Network:- A Network is a collection of Interconnected components.

Network Analysis:- Network Analysis is the process of finding the voltage across, and the currents through, every component in the network.

voltage:- According to the structure of an atom, we know that there are two types of charges.

(a) positive charge (2) Negative charge.

A force of attraction exists between these two charges. A certain amount of energy required to overcome the force. The difference in potential energy of the charges is called the potential difference.

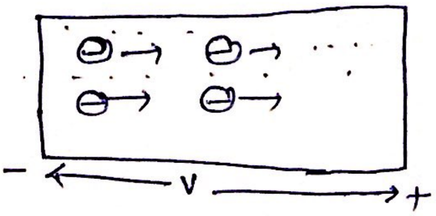
potential difference in electrical Terminology is Known as voltage, and denoted by 'V'.

$$V = \frac{W}{Q}$$

where w is energy expressed in Joul (J)

Q charge in coloumbs (C)

current:- There are free $\bar{e}$ available in all semi conductive and conductive materials. These free $\bar{e}$ move random in all directions within the structure in the absense of external pressure or voltage. If a certain amount of voltage is applied across the material, all the free $\bar{e}$ move in one direction depending on the polarity of the applied voltage.



This movement of $\bar{e}$ from one end of the material to th other constitutes an electric current, denoted by I or i, units for current is 'A'

The current is defined as the rate of flow of electrons in a conductive or semiconductive material.

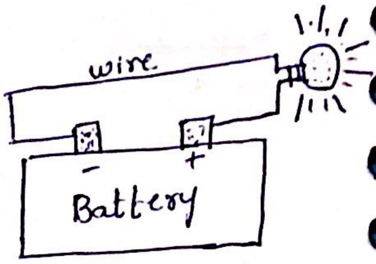
$$I = \frac{Q}{t}$$

Circuit :- an electronic circuit consist of three parts.

(a) energy source, such as battery or generator.

(b) The load or sink, such as lamp or motor

(c) connecting wires.



The battery connected to a lamp with two wires. The purpose of the circuit is to transfer energy from source to the battery

① Network Topology

Network Analysis is finding unknown branch currents (or) voltages across various branches by using different techniques such as mesh analysis, nodal analysis, $\Delta-\lambda$ transformation, source transformation and so on. The no of unknown variables increases as the network becomes complicated. In such cases, network analysis is made simpler by using the concepts of network topology.

In network topology only geometrical pattern of network is considered.

Linear graph:- It is defined as a collection of various nodes & branches. It describes the relation b/w nodes & branches & their interconnection.

Node:- It is defined as a common point at which two (or) more branches meet together. Degree of node: No of branches incident

Branch:- It is a line joining two nodes which represents a circuit element.

A graph of any network can be drawn by placing all the nodes where two (or) more branches intersect. The network elements are represented by the lines joining nodes.

The voltage & current sources are replaced by their internal impedences. If impedences are unknown the voltage source is short circuited & current source is open circuited.

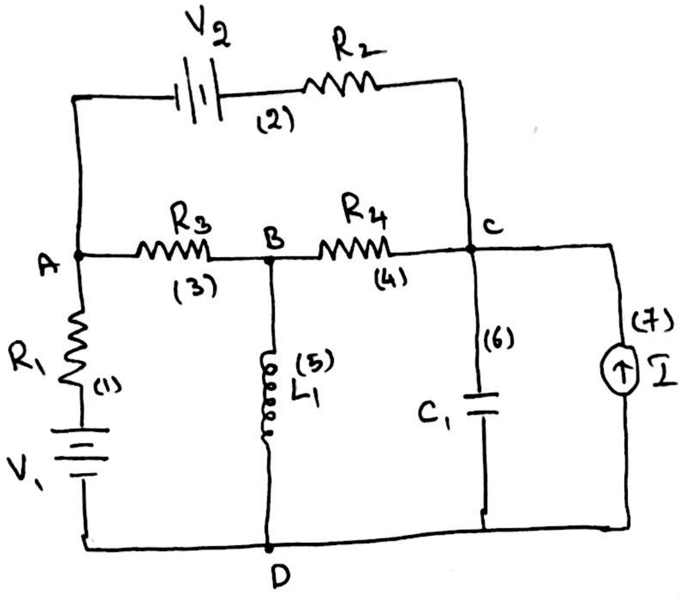
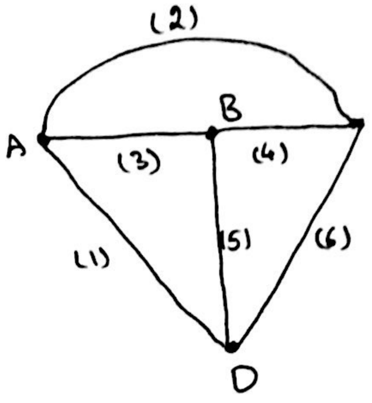


fig (a)



fig(b)

* consider a simple electrical network as shown in fig(a). Identify the nodes & branches in the network.

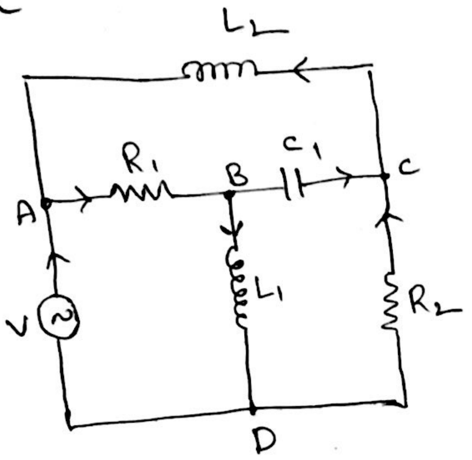
In given n/w there are 4 nodes & 7 branches. But in the graph we have only 6 branches because the current source is replaced by the open circuit & hence not considered.

The graph of given n/w is shown in fig (b).

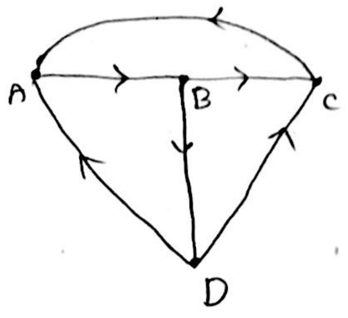
* Oriented Graph:-

It is a graph in which the orientations of the ~~graph~~ branches are known in the given network and the same are transferred on the graph. The oriented graph is also called as "Directed Graph."

Ex:-



(a) Network



(b) Oriented Graph

* Unoriented Graph:-

When the directions of the currents are not given in the network and the graph is without such orientations, such a graph is called "Unoriented graph."

* Planar Graph:-

A planar graph is a graph drawn on a two-dimensional plane so that no two branches intersect at a point which is not a node.

Ex:-

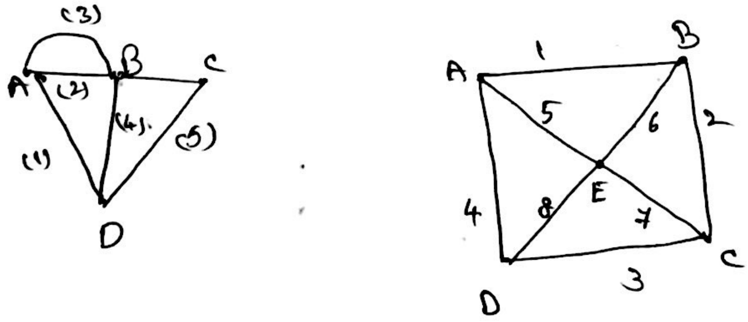


fig: Planar Graph

* Non-planar Graph:-

It is a graph drawn on a two dimensional plane so that two (or) more branches interesect at a point other than a node on a graph.

Ex:-

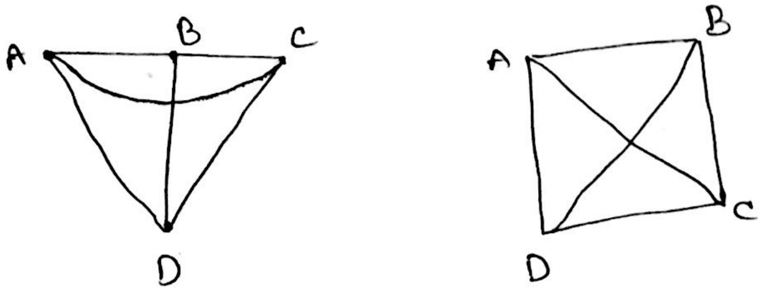
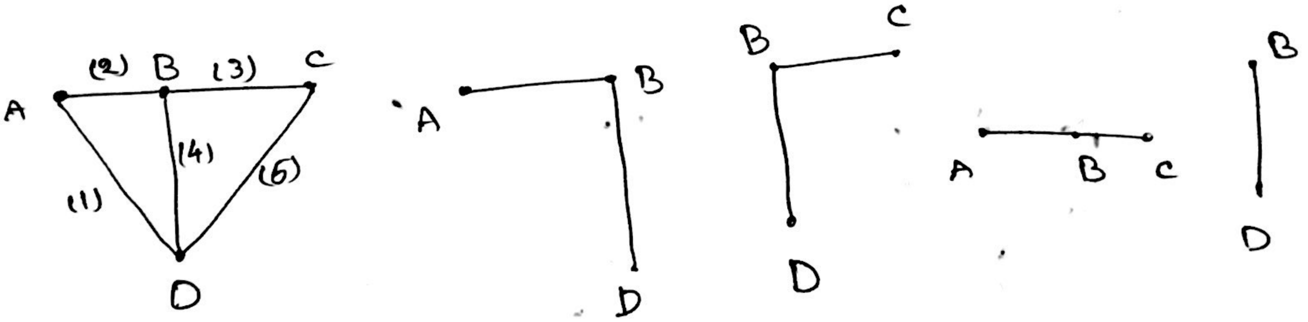


Fig: Non-Planar Graph

* Sub Graph:-

A subgraph is a subset of branches & nodes of a graph. There are two types of subgraphs.

If the subgraph contains branches & nodes less in number than those on the graph then it is called as "Proper Sub Graph."

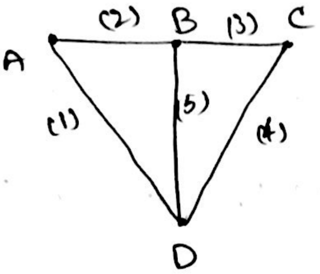


(a) Graph (b) (c) (d) (e)

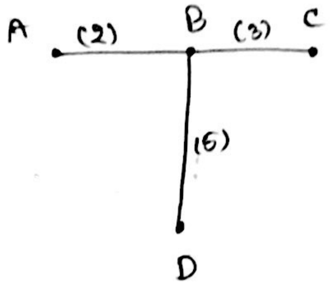
Proper Sub Graphs

If the subgraph contains all the nodes of a graph and the branches, then a subgraph is called "Improper Subgraph."

Ex:-



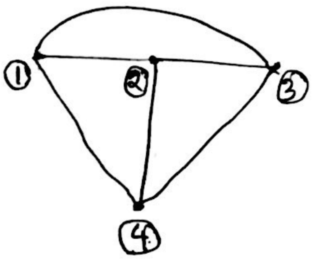
(a) Graph



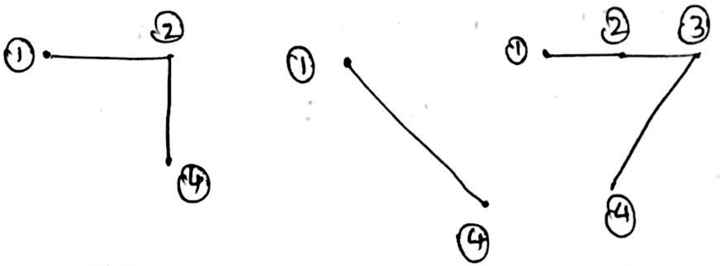
(b) Improper Subgraph

Path:- A path is a transversal from one node to another node in a graph along with the branches such that no node is encountered twice.

from the graph shown in fig(a), path exists between (1) & (4) along the branches shown.



(a) Graph

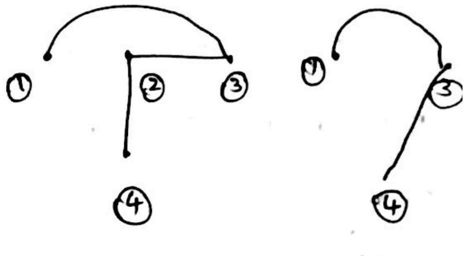


(b) (c) (d)

Rank of a Graph:-

If there exists 'n' no of nodes then rank 'R' of graph is

$$R = (n-1)$$



(e) (f)

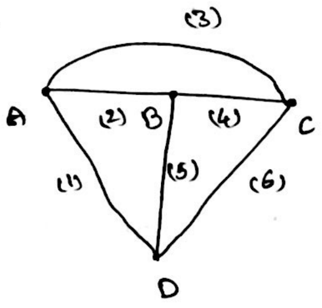
Paths from ① to ④

* Tree:- It is a set of branches with every node connected to every other node, such that any one of the branches removed changes this property.

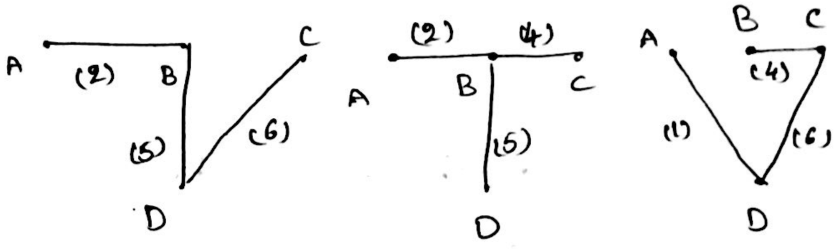
(or)

It is a connected subgraph containing all the nodes of the graph not forming any loop (or) closed path.

Ex:-



(a) Graph



(b) (c) (d)

Trees

Properties of a ~~graph~~ tree:-

* Tree contains all the nodes on the graph
* It does n't contain any closed path
* In a tree minimum end nodes are two
* Tree contains (n-1) branches if 'n' nodes are present in a tree.

* Branch of a tree (Twig):-

A branch of a tree is called Twig. If there are n no of nodes on a ~~tree~~ graph then the tree contains (n-1) twigs.

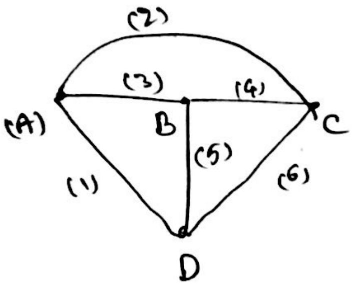
Ex:- for tree in fig (b), Twigs are T = {(2),(5),(6)}

(C), Twigs are T : {(2),(4),(5)}

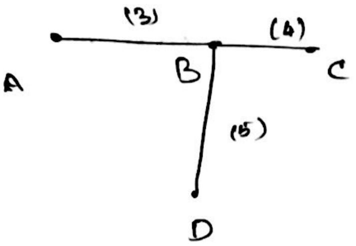
* **Cotree** :- A set of branches forming a complement of tree is called "Co-tree".

No. of branches of a cotree = $b-(n-1)$

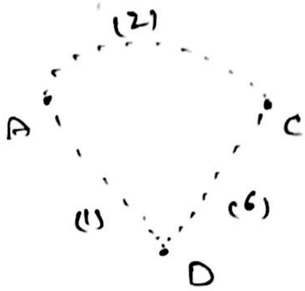
b: no of branches of a graph.



(a) Graph



(b) Tree



(c) Cotree

No. of branches of cotree = $6-(4-1) = 6-3 = 3$

* **Chord (or) Link** :-

The branches which are not in a tree are called chords (or) links.

Ex:- for the tree in fig (b), the links are $L = \{(1), (2), (6)\}$

* **Loop** :- If a network consists of 'n' nodes, which are interconnected in some way by 'b' branches, then it is possible to traverse adjacent branches starting at any node & returning to the original starting node in different ways. such a closed path formed by the network branches is called a "loop".

closed path formed by the network branches is called loop.

Properties of a loop:-

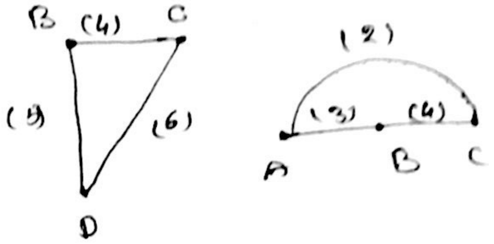


Fig: loops

1) There are exactly two paths between any pair of nodes in the circuit.

2) There exists atleast two branches in a loop

3) The maximum possible branches in a loop are equal to number of nodes.

Incidence Matrix:-

An Oriented graph can be explained completly (or) represented with the help of a matrix known as incidence matrix. It gives information about the branches ~~associated~~ incident on the nodes & their orientations at the node.

There are two types of incident matrix.

1) Complete incident matrix (A_a)

2) Reduced incident matrix (A)

* Complete Incident Matrix (A_a):-

Incident matrix is nothing but a mathematical model to represent the given network with all the information available. All the information is written in a matrix form which is called complete incidence matrix(A

E(4) If there are 'n' nodes & 'b' branches, the order of complete incidence matrix is $(n \times b)$.

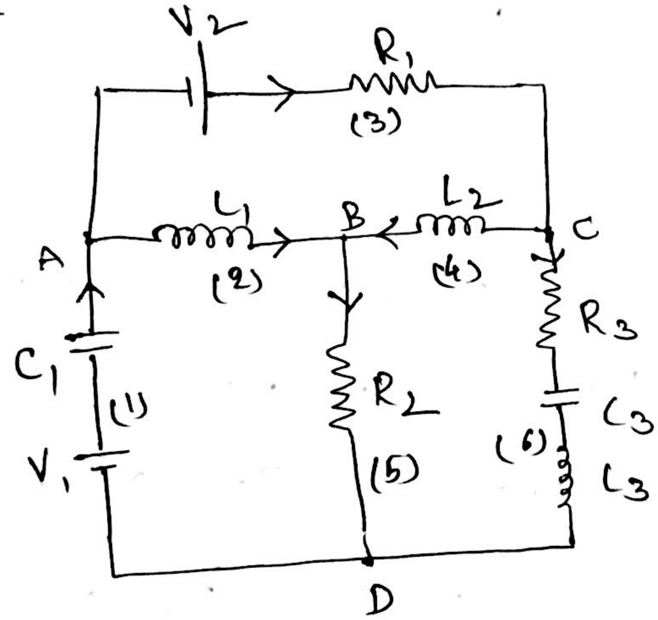
The standard convention for writing the incidence matrix is

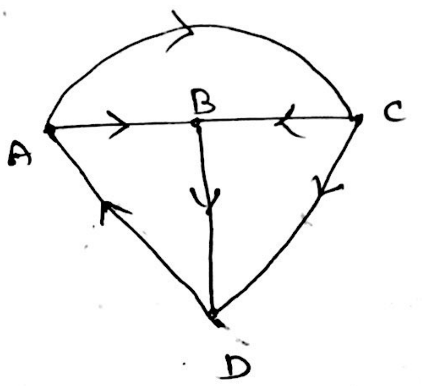
a_{nb} = 1, if 'b' branch is associated with node 'n' & oriented away from node 'n'.

= -1, if 'b' branch is associated with node 'n' & oriented towards node 'n'

= 0, if branch 'b' is not associated with node 'n'.

Ex:-





$n \times b$

Nodes = A, B, C, D = 4

Branches = 1, 2, 3, .. 6 = 6

Order : 4×6.

$$A_a = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline A & -1 & +1 & +1 & 0 & 0 & 0 \\ B & 0 & -1 & 0 & -1 & +1 & 0 \\ C & 0 & 0 & -1 & +1 & 0 & +1 \\ D & +1 & 0 & 0 & 0 & -1 & -1 \end{array} \leftarrow \text{Branches}$$

↑ Nodes

Properties of complete Incidence Matrix:-

1) The sum of entries in any column is zero

2) Rank of complete incidence matrix is (n-1)

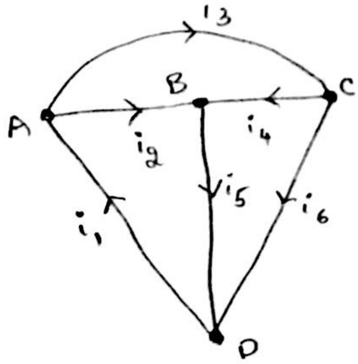
3) Determinant of a loop of complete incident matrix is always zero.

Reduced Incidence Matrix:-

When any one row from the complete incidence matrix is: eliminated by using mathematical manipulations, then the matrix is called reduced incidence matrix. It is denoted by A.

The order of reduced incidence matrix is (n-1) x b.

- Given A, A_a is easily obtained by using the property that each column representing a branch contains two non-zero entries +1 & -1, the rest being zero.
- The no of trees in a graph is given by $\det|A.A^T|$
- Reduced incidence matrix can also be found by applying KCL to the nodes of the graph.



KCL at node A:-

$$i_1 = i_2 + i_3 \longrightarrow ①$$

$$\Rightarrow -i_1 + i_2 + i_3 = 0$$

KCL at node B:-

$$i_5 = i_2 + i_4 \Rightarrow +i_5 - i_2 - i_4 = 0 \longrightarrow ②$$

KCL at node C:-

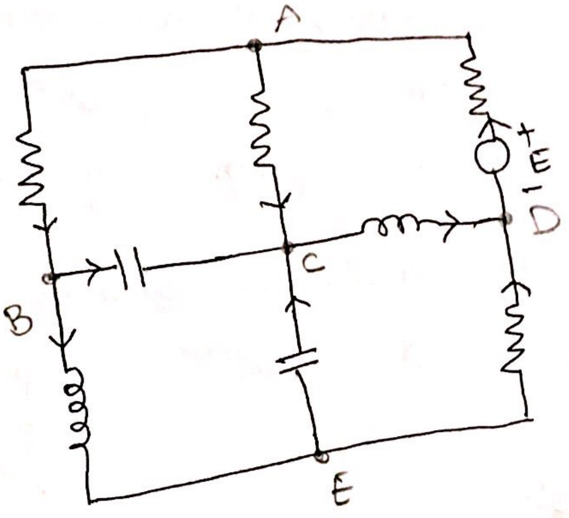
$$i_4 + i_6 = i_3 \Rightarrow i_4 + i_6 - i_3 = 0 \longrightarrow ③$$

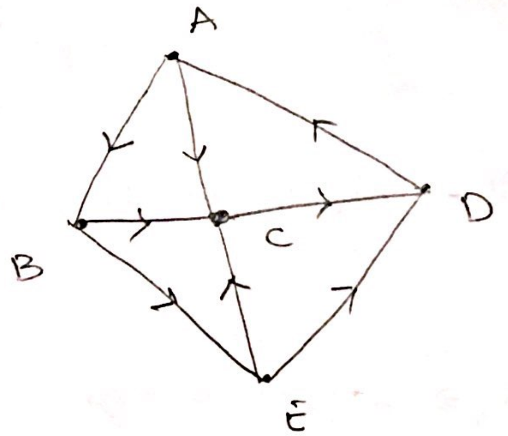
$$\begin{array}{cc} & \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 1 \end{bmatrix}_{3\times6} \end{array} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix}_{6\times1} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}_{3\times1}$$

$\Rightarrow [A][i_b] = [0]$; where A = Reduced incidence matrix

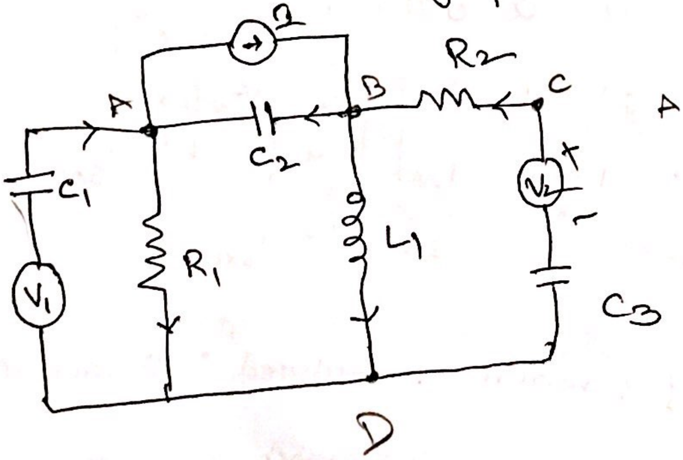
i_b = branch currents

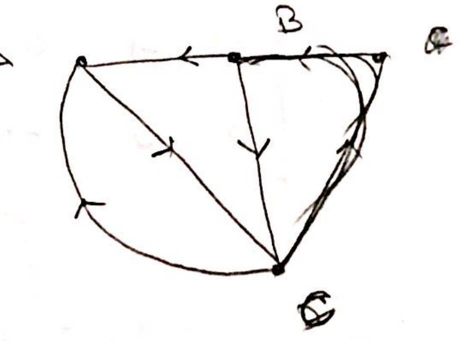
① Draw the directed graph for the given n/w.





② Draw the oriented graph for the network shown

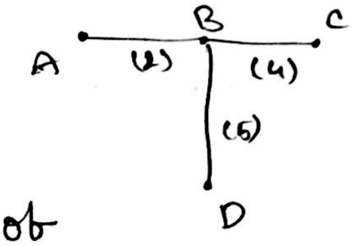




In above matrix, the fourth row is negative of sum of three rows. Hence 4th row is eliminated. The reduced incident matrix is

$$A = \begin{array}{c} \\ \\ \\ \\ \end{array}\begin{array}{cccccc|l} 1 & 2 & 3 & 4 & 5 & 6 & \rightarrow \text{branches} \\ -1 & 1 & 1 & 0 & 0 & 0 & A \leftarrow \text{nodes} \\ 0 & -1 & 0 & -1 & 1 & 0 & B \\ 0 & 0 & -1 & +1 & 0 & +1 & C \end{array}$$

for given graph, consider a tree as[1] shown in fig



The reduced incident matrix, can be written as writting columns corres-ponding to twigs first & columns of links atter that

$$A = \begin{array}{ccc|ccc} 2 & 4 & 5 & 1 & 3 & 6 \\ 1 & 0 & 0 & -1 & 1 & 0 \\ -1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & +1 \\ \multicolumn{3}{c|}{\underbrace{\qquad\qquad}_{\text{Twigs}}} & \multicolumn{3}{c}{\underbrace{\qquad\qquad}_{\text{links}}} \end{array}$$

$$\Rightarrow A = [A_T \; A_L]$$

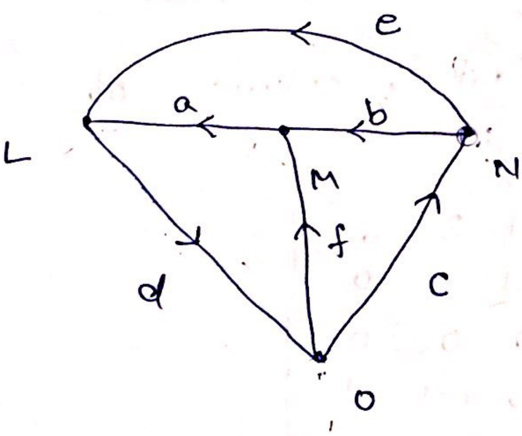
A_T = Twig Matrix of order $(n-1) \times (n-1)$

A_L = link Matrix of order $(n-1) \times (b-(n-1))$

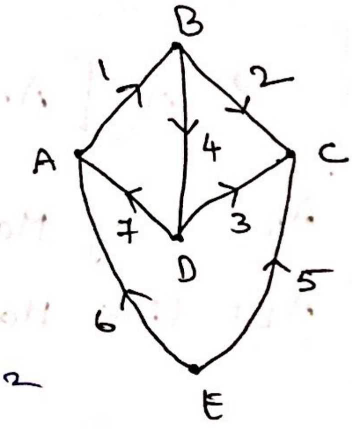
Ex:- Draw the oriented graph from complete incidence matrix given below

Nodes	Branches					
	a	b	c	d	e	f
L	-1	0	0	+1	-1	0
M	1	-1	0	0	0	-1
N	0	1	-1	0	1	0
O	0	0	1	-1	0	1

4 - nodes, 6 - branches



2 Write the a) complete incidence matrix for the graph
b) Reduced " "
shown in fig.



Sol:- Order = 5 × 7

$$A_a = \begin{array}{c} \\ A \\ B \\ C \\ D \\ \text{Ⓔ} \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \left[\begin{array}{ccccccc} +1 & 0 & 0 & 0 & 0 & -1 & -1 \\ -1 & +1 & 0 & +1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & +1 & -1 & 0 & 0 & +1 \\ 0 & 0 & 0 & 0 & +1 & +1 & 0 \end{array}\right] \end{array} \begin{array}{l} \\ \cancel{-1} \\ \cancel{+1} \\ -3 \\ +1 \Leftarrow -2 \\ = +2 \end{array}$$

Reduced incidence Matrix

$$A = \begin{array}{c|ccccccc|} & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ A & +1 & 0 & 0 & 0 & 0 & -1 & -1 \\ B & -1 & +1 & 0 & +1 & 0 & 0 & 0 \\ C & 0 & -1 & -1 & 0 & -1 & 0 & 0 \\ D & 0 & 0 & +1 & -1 & 0 & 0 & +1 \end{array}$$

Here last row 'E' is eliminated because the sum of remaining row is -2 & the sum of last row is $+2$. i.e., last row is negative of sum of remaining 4 rows.

Number of possible trees of a graph:-

No of possible trees of a graph can be calculated by using reduced incident matrix.

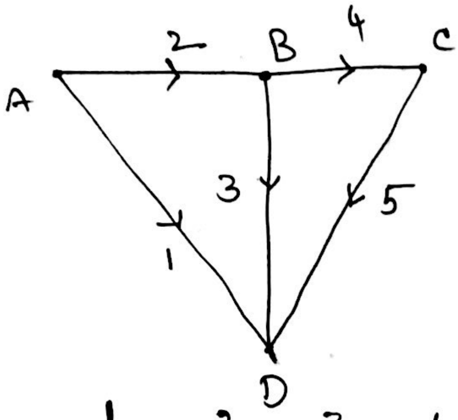
No of trees is given by

$$\text{No. of trees of linear graph} = \det\{[A][A^{t}]\}$$

$[A]$ — Reduced incident matrix

$[A^{t}]$ - Transpose of reduced incident matrix.

1) for the given graph. Write a) Complete incidence matrix
b) Reduced " "
c) No of trees.



Sol:-

$$A_a = \begin{array}{c|ccccc|} & 1 & 2 & 3 & 4 & 5 \\ A & +1 & +1 & 0 & 0 & 0 \\ B & 0 & -1 & +1 & +1 & 0 \\ C & 0 & 0 & 0 & -1 & +1 \\ D & -1 & 0 & -1 & 0 & -1 \end{array}$$

← Branches

↑ Nodes

In above matrix, 4th row is negative sum of first three rows. Hence it can be eliminated.

$$A = \begin{array}{c|ccccc|} & 1 & 2 & 3 & 4 & 5 \\ A & +1 & +1 & 0 & 0 & 0 \\ B & 0 & -1 & +1 & +1 & 0 \\ C & 0 & 0 & 0 & -1 & +1 \end{array}_{3\times5}$$

← Branches

↑ Nodes

Scanned by CamScanner

$$A^T = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}_{5\times3}$$

No of possible trees = $\det\{[A][A^T]\}$

$$[A][A^T] = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

$$\det\{[A][A^T]\} = 2\{6-(1)\} - (-1)\{-2-0\} - 0\{1-0\}$$

$$= 2\times5 + 1(-2) = 10 - 2 = 8.$$

$\therefore$ No of trees = $\underline{8}$.

2) The reduced incident matrix is given. a) calculate No of trees. (b) Write complete incident matrix (c) Draw the graph.

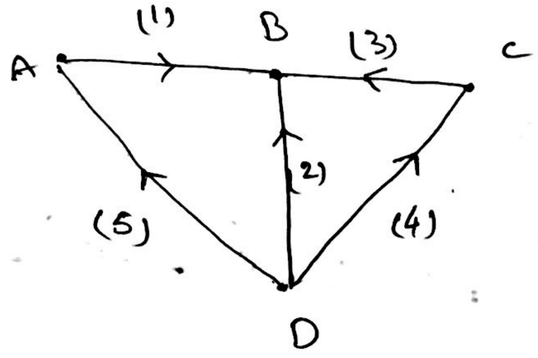
$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

sol:- Given $$A = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

Construct 4th row to obtain complete incidence matrix.

$$A_a = \begin{array}{c|ccccc|c} & 1 & 2 & 3 & 4 & 5 \leftarrow \text{Branches} & \\ \hline & 1 & 0 & 0 & 0 & -1 & A \\ & -1 & -1 & -1 & 0 & 0 & B \\ & 0 & 0 & 1 & -1 & 0 & C \\ & 0 & 1 & 0 & 1 & 1 & D \leftarrow \text{Nodes} \end{array}$$

Graph:-



NO of trees = $\det\{[A][A^T]\}$.

$$[A][A^T] = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix}_{3\times5} \begin{bmatrix} 1 & -1 & 0 \\ 0 & -1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}_{5\times3}$$

$$= \begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

$$\det\left[[A][A^T]\right] = 2\{6-(1)\} - (-1)\{-2-(0)\} - 0\{1-0\}$$

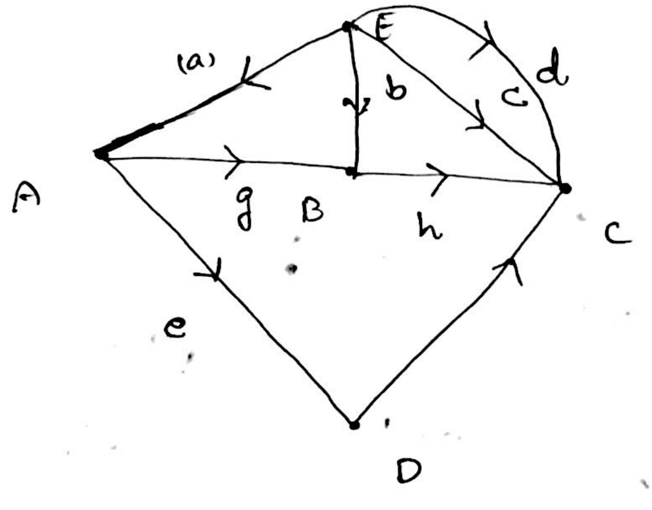
$$= 2\times5+1(-2) = 10-2 = 8$$

$\therefore$ No of trees = $\underline{\underline{8}}$

3) Draw the graph for the given incidence matrix. And also calculated

$$A_a = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{cccccccc|c} a & b & c & d & e & f & g & h & \\ -1 & 0 & 0 & 0 & +1 & 0 & +1 & 0 & A \\ 0 & -1 & 0 & 0 & 0 & 0 & -1 & +1 & B \\ 0 & 0 & -1 & -1 & 0 & -1 & 0 & -1 & C \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & D \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & E \end{array}$$

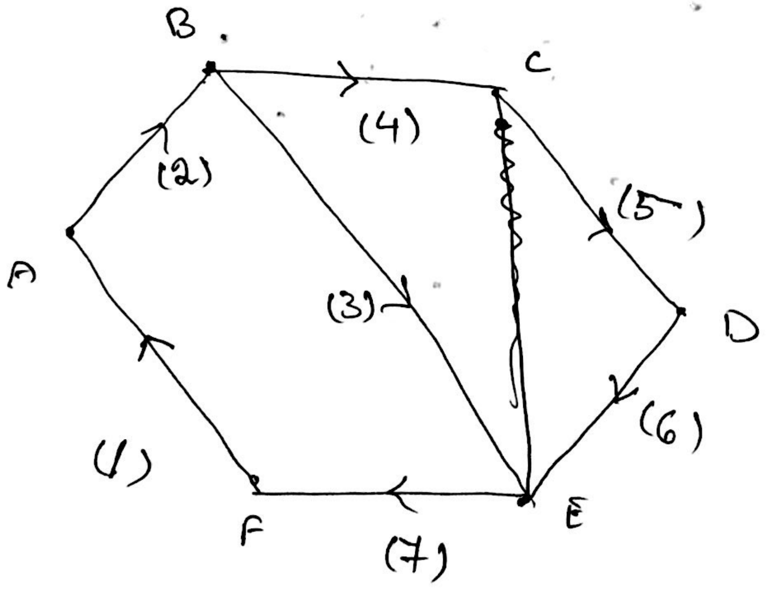
sol:-



4) Obtain the complete incidence matrix from Reduced inciden matrix & draw graph.

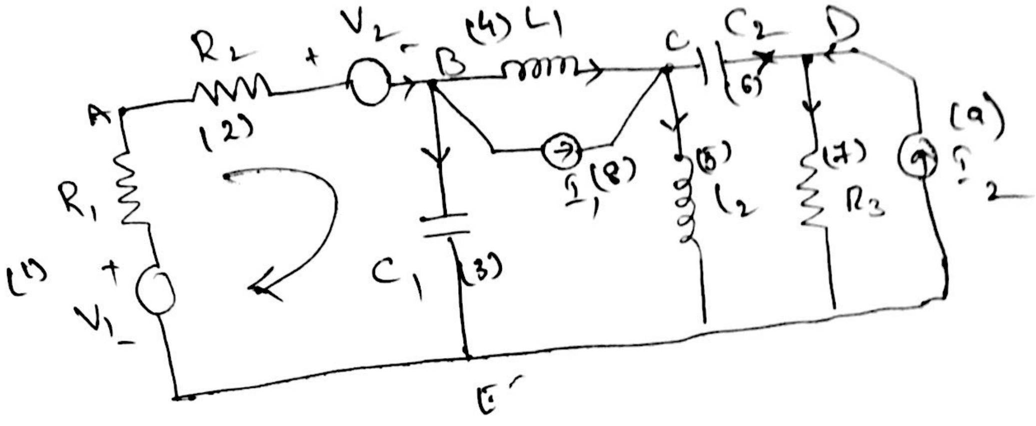
$$A = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 & 1 \end{bmatrix}$$

$$A_a = \begin{array}{c} \\ \\ \end{array} \begin{array}{ccccccc|c} 1 & 2 & 3 & 4 & 5 & 6 & 7 & \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & A \\ 0 & -1 & 1 & 1 & 0 & 0 & 0 & B \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & C \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & D \\ 0 & 0 & -1 & 0 & 0 & -1 & 1 & E \\ 1 & 0 & 0 & 0 & 0 & 0 & -1 & f \end{array}$$

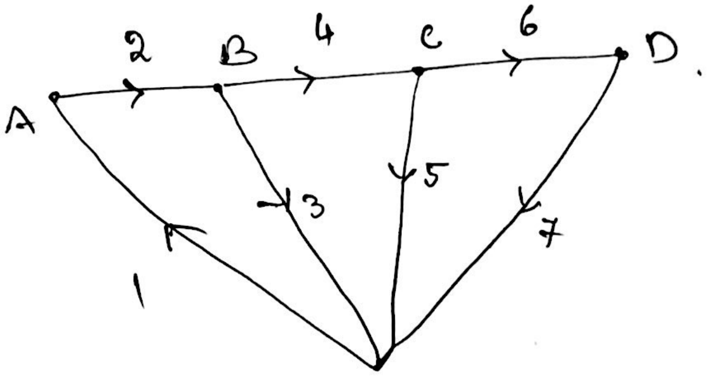


11 (10)

Tie-set Matrix(KS):- Draw oriented graph & write incidence matrix for the network shown in fig.



Oriented Graph:-



Complete Incidence Matrix:-

$$A_a = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \left[\begin{array}{ccccccc} -1 & +1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & +1 & +1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & +1 & +1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & +1 \\ 1 & 0 & -1 & 0 & -1 & 0 & -1 \end{array}\right] \end{array}$$

Tieset Matrix (or) fundamental loop (B):

for a given tree of a graph, addition of each link b/w any two nodes forms a loop called the 'fundamental loop'. In a loop there exists a closed path & the circulating current is called 'link current'. The current inv any branch of a graph can be found by using link currents.

* The fundamental loop formed by one link has a unique path in the tree joining the two nodes of the link. This loop is called f loop (or) Best tie set.

* A tie set is a set of branches which forms a closed loop. The tie set matrix is formed by taking the branches of a given graph as columns & fundamental loops forms as rows.

Steps involved in Tieset Matrix:-

1) Select a tree

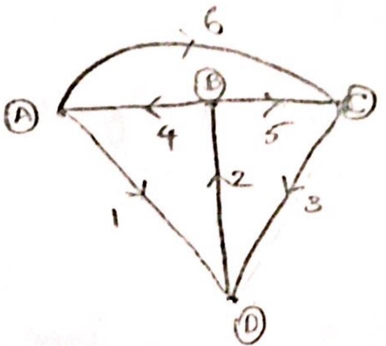
2) On adding a link to a tree at a time will result in the formation of one fundamental loop.

3) The direction of current in f-loop is selected as the direct of the link current.

4) The no. of fundamental loops formed will be equal to the no of links present..

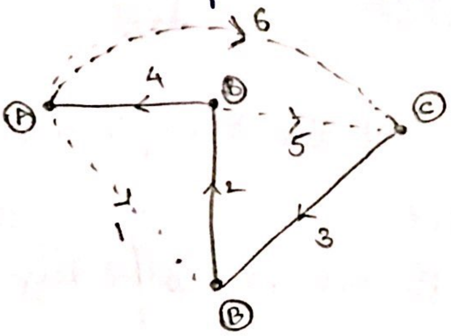
5) Apply KVL to the f-loops formed. These equations can be written in matrix form. The matrix obtained is called tie-set matrix.

Consider the following graph:



Step 1 :-

Consider a tree & represent the links



No. of nodes = 4

No of branches = 6

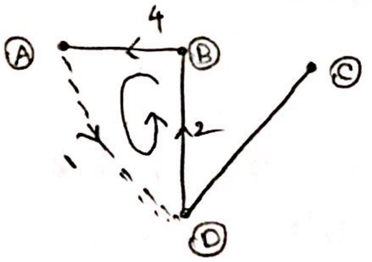
No. of twigs = n-1
= 4-1 = 3

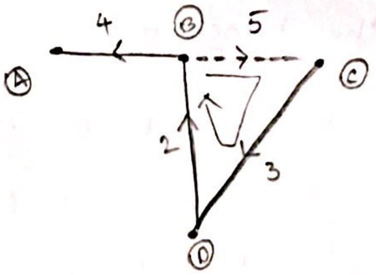
No. of links = 6-(n-1)
= 6-3 = 3

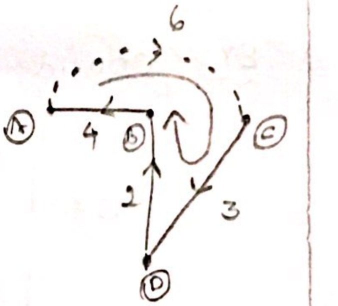
There are three twigs & three links.

No of f-loops will be three = No. of links

let branch voltages be $V_1, V_2, V_3 \dots V_6$. Writing KVL eqs for F-loops







$V_1 + V_2 + V_4 = 0 \rightarrow ①$

$V_5 + V_2 + V_3 = 0 \rightarrow ②$

$V_6 + V_3 + V_2 + V_4 = 0 \rightarrow ③$

Arranging these eq's in matrix form, we get tie set matrix

$$\begin{array}{c|cccccc|l} \text{floop} & 1 & 2 & 3 & 4 & 5 & 6 & \leftarrow \text{branches} \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & \\ 2 & 0 & 1 & 1 & 0 & 1 & 0 & \\ 3 & 0 & 1 & 1 & 1 & 0 & 1 & \end{array}_{3\times6} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix}_{6\times1} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}_{3\times1} \Rightarrow BV_b = 0$$

B = [ ]

The entries on the tieset matrix can also be made by the following rule.

$b_{ij} = 1$, if branch j is in f-loop i and their directions coincide

$b_{ij} = -1$, if branch j is in the f loop i and their current directions are opposite

$b_{ij} = 0$, if branch j is not in the f loop i.

Relation between branch currents and link currents:-

$$[I_b] = [B^T][I_L]$$

Where I_b & I_L are the branch current matrix and loop current matrix.

Ex:- Consider the tieset matrix

$$B = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix} \Rightarrow B^T = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[I_b] = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} \qquad [I_L] = \begin{bmatrix} I_1 \\ I_5 \\ I_6 \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_5 \\ I_6 \end{bmatrix} \Rightarrow$$

$i_1 = I_1$

$i_2 = I_1 + I_5 + I_6$

$i_3 = I_5 + I_6$

$i_4 = I_1 + I_6$

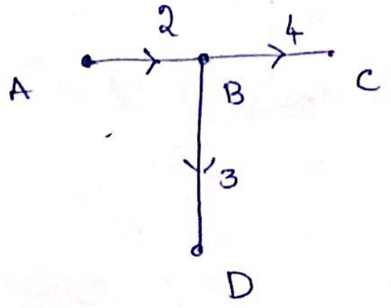
$i_5 = I_5$

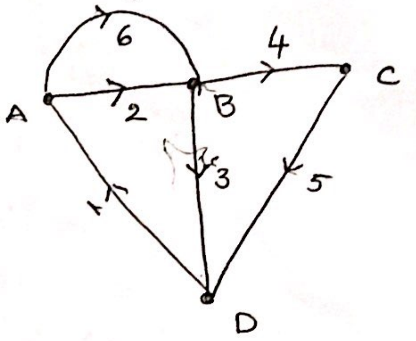
$i_6 = I_6$

Ex:-

Write the tieset schedule for the given graph

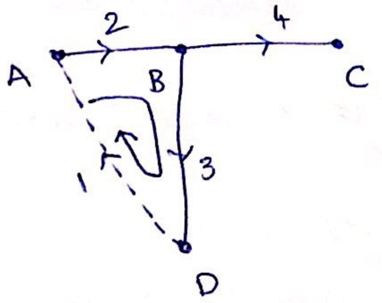
① Select a tree

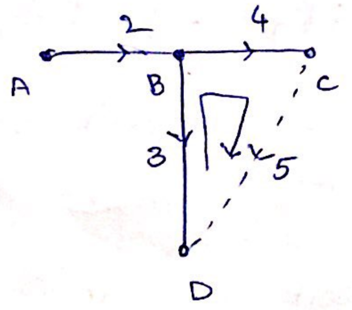


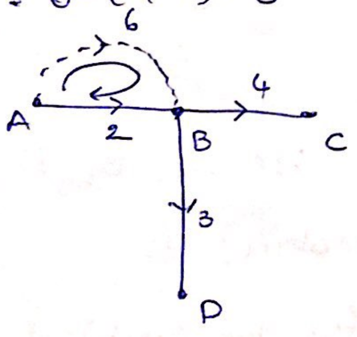


② No of fundemental loops = no of links = $b-(n-1)$

$= 6-(4-1) = 3$







f loop-1 : [1,2,3] f loop 2 [5,3,4] f loop 3 [6,2]

writing the information in the tabular column is called tieset schedule

f loop	Branches					
	1	2	3	4	5	6
1	1	1	1	0	0	0
2	0	0	-1	1	1	0
3	0	-1	0	0	0	1

$$B = \begin{array}{c|cccccc} \text{f-loop} \searrow & 1 & 2 & 3 & 4 & 5 & 6 \rightarrow \text{branches} \\ \hline 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 0 & 0 & -1 & 1 & 1 & 0 \\ 3 & 0 & -1 & 0 & 0 & 0 & 1 \end{array}$$

In general, matrix B is rearranged such that first columns corresponds to entries of links and then columns corresponding to entries of twigs as shown

$$B = \begin{array}{c|ccc:ccc} & 1 & 5 & 6 & 2 & 3 & 4 \\ \hline 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 2 & 0 & 1 & 0 & 0 & -1 & 1 \\ 3 & 0 & 0 & 1 & -1 & 0 & 0 \end{array}$$

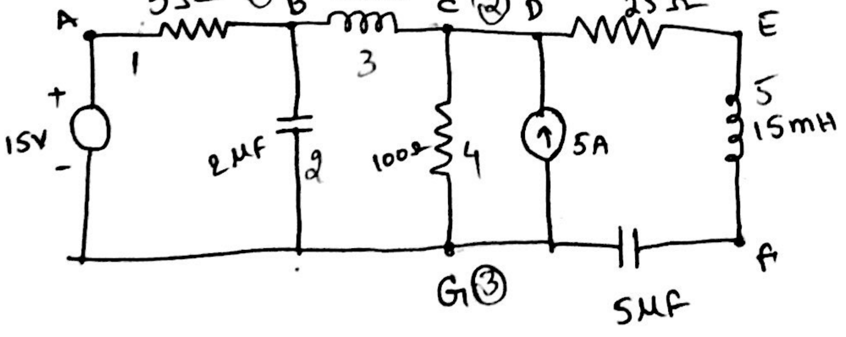
f loops $\underbrace{\qquad\qquad}_{\text{links}}$ $\underbrace{\qquad\qquad}_{\text{Twigs}}$

~~Twigs~~

$$\Rightarrow \quad B = [B_L : B_T] = [U : B_T]$$

Where B_L is the link matrix which is always identity matrix, B_T is called Twig matrix.

* for the electrical network shown in figure, draw its topological graph and write the incidence matrix, tieset matrix, link current transformation equation & branch currents.



5) The cutset is oriented in the same direction as twig

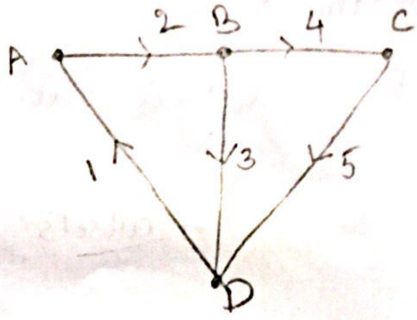
6) Assign +1 in matrix if direction of cutset & branch are same

7) Assign −1 in matrix if direction of cutset & branch are not same.

8) Assign 0, if cutset do not cover the branch.

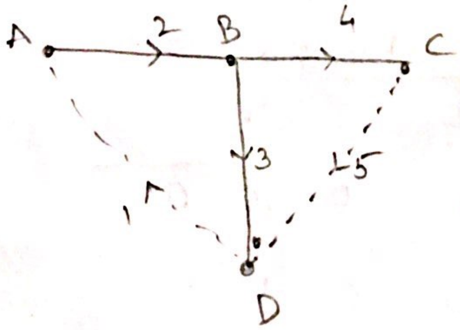
9) Applying KCL to the cutsets. These eq's can be written in matrix form and the matrix is called cutset matrix

Example :- Consider the graph.



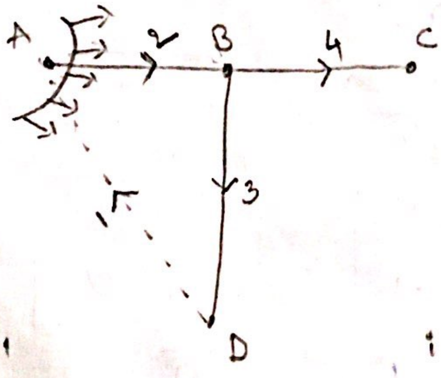
Graph

* Select a tree



Tree & cotree

* No of cutsets = no of twigs = $n-1 = 4-1 = 3$

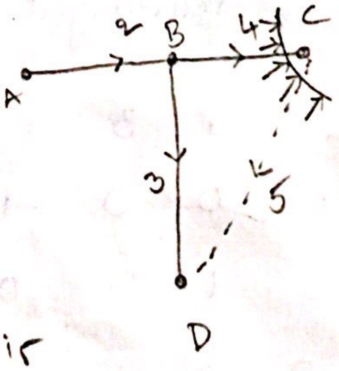


$i_2 = i_1$

$i_2 - i_1 = 0$

Cutset1

$i_2 - i_1 = 0 \rightarrow$ ①

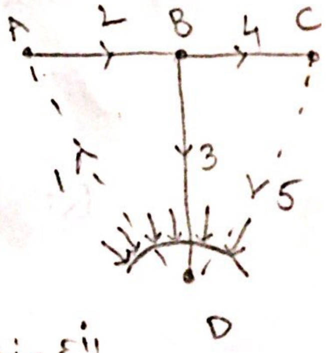


$i_4 = i_5$

$i_4 - i_5 = 0$

Cutset 2

$i_4 - i_5 = 0 \rightarrow$ ②



$i_3 + i_5 = i_1$

cutset 3

$\Rightarrow i_3 + i_5 - i_1 = 0$

$i_3 + i_5 - i_1 = 0 \rightarrow$ ③

*Arranging these eq's in form of matrix, we get cutset matrix

$$\begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \leftarrow \text{branches} \end{array} \\ \begin{array}{r} \text{cutset } 1 \\ 2 \\ 3 \end{array} & \left[\begin{array}{ccccc} -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ -1 & 0 & 1 & 0 & 1 \end{array}\right]_{3\times5} \end{array} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix}_{5\times1} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}_{3\times1}$$

$$Q I_b = 0$$

* The information about fundamental cutsets in the tabular form called cutset schedule.

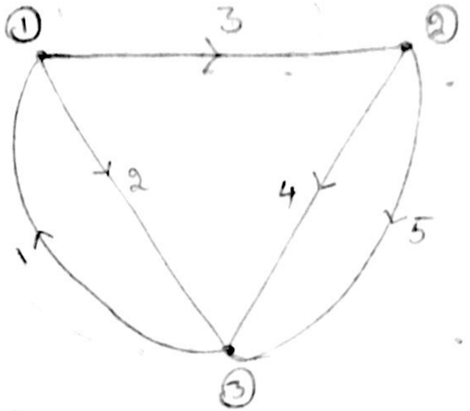
f cutsets	Branches				
	1	2	3	4	5
1	-1	1	0	0	0
2	0	0	0	1	-1
3	-1	0	1	0	1

* cutset matrix is rearranged as

$$Q = \begin{array}{cc} & \begin{array}{ccc|cc} 2 & 4 & 3 & 1 & 5 \end{array} \\ \begin{array}{r} 1 \\ 2 \\ 3 \end{array} & \left[\begin{array}{ccc|cc} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 & 1 \end{array}\right] \\ & \begin{array}{ccc|cc} & \text{Twigs} & & \text{links} & \end{array} \\ & \begin{array}{ccc} Q_T & : & Q_L \end{array} \end{array} = [Q_T : Q_L]$$

$$[U : Q_L]$$

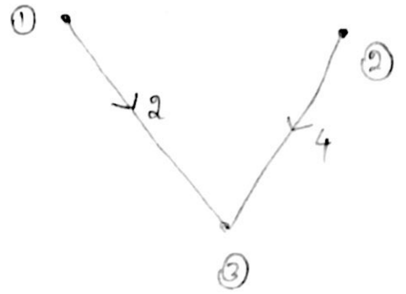
sol:- The simplified graph consists of three nodes.



No of nodes = 3

No. of branches = 5

Select a tree :-

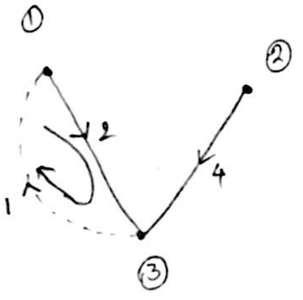


no. of f loops = no of links

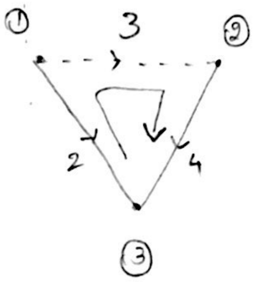
= b − (n−1)

= 5 − (3−1) = 3

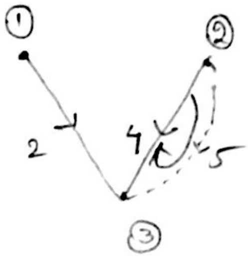
f loops :-



f loop ①



f loop ②



f loop ③

Tieset matrix, B =

$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \leftarrow \text{Branches} \\ \hline 1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 0 & -1 & 1 & 1 & 0 \\ 3 & 0 & 0 & 0 & -1 & 1 \end{array}$$

↗ f loop

link current transformation:- $[I_B] = [B_T][I_L]$

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} \Rightarrow \begin{array}{l} i_1 = I_1 \\ i_2 = I_1 - I_2 \\ i_3 = I_2 \\ i_4 = I_2 - I_3 \\ i_5 = I_3 \end{array}$$

Incidence matrix :-

$$A_a = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \rightarrow \text{branches} \\ \left[\begin{array}{ccccc} -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 \\ 1 & -1 & 0 & -1 & -1 \end{array}\right]_{3\times5} \end{array}$$

↑ nodes

Reduced Incidence matrix :-

$$A = \left[\begin{array}{ccccc} -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 \end{array}\right]_{2\times5}$$

Cutset Matrix :-

* The cutset is a set of branches whose removal cuts the graph into two parts.
* A Basic cutset is a set of branches that has only one twig in its set and any no of links.
* The no of cutsets are equal to no. of twigs (n-1)
* The direction of cutset should be taken along the direction of twig it is cutting.

Procedure :-

1) Select a tree and cotree from the graph.
2) Determine no of twigs (tree branches)
3) No of cutsets equal to the no of twigs.
4) Cutset can contain any no of links but only one twig

Relation between tree branch voltages & f cutset (branch voltages :-

$$[V_b] = [Q^T][V_t]$$

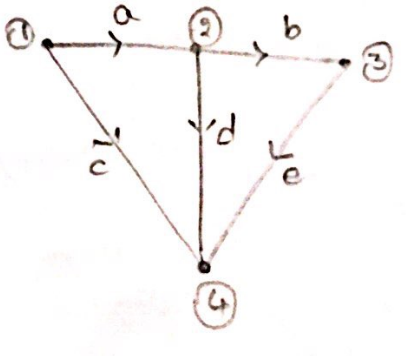
where V_b is the column matrix of branch voltages & V_t is the column matrix of twig voltages.

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}_{5\times 3} \begin{bmatrix} V_2 \\ V_4 \\ V_3 \end{bmatrix}$$

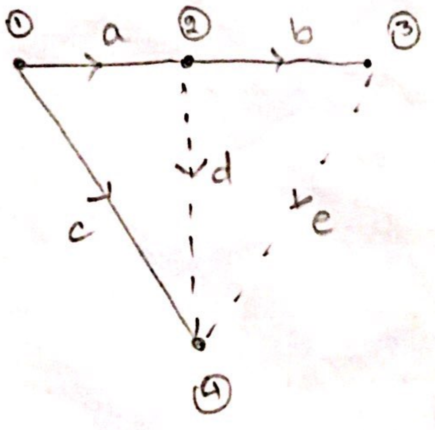
$\Rightarrow$ $V_1 = -V_2 + V_3$; $V_2 = V_2$; $V_3 = V_3$; $V_4 = V_4$; $V_5 = -V_4 + V_3$

Ex:-

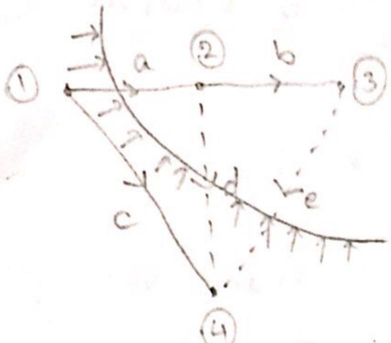
Write the cutset matrix for the directed graph



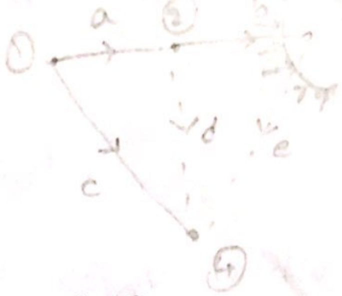
Select a tree



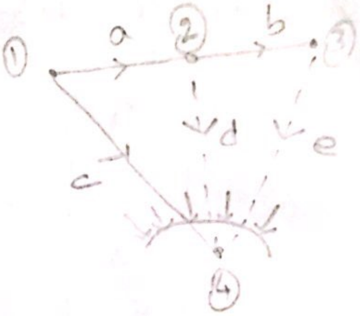
No of cutsets = no of twigs = n-1 = 4-1 = 3



f cut-set ①



f cutset ②



f cutset ③

$$Q = \left[\begin{array}{c|ccc:cc} & a & b & c & d & e \\ \hline 1 & 1 & 0 & 0 & -1 & -1 \\ 2 & 0 & 1 & 0 & 0 & -1 \\ 3 & 0 & 0 & 1 & 1 & 1 \end{array}\right]$$

Q_T $\qquad$ Q_L

current equations are

$i_a - i_d - i_e = 0$; $i_b - i_e = 0$; $i_c + i_d + i_e = 0$

Magnet :-

It is a piece of solid body which possess a property of attracting iron pieces & pieces of other metals. This is called natural magnet.

Magnetic Induction :-

~~Any current~~ The phenomenon due to which a magnet can induce magnetism in a iron (or) steel piece of magnetic material placed near to it is called magnetic induction.

Laws of magnetism :-

Law1:- It states that "like magnetic poles repel & unlike poles attract each other."

Law2:- The force exerted by one pole on the other pole is

a) directly proportional to the product of pole strengths.

b) inversly proportional to the square of the distance b/w them &

c) nature of medium surrounding the poles.

Mathematically,

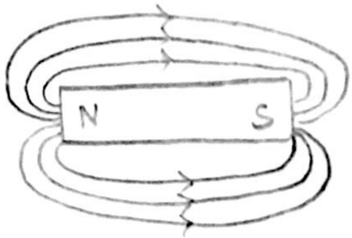
$$f \propto \frac{M_1M_2}{\mu d^2}$$

Where M_1 & M_2 are pole strengths of the poles & d is the distance b/w poles

$$f = \frac{kM_1M_2}{\mu d^2} = \frac{m_1m_2}{4\pi\mu d^2} \quad \therefore k = 1/4\pi$$

Where K depends on the nature of the sorroundings & called permeability.

Magnetic field :- The area around a magnet where the effect of the magnetic force produced by the magnet can be detected is called the magnetic field.



The no of lines of force existing in particular magnetic field is called "**magnetic flux**"(ϕ). units are weber (wb).

Magnetic field Intensity :- It is the amount of magnetising force. It is defined as the magnetic force experienced by a N-pole of one weber placed at a point within a magnetic field.

Let a pole of strength m webers lies in air. The magnetic field intensity (H) at a point 'p' at a distance 'd' meters from the pole is the force experienced by the one weber pole at the point.

w.k.t $$F = \frac{m_1 m_2}{4\pi\mu_o d^2} \quad (m = 1)$$

$$\Rightarrow \boxed{F = \frac{m \times 1}{4\pi\mu_o d^2}} \text{ Newtons}$$

& $$H = \frac{m}{4\pi\mu_o d^2} \; N/Wb$$

Magnetic flux Density :-

Magnetic flux density at a point is the amount of flux passing per unit cross sectional area.

$$\text{Magnetic flux density} = \frac{\text{Magnetic flux}}{\text{Area}}$$

$$\boxed{B = \frac{\phi}{A}} \ Wb/m^2$$

Permeability :- It is the property of the medium in which magnet is placed.

for any magnetic material there are two permeabilities

1) **Absolute Permeability** :-

The ratio of magnetic flux density (B) in particular medium to the magnetic field strength producing that flux density is called absolute permeability. Denoted by 'μ'.

$$\boxed{\mu = \frac{B}{H}} \Rightarrow B = \mu H \ (H/m)$$

If the magnet is placed in free space, then permeability is denoted by 'μ_0', which is given by

$$\boxed{\mu_0 = \frac{B_0}{H} = 4\pi \times 10^{-7}} \ H/m$$

2) **Relative Permeability** :- It is the ratio of flux density produced in a medium to the flux density produced in free space under same magnetic field strength (H).

$$\boxed{\mu_r = \frac{B}{B_0}}$$

But we know that

$$\mu = \frac{B}{H} \quad \& \quad \mu_0 = \frac{B_0}{H}$$

$$\Rightarrow \frac{\mu}{\mu_0} = \frac{B}{\cancel{H}} \Big/ \frac{B_0}{\cancel{H}} = \frac{B}{B_0}$$

$$\Rightarrow \boxed{\frac{\mu}{\mu_0} = \mu_r} \Rightarrow \boxed{\mu = \mu_0 \mu_r} \ H/m$$

Magnetic circuit :- It is a closed path containing a magnetic flux. It contains magnetic elements that have different permiability lengths & areas. It can also contain air gap & other materials.

Magnetomotive force :- It is the force behind the flow of flux in a magnetic current. It is caused by a current flowing through one (or) more turns.

If I is current flowing through a coil of N turns, then $\boxed{mmf = NI}$ Ampere-turns (AT)

If the mean length of magnetic circuit 'l' m & cross sectional area A m^2, then the magnetic field strength is given by

$$\boxed{H = \frac{NI}{l}} \ AT/m$$

Reluctance (S):- The opposition by the material to the flow of flux is called reluctance.

It is directly proportional to the length of the magnetic circuit & inversly propoitional to the area of cross-sec

$$S \propto \frac{l}{a} \Rightarrow S = Kl/a$$

Where K is constant which is reciprocal of permeability

$$\Rightarrow \boxed{S = \frac{l}{\mu a} = \frac{l}{\mu_0 \mu_r a}} \; A/Wb$$

It can also be expressed as ratio of mmf to the flux produced

$$\boxed{S = \frac{m.m.f}{\phi} = \frac{NI}{\phi}} \; AT/Wb$$

Permeance:- It is the property of the magnetic circuit due to which it allows the flow of flux through it. It is reciprocal of reluctance.

$$\boxed{\text{Permeance} = \frac{1}{\text{Reluctance}}} \; Wb/A$$

Magnetic Circuits:-

Series magnetic circuits:-

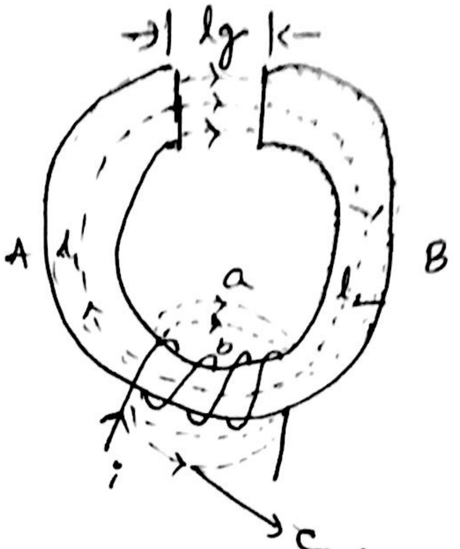
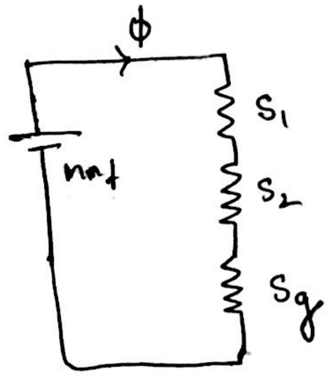


figure shows the composite magnetic circuits with two dibberent magnetic materials A & B, air gap in series.

Let l_1 is the length of magnetic material A & l_2 is length of B, l_g is length of air gap. Let A_1, A_2 & A_g are the cross sectional areas of magnetic materials A, B & air gap restively.

If μ_1 & μ_2 is the absolute permeability of A & B then the total reluctance of the circuit is given by sum of the individual reluctances.



Reluctance of A, $S_1 = \dfrac{l_1}{\mu_1 A_1}$

Reluctance of B, $S_2 = \dfrac{l_2}{\mu_2 A_2}$

Reluctance of air gap, $S_g = \dfrac{l_g}{\mu_0 A_g}$

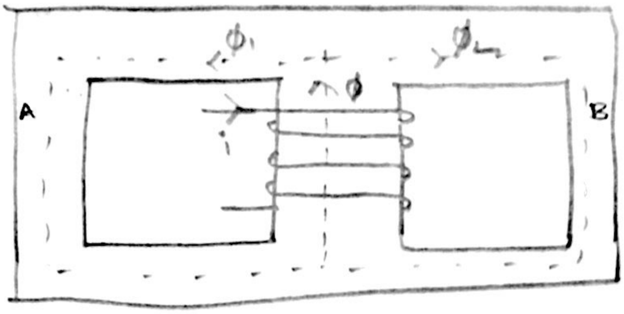
Total reluctance of magnet circuit is

$$S = \frac{l_1}{\mu_1 A_1} + \frac{l_2}{\mu_2 A_2} + \frac{l_g}{\mu_0 A_g}$$

$\Rightarrow$ Total flux $\phi = \dfrac{mmf}{S} \Rightarrow \phi = \dfrac{mmf}{S_1 + S_2 + S_g}$

$$\Rightarrow mmf = S_1\phi + S_2\phi + S_g\phi \Rightarrow \boxed{mmf = mmf_1 + mmf_2 + mmf_g}$$

Parallel Magnetic circuit :-



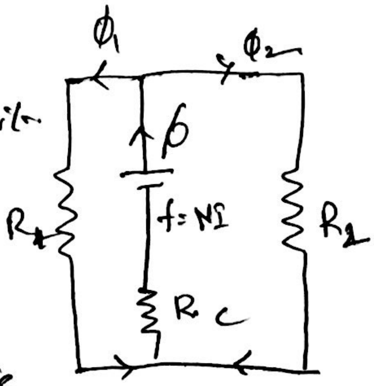
In series magnetic circuits, all the flux is confined to a single closed loop.

If the flux is divided so that part of it is confined to a portion of device & part to another, the magnetic circuit is called parallel magnetic circuit.

consider an iron core as shown in fig. The coil is wound which carries current 'i' & produces flux ϕ. This ϕ is divided. Some flux passes through limb A & remaining through limb B.

$$\Rightarrow \phi = \phi_1 + \phi_2$$

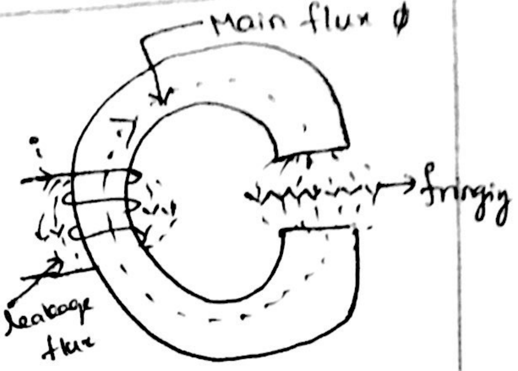
The equivalent electric circuit is shown in figure.



$$\text{Total flux} = \frac{mmf}{\text{Total reluctance}}$$

$$\boxed{\phi = \frac{mmf}{(S_1 \parallel S_2) + S_c}}$$

Magnetic leakage & fringing:-

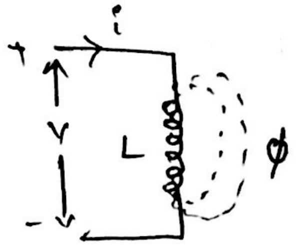


Fringing results due to air gap in magnetic circuits. It makes flux density differrent in air compared to core. Increase the area & decreases density of flux

i.e., $\phi_c = \phi_g$ But $B_c \neq B_g$

When 'i' current flows through the coil flux is produced. But all flux is not confined to ironcome, some flux leavs through air surrounding the coil. The flux which are not intended is known as leakage flux. leakage factor is given by $\boxed{\lambda = \frac{\text{Total flux}}{\text{useful flux}}}$

Self inductance & mutual inductance:-



When the current flows through the coil, magnetic flux is produced. If the no of turns in the coil is N, the flux linkage is given by

$$\boxed{\lambda = N\phi}$$

$N\phi = Li \Rightarrow L = \frac{N\phi}{i}$

In a linear inductor, the flux produced is proportional to current i, we have

$$\boxed{\lambda = Li}$$

Where the constant of proportionality 'L' is the inductance of the coil.

According to faraday's law, a voltage induced across the terminals of the coil is equal to rate of change of its flux linkage, λ. Therefore

$$V = \frac{d\lambda}{dt} = L\frac{di}{dt} \quad (8)$$

$$V = N\frac{d\phi}{dt}$$

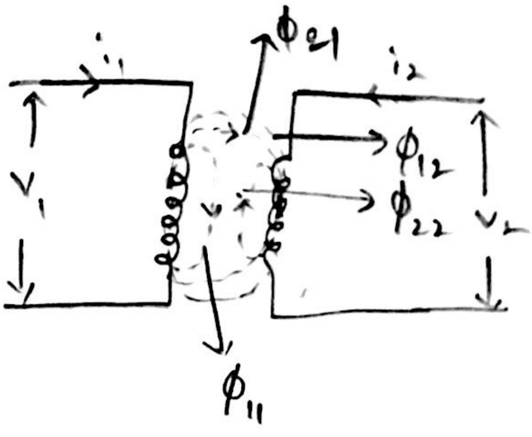
$$\Rightarrow \quad N\frac{d\phi}{dt} = L\frac{di}{dt}$$

$\Rightarrow L = N\frac{d\phi}{di}$ is the coebbicient of self inductance.

The property of the coil that opposes any change in the current flowing through it is called self inductance.

Mutual Inductance :-

If any other coil is placed adjacent to the existing coil, then the flux from coil ① cuts coil ②, inducing an emf in coil②.



According to lenz's law the emf induced in coil② sets up a flux that opposes the flux from coil① i.e., induced emf is the counter emf & the inductive effect is referred as mutual inductance.

Consider two coils with inductance L_1 & L_2 placed together. Let currents are i_1 & i_2. The current i_1 in coil ① produces a flux ϕ_1; part of which links with coil ① & remaining flux links with coil②

let flux linking with coil ① is ϕ_{11} & with coil② is ϕ_{21}

$$\Rightarrow \boxed{\phi_1 = \phi_{11} + \phi_{21}}$$

Similarly when i_2 flows through coil②, flux is given by $\boxed{\phi_2 = \phi_{12} + \phi_{22}}$

The emf induced in coil ① is due to ϕ_{11} & ϕ_{21}.

Total flux linkage is given by

$$\phi_{L1} = \phi_{11} + \phi_{12}$$

The net flux linkage is given by

$$\lambda = N_1\phi_{L1} = N_1\phi_{11} + N_1\phi_{12}$$

But we have flux linkage ϕ_1 in coil is due to its own current i_1, $\boxed{N_1 \phi_{11} = L_1 i_1}$

L_1 = self inductance

lly $\boxed{N_1 \phi_{12} = \pm M_{12} i_2}$ linkage flux in ① due to i_2

A/c faraday's law, emf induced in coil ① is

$$V_1 = \frac{d\lambda_1}{dt} = \frac{d}{dt}(N_1 \phi_{L1})$$

$$= \frac{d}{dt}\left[N_1 \phi_{11} + N_1 \phi_{12}\right]$$

$$= \frac{d}{dt}\{L_1 i_1 \pm M_{12} i_2)$$

$$V_1 = L_1 \frac{di_1}{dt} \pm M_{12} \frac{di_2}{dt}$$

lly for coil ② :

$$V_2 = L_2 \frac{di_2}{dt} \pm M_{21} \frac{di_1}{dt}$$

$$\lambda_2 = N_2 \phi_{L2}$$

$$\Rightarrow \lambda_2 = \cancel{N_2}\left[N_2 \phi_{21} + N_2 \phi_{22}\right]$$

But, $N_2 \phi_{21} = M_{21} i_1$ & $N_2 \phi_{22} = L_2 i_2$.

$\Rightarrow$ ~~$d\lambda_2 =$~~ The emf induced in coil ② is

$$V_2 = \frac{d\lambda_2}{dt} = \frac{d}{dt}\left[N_2 \phi_{21} + N_2 \phi_{22}\right]$$

$$= \frac{d}{dt}\left[\pm M_{21} i_1 + L_2 i_2\right]$$

$$= \pm M_{21} \frac{di_1}{dt} + L_2 \frac{di_2}{dt} = L_2 \frac{di_2}{dt} \pm M_{21} \frac{di_1}{dt}$$

If $M_{12} = M_{21} = M$,

$$\boxed{V_1 = L_1 \frac{di_1}{dt} \pm M \frac{di_2}{dt}} \; ; \; \boxed{V_2 = L_2 \frac{di_2}{dt} \pm M \frac{di_1}{dt}}$$

Coebbicient of coupling:-

In a coupled circuit, the ratio between the flux linking other coil to the total flux is known as coebbicient of coupling. Denoted by K.

$$K = \frac{\phi_{21}}{\phi_1} = \frac{\phi_{12}}{\phi_2}$$

K is maximum if $\phi_{21} = \phi_1$ & $\phi_{12} = \phi_2$; Max value of $K = 1$

we have

$$M = \frac{N_1 \phi_{12}}{i_2} \; \& \; M = \frac{N_2 \phi_{21}}{i_1}$$

$$\Rightarrow M^2 = \frac{N_1 \phi_{12}}{i_2} \times \frac{N_2 \phi_{21}}{i_1}$$

$$= \frac{N_1 K \phi_2}{i_2} \times \frac{N_2 K \phi_1}{i_1}$$

$$= K^2 \cdot \frac{N_1 \phi_1}{i_1} \cdot \frac{N_2 \phi_2}{i_2}$$

$$\Rightarrow M^2 = K^2 L_1 L_2$$

$$\Rightarrow \boxed{M = K\sqrt{L_1 L_2}}$$

Dot Convention :-

The sign of mutually induced voltage depends on direction of winding of the coils. But it is very inconvenien to supply the information about winding direction of the coils. Hence dot conventions are used for purpose of indicating direction of windings.

The dot conventions are interpreted as follows.

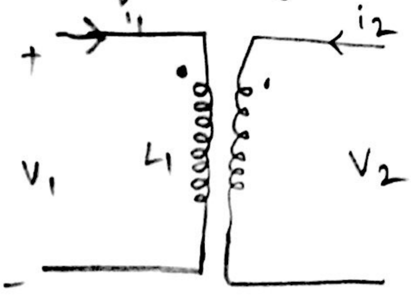
* If a positive currents enters into the dots of both the coils (or) out of dots of both the coils, then mutually induced voltages for both the coils add to the self induced voltages. Hence mutually induced voltages will have same polarity as that of self induced voltages.

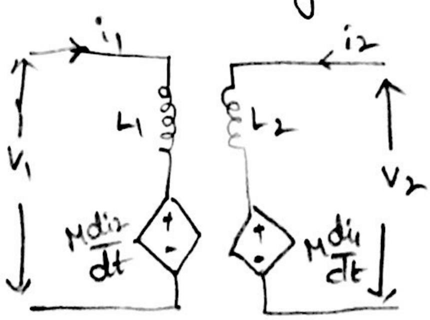
* If a positive current enters into (or out of) the dot in one coil & in other coil current flows out of (or into the dot, then mutually induced voltages will have polarity opposite to that of self induced voltages.

In simple words it can be given as:-

1) If a current enters a dot in one coil, then mutually induced voltage in other coil is positive at the dotted end.

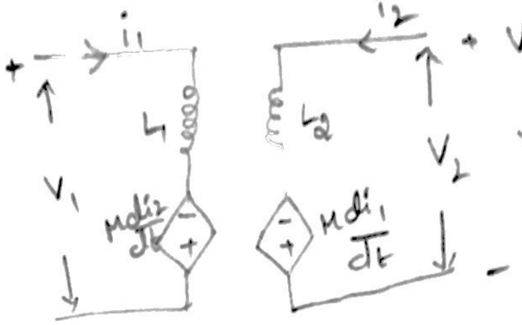
2) If a current leaves a dot in the coil, then mutually induced voltage in other coil is negative at the dotted end.





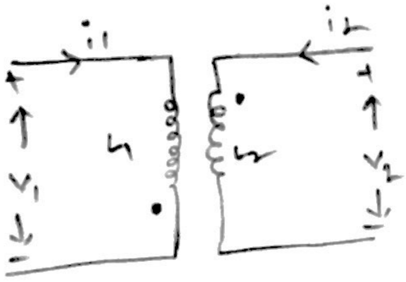
$V_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$

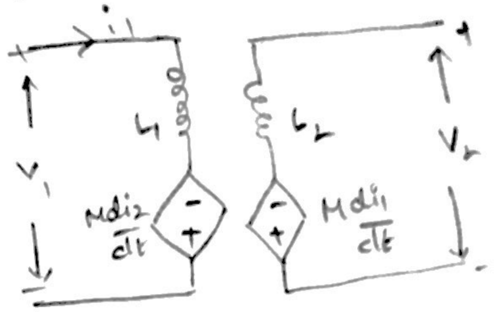
$V_2 : L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$



$$V_1 = L_1\frac{di_1}{dt} - M\frac{di_2}{dt}$$

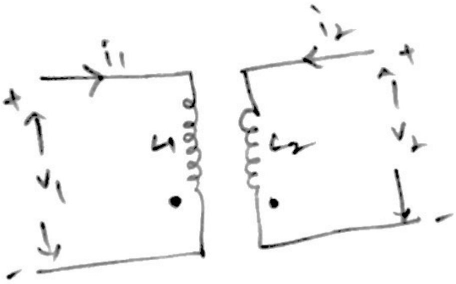
$$V_2 = L_2\frac{di_2}{dt} - M\frac{di_1}{dt}$$

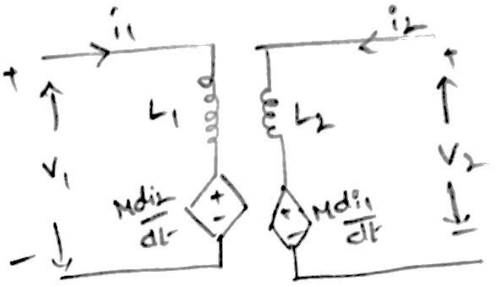




$$V_1 = L_1\frac{di_1}{dt} - M\frac{di_2}{dt}$$

$$V_2 = L_2\frac{di_2}{dt} - M\frac{di_1}{dt}$$





$$V_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$V_2 = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

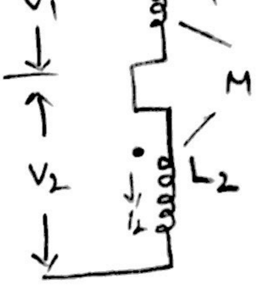
* The analysis of multiwinding inductor n/w's can be carried out for each pair of windings using same dot convention. The symbols used for representing diff pairs are □, Δ, •, ★ etc

Inductive coupling in series :-

when two inductors having self inductances L_1 & L_2 are coupled in series, ~~such that the~~ mutual inductance M exists between them. Two kinds of series connection are possible.

Series Aiding :-

In this connection, two coils are connected in series such that their induced fluxes (or) voltages are addition in nature.



* Here i_1 & I_2 are nothing but current's which are entering at dots for both the coils.

Self induced voltage in coil ① is $v_1 = L_1 \frac{di_1}{dt}$

Self induced voltage in coil ② is $v_2 = L_2 \frac{di_2}{dt}$

Mutually induced voltage in coil ① due to change in coil ② is

$$v_1' = M \frac{di_2}{dt}$$

Mutually induced voltage in coil ② due to change in coil ① is

$$v_2' = M \frac{di_1}{dt}$$

∴ Total induced voltage $= v_1 + v_2 + v_1' + v_2'$

$$V = L_1 \frac{di_1}{dt} + L_2 \frac{di_2}{dt} + M \frac{di_2}{dt} + M \frac{di_1}{dt}$$

In series as same current flows, $i_1 = i_2 = i$

$$\Rightarrow V = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + M \frac{di}{dt} + M \frac{di}{dt}$$

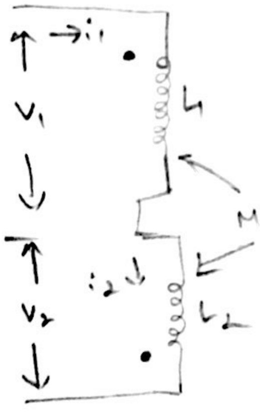
$$\Rightarrow V = \frac{di}{dt} \left[L_1 + L_2 + 2M \right]$$

$$\boxed{L_{eff} = L_1 + L_2 + 2M}$$

Series opposing:-

* In this connection, two coils are connected in such a way that, the induced fluxes (or) voltages are of opposite polarities.

* Here i_1 & i_2 are in series in which current is entering at dot in coil ① & leaving at dot in coil ②



self induced voltage in coil ①, $v_1 = L_1 \frac{di_1}{dt}$

self induced voltage in coil②, $v_2 = L_2 \frac{di_2}{dt}$

Mutually induced voltage in coil① due to change in coil②,

$$v_1' = -M\frac{di_2}{dt}$$

Mutually induced voltage in coil② due to change in coil①,

$$v_2' = -M\frac{di_1}{dt}$$

∴ Total induced voltage $V = v_1+v_2+v_1'+v_2'$

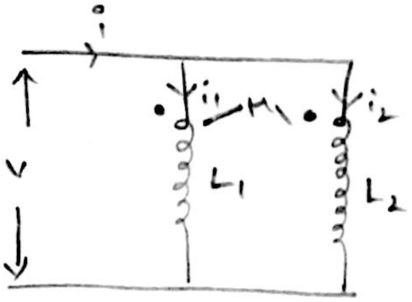
$$\Rightarrow V = L_1\frac{di_1}{dt}+L_2\frac{di_2}{dt}-M\frac{di_2}{dt}-M\frac{di_1}{dt} \qquad (\because i_1=i_2=i)$$

$$\Rightarrow v = \frac{di}{dt}(L_1+L_2-2M)$$

$$\boxed{L_{eff} = L_1+L_2-2M}$$

Inductive coupling in parallel :-

Parallel Aiding:-



In parallel magnetic circuits, there will be same voltage across all parallel branches.

$$\boxed{V = V_1 = V_2}$$

The total voltage across L_1 is given by

$$v_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt} \longrightarrow ①$$

lly across L_2 is

$$v_2 = L_2\frac{di_2}{dt} + M\frac{di_1}{dt} \longrightarrow ②.$$

$$L_1\frac{di_1}{dt}+M\frac{di_2}{dt}=L_2\frac{di_2}{dt}+M\frac{di_1}{dt}$$

$$\Rightarrow \frac{di_1}{dt}(L_1-M)=\frac{di_2}{dt}(L_2-M)$$

$$\Rightarrow \frac{di_1}{dt}=\frac{di_2}{dt}\left(\frac{L_2-M}{L_1-M}\right)$$

We know that $i=i_1+i_2$

$$\Rightarrow \frac{di}{dt}=\frac{di_1}{dt}+\frac{di_2}{dt}$$

$$\Rightarrow \frac{di}{dt}=\frac{di_2}{dt}\left[\frac{L_2-M}{L_1-M}+1\right]$$

$$\Rightarrow \frac{di}{dt}=\frac{di_2}{dt}\left[\frac{L_2+L_1-2M}{L_1-M}\right]$$

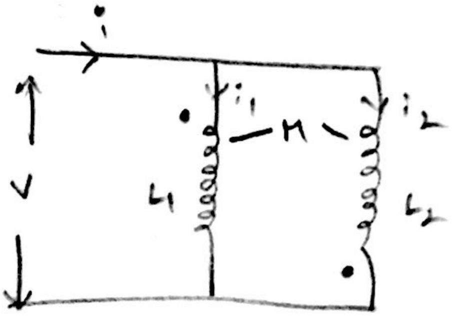
lly total voltage $V=L_{eq}\frac{di}{dt}=V_1=V_2$

$$\Rightarrow L_{eq}\frac{di_2}{dt}\left[\frac{L_2+L_1-2M}{L_1-M}\right]=L_1\frac{di_1}{dt}+M\frac{di_2}{dt}$$

$$\Rightarrow L_{eq}\frac{di_2}{dt}\left[\frac{L_1+L_2-2M}{L_1-M}\right]=L_1\left[\frac{L_2-M}{L_1-M}\right]\frac{di_2}{dt}+M\frac{di_2}{dt}$$

$$\Rightarrow L_{eq}\cancel{\frac{di_2}{dt}}\left[\frac{L_1+L_2-2M}{\cancel{L_1-M}}\right]=\cancel{\frac{di_2}{dt}}\left[\frac{L_1L_2-M\cancel{L_1}+M\cancel{L_1}-M^2}{\cancel{L_1-M}}\right]$$

$$\Rightarrow \boxed{L_{eq}=\frac{L_1L_2-M^2}{L_1+L_2-2M}}$$



Parallel Opposing:-

$$\boxed{L_{eq}=\frac{L_1L_2-M^2}{L_1+L_2+2M}}$$

Ideal transformer :-

Transfer of energy from one circuit to another circuit through mutual inductance is widely used in power systems. This is done by transformer.

An ideal transformer is characterised by assuming

- No power dissipation in primary & secondary windings (R=0)
- self inductance of primary & secondary are extremely large in comparision with load impedence
- Coefficient of coupling is unity.
- NO leakage flux & no iron losses.

we know that

$$v = L\frac{di}{dt}$$

$$\Rightarrow L\frac{di}{dt} = N\frac{d\phi}{dt}$$

$$\Rightarrow L = N\frac{d\phi}{di}$$

But $\phi = \frac{Ni}{S}$

$$\Rightarrow L = N.\frac{d}{di}\left(\frac{Ni}{S}\right)$$

$$\Rightarrow L = N.\frac{N}{S}.\frac{di}{di}$$

$$\Rightarrow L = \frac{N^2}{S} \Rightarrow L \propto N^2$$

Use of transformer for voltage level adjustment :-

If the magnetic permeability of core is infinitely large then flux will be confined to the core. If ϕ is the flux in single turn on the core & N_1, N_2 are the no of turns of primary & secondary, then total flux through windings ① & ② are given as

$$\phi_1 = N_1 \phi \; ; \; \phi_2 = N_2 \phi$$

w.k.t
$$V_1 = \frac{d\phi_1}{dt} = N_1 \frac{d\phi}{dt}$$

$$V_2 = \frac{d\phi_2}{dt} = N_2 \frac{d\phi}{dt}$$

$$\Rightarrow \frac{V_2}{V_1} = \frac{N_2 \frac{d\phi}{dt}}{N_1 \frac{d\phi}{dt}} \Rightarrow \boxed{\frac{V_2}{V_1} = \frac{N_2}{N_1} = a}$$

Relation b/w currents in primary & secondary windings:-

from Eq ②, we have

$$I_1 j\omega M = I_2 (Z_L + j\omega L_2)$$

$$\Rightarrow \frac{I_2}{I_1} = \frac{j\omega M}{Z_L + j\omega L_2}$$

As L_2 tends to infinity ; $Z_L + j\omega L_2 = j\omega L_2$

$$\Rightarrow \frac{I_2}{I_1} = \frac{j\omega M}{j\omega L_2} \Rightarrow \frac{I_2}{I_1} = \frac{M}{L_2} = \frac{\sqrt{L_1 L_2}}{L_2} = \sqrt{\frac{L_1}{L_2}}$$

$$\Rightarrow \frac{I_2}{I_1} = \sqrt{1/a^2} \Rightarrow \boxed{\frac{I_2}{I_1} = \frac{1}{a}} = \frac{N_1}{N_2}$$

$$\Rightarrow \boxed{N_1 I_1 = N_2 I_2}$$

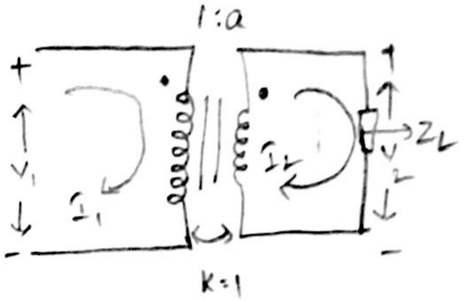
Ideal Transformer:-

An ideal transformer is one that has

- No winding resistance
- No leakage flux
- No iron losses

It is very tightly coupled transformer in which the coupling coefficient is essentially unity.

Turns ratio of an ideal transformer:-



The self inductance of the coil is proportional to the square of the number of wire forming the coil.

$$L \propto N^2 \Rightarrow \frac{L_2}{L_1} = \frac{N_2^2}{N_1^2} = a^2 \longrightarrow \text{ⓐ}$$

$$\Rightarrow \boxed{a = \frac{N_2}{N_1}}$$

figure shows an ideal transformer to which a secondary load is connected. It has an unity value of coupling coefficient & 1:a ratio of N_1 & N_2

For analysing the ideal transformer, mesh equations are given as

$$V_1 = j\omega L_1 I_1 - j\omega M I_2 \longrightarrow ①$$

$$\Rightarrow 0 = -j\omega M I_1 + j\omega L_2 I_2 + Z_L I_2 \longrightarrow ②$$

$$\Rightarrow I_2 = \frac{j\omega M I_1}{(j\omega L_2 + Z_L)} \longrightarrow ③$$

By substituting ③ in ①, we can determine the valu of input impedence of an ideal transformer

$$\Rightarrow V_1 = j\omega L_1 I_1 - j\omega M\left[\frac{j\omega M I_1}{j\omega L_2 + Z_L}\right]$$

$$\Rightarrow V_1 = j\omega L_1 I_1 + I_1\left(\frac{\omega^2 M^2}{j\omega L_2 + Z_L}\right)$$

$$\Rightarrow \frac{V_1}{I_1} = j\omega L_1 + \frac{\omega^2 M^2}{j\omega L_2 + Z_L}$$

$$\Rightarrow \boxed{Z_{in} = j\omega L_1 + \frac{\omega^2 M^2}{j\omega L_2 + Z_L}}$$

Since $K = 1$; $M = K\sqrt{L_1 L_2} \Rightarrow M^2 = L_1 L_2$

$$\Rightarrow Z_{in} = j\omega L_1 + \frac{\omega^2 L_1 L_2}{Z_L + j\omega L_2}$$

from Eq ⓐ, we have $\frac{L_2}{L_1} = a^2 \Rightarrow L_2 = a^2 L_1$

$$\Rightarrow Z_{in} = j\omega L_1 + \frac{\omega^2 a^2 L_1^2}{Z_L + j\omega a^2 L_1}$$

$$\Rightarrow Z_{in} = \frac{j\omega L_1 Z_L - \cancel{\omega^2 a^2 L_1^2} + \cancel{\omega^2 a^2 L_1^2}}{Z_L + j\omega a^2 L_1}$$

$$\Rightarrow Z_{in} = \frac{j\omega L_1 Z_L}{Z_L + j\omega a^2 L_1}$$

$$\Rightarrow \boxed{Z_{in} = \frac{Z_L}{Z_L/j\omega L_1 + a^2}}$$

An ideal transformer will always have high impedence for both primary & secondary coils.

i.e., $L_1 \to \propto$; $1/L_1 \to 0$

$$\Rightarrow \boxed{Z_{in} = \frac{Z_L}{a^2}} \longrightarrow \text{Impedence matching}$$

The input impedence is proportional to the load impedence, the proportionality constant is reciprocal of the square of the turns ratio.

In a pair of coupled coils coil ① has a continuous current of 2A, and the corresponding fluxes ϕ_{11} & ϕ_{21} are 0.3 & 0.6 mwb. If $N_1 = 500$ & $N_2 = 1500$. find L_1, L_2, M & K.

Given:- $N_1 = 500$; $N_2 = 1500$, $I_1 = 2A$

$\phi_{11} = 0.3m$; $\phi_{21} = 0.6 mwb$

Total flux

$$\Rightarrow \phi_1 = \phi_{11} + \phi_{21} = 0.3 + 0.6 = 0.9 mwb$$

Self inductance of coil ① is

$$L_1 = \frac{N_1 \phi_1}{i_1} = \frac{500 \times 0.9 \times 10^{-3}}{2} = 0.225 H$$

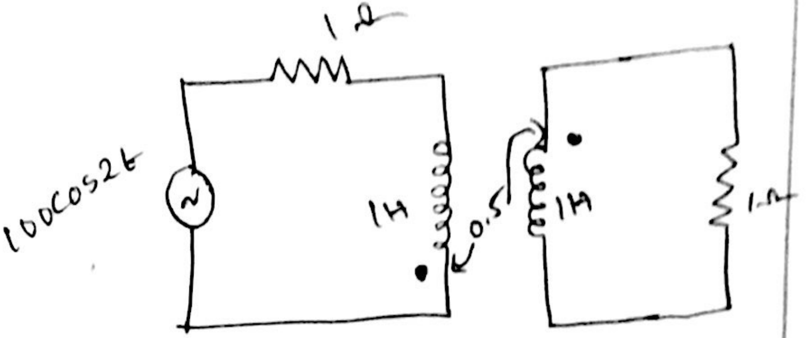
The coupling coefficient

$$K = \frac{\phi_{21}}{\phi_1} = \frac{0.6}{0.9} = 0.667$$

Mutual inductance $$M = \frac{N_2 \phi_{21}}{i_1} = \frac{1500 \times 0.6 \times 10^{-3}}{2} = 0.45H$$

we have $$M = K\sqrt{L_1 L_2} \Rightarrow L_2 = \frac{M^2}{K^2 L_1} = \frac{(0.45)^2}{0.667^2 \times 0.225} = 2.023H$$

find the power dissipated in resistor for the circuit shown in fig.



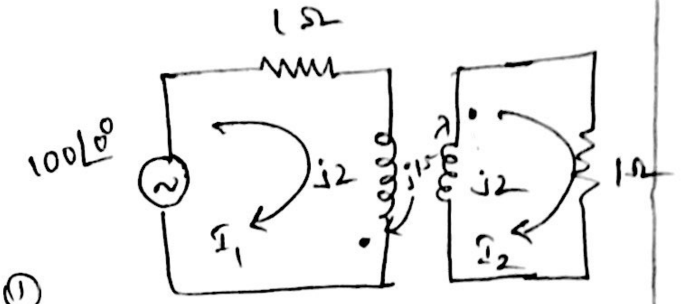
Sol:- Given

$\omega = 2$

$\Rightarrow X_L = j\omega L = j2\Omega$

$X_M = j\omega M = j1\Omega$

Applying KVL for loop ①



$\Rightarrow 100\angle 0° = 1I_1 + j2I_1 + j1I_2$

$\Rightarrow 100\angle 0° = I_1(1+j2) + j1I_2 \longrightarrow$ ①

Applying KVL for loop ②

$0 = 1.I_2 + j2I_2 + j1I_1$

$\Rightarrow 0 = I_2(1+j2) + j1 I_1 \longrightarrow$ ②

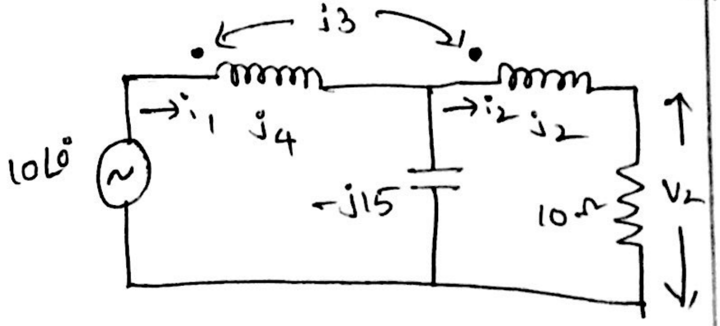
Solving ① & ②, we get

$I_1 = 60\angle -53.13°$

$\text{power} = I_{rms}^2 R = \left(\frac{50}{\sqrt{2}}\right)^2 \times 1 = 1250W$

find the voltage across 10Ω resistor for the network shown

Sol:- from fig



$V_2 = I_2 10$

KVL for loop ①

$\Rightarrow 10\angle 0^\circ = j4I_1 - j15(I_1 - I_2) + j3I_2$

$\Rightarrow -j11I_1 + j18I_2 = 10\angle 0^\circ \longrightarrow$ ①

KVL for loop ②

$0 = 10I_2 + j2I_2 - j15(I_2 - I_1) + j3I_1$

$\Rightarrow j18I_1 - j13I_2 + 10I_2 = 0 \longrightarrow$ ③

solving ① & ②, we get

$I_2 = -0.578\angle 110.7^\circ$

⇒ Voltage across 10Ω resistor is

$V_2 = 10I_2 = -5.78\angle 110.7^\circ$

$|V_2| = 5.78$