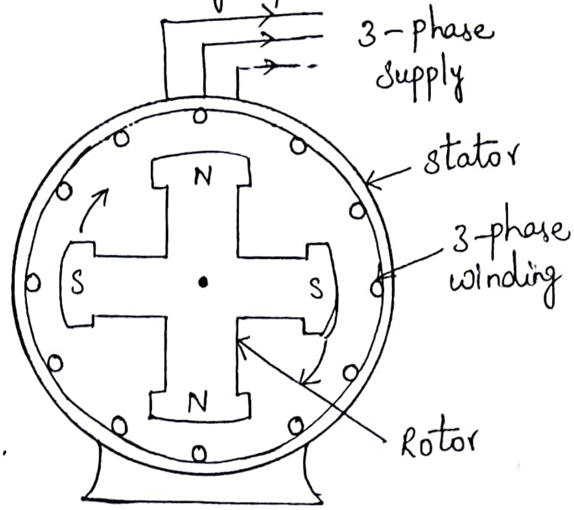
ALTERNATORS

The machine which produces 3-phase power from mechanical power is called an alternator or Synchronous generator. Alternators are the primary source of all the Electrical Energy we Consume. They convert mechanical Energy into a.c Energy.

Principle :-

An alternator operates on the same fundamental principle of Electromagnetic induction as a d.c generator i.e, when the flux linking a Conductor changes, an Emf is induced in the conductor. Like a d.c generator, an alternator also has an armature winding and a field winding. But there is one important difference between the two. In a d.c generator, the armature winding is placed on the rotor in order to provide a way of converting alternating voltage generated in the winding to a direct voltage at the terminals through the use of a rotating commutator. The field poles are placed on the stationary part of the machine. Since no commutator is required in an alternator, it is usually more convenient and advantageous to place the field winding on the rotating part (i.e., rotor) and armature winding on the stationary part (i.e., stator).



Advantages of Stationary armature :-

The field winding of an alternator is placed on the rotor and is Connected to d.c supply through two slip rings. The 3-phase armature winding is placed on the stator.

This arrangement has the following advantages :

(ii) It is easier to insulate stationary winding for high voltages for which the alternators are usually designed. It is because they are not subjected to centrifugal forces and also extra space is available due to the stationary arrangement of the armature.

(iii) The stationary 3-phase armature can be directly connected to load without going through large, unrealiable slip rings & brushes.

(iv) only two slip rings are required for d.c supply to the field winding on the rotor. Since the exciting current is small, the slip rings and brush gear required are of light construction.

(v) Due to simple and robust construction of the rotor, higher speed of rotating d.c field is possible. This increases the output obtainable from a machine of given dimensions.

<u>Construction of Alternator</u> :-

An alternator has 3-phase winding on the stator and a d.c field winding on the rotor.

1) <u>Stator</u> : It is the stationary part of the machine and is builtup of sheet-steel laminations having slots on its inner periphery. A 3-phase winding is placed in these slots and serves as the armature winding of the alternator. The armature winding is always connected in star and the neutral is connected to ground.

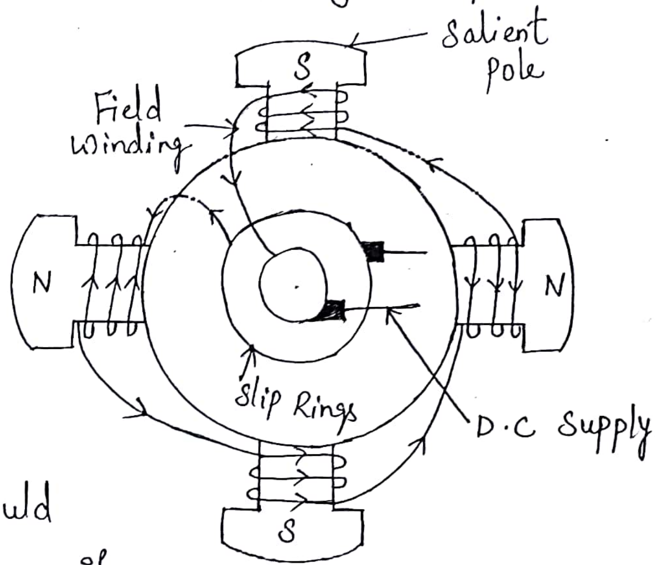
2) <u>Rotor</u> :- The rotor carries a field winding which is supplied with direct current through two slip rings by a seperate dc source. This d.c source is generally a small d.c shunt or compound generator mounted on the shaft of the alternator. Rotor construction is of two types, namely;

(i) Salient (or projecting) pole type

(ii) Non-Salient (or cylindrical) pole type

(i) Salient pole type :-

In this type, salient or projecting poles are mounted on a large circular steel frame which is fixed to the shaft of the alternator as shown. The individual field pole windings are connected in series in such a way that when the field winding is energised by the d.c exciter, adjacent poles have opposite polarities.



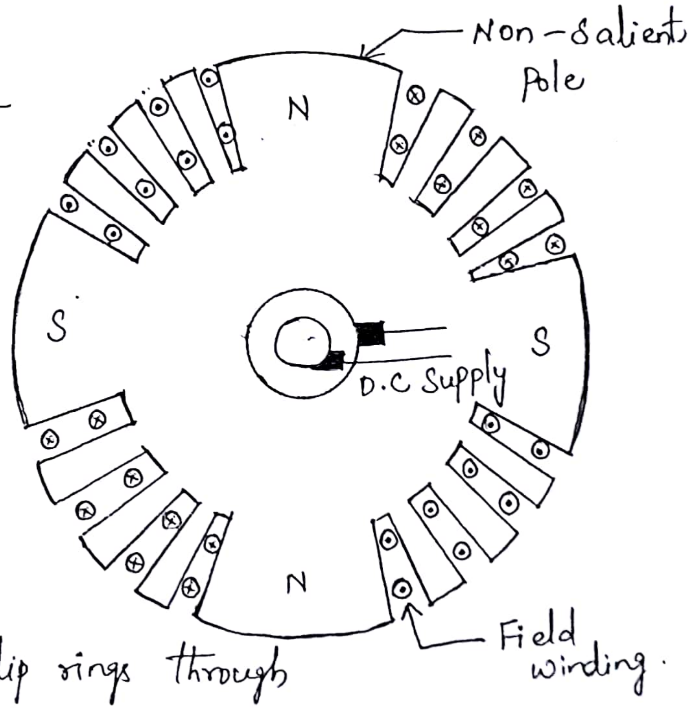
Low and medium – speed alternators (120 – 400 r.p.m) such as those driven by diesel engines or water turbines have salient pole type rotors due to the following reasons :

(a) The salient field poles would cause an excessive windage loss if driven at high speed and would tend to provide noise.

(b) Salient – pole construction cannot be made strong enough to withstand the mechanical stress to which they may be subjected at higher speeds.

(ii) Non – salient pole type :-

In this type, the rotor is made of smooth solid forged – steel radial cylinder having a no. of slots along the outer periphery. The field windings are embedded in these slots and are connected in series to the slip rings through



which they are energised by the d.c exciter. The regions forming the poles are usually left unslotted as shown. It is clear that the poles formed are non-salient i.e., they do not project out from the rotor surface.

High speed alternators (1500 or 3000 rpm) are driven by steam turbines and use non-salient type rotors due to the following reasons :

(a) This type of construction has mechanical robustness & gives noiseless operation at high speeds.

(b) The flux distribution around the periphery is nearly a sine wave and hence a better emf waveform is obtained than in the case of salient-pole type.

Since steam turbines run at high speed and a frequency of 50Hz is required, we need a small no. of poles on the rotor of high-speed alternators. we can use no less than 2 poles and this fixes the highest possible speed. The next lower speed is 1500 rpm for a 4-pole machine.

Alternator operation :-

The rotor winding is energised from the d.c exciter and alternate N and S poles are developed on the rotor. when the rotor is rotated in anticlockwise direction by a prime mover, the stator or armature conductors are cut by the magnetic flux of rotor poles. Consequently, emf is induced in the armature conductors due to electromagnetic induction. The induced emf is alternating since N and S poles of rotor alternately pass the armature conductors. The direction of induced emf can be found by Fleming's right hand rule and frequency is given by:

$$f = \frac{NP}{120}$$

where N = speed of rotor in rpm
P = no. of rotor poles.

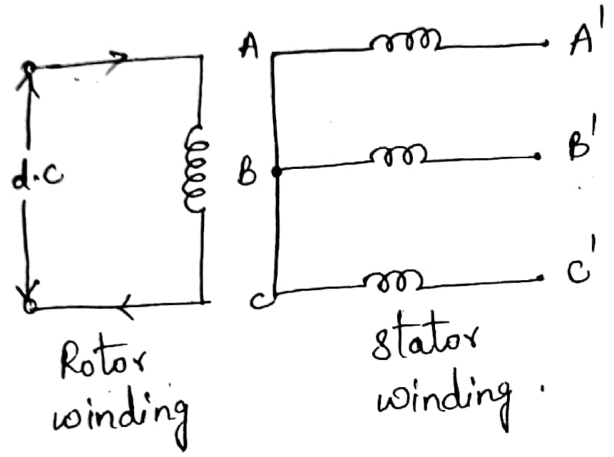


fig (i)

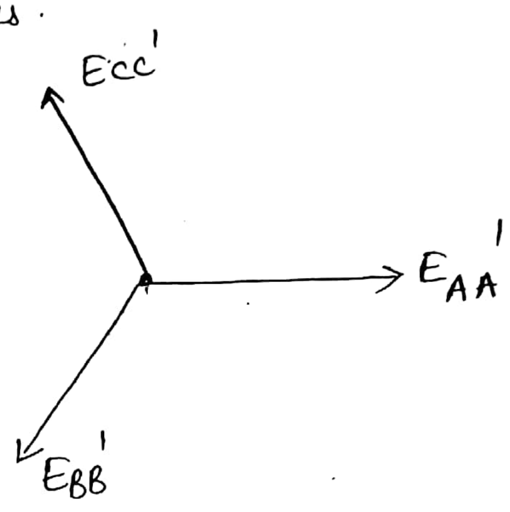


fig (ii)

Fig (i) shows star-connected armature winding and d.c field winding. When the rotor is rotated, a 3-phase voltage is induced in the armature winding. The magnitude of induced emf depends upon the speed of rotation and the d.c. exciting current. The magnitude of emf in each phase of the armature winding is the same. However, they differ in phase by 120° electrical as shown in the phasor diagram in fig: (ii).

Frequency :-

The frequency of induced emf in the armature conductor depends upon speed and the number of poles.

Let N = rotor speed in rpm

P = no. of rotor poles

f = freq. of emf in Hz.

Consider a stator conductor that is successively swept by the N and S poles of the rotor. If a +ve voltage is induced when a N-pole sweeps across the conductor, a similar -ve voltage is induced when a S-pole sweeps by. This means that one complete cycle of emf is generated in the conductor as a pair of poles passes it i.e., one N-pole and the adjacent following S-pole. The same is true for every other armature conductor.

$\therefore$ No. of cycles/revolution = No. of pairs of poles = $P/2$

No. of revolutions/second = $N/60$.

$\therefore$ No. of cycles/second = $(P/2)(N/60) = \frac{NP}{120}$.

But no. of cycles of e.m.f per second is its frequency.

$$\therefore f = \frac{NP}{120}.$$

It may be noted that N is the synchronous speed. For a given alternator, the no. of rotor poles is fixed and, therefore, the alternator must be run at synchronous speed to give an output of desired frequency. For this reason, an alternator is sometimes called synchronous generator.

pitch factor and Distribution factor :-

The armature winding of an alternator is distributed over the entire armature. The distributed winding produces nearly a sine waveform and the heating is more uniform. Likewise, the coils of armature winding are not full pitched ie., the two sides of a coil are not at corresponding points under adjacent poles.

The fractional pitched armature winding requires less copper per coil and at the same time waveform of output voltage is improved.

(i) pitch factor (K_p) :

$$\text{pitch factor},\ K_p = \frac{\text{Emf with short-pitch coil}}{\text{Emf with full-pitch coil}}$$

Pole pitch is the distance between the centre lines of adjacent N and S poles measured along the circumference of armature surface. When the two sides of a coil are full pole pitch apart, it is called full-pitched coil. The Emf's in the coil sides of a full-pitched coil are in phase. In practice, coil pitch is less than pole pitch and hence Emfs in the coil sides have a phase difference. The resultant Emf in the coil will be less than that of full-pitched coil. Therefore, for full-pitch coil, $K_p = 1$ while for short pitch coil, $K_p < 1$.

(ii) Distribution factor (K_d) :

$$\text{Distribution factor},\ K_d = \frac{\text{Emf with distributed winding}}{\text{Emf with concentrated winding}}$$

Since the conductors are spread over the surface of the armature in slots, their Emf's differ in phase and the total Emf is the vector sum and not arithmetic sum. Hence $K_d = 1$ for concentrated winding but it is less than 1 for distributed winding.

EMF Equation of an Alternator :-

Let Z = No. of Conductors or coil sides in series per phase

ϕ = flux per pole in webers.

P = No. of rotor poles.

N = Rotor speed in rpm.

In one revolution (i.e., $60/N$ speed), each stator conductor is cut by $P\phi$ webers i.e.,

$$d\phi = P\phi \; ; \; dt = 60/N .$$

$\therefore$ Average emf induced in one stator conductor

$$= \frac{d\phi}{dt} = \frac{P\phi}{60/N} = \frac{P\phi N}{60} \text{ volts.}$$

Since there are Z conductors in series per phase,

$$\therefore \text{Average emf/phase} = \frac{P\phi N}{60} \times Z$$

$$= \frac{P\phi Z}{60} \times \frac{120f}{P} \qquad \Big|\; \because N = \frac{120f}{P}$$

$$= 2f\phi Z \text{ volts.}$$

RMS value of emf/phase = Average value/phase x form factor

$$= 2f\phi Z \times 1.11 = 2.22 f\phi Z \text{ volts.}$$

$$E_{rms}/\text{phase} = 2.22 f\phi Z \text{ volts.} \quad \text{—— (1)}$$

If K_p and K_d are the pitch factor and distribution factor of the armature winding, then

$$E_{rms}/\text{phase} = 2.22\, K_p K_d Z f \phi \text{ volts} \quad \text{—— (2)}$$

Problem :- A 3-phase, 50 HZ, star-connected alternator has 5 180 conductors per phase and flux per pole is 0.0543 wb. Find (i) emf generated per phase and (ii) emf between line terminals. Assume the winding to be full pitched and distribution factor to be 0.96.

Sol :- (i) Generated emf / phase, $E_{ph} = 2.22 K_p K_d Z f \phi$

$$= 2.22 \times 1 \times 0.96 \times 180 \times 50 \times 0.0543$$

$$= 1041.5 \text{ V}$$

(ii) Line voltage, $E_L = \sqrt{3} E_{ph} = \sqrt{3} \times 1041.5 = 1803.19 \text{V}.$

Problem :- Find the no. of armature conductors in series per phase required for the armature of a 3-phase, 50 HZ, 10-pole alternator. The winding is star-connected to give a line voltage of 11000V. The flux per pole is 0.16 wb. Assume $K_p = 1$ and $K_d = 0.96$.

Sol :- Generated emf / phase, $E_{ph} = E_L/\sqrt{3} = 11000/\sqrt{3} = 6352V$

Let Z be the no. of conductors in series per phase.

$$E_{ph} = 2.22 K_p K_d Z f \phi.$$

$$Z = \frac{E_{ph}}{2.22 K_p K_d f \phi} = \frac{6352}{2.22 \times 1 \times 0.96 \times 50 \times 0.16} = 372.5$$

Problem :- The armature of an 8-pole, 3-phase, 50HZ alternator has 18 slots and 10 conductors/slot. A flux of 0.04 wb is entering the armature from one pole. Calculate the induced emf per phase.

Sol :- Since the values of K_p and K_d are not given, they will be assumed 1.

Total no. of conductors $= 18 \times 10 = 180$.

No. of conductors / phase $= Z = 180/3 = 60$.

Induced emf / phase, $E_{ph} = 2.22\, K_p K_d Z f \phi$.

$$= 2.22 \times 1 \times 1 \times 60 \times 50 \times 0.04$$

$$= 266.4\ V.$$

problem :- A 3-phase, star-connected alternator on open circuit is required to generate a line voltage of 3600V at 50 Hz when driven at 500 rpm. The stator has 3 slots per pole per phase and 10 conductors per slot. Calculate (i) the no. of poles and (ii) useful flux per pole. Assume all the conductors per phase to be connected in series and the coils to be full-pitched with $K_d = 0.96$.

Sol :- (i)

$$f = \frac{N_s P}{120}$$

$$50 = \frac{500 \times P}{120}$$

$$\therefore P = 12.$$

(ii) No. of slots / phase $= 3 \times 12 = 36$

No. of conductors / phase, $Z = 36 \times 10 = 360$

EMF / phase, $E_{ph} = \dfrac{3600}{\sqrt{3}} = 2080V$

Distribution factor $K_d = 0.96$, pitch factor $K_p = 1$. (full pitch)

Induced emf / phase, $E_{ph} = 2.22\, K_p K_d Z f \phi$.

$$2080 = 2.22 \times 1 \times 0.96 \times 360 \times 50 \times \phi$$

$$\therefore \phi = 0.0543\ wb$$

Alternator on No load :-

When the rotor is rotating and energised, the circuit diagram for an alternator with its stator circuit open (i.e. no load condition) is as shown in fig (i). Each phase generates an emf E. The per phase armature resistance and leakage reactance are R_a and X_L respectively.

It is clear from the emf equation that the generated emf E in an alternator depends upon speed and flux per pole. Since most of the alternators are operated at constant speed, the generated emf would depend upon field excitation. By including a rheostat in the field circuit, the field excitation and hence generated emf can be changed. It is always convenient to analyse an alternator on single-phase basis; the conditions in the other two phases being similar. Fig. (ii) shows one phase of the alternator. It is clear that at no load, terminal voltage V per phase is equal to the generated emf E per phase i.e.

$$\boxed{V = E} \text{ ------ at no load.}$$

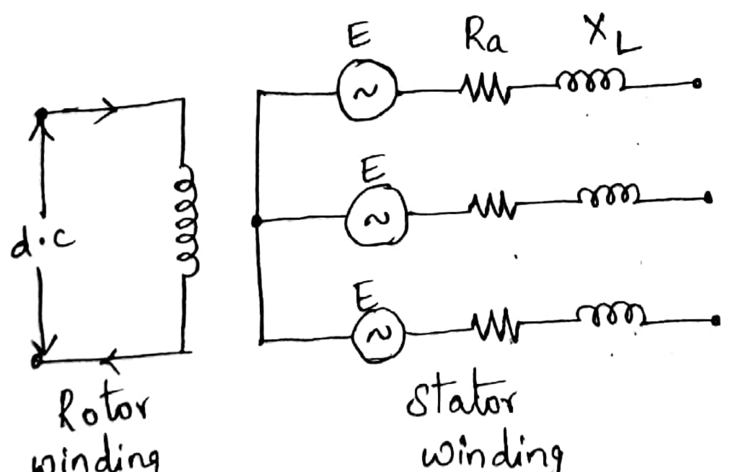


fig : (i)

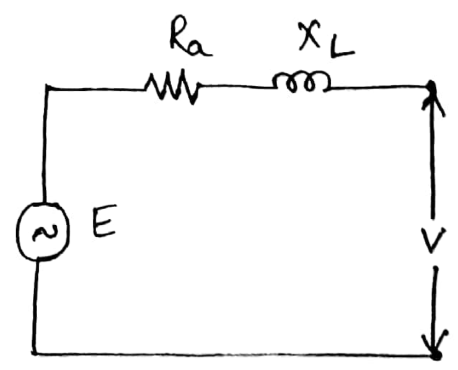
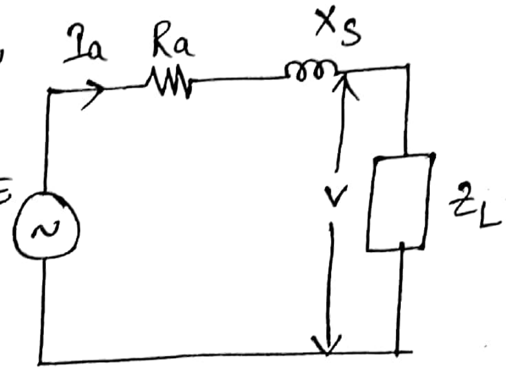


fig : (ii)

Alternator on Load :-

When load on an alternator changes, the terminal voltage V also changes. The change in V is due to the following three effects:



i) Voltage drop in armature resistance R_a.

ii) Voltage drop in armature leakage reactance X_L.

iii) Voltage drop due to armature reaction.

The voltage drop due to armature reaction is accounted for by assuming a fictitious reactance X_a in the armature winding. The phasor sum of X_L and X_a gives synchronous reactance X_S. The per phase equivalent circuit of an alternator on load can be represented as shown above.

Synchronous impedance/phase,

$$Z_S = \sqrt{R_a^2 + X_S^2}.$$

If V is the terminal voltage/phase and E is the generated emf/phase, then,

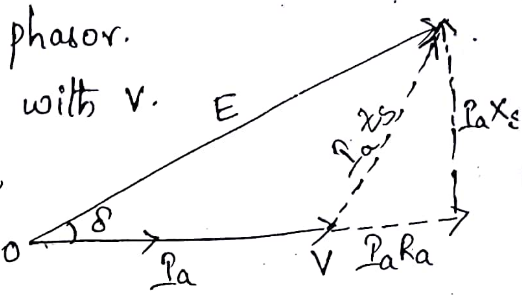
$$E = V + I_a(R_a + jX_S) = V + I_a Z_S.$$

Phasor diagram of a loaded Alternator :-

Let
E = no-load emf/phase
V = terminal voltage/phase
I_a = armature current/phase
Z_a = Synchronous impedance/phase
ϕ = load p.f angle.

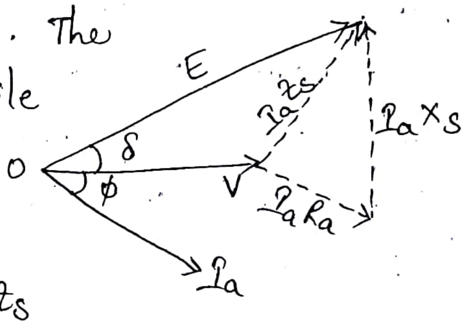
(i) Unity p.f load :

Here V is taken as the reference phasor. The current phasor I_a is in phase with V. The voltage drop I_aR_a is in phase with I_a while the voltage drop I_aX_s leads I_a by 90°; the phasor sum of the two giving I_aZ_s. The phasor sum of V and I_aZ_s gives E.



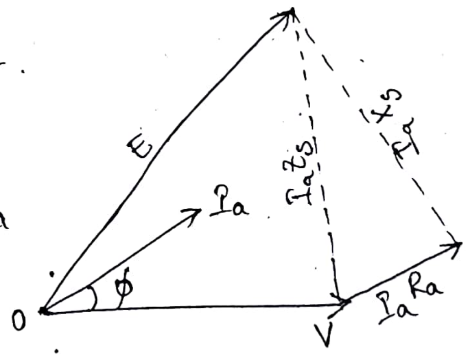
(ii) Lagging p.f load :

Here again V is taken as the reference phasor. The current phasor I_a lags V by ϕ. The I_aR_a drop is in phase with I_a while the drop I_aX_s leads I_a by 90°; the phasor sum of the two giving I_aZ_s. The phasor sum of V and I_aZ_s gives E.



(iii) Leading p.f load :

V is taken as the reference phasor. The current phasor I_a leads V by ϕ. The I_aR_a drop is in phase with I_a while the drop I_aX_s leads I_a by 90°; the phasor sum of the two giving I_aZ_s. The phasor sum of V and I_aZ_s gives E.



Voltage Regulation :-

The voltage regulation of an alternator is defined as the percentage rise in terminal voltage when full-load is removed i.e.,

$$\%\ \text{vol. reg.} = \frac{E - V}{V} \times 100$$

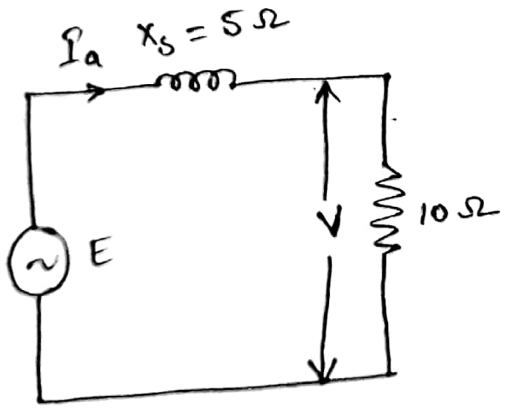
where E = no-load voltage per phase

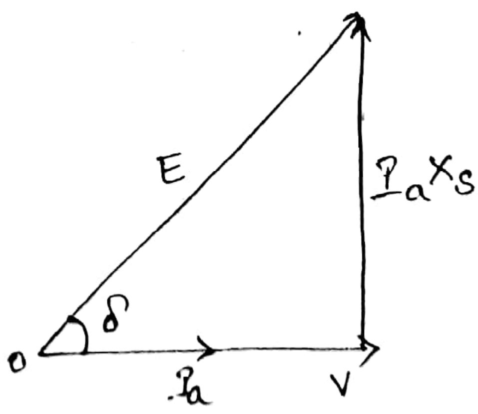
V = full-load voltage per phase.

Its value depends upon load current and load p.f.

Problem :- A 3-phase, star-connected, 50 Hz alternator has 96 conductors per phase and a flux/pole 0.1 wb. The alternator winding has a synchronous reactance of 5Ω/phase and negligible resistance. The distribution factor for the stator winding is 0.96.
Calculate the terminal voltage when three non-inductive resistors, of 10Ω/phase, are connected in star across the terminals.

Sol :- Fig (i) shows the equivalent circuit for one phase while fig (ii) shows its phasor diagram.





Generated emf / phase, $E = 2.22\, K_p K_d\, Z f \phi$ (3)

$= 2.22 \times 1 \times 0.96 \times 96 \times 50 \times 0.1$

$= 1023\ V$.

Impedance / phase, $Z = \sqrt{R_a^2 + X_s^2} = \sqrt{5^2 + 10^2} = 11.18\ \Omega$

current / phase, $I_a = E/Z = 1023/11.18 = 91.5\ A$

From phasor diagram,

Terminal voltage / phase, $V = \sqrt{E^2 - (I_a X_s)^2}$

$= \sqrt{(1023)^2 - (91.5 \times 5)^2} = 915\ V$

Terminal line voltage $= \sqrt{3} \times 915 = 1585\ V$.

Problem :- A 1500 KVA, 6.6KV, 3-phase, star-connected alternator has a resistance of $0.5\ \Omega$/phase and a synchronous reactance of $5\ \Omega$/ phase. Find its voltage regulation for (i) unity p.f (ii) 0.8 lagging p.f and (iii) 0.8 leading p.f.

o/p power in VA = $\sqrt{3}\, V_L I_L$

Sol :- Line current, $I_L = \dfrac{1500 \times 10^3}{\sqrt{3} \times 6600} = 131\ A$

Armature current / phase, $I_a = I_L = 131\ A$

voltage / phase, $V = 6600/\sqrt{3} = 3810\ V$

$I_a R_a = 131 \times 0.5 = 65.5\ V$

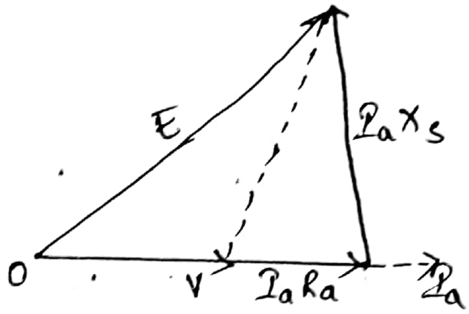
$I_a X_s = 131 \times 5 = 655\ V$

(i) Unity p.f : In drawing the phasor diagram for unity p.f, voltage has been taken as the reference phasor.

$$E = \sqrt{(V + I_a R_a)^2 + (I_a X_s)^2}$$

$$= \sqrt{(3810 + 65.5)^2 + (655)^2}$$

$$= 3930\,V$$

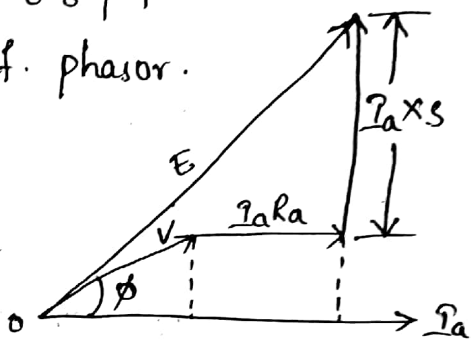


$$\therefore \%\ \text{vol. reg.} = \frac{3930 - 3810}{3810} \times 100 = 3.15\%$$

(ii) 0.8 p.f lagging :

In drawing the phasor diagram for 0.8 p.f lagging current has been taken as ref. phasor.

$$\cos\phi = 0.8, \quad \sin\phi = 0.6$$



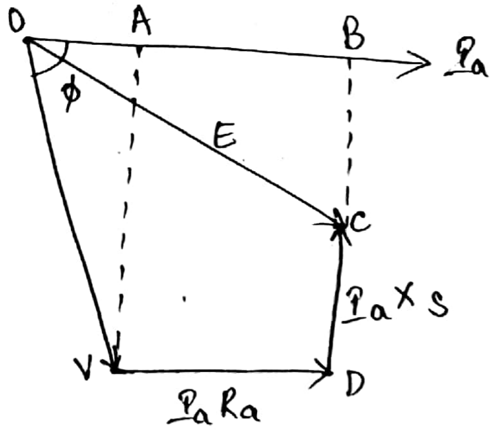
$$E = \sqrt{(V\cos\phi + I_a R_a)^2 + (V\sin\phi + I_a X_s)^2}$$

$$= \sqrt{(3810 \times 0.8 + 65.5)^2 + (3810 \times 0.6 + 655)^2} = 4283\,V.$$

$$\therefore \%\ \text{vol. reg} = \frac{4283 - 3810}{3810} \times 100 = 12.4\%$$

(iii) 0.8 p.f leading :

Here current has been taken as ref. phasor.



$$OB = OA + AB = V\cos\phi + I_a R_a$$

$$BC = BD - CD = V\sin\phi - I_a X_s.$$

In right angled triangle OBC

$$E = \sqrt{(OB)^2 + (BC)^2}$$

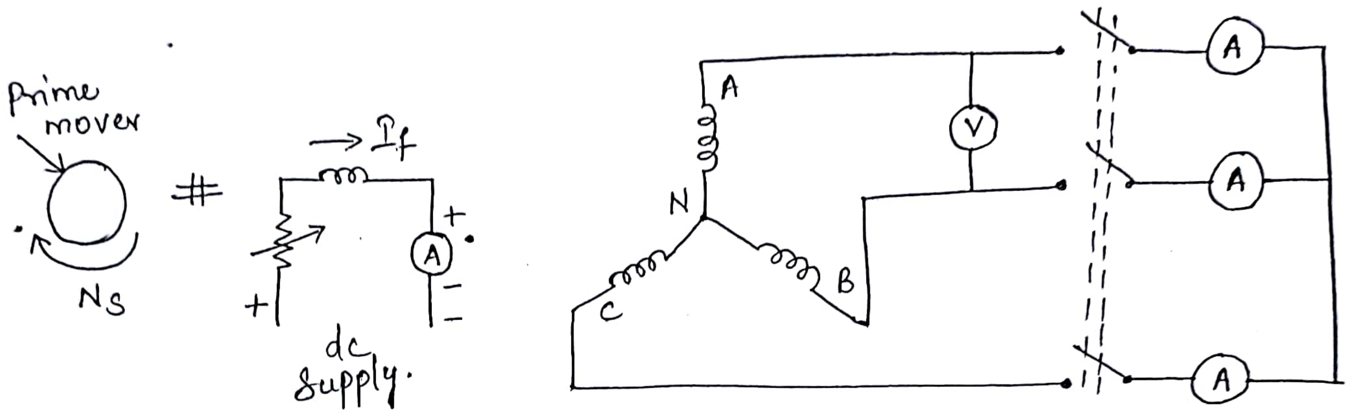
$$= \sqrt{(V\cos\phi + I_a R_a)^2 + (V\sin\phi - I_a X_s)^2}$$

$$= \sqrt{(3810 \times 0.8 + 65.5)^2 + (3810 \times 0.6 - 655)^2} = 3515\,V$$

$$\therefore \%\ \text{vol. reg} = \frac{3515 - 3810}{3810} \times 100 = -7.7\%$$

Predetermination of Regulation by Synchronous Impedance method – OC and SC tests :

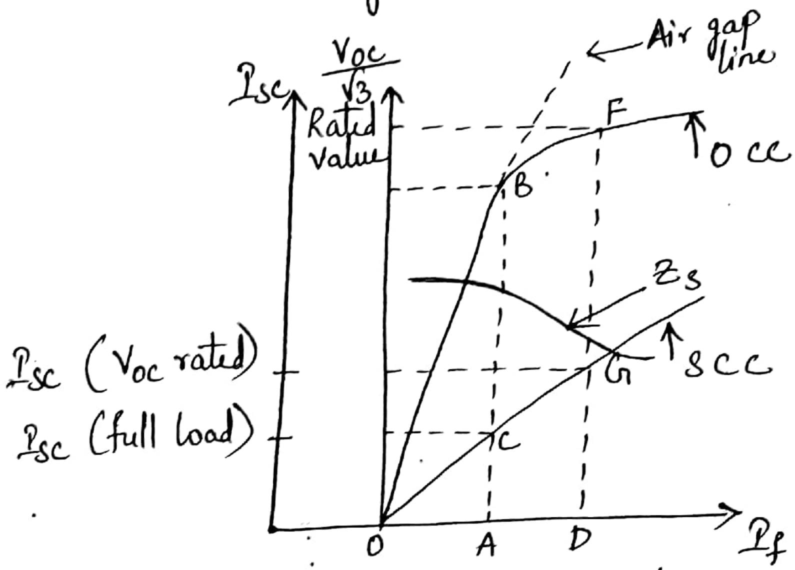
Open-circuit (OC) and short-circuit (SC) tests are perform on a three-phase generator to determine its synchronous reactance. Figure shows the connection diagram for performing the OC and SC tests.



In the connection diagram, the rotor field winding is connected to a dc voltage source through an ammeter and a rheostat. The three-phase stator winding is connected to a three-pole switch for short-circuiting the winding. The rotor is driven by a prime mover at the synchronous speed. The open-circuit voltage is measured by the voltmeter connected between two lines and the three ammeters measure the short-circuit current.

For the OC test, the switch is kept open and the open-circuit line voltage V_{oc} is read from the voltmeter for different values of the field excitation current I_f. The excitation current can be varied using the rheostat in the field circuit. The field current is increased in steps up to an open-circuit voltage, which is approximately 20–25% above the rated voltage.

The open-circuit characteristic (OCC) is plotted with the phase voltage $V_{oc}/\sqrt{3}$ on the y-axis against I_f on the x-axis as shown. The OCC is the magnetization characteristic of the generator.



For the sc test, the field current is adjusted to a low value such that when the switch is closed the rated full-load current flows in the stator armature winding and the terminal voltage is zero. Since I_f is adjusted to a low value, the magnetic circuit is unsaturated. Therefore, the variation of the armature current under short circuit, I_{sc}, with the variation in the field current is linear. Only one reading is sufficient to obtain the short-circuit characteristic (SCC). The SCC is also plotted with the current I_{sc} on the y-axis and the field current I_f on the x-axis as shown.

Under the sc condition of the synchronous generator $V_t = 0$, and it follows from equation $E = V_t + I_a Z_s$ that,

$$Z_s = \frac{V_{oc}/\sqrt{3}}{I_{sc,\text{full load}}} \quad \text{at } I_f \text{ corresponding to } I_{sc,\text{full load}}.$$

As shown in fig. at a field current OA, The short AY circuit current is AC, which is equal to the full-load current, and AB is the induced line voltage of the synchronous generator. Then

$$Z_s = \frac{AB}{AC}$$

Because of the non-linear nature of OCC shown, the ratio of the open-circuit voltage to the short-circuit current at different values of excitation currents is different. Hence, Z_s when plotted at different excitation currents, as shown, indicates a higher value at lower excitations as compared to higher excitations. When the machine is loaded, it operates under the condition of magnetic saturation and the value of Z_s will be lower than that obtained by using equation $Z_s = \dfrac{V_{oc}/\sqrt{3}}{I_{sc,\text{ full load}}}$

A more realistic value of Z_s may be computed as,

$$Z_s = \frac{V_{oc,\text{ rated}} / \sqrt{3}}{I_{sc}} \quad \text{at } I_f \text{ corresponding to } V_{oc,\text{ rated value}}$$

As shown in figure, at field current OD, the induced rated line voltage of the synchronous generator is DF and DG is the short-circuit current. Then

$$Z_s = \frac{DF}{DG}.$$

For a synchronous machine, the dc resistance of the stator winding can be determined by the ammeter-voltmeter method. The value of ac resistance is higher than dc resistance.

The value of dc resistance may be multiplied by a factor 1.4 to get the ac resistance value R_a. The synchronous reactance then can be calculated using equation $Z_s = \sqrt{R_a^2 + X_s^2}$ as

$$\boxed{X_s = \sqrt{Z_s^2 - R_a^2}}$$

Problem :- Determine the voltage regulation of a 2200-V, single-phase alternator giving a current of 120A at ⓐ unity power factor, ⓑ power factor 0.8 leading, and ⓒ power factor 0.707 Lagging. From the oc and sc test results, full-load current of 120A is produced on short circuit by a field excitation of 2.5A and an electromotive force of 480V is produced on open circuit by the same excitation. The armature resistance is $0.4\,\Omega$.

Sol :- From oc and sc test, the synchronous impedance Z_s can be given as,

$$Z_s = \frac{V_{oc}}{I_{sc}}$$

$$= \frac{480}{120} = 4\,\Omega$$

Using equation, $Z_s = \sqrt{R_a^2 + X_s^2}$

$$X_s = \sqrt{4^2 - 0.4^2} = 3.98\,\Omega$$

The impedance angle,

$$\tan\theta = \frac{X_s}{Z_s}$$

$$\theta = \tan^{-1}\left(\frac{3.98}{0.4}\right) = 84.26^\circ$$

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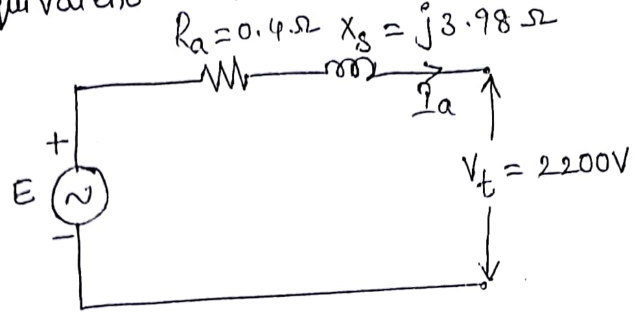
The Equivalent Circuit of the alternator is as shown ||



ⓐ At unity p.f, $\cos\phi = 1$, then $\phi = 0°$.

Let the terminal voltage V_t be the reference phasor. Then

$$V_t = 2200 + j0$$

$$I_a = 120\angle 0°.$$

Using Equation,

$$E = V_t + I_a Z_s$$

$$= 2200 + 120\angle 0° \times 4\angle 84.26°.$$

$$= 2200 + 480\angle 84.26°$$

$$= 2200 + 48 + j477.6$$

$$= 2248 + j477.6$$

$$= 2298.17\angle 12°.$$

$$\therefore \%\ \text{vol. reg} = \frac{2298.17 - 2200}{2200} \times 100 = 4.46.$$

ⓑ At 0.8 leading p.f, $\cos\phi = 0.8$, then $\phi = \cos^{-1} 0.8 = \angle 36.87°$.

Then $I_a = 120\angle 36.87°$

$$E = V_t + I_a Z_s$$

$$= 2200 + 120\angle 36.87° \times 4\angle 84.26°$$

$$= 2200 + 480\angle 121.13°.$$

$$= 2200 - 248.15 + j410.88 = 1951.85 + j410.88$$

$$= 1994.63\angle 12'.$$

$\therefore$ % vol. reg $= \dfrac{1994.63 - 2200}{2200} \times 100 = -9.335$

ⓒ At 0.707 lagging p.f, $\cos\phi = 0.707$, then $\phi = \cos^{-1} 0.707$
$= \angle 45^\circ$.

Then $I_a = 120 \angle -45^\circ$

$E = V_t + I_a \cdot Z_s$

$= 2200 + 120 \angle -45^\circ \times 4 \angle 84.26^\circ$

$= 2200 + 480 \angle 39.26^\circ$

$= 2200 + 371.65 + j303.76$

$= 2571.65 + j303.76$

$= 2589.53 \angle 6.74^\circ$.

% vol. reg $= \dfrac{2589.53 - 2200}{2200} \times 100 = 17.706$.