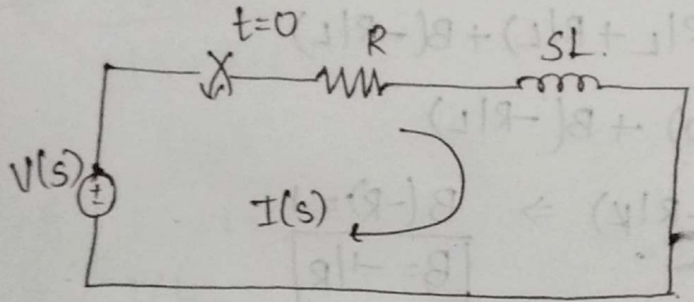
Analysis of R-L series circuit for various input.

Time-domain



In S-domain



Applying KVL for the above circuit

$$V(S) = I(S)R + SL\,I(S)$$

$$V(S) = I(S)[R + SL]$$

$$\frac{V(S)}{I(S)} = R + SL \Rightarrow \frac{I(S)}{V(S)} = \frac{1}{R+SL}$$

$$\frac{I(S)}{V(S)} = \frac{1}{L(S + R/L)}$$

Unit step

$$u(t) = 1, \quad t \geq 0$$
$$\quad\quad\; 0, \quad t < 0$$

$$V(S) = U(S) = 1/S.$$

$$I(S) = \frac{V(S)}{L(S+R/L)} = \frac{1/S}{L(S+R/L)}$$

$$= \frac{1/L}{S(S+R/L)}$$

Consider

$$\frac{1/L}{s(s+R/L)} = \frac{A}{s} + \frac{B}{s+R/L} \rightarrow (I)$$

$$1/L = A(s+R/L) + B(s) \rightarrow (1)$$

Let $s=0$.

$$1/L = A(0+R/L) + B(0)$$

$$1/\cancel{L} = A(R/\cancel{L})$$

$$AR = 1 \Rightarrow \boxed{A = 1/R}$$

Substitute $s=-R/L$ in eqn (1).

$$1/L = A(-R/L+R/L)+B(-R/L)$$

$$1/L = A(0) + B(-R/L)$$

$$1/\cancel{L} = B(-R/\cancel{L}) \Rightarrow B(-R)=1$$

$$\boxed{B=-1/R}$$

Substitute A, B in (I)

$$\frac{1/L}{s(s+R/L)} = \frac{1/R}{s} + \frac{-1/R}{s+R/L}$$

$$I(s) = \frac{1/R}{s} + \frac{-1/R}{s+R/L} \rightarrow (2)$$

Apply Inverse Laplace transform for the eqn (2)

$$i(t) = \frac{1}{R}(1) + -1/R\left(e^{-Rt/L}\right)$$

$$\boxed{i(t) = \frac{1}{R}\left(1-e^{-Rt/L}\right)}$$

<u>Unit Impulse</u>:-

$\delta(t) = 1,$

$V(s) = 1$

$$I(s) = \frac{V(s)}{L(s+R/L)} = \frac{1}{L(s+R/L)}$$

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Sk. Zeba No (4)

$$I(s) = \frac{1}{L(s+R/L)} \rightarrow (1)$$

Apply Inverse Laplace transform for equation (1)

$$i(t) = \frac{1}{L} e^{-Rt/L}.$$

Unit Ramp:-

$$r(t) = t,\ t \geq 0$$
$$0,\ t < 0$$

$$V(s) = 1/s^2.$$

$$I(s) = \frac{V(s)}{L(s+R/L)}$$

$$= \frac{1/s^2}{L(s+R/L)}$$

$$I(s) = \frac{1/L}{s^2(s+R/L)}$$

Consider $\frac{1/L}{s^2(s+R/L)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+R/L}. \rightarrow (2)$

$$1/L = A(s)(s+R/L) + B(s+R/L) + C(s^2)$$

let $s = 0$

$$1/L = A(0)(0+R/L) + B(0+R/L) + C(0)$$

$$1/\cancel{L} = B(R/\cancel{L})$$

$$B = 1/R.$$

Let $s = -R/L$

$$1/L = A(-R/L)(-R/L+R/L) + B(-R/L+R/L) + \left(\left(-\frac{R}{L}\right)^2 \times C\right)$$

$$1/\cancel{L} = C\left(\frac{R^2}{L^{\cancel{2}}}\right)$$

$$\frac{R^2C}{L} = 1 \Rightarrow C = \frac{L}{R^2}.$$

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$$1/L = A(s^2 + R/L\,s) + B(s + R/L) + Cs^2$$

$$1/L = s^2(A + C) + s(AR/L + B) + BR/L$$

Comparing s^2 coefficients on b.s

$$A + C = 0$$

$$A + \frac{L}{R^2} = 0$$

$$A = -L/R^2$$

Sub A, B, C values $\rightarrow$ ①

$$\frac{1/L}{s^2(s+R/L)} = \frac{-L/R^2}{s} + \frac{1/R}{s^2} + \frac{L/R^2}{s+R/L}$$

$$I(s) = \frac{-L/R^2}{s} + \frac{1/R}{s^2} + \frac{L/R^2}{s+R/L}$$

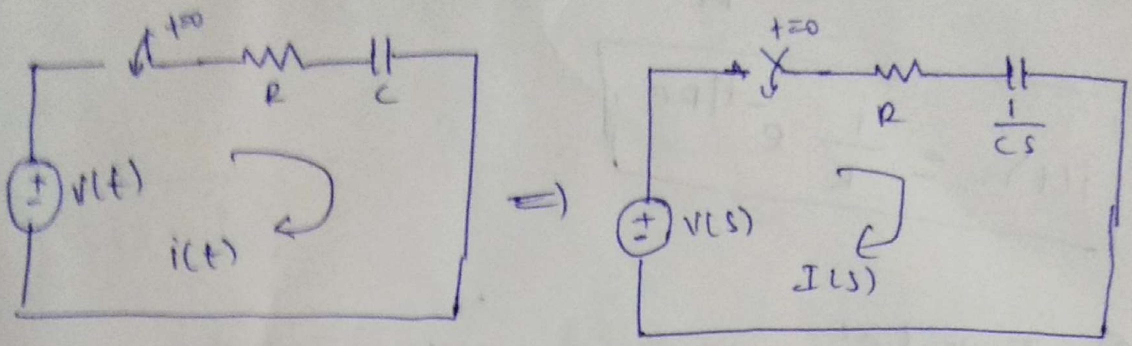
Apply Inverse Laplace Transform for eqn ③

$$i(t) = -\frac{L}{R^2}(1) + \frac{1}{R}(t) + \frac{L}{R^2}(e^{-Rt/L})$$

$$\boxed{i(t) = \frac{L}{R^2}(e^{-Rt/L} - 1) + \frac{1}{R}(t)}$$

Trupthi Podda (8)

Analysis of series R-C circuit



$$V(s) = I(s)R + \frac{I(s)}{Cs}$$

$$V(s) = I(s)\left(R + \frac{1}{Cs}\right)$$

$$\frac{V(s)}{I(s)} = R + \frac{1}{Cs}$$

$$\frac{V(s)}{I(s)} = \frac{CsR+1}{Cs}$$

$$\frac{I(s)}{V(s)} = \frac{Cs}{RCs+1}$$

$$\frac{I(s)}{V(s)} = \frac{\cancel{C}s}{R\cancel{C}\left(s+\frac{1}{RC}\right)}$$

$$\frac{I(s)}{V(s)} = \frac{s}{R\left(s+\frac{1}{RC}\right)}$$

For unit step funⁿ

$$V(s) = \frac{1}{s}$$

$$I(s) = \frac{V(s) \times s}{R\left(s+\frac{1}{RC}\right)}$$

$$= \frac{\frac{1}{\cancel{s}} \times \cancel{s}}{R\left(s+\frac{1}{RC}\right)}$$

$$\Rightarrow I(s) = \frac{1}{R\left(s+\frac{1}{RC}\right)}$$

$$\boxed{i(t) = \frac{1}{R} e^{-t/RC}}$$

unit Impulse

$$V(s) = 1$$

$$I(s) = \frac{s}{R\left(s+\frac{1}{RC}\right)}$$

$$\frac{s}{R\left(s+\frac{1}{RC}\right)} = \frac{1}{R}\left[\frac{s+\frac{1}{RC}-\frac{1}{RC}}{\left(s+\frac{1}{RC}\right)}\right]$$

$$= \frac{1}{R}\left[\frac{s+\cancel{\frac{1}{RC}}}{s+\cancel{1/RC}} - \frac{\frac{1}{RC}}{s+1/RC}\right]$$

$$= \frac{1}{R}\left[1 - \frac{1/RC}{s+1/RC}\right]$$

$$\boxed{i(t) = \frac{1}{R}\left[\delta(t) - \frac{1}{RC} e^{-t/RC}\right]}$$

ramp function

$$V(s) = \frac{1}{s^2}$$

$$I(s) = \frac{\frac{1}{\cancel{s^2}} \times \cancel{s}}{R\left(s+\frac{1}{RC}\right)}$$

$$= \frac{1}{Rs\left(s+\frac{1}{RC}\right)}$$

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$$= \frac{\frac{1}{R}}{s(s + 1/RC)}$$

$$\frac{\frac{1}{R}}{s(s + \frac{1}{RC})} = \frac{A}{s} + \frac{B}{s + \frac{1}{RC}}$$

At $s = 0$

$$\frac{1}{R} = A\left(0 + \frac{1}{RC}\right)$$

$$\frac{1}{\cancel{R}} = A\left(\frac{1}{\cancel{R}C}\right)$$

$$A = C$$

At $s = -1/RC$

$$\frac{1}{R} = B \times \left(-\frac{1}{RC}\right)$$

$$\frac{1}{\cancel{R}} = \frac{-B}{\cancel{R}C}$$

$$B = -C$$

$$= \frac{C}{s} - \frac{C}{s + 1/RC}$$

$$= C\left[\frac{1}{s} - \frac{1}{s + 1/RC}\right]$$

$$\boxed{i(t) = C\left[1 - e^{-t/RC}\right]}$$

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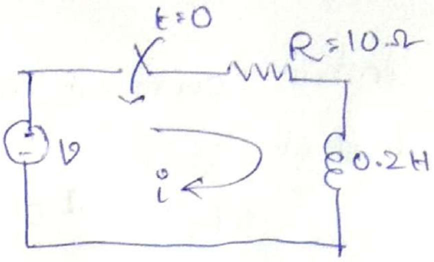
1) In series RL circuit, an exponential voltage $v = 50e^{-100t}$ V is applied by closing switch at $t=0$. find the current.



Sol:- Applying KVL for the circuit

$$v = iR + L\frac{di}{dt}$$

& $i(0^-) = 0$ in the circuit

In s-domain

$v(t) = 50e^{-100t}$

$\Rightarrow V(s) = \frac{50}{s+100}$

$$\Rightarrow V(s) = RI(s) + sLI(s) - Li(0^-)$$

$$\Rightarrow \frac{50}{s+100} = 10I(s) + 0.2sI(s)$$

$$\Rightarrow I(s) = \frac{50}{(s+100)(0.2s+10)} = \frac{50/0.2}{(s+100)(s+\frac{10}{0.2})}$$

$$\Rightarrow I(s) = \frac{250}{(s+100)(s+50)} = \frac{A}{s+100} + \frac{B}{s+50}$$

$$\Rightarrow 250 = A(s+50) + B(s+100)$$

At $s=-50 \Rightarrow 250 = B(-50+100)$

$$\Rightarrow B = \frac{250}{50} = 5$$

At $s=-100 \Rightarrow 250 = A(-100+50)$

$$\Rightarrow A = \frac{-250}{50} = -5$$

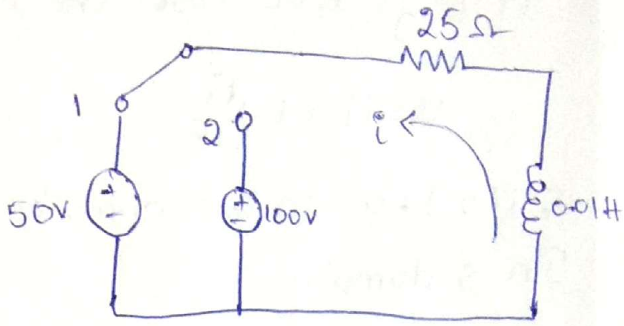
$$\Rightarrow I(s) = -\frac{5}{s+100} + \frac{5}{s+50} = 5\left(\frac{1}{s+50} - \frac{1}{s+100}\right)$$

Inverse laplace,

$$\boxed{i(t) = 5\left(e^{-50t} - e^{-100t}\right) A}$$

2) In RL circuit shown in fig. the switch is in position 1. At $t=0$ it is switched to position 2. find the current.

Sol:- Initial current in the circuit is



$$i(0^-) = -\frac{50}{25} = -2A$$

At $t=0^+$, switch is at position 2

$$\Rightarrow 100 = 25i + 0.01\frac{di}{dt}$$

In s domain

$$\frac{100}{s} = 25I(s) + s\,0.01I(s) - 0.01(-2)$$

$$\Rightarrow I(s)\left[25 + 0.01s\right] = \frac{100}{s} - 0.02$$

$$\Rightarrow I(s) = \frac{100}{s(25+0.01s)} - \frac{0.02}{(25+0.01s)}$$

$$\Rightarrow I(s) = \frac{10^4}{s(s+2500)} - \frac{2}{(2500+s)}$$

$$\frac{10^4}{s(s+2500)} = \frac{A}{s} + \frac{B}{s+2500}$$

$$\Rightarrow 10^4 = A(s+2500) + Bs$$

$$s=0, \Rightarrow 10^4 = A.2500 \Rightarrow A = \frac{100}{25} = 4$$

$$s=-2500 \Rightarrow 10^4 = B(-2500) \Rightarrow B = \frac{-100}{25} = -4$$

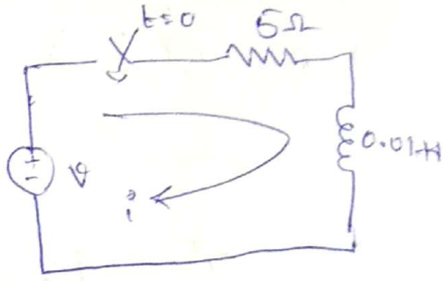
$$\Rightarrow \frac{10^4}{s(s+2500)} = \frac{4}{s} - \frac{4}{s+2500}$$

$$\Rightarrow I(s) = \frac{4}{s} - \frac{4}{s+2500} - \frac{2}{s+2500} = \frac{4}{s} - \frac{6}{s+2500}$$

Inverse laplace

$$\boxed{i(t) = 4 - 6e^{-2500t} \text{ A}}$$

3 In series RL circuit, the source is $v = 100\sin(500t)$. Determine the current.



Sol:-

$$V(s) = 5I(s) + 0.01sI(s) - 0.01i(0^-)$$

$$\Rightarrow V(s) = 5I(s) + 0.01sI(s) \quad \because i(0^-) = 0$$

$v(t) = 100\sin 500t$

$$\Rightarrow V(s) = \frac{100 \cdot 500}{s^2 + 500^2}$$

$$\Rightarrow I(s)[5 + 0.01s] = \frac{5 \times 10^4}{s^2 + 500^2}$$

$$\Rightarrow I(s) = \frac{5 \times 10^4 \times 10^2}{(s^2 + 500^2)(s+500)}$$

$$\Rightarrow I(s) = \frac{5 \times 10^6}{(s+j500)(s-j500)(s+500)} = \frac{A}{s+j500} + \frac{B}{s-j500} + \frac{C}{s+500}$$

$$\Rightarrow 5 \times 10^6 = A(s-j500)(s+500) + B(s+j500)(s+500) + C(s+j500)(s-j500)$$

At $s = +j500$

$$\Rightarrow 5 \times 10^6 = B(j500 + j500)(j500 + 500)$$

$$\Rightarrow \cancel{5} \times 10^{\cancel{6}} = B \cdot j\cancel{10^3} \cdot \cancel{5} \times \cancel{10^2}(1+j) \Rightarrow B = \frac{10}{j(1+j)} = \frac{10}{(j-1)} \times \frac{j+1}{j+1}$$

$$\Rightarrow B = \frac{10(j+1)}{-1-1} = -5(j+1)$$

At $s = -j500$

$\Rightarrow 5\times10^6 = A(-j500-j500)(-j500+500)$

$\Rightarrow \cancel{5}\times10^{\cancel{6}} = A.-j.\cancel{10^3}.\cancel{5}\times\cancel{10^2}(1-j)$

$\Rightarrow A = \dfrac{10}{-j(1-j)} = \dfrac{10}{1+j}\times\dfrac{1-j}{1-j} = \dfrac{10(1-j)}{-2} = -5(1-j)$

$\Rightarrow A = 5(j-1)$

At $s = -500$

$\Rightarrow 5\times10^6 = C((-500)^2+(500)^2) = C(250000+250000)$

$\Rightarrow \cancel{5}\times10^{\cancel{6}} = C.\cancel{50}\times10^{4} \Rightarrow C = 10$

$\Rightarrow I(s) = \dfrac{5(j-1)}{s+j500} - \dfrac{5(j+1)}{s-j500} + \dfrac{10}{s+500}$

Inverse laplace

$\Rightarrow I(s) = \dfrac{(5j-5)(s-j500)-(5j+5)(s+j500)}{(s+j500)(s-j500)} + \dfrac{10}{s+500}$

$= \dfrac{\cancel{5js} - 5j.j500 - 5s + j\cancel{2500} - \cancel{5js} - 5j.j500 - 5s - j\cancel{2500}}{s^2+500^2}$

$= \dfrac{5(500) - 10s + 5(500)}{s^2+500^2} = \dfrac{10(500-s)}{s^2+500^2}$

$= 10\left[\dfrac{500}{s^2+500^2} - \dfrac{s}{s^2+500^2}\right] + \dfrac{10}{s+500}$

Inverse laplace

$\Rightarrow \boxed{i(t) = 10\sin 500t - 10\cos 500t + 10e^{-500t}\ A}$

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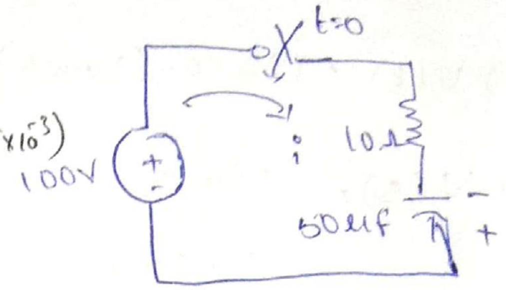
4. In series RC circuit, the capacitor has an initial charge of 2.5mC. At $t=0$, switch is closed. Determine the current.

Sol:- In time domain eq is



$$100 = 10i + \frac{1}{50\times10^{-6}}\int i\,dt + \frac{1}{50\times10^{-6}}(-2.5\times10^{-3})$$

$$\Rightarrow 100 = 10i + \frac{1}{50\times10^{-6}}\int i\,dt - \frac{25\times10^{-4}}{50\times10^{-6}}$$

In sdomain

$$\Rightarrow \frac{100}{s} = 10I(s) + \frac{I(s)}{50\times10^{-6}\cdot s} - \frac{1}{2\times10^{-2}\cdot s}$$

$$\Rightarrow I(s)\left[10 + \frac{1}{s\cdot 50\times10^{-6}}\right] = \frac{100}{s} + \frac{100}{2s} = \frac{100}{s} + \frac{50}{s} = \frac{150}{s}$$

$$\Rightarrow I(s) = \frac{150}{s\left[10 + \frac{1}{s\cdot50\times10^{-6}}\right]} = \frac{150}{10s + \frac{1}{50\times10^{-6}}} = \frac{15\cancel{0}}{1\cancel{0}\left[s + \frac{1}{50\times10^{-5}}\right]}$$

$$\Rightarrow I(s) = \frac{15}{s + \frac{\cancel{100}\times10^{3}}{\cancel{50}}} = \frac{15}{s+2000}$$

Inverse laplace

$$\Rightarrow \boxed{i(t) = 15e^{-2000t}\text{ A}}$$

5) The series RC circuit has a sinusoidal voltage source $v = 180\sin(2000t+\phi)$ V and an initial charge of 1.25mC. Determine current if switch is colsed at $t=0$ & $\phi = 90°$.

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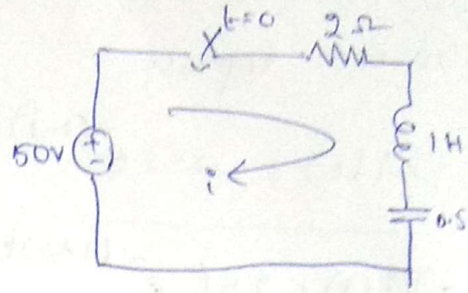
6. In series RLC circuit, there is no initial charge in capacitor. If switch is closed at t=0, Calculate current.



Sol:- Time domain equation

$$50 = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt$$

$$\Rightarrow 50 = 2i + 1\frac{di}{dt} + \frac{1}{0.5}\int i dt$$

S-Domain

$$\Rightarrow \frac{50}{s} = 2I(s) + I(s).s + \frac{1}{0.5}\frac{I(s)}{s}$$

$$\Rightarrow I(s)\left[2 + s + \frac{1}{0.5s}\right] = \frac{50}{s}$$

$$\Rightarrow I(s)\left[\frac{2\times0.5s + 0.5s^2 + 1}{0.5\cancel{s}}\right] = \frac{50}{\cancel{s}}$$

$$\frac{-2 \pm \sqrt{4-4\times2}}{2}$$

$$= \frac{-2 \pm \sqrt{-4}}{2}$$

$$= \frac{-2 \pm j2}{2}$$

$$= -1 \pm j$$

$$\frac{1}{0.5} = 10 \; \frac{10}{5} = 2$$

$$\Rightarrow I(s) = \frac{50\times0.5}{0.5s^2 + 2\times0.5s + 1} = \frac{50}{s^2 + 2s + 2}$$

$$= \frac{50}{(s-(-1+j))(s-(-1-j))} = \frac{50}{(s+1-j)(s+1+j)}$$

$$\Rightarrow \frac{50}{(s+1-j)(s+1+j)} = \frac{A}{s+1-j} + \frac{B}{s+1+j}$$

$$\Rightarrow 50 = A(s+1+j) + B(s+1-j)$$

At $s = -1-j \Rightarrow 50 = B(\cancel{-1}-j+\cancel{1}-j)$

$$\Rightarrow B = \frac{50}{-2j} = \frac{-25}{j} = 25j$$

At $s = -1+j \Rightarrow 50 = A(-1+j+1+j) = 2jA$

$$\Rightarrow A = \frac{50}{2j} = -25j$$

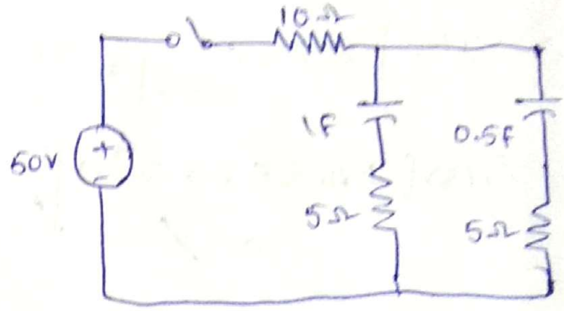
$$\Rightarrow I(s) = \frac{-25j}{s+1-j} + \frac{25j}{s+1+j}$$

Inverse Laplace

$$\Rightarrow i(t) = -25j\, e^{-(1-j)t} + 25j\, e^{-(1+j)t}$$

$$\Rightarrow \boxed{i(t) = 25\left(e^{-(1+j)t} - e^{-(1-j)t}\right) A}$$

7. In the network, the switch is closed at t=0. find current.



Sol:- $$Z(s) = 10 + \frac{\left(5+\frac{1}{s}\right)\left(5+\frac{1}{0.5s}\right)}{5+\frac{1}{s}+5+\frac{1}{0.5s}}$$

$$= 10 + \frac{25 + \frac{5}{0.5s} + \frac{5}{s} + \frac{1}{0.5s^2}}{10 + \frac{1}{s}\left(1+\frac{1}{0.5}\right)}$$

$$= 10 + \frac{25 + \frac{10}{s} + \frac{5}{s} + \frac{2}{s^2}}{10 + \frac{1}{s}(3)} = 10 + \frac{25 + \frac{15}{s} + \frac{2}{s^2}}{10 + 3/s}$$

$$= 10 + \frac{25s^2+15s+2}{10s^2+3s} = \frac{100s^2+30s+25s^2+15s+2}{s(10s+3)}$$

$$= \frac{125s^2+45s+2}{s(10s+3)}$$

$$I(s) = \frac{V(s)}{Z(s)} = \frac{50/\cancel{s} \times \cancel{s}(10s+3)}{125s^2+45s+2} = \frac{500(s+0.3)}{125s^2+45s+2}$$

$$= \frac{\overset{4}{\cancel{500}}(s+0.3)}{\cancel{125}\left(s^2+\frac{45s}{125}+\frac{2}{125}\right)} = \frac{4(s+0.3)}{(s+0.308)(s+0.052)}$$

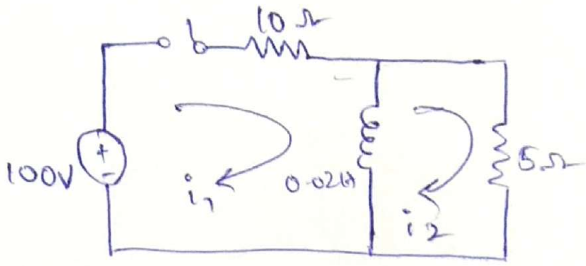
on solving, we get

$$I(s) = \frac{1/8}{s+0.308} + \frac{31/8}{s+0.052}$$

$$\Rightarrow \boxed{i(t) = 1/8\, e^{-0.308t} + 31/8\, e^{-0.052t}}\ A$$

8. find the currents when switch is closed in both loops

KVL for loop (1)



$$100 = 10 i_1 + 0.02 \frac{di_1}{dt} - 0.02 \frac{di_2}{dt}$$

$$\Rightarrow \frac{100}{s} = 10 I_1(s) + 0.02s\,(I_1(s) - I_2(s))$$

$$\Rightarrow [10 + 0.02s]\, I_1(s) - 0.02s\, I_2(s) = \frac{100}{s} \quad \longrightarrow (1)$$

KVL for loop (2)

$$5 I_2(s) + 0.02s\,(I_2(s) - I_1(s)) = 0$$

$$\Rightarrow I_2(s)\,[5 + 0.02s] - 0.02s\, I_1(s) = 0$$

$$\Rightarrow I_2(s) = \frac{0.02s\, I_1(s)}{5 + 0.02s} \quad \longrightarrow (2) \quad = I_1(s)\left[\frac{s}{s+250}\right]$$

(2) in (1)

$$\Rightarrow [10 + 0.02s]\, I_1(s) - 0.02s\left[\frac{0.02s\, I_1(s)}{5 + 0.02s}\right] = \frac{100}{s}$$

on solving

$$I_1(s) = 6.67\left[\frac{s+250}{s(s+166.7)}\right] = \frac{10}{s} - \frac{3.33}{s+166.7}$$

$\Rightarrow i_1 = 10 - 3.33 e^{-166.7t}$

from (2)

$$I_2(s) = 6.67 \left[\frac{1}{s+166.7} \right] =$$

$\Rightarrow i_2 = 6.67 . e^{-166.7t}$