# THREE PHASE INDUCTION MOTORS.

The most common type a.c motor being used is the Induction motor. An induction motor is simply an electric transformer whose magnetic circuit is separated by an air gap into two movable positions, one carrying primary and other was secondary winding. Alternating current supplied to the primary winding from an electric power system induces an opposing current in the secondary winding, when latter is short circuited (or) close through external impedance. The energy is transferred to the rotor windings through electromagnetic induction and hence such machines are called "Induction motors".

An induction motor can be treated as a rotating transformer i.e., one in which primary winding is stationary but the secondary is free to rotate. Polyphase Induction motors is extensively used for industrial drives as it has following advantages.

**ADVANTAGES:-**

1. It has very simple design.
2. Rugged construction.
3. Reliable operation.
4. Low Initial cost.
5. Easy operation and simple maintanance.

contd..

6. It starts up from rest and needs no extra starting motor and has not to be synchronised. Its starting arrangement is simple especially for squirrel cage type motor.

7. It has self-starting torque.

## DISADVANTAGES :-

1. Its speed can't be varied without sacrificing some of its efficiency.
2. Its speed decreases with increases in load.
3. Its starting torque is inferior to d.c shunt motor.

## TYPES AND CONSTRUCTIONAL DETAILS OF INDUCTION MOTORS :-

The main parts of an induction motors are
(i) stator (ii) Rotor.

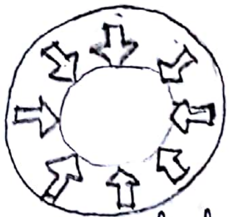
Induction motors are classified depending on the construction of the rotor. There are two types of induction motors.

(a) wound-rotor (or) slip ring induction motors
(b) squirrel cage induction motor.

(i) **Stator :-**

The stator is stationary, hollow cylindrical structure made of slotted laminations of sheet steel punching. The stator core is made of laminations which are 0.03 to 0.06 cm thick. These laminations are insulated by a coating of an insulating varnish. [oxide coating applied by heat treatment.

of 5 to 7 cm
· Ventilating ducts are provided at intervals
along the length of the core.
· The stator carries a 3-$\phi$ winding and is fed from
a 3-$\phi$ supply. It is wound for a definite number
of poles [ poles determined by requirement of speed].
Greater the no. of poles, lesser the speed.
· These ~~three~~ single phase windings are insulated from
each other and separated in space by 120°. These
windings are either connected in stator in Delta
internally.
The stator windings when supplied with 3-$\phi$ current
produce a magnetic field or flux which is of
constant value but which reduces (or) rotates at
synchronous speed $[N_S = \frac{120f}{P}]$. This revolving magnetic
flux induces an emf in the rotor by mutual
Induction.


stator lamination

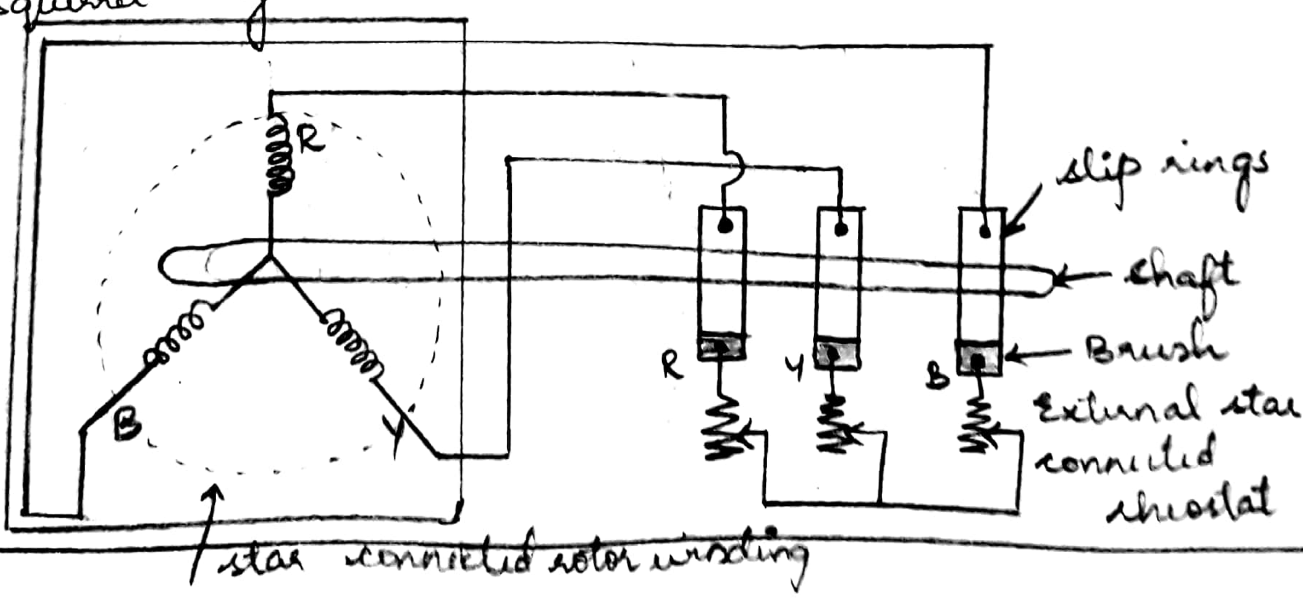
(ii) **Rotor :-**

The rotor core is of laminated construction with
slots punched in for accommodating the rotor
winding (or) rotor bars.
· In smaller machines, the punched laminations are
stacked and fitted directly into a shaft, in large
machines, a stack of annular punching of suitable
crosssectional area are fitted into a spider web
arrangement on the shaft.

a) Wound rotor / slip ring rotor :-

The winding of the rotor is similar to the winding of the stator and is placed in the slots of motor.

- The no of slots on the rotor is small and also the turns per phase are fewer.
- The wound rotor is connected in star, and the three ends of the winding of the three phases are brought out through slip rings. ∴ These type of rotor is known as slip ring induction motor.
- These type of motors is preferred generally for large machines, where starting torque can be controlled through an external resistance connected to the rotor circuit.
- The introduction of additional resistance in the rotor circuit during the starting period for increasing the starting torque of the motor for changing its speed / torque / current characteristics.
- When running under normal conditions, the wound rotor is short circuited on itself just like the squirrel cage rotor.

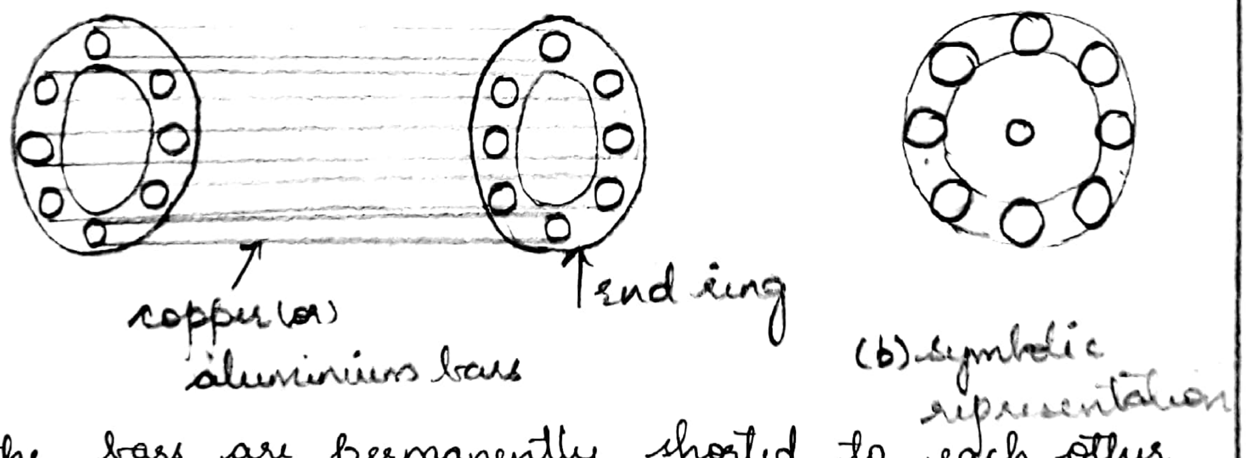


b. Squirrel cage Rotor :-

The rotor core is cylindrical and slotted on its pheriphery. The rotor consists of uninsulated copper or alluminium bars called rotor conductors. The ~~p~~ bars are placed in the slots. These bars are permanently shorted at each end with the help of conducting copper ring called end ring.

- The bars are usually brazed to the end rings to provide good mechanical strength.
- The entire structure looks like a cage, forming a closed electrical circuit. So the rotor is called squirrel ~~sq~~ cage rotor.

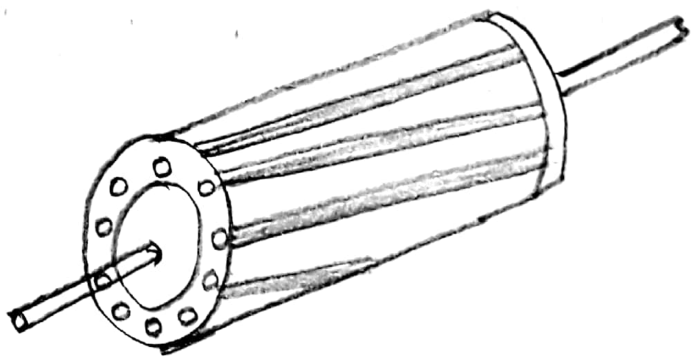
Squirrel cage rotors.




- As the bars are permanently shorted to each other through end ring, the entire rotor resistance is very very small. Hence this rotor is also called short circuited rotor.
- As rotor itself is short circuited, no external resistance can have any effect on the rotor resistance. Hence no external resistance can be introduced in the rotor circuit. So slip ring and brush assembly

is not required for this rotor. Hence the construction of this rotor is very simple.

- Fan blades are generally provided at the ends of the rotor core. This circulates the air through the machine while operation, providing the necessary cooling. The air gap between stator and rotor is kept uniform and small as possible.
- In this type of rotor, the slots are not arranged parallel to the shaft axis but are skewed as shown in the fig.



The advantages of skewing are

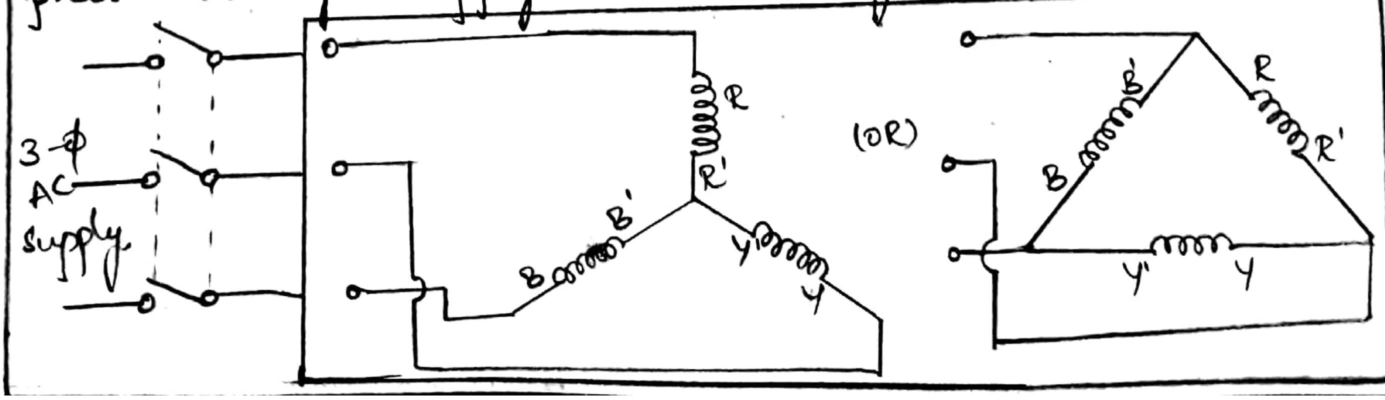
1. The magnetic hum i.e, noise get reduced due to skewing hence makes the motor operation quiter.
2. It makes motor operation smooth.
3. The stator and rotor may get magnetically locked. Such a tendency of magnetic locking gets reduced due to skewing.
4. It increases the effective transformation ratio between stator and rotor.

## ROTATING MAGNETIC FIELD (RMF) :-

- The rotating magnetic field can be defined as the field or flux having constant amplitude but whose axis is continously rotating in a plane with a certain speed.
- If the arrangement is made to rotate a permanent magnet, then the resulting field is a rotating magnetic field. But in this method, it is necessary to rotate a magnet physically to produce rotating magnetic field.
- In three phase induction motors such a rotating magnetic field is produced by suppling currents to a set of stationary windings with the 3-$\phi$ a.c ~~sys~~ supply.
- The current carrying windings produce the magnetic flux. Due to interaction of these fluxes produced due to 3-$\phi$ supply resultant flux has a constant magnitude and its axis rotating in space, without physically rotating the windings. This type of field is rotating magnetic field.

## PRODUCTION OF RMF :-

- A three phase induction motor consists of three phase windings as its stationary part called stator. The three phase windings are displaced from each other by 120°. These windings are connected in star (or) delta and are supplied by a three phase ac ~~supply~~ supply. These windings are denoted as RR', YY', BB'.



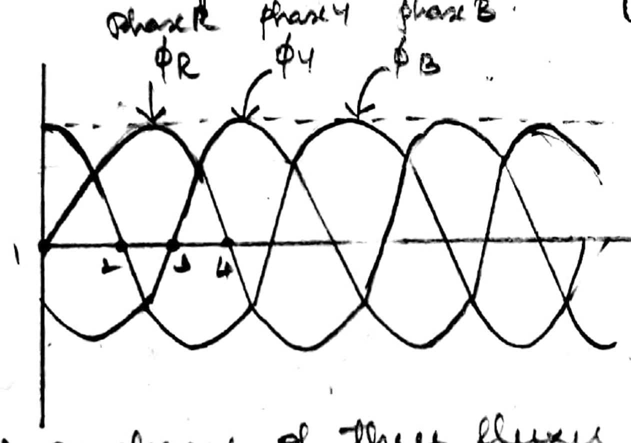
- The three phase currents flow simultaneously through the windings and are displaced from each other by 120° electrically. Each alternating phase current produces its own flux which is sinusoidal. All three fluxes are sinusoidal and are separated by 120°. If the phase sequence of windings is RYB, the mathematical equations for instantaneous values of three fluxes $\phi_R$, $\phi_Y$ and $\phi_B$ can be written as

$$\phi_R = \phi_m \sin\omega t = \phi_m \sin\theta \qquad \text{—— (1)}$$

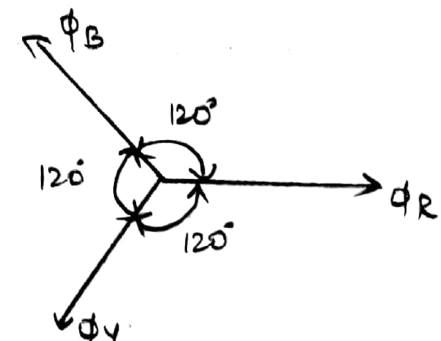
$$\phi_Y = \phi_m \sin(\omega t - 120°) = \phi_m \sin(\theta - 120°) \qquad \text{—— (2)}$$

$$\phi_B = \phi_m \sin(\omega t - 240) = \phi_m \sin(\theta - 240) \qquad \text{—— (3)}$$

- As windings are identical and supply is balanced, the magnitude of each flux is $\phi_m$. Due to phase difference sequence R-Y-B, flux $\phi_Y$ lags behind $\phi_R$ by 120° and $\phi_B$ lags $\phi_Y$ by 120°, $\phi_B$ lags $\phi_R$ by 240°. The flux $\phi_R$ is taken as reference while writing equations.




(a) waveforms of three fluxes




(b) Assumed +ve directions

The resultant flux $\phi_T$ is the phasor addition of $\phi_R, \phi_Y, \phi_B$.

$$\bar{\phi}_T = \bar{\phi}_R + \bar{\phi}_Y + \bar{\phi}_B$$

Find $\phi_T$ at instants 1, 2, 3, 4 as shown in fig (b) which represents the values of $\theta$ as 0°, 60°, 120°, 180° respectively. The phasor addition can be performed by obtaining values of $\phi_R$, $\phi_Y$ and $\phi_B$ by substituting values of $\theta$ in eq(1), (2), (3)

## CASE 1:- $\theta = 0°$

$\phi_R = \phi_m \sin 0 = 0$

$\phi_Y = \phi_m \sin(-120°) = -0.866\phi_m$

$\phi_B = \phi_m \sin(-240°) = +0.866\phi_m$

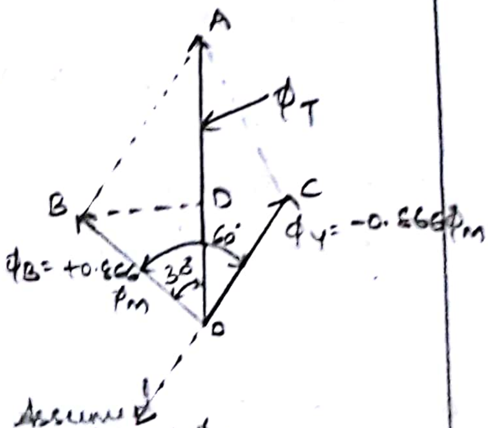



fig (c). Vector diagram for $\theta = 0°$

The phasor addition is shown in fig. The +ve values are shown in ~~the~~ assumed +ve direction while negative values are shown in opposite direction to the +ve direction of resp. fluxes. Refer to assumed positive directions shown in fig (b).

BD is drawn perpendicular from B on $\phi_T$. It bisects $\phi_T$.

$$OD = DA = \frac{\phi_T}{2}$$

In $\triangle OBD$,

$$\angle BOD = 30°$$

$$\cos 30° = \frac{OD}{OB} = \frac{\phi_T/2}{0.866\phi_m}$$

$$\phi_T = 2 \times 0.866\phi_m \times \cos 30°$$

$$= 1.5\phi_m$$

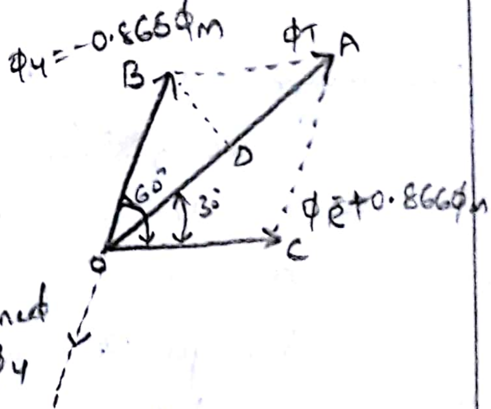
$\therefore$ magnitude of $\phi_T$ is $1.5\phi_m$ and its position is vertically upwards at $\theta = 0°$.

## CASE 2:- $\theta = 60°$

$\phi_R = \phi_m \sin 60° = +0.866\phi_m$

$\phi_Y = \phi_m \sin(-60°) = -0.866\phi_m$

$\phi_B = \phi_m \sin(-180°) = 0$



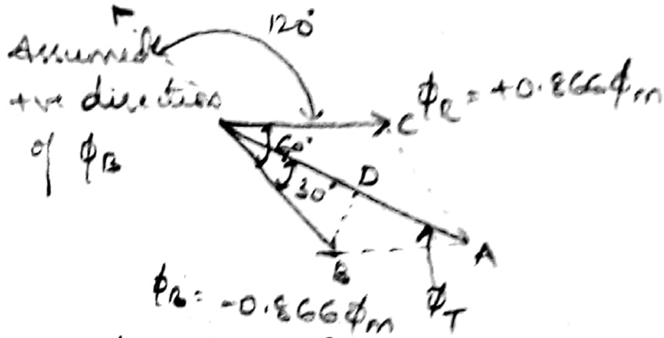

(d) Vector diagram for $\theta = 60°$

Doing the same construction, drawing perpendicular from B on $\phi_T$ we get the same result,

$$\phi_T = 1.5\phi_m$$

But it can be seen that though its magnitude is $1.5\phi_m$ it has rotated through $60^\circ$ in space, in clockwise direction from its previous position.

fig (d) :- Vector diagram $\theta = 120^\circ$




**CASE 3 :-** $\theta = 120^\circ$

$$\phi_R = \phi_m \sin 120 = +0.866\phi_m$$

$$\phi_Y = \phi_m \sin 0 = 0$$

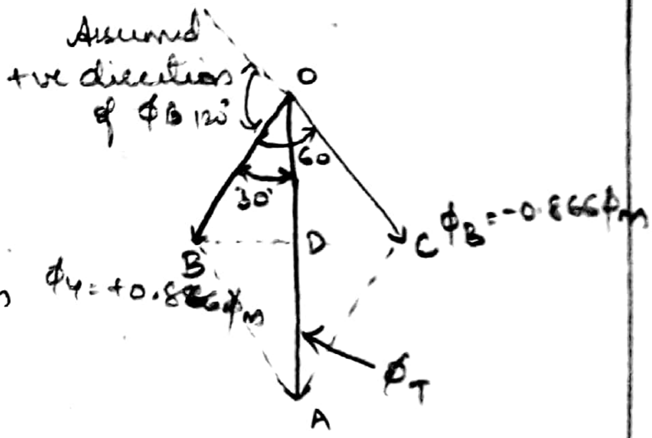
$$\phi_B = \phi_m \sin(-120) = -0.866\phi_m$$

Similarly drawing $\perp^{r}$ from B on $\phi_T$, we have

$$\phi_T = 1.5\phi_m$$

But the position of $\phi_T$ is such that it has further rotated through $60^\circ$ from its previous position in clockwise direction

**CASE 4 :-** $\theta = 180^\circ$




$$\phi_R = \phi_m \sin 180 = 0$$

$$\phi_Y = \phi_m \sin 60 = +0.866\phi_m$$

$$\phi_B = \phi_m \sin(-60) = -0.866\phi_m$$

$|||^{ly}$ $\phi_T = 1.5\phi_m$

fig(e) Vector diagram for $\theta = 180^\circ$

The magnitude of $\phi_T$ remains same. But it can be seen that it has further rotated through $60^\circ$ from its previous position in clockwise direction.

So for an electrical half cycle of $180^\circ$, resultant $\phi_T$ has rotated through $180^\circ$. This is applicable for the windings wound for 2 poles.

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The following conclusions are made: for.

a) The resultant of the three alternating fluxes, separated from each other by 120°, has a constant amplitude of 1.5 $\phi_m$ where $\phi_m$ is maximum amplitude of an individual flux due to any phase.

b) The resultant always keeps on rotating with a certain speed in space.

* This shows that when a three phase windings are excited by balanced 3-$\phi$ ac supply then the resulting field produced is RMF. ~~For~~ Though nothing is rotating physically, the field is rotating in space having constant amplitude.

SPEED OF RMF :-

There exists the fixed relation between frequency f of a.c supply to the windings, the no. of poles P for which winding is wound and speed N r.p.m of rotating magnetic field. For standard frequency, whatever speed of RMF results is called synchronous speed, in case of induction motors denoted as $N_s$.

$$N_s = \frac{120f}{P} = \text{speed in RMF}$$

This is the speed with which RMF rotates in space.

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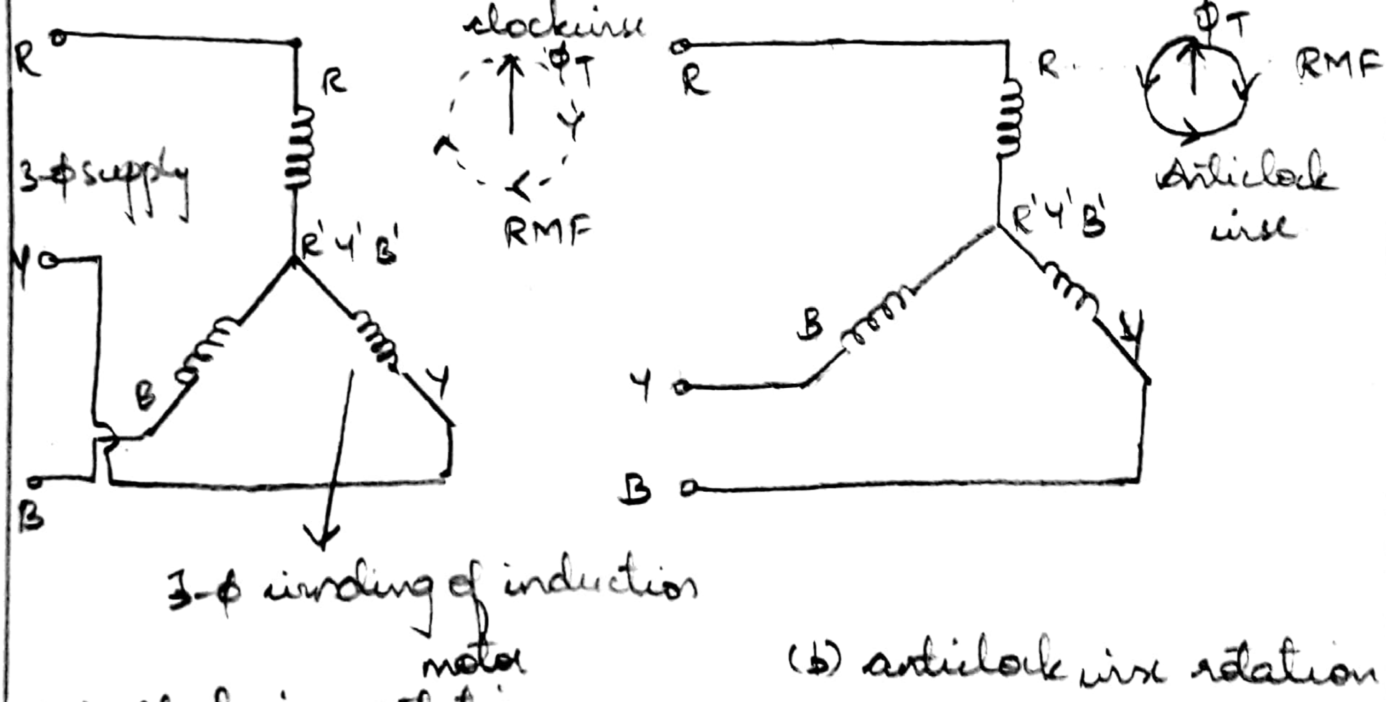
DIRECTION OF RMF :-

The direction of RMF is always from the axis of the leading phase of the three phase winding towards the lagging phase of the winding. In the phase sequence of RYB, phase R leads Y by 120°, Y leads B by 120°. So RMF rotates from axis of R to axis of Y then to axis of B and soon. So the direction is clockwise.

- The direction can be reversed by interchanging any two terminals of the three phase windings while connecting to the 3-ϕ supply. The terminals Y and B are shown interchanged. In such case RMF direction will be anticlockwise.




(a) clockwise rotation

(b) anticlock wise rotation

As Y and B of windings are connected to B and Y from winding point of view the phase sequence becomes R-B-Y. Thus RMF axis follows the direction from R to B to Y. i.e anticlock wise.

## COMPARISON OF SQUIRREL CAGE AND WOUND ROTOR :-

| WOUND (OR) SLIP RING ROTOR | SQUIRREL CAGE ROTOR |
|---|---|
| 1. Rotor consists of a three phase winding similar to stator wdg. | 1. Rotor consists of bars which are shorted at the ends with the help of end rings. |
| 2. Construction is complicated. | 2. Construction is very simple. |
| 3. Resistance can be added externally | 3. As permanently shorted, external resistance cannot be added |
| 4. Slip rings and brushes are present to add external resistance. | 4. Slip rings and brushes are absent. |
| 5. The construction is delicate and due to brushes, frequent maintenance is necessary. | 5. The construction is robust and maintenance free. |
| 6. The rotors are very costly. | 6. Due to simple construction, the rotors are cheap. |
| 7. Only 5% of induction motors in industry use slip ring rotor. | 7. Very common almost 95% induction motors use this type of rotor |
| 8. High starting torque can be obtained | 8. Moderate starting torque which cannot be controlled. |
| 9. Rotor resistance starter can be used | 9. Rotor resistance starter cannot be used. |
| 10. Rotor must be wound for the same number of poles as stator | 10. The rotor automatically adjusts itself for same number of poles as stator. |
| 11. speed control by rotor resistance is possible. | 11. speed control by rotor resistance is not possible. |
| 12. Rotor copper losses are high hence efficiency is less. | 12. Rotor copper losses are less hence have higher efficiency. |
| 13. Used for lifts, hoists, cranes, elevators, compressors etc. | 13. used for lathes, drilling machines, fans, blowers, water pumps, grinders, printing m/c etc. |

# WORKING PRINCIPLE OF AN INDUCTION MOTOR :-

Induction motor works on the principle of electromagnetic induction

- When a three phase supply is given to the three phase stator winding, a rotating magnetic field of constant magnitude is produced. The speed of this rotating magnetic field is synchronous speed $N_s$ r.p.m.

$$N_s = \frac{120f}{P} = \text{speed of rotating magnetic field.}$$

where $f$ = supply frequency

$P$ = No. of poles for which stator winding is wound.

- This rotating field produces an effect of rotating poles around a rotor. Let the direction of rotation of this rotating magnetic field is clockwise as shown is fig (a)

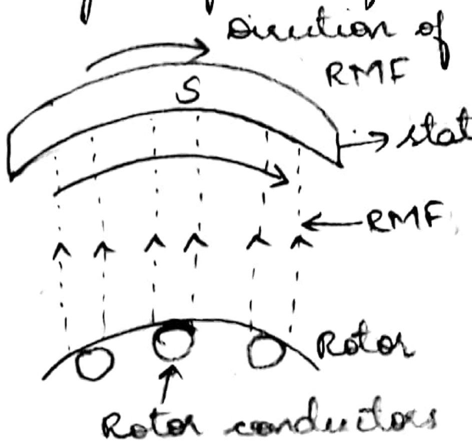



fig (a)

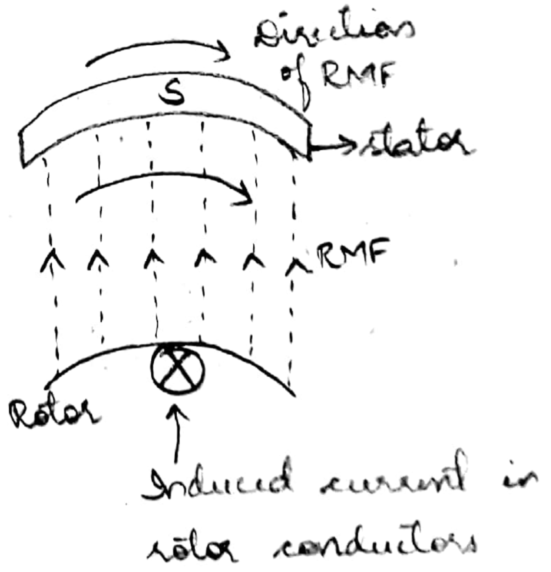



fig. (b)

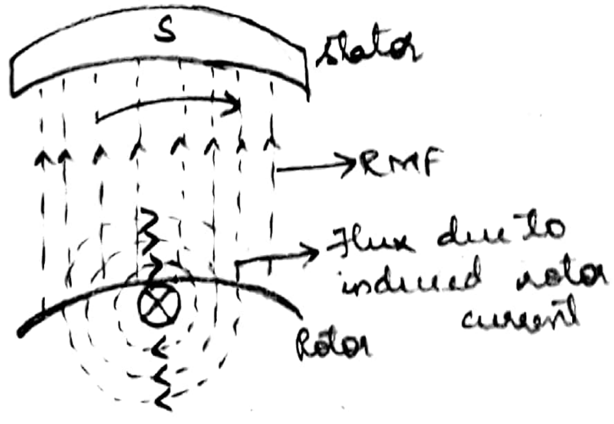



fig. (c)

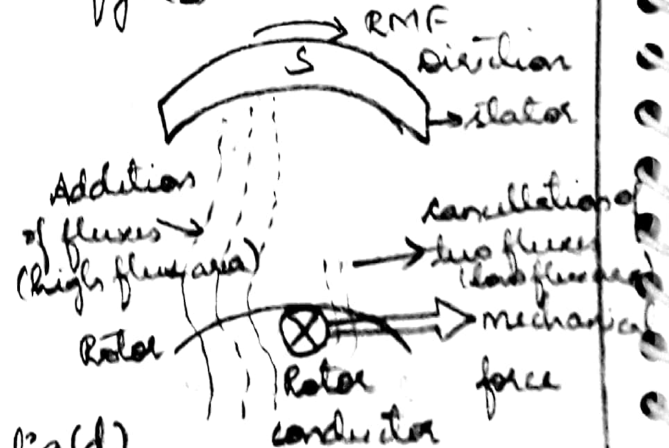



fig (d)

- At this instant rotor is stationary and stator flux RMF is rotating ∴ There exists a relative motion between the RMF and rotor conductors. This RMF gets cut by rotor conductors as RMF sweeps over rotor conductors.
- Whenever conductors cut the flux, emf gets induced in it. The emf induced in the rotor conductors is called rotor induced emf. (electromagnetic induction)
- As rotor forms closed circuit, induced emf circulates current through rotor called rotor current as shown in fig (b). Let direction of this current is going into the paper denoted by a cross as shown in fig (b).
- Any current carrying conductor produces its own flux. So rotor produces its flux called rotor flux. For assumed direction of rotor current, the direction of rotor flux is clockwise as shown in fig (c). This direction can be easily determined by using right hand thumb rule.
- There are two fluxes, one RMF and other rotor flux. Both fluxes interact with each other as shown in fig(d). On the left of rotor conductors, two fluxes are in same direction hence add up to high flux area. On the right side, two fluxes cancel each other to produce low flux area.
- As flux lines act as streched rubber band, high flux density area exerts a push on rotor conductor towards low flux density area. So rotor conductor experiences a force from left to right in this case as shown in fig(d), due to interaction of the two fluxes.
- As all the rotor conductors experiences a force, the overall rotor experiences a torque and starts rotating. So interaction

of three fluxes is very essential for a motoring action.

- As seen from (d), the direction of force experienced is same as that of rotating magnetic field. Hence rotor starts rotating in the same direction as that of rotating magnetic field.
- According to Lenz's law, the direction of induced current in the rotor is so as to oppose the cause producing it. The cause of rotor current is the induced emf which is induced due to the relative motion between RMF and rotor conductors.
- Hence to oppose the relative motion i.e to reduce the relative speed, the rotor experiences a torque in the same direction as RMF and tries to catch up the speed of RMF.

So, $N_s$ = speed of rotating magnetic field in rpm

$N$ = speed of rotor i.e, motor in r.pm

$N_s - N$ = relative speed b/w RMF & rotor conductor

∴ The rotor always rotates in same direction of RMF.

→ When rotor starts rotating, it tries to catch speed of RMF. If it catches the speed, the relative motion b/w rotor and RMF will vanish [$N_s - N = 0$]. The relation motion is the main cause for the induced emf in rotor. ∴ emf vanish and there cannot be rotor current and flux to produce torque on rotor. ∴ Motors will stop.

- Then there exists relative motion b/w rotor and RMF and motor will start. But due to inertia of rotor, this does not happen in practice and rotor continues to rotate with a speed slightly less than the synchronous speed of RMF.
- The induction motor never rotates at synchronous speed hence called subsynchronous speed and motor as asynchronous motor

$N < N_s$

∴ Rotor slips behind the RMF produced by stator.

* The difference between the two is called slip speed of motor

$N_s - N$ = slip speed of the motor in r.p.m.

This speed decides the magnitude of the induced emf and the rotor current which decides the torque produced. The torque produced is as per requirements of overcoming the friction and iron losses of the motor along with torque demanded by load.

## SLIP OF INDUCTION MOTOR :-

- Slip speed is the difference between the two speeds i.e, RMF ($N_s$) and rotor speed (N). This slip speed is expressed as the percentage of synchronous speed.
- Slip of the motor is defined as difference b/w the synchronous speed ($N_s$) and actual speed of rotor (N), expressed as a fraction of the synchronous speed $N_s$.
- Also called as absolute slip or fractional slip denoted as 's'

$$s = \frac{N_s - N}{N}$$

Percentage slip is expressed as,

$$\% \; s = \frac{N_s - N}{N_s} \times 100$$

In terms of slip, the actual speed of motor (N) is

$$N = N_s(1-s)$$

At start, motor is at rest and hence speed N is zero.

$s = 1$ at start.

- This is the maximum value of slip s possible for induction motor which occurs at start.
- While $s = 0$ gives $N = N_s$ which is not possible for an

induction motor. So slip cannot be zero at any circumsta[nce]

- Practically motor operates in the slip range of 0.01 to 0.05 i.e 1% to 5%. The slip corresponding to full load speed of the motor is called full load slip.

PROBLEMS :-

1. A 4 pole, 3-$\phi$ induction motor is supplied from 50Hz supply. Determine its synchronous speed. On full load, its speed is observed to be 1410 r.p.m. Calculate its full load slip.

Sol. Given data :-

$$P = 4, \quad f = 50Hz, \quad N = 1410 \text{ r.p.m.}$$

$$N_S = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ r.p.m}$$

Full load absolute slip is

$$S = \frac{N_S - N}{N_S} = \frac{1500 - 1410}{1500} = 0.06$$

$$\% S = 0.06 \times 100 = 6\%$$

$$\boxed{N_S = 1500 \text{ r.p.m}, \quad \% S = 6\%}$$

2. A 4 pole, 3-$\phi$, 50Hz, star connected induction motor has a full load slip of 4%. Calculate full load speed of motor.

Sol. Given data :-

$$P = 4, \quad f = 50Hz, \quad \% S_{fl} = 4\% = 0.04 = S_{fl}$$

$$N_S = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ r.p.m}$$

$$S_{fl} = \frac{N_S - N_{fl}}{N_S} \Rightarrow 0.04 = \frac{1500 - N_{fl}}{1500}$$

$\boxed{N_{fl} = 1440 \text{ r.p.m}}$ $N_{fl}$ = full load speed of motor.

## EFFECT OF SLIP ON ROTOR PARAMETERS :-

- In case of a transformer, frequency of the induced emf in the secondary is same as the voltage applied to primary.
- In case of induction motor at start $N=0$ and slip $s=1$. Under this condition as long as $s=1$, the frequency of induced emf in rotor is same as the voltage applied to the stator.
- But as motor gathers speed, induction motor has some slip corresponding to speed N. In such case, the frequency of induced emf in rotor is no longer same as that of stator voltage. Slip affects the frequency of rotor induced emf.
- Due to this some other parameters also gets affected.

(i) Rotor frequency
(ii) magnitude of rotor induced emf
(iii) Rotor reactance
(iv) Rotor power factor
(v) Rotor current

(i) **EFFECT ON ROTOR FREQUENCY :-**

The speed of RMF is

$$N_s = \frac{120f}{P} \quad \text{——(1)}$$

At start when $N=0$, $s=1$ and stationary rotor has maximum relative motion with r.t. RMF. ∴ maximum emf gets induced in the rotor at start. The frequency of this induced emf at start is same as that of supply frequency.

- As motor actually rotates with speed N, the relative speed of rotor w.r.t RMF decreases and becomes equal to slip speed of $N_s - N$. The induced emf in rotor depends on rate of cutting flux i.e $N_s - N$. Hence in running condition magnitude of induced emf decreases so as its frequency.

- The rotor is wound for same number of poles as that of stator i.e P
- If $f_r$ is the frequency of rotor induced emf in running condition at slip speed $N_s - N$ then there exists relation b/w $N_s - N$, $f_r$ and P similar to eq(1). Rotor in running condⁿ,

$$N_s - N = \frac{120 f_r}{P} \quad , \text{ rotor poles} = \text{stator poles} = P$$

Dividing (2)/(1)

$$\frac{N_s - N}{N_s} = \frac{120 f_r / P}{120 f / P} \quad , \text{ but } \frac{N_s - N}{N_s} = s$$

$$s = \frac{f_r}{f} \Rightarrow \boxed{f_r = sf}$$

At start $s=1$, hence rotor freq is same as supply freq. As slip of the induction motor is in the range 0.01 to 0.05, rotor freq is very small in running condition.

(ii) EFFECT ON MAGNITUDE OF ROTOR INDUCED EMF :-

When rotor is standstill, $s=1$, relative speed is maximum and maximum emf gets induced in the rotor. Let this emf be

$E_2$ = Rotor induced e.m.f per phase on standstill condition

As rotor gains speed, the relative speed b/w rotor and RMF decreases and hence e.m.f in rotor also decreases as it is proportional to the relative speed $N_s - N$. Let this emf be

$E_{2r}$ = Rotor induced e.m.f per phase in running condition

Now, $E_2 \propto N_s$ while $E_{2r} \propto N_s - N$

Dividing two proportionality eqn.

$$\frac{E_{2r}}{E_2} = \frac{N_s - N}{N_s} \quad \text{but } \frac{N_s - N}{N_s} = s.$$

$$\frac{E_{2r}}{E_2} = s$$

$$\boxed{E_{2r} = sE_2}$$

The magnitude of the induced emf in the rotor also reduces by slip times the magnitude of induced emf at standstill condⁿ.

(iii) EFFECT ON ROTOR RESISTANCE AND REACTANCE :-

The rotor winding has its own resistance and the inductance. In a case of squirrel cage rotor, the rotor resistance is very small and neglected but slip ring rotor has its own resistance which can be controlled by adding external resistance through slip rings.

Let,

$R_2$ = rotor resistance per phase on standstill

$X_2$ = rotor reactance per phase on standstill

At standstill.

$f_r = f$ [hence if $L_2$ is the inductance of rotor per phase)

$X_2 = 2\pi f_r L_2 = 2\pi f L_2 \ \Omega/ph$

while $R_2$ = Rotor resistance in $\Omega/ph$

In running condition,

$f_r = sf$ hence

$$X_{2r} = 2\pi f_r L_2 = 2\pi f s L_2 = s\cdot(2\pi f L_2)$$

$$\boxed{X_{2r} = sX_2}$$

where $X_{2r}$ = Rotor reactance in running condition.

As resistance is independent of freq remains same at standstill and in running condition. While rotor reactance decreases by slip times the rotor reactance at standstill.

Hence rotor impedence per phase is

$Z_2$ = Rotor impedance on standstill $(N=0)$ condition

$= R_2 + jX_2 \ \Omega/ph$

$Z_2 = \sqrt{R_2^2 + X_2^2} \ \Omega/ph$

while $Z_{2r}$ = Rotor impedance in running condition

$= R_2 + jX_{2r} = R_2 + j(sX_2) \ \Omega/ph$

$$\boxed{Z_{2r} = \sqrt{R_2^2 + (sX_2)^2} \ \Omega/ph}$$

(iv) EFFET ON ROTOR POWER FACTOR:-

From rotor impedance, the expression for the power factor of rotor at standstill and also in running condition can be written.

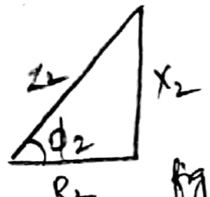



fig (a)

• The impedance triangle on standstill rotor condition is shown in fig(a)

From fig, $\cos\phi_2$ = Rotor P.F on standstill

$$\boxed{\cos\phi_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{R_2^2 + X_2^2}}}$$ At standstill

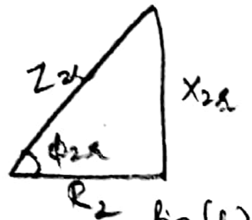



fig (b)

The impedance in running condition becomes $Z_{2r}$ and impedance $\Delta^{le}$ is shown in fig(b)

From fig,

$$\cos\phi_{2r} = \frac{R_2}{Z_{2r}} = \frac{R_2}{\sqrt{R_2^2 + X_{2r}^2}}$$

$$\boxed{\cos\phi_{2r} = \frac{R_2}{Z_{2r}} = \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}}$$ — Running

* As rotor winding is inductive, the rotor pf is always lagging in nature.

(iv) EFFECT ON ROTOR CURRENT :-

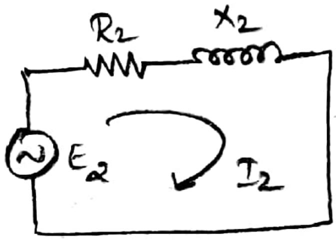
Let $I_2$ = rotor current per phase on standstill condition.
The magnitude of $I_2$ depends on magnitude of $E_2$ and impedance $Z_2$ per phase.

$$I_2 = \frac{E_2 \text{ per phase}}{Z_2 \text{ per phase}} \text{ A.}$$

sub $Z_2$

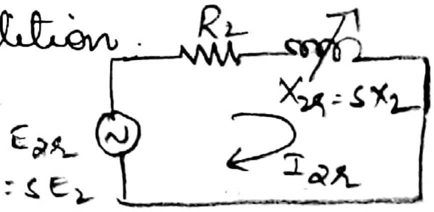
$$I_2 = \frac{E_2}{\sqrt{R_2^2 + X_2^2}} \text{ A.}$$

The equivalent rotor circuit on standstill is shown in fig (a)




$\phi_2$ is the angle b/w $E_2$ and $I_2$ which determines rotor p.f on standstill.

- In running condition, $Z_2$ changes to $Z_{2r}$ while induced e.m.f changes to $E_{2r}$. Hence the magnitude of current in running condition is also different than $I_2$ on standstill.
- The eq. rotor circuit on running condition is shown in fig (b).




$I_{2r}$ = Rotor current per phase in running condition

- The value of slip depends on speed which inturn depends on load on motor. Hence $X_{2r}$ is shown variable in eq. ckt.

From ckt,

$$I_{2r} = \frac{E_{2r}}{Z_{2r}} = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

$\phi_{2r}$ is the angle b/w $E_{2r}$ and $I_{2r}$ which decides pf in running condition.

* Putting s=1, ex obtained in running cond$^n$, the values at standstill can be obtained.

## INDUCTION MOTOR AS TRANSFORMER:-

W.K.T, transformer is a device in which two windings are magnetically coupled and when one winding is excited by a.c supply of certain frequency, the emf gets induced in the second winding having same frequency as that of supply given to the first windings. The winding to which supply is given is called primary winding and winding in which emf gets induced is called secondary winding. The induction motor can be regarded as the transformer.

Differences between normal transformer and induction motor

→ Normal transformer is the alternating flux transformer. while induction motor is a rotating flux transformer.

→ Normal transformer has no air gap where as induction motor has distinct air gap between its stator & rotor.

→ In normal transformer, the frequency of induced emf and current in primary and secondary is always same. In induction motor frequency of emf and current on ~~rotor~~ stator side remains same but frequency of rotor emf and current depends on slip and slip depends on the motor. So there is a variable frequency on rotor side. [But when N=0, S=1, then frequency of supply to the stator and of induced emf in rotor is same.

→ In normal transformer, the entire energy present in its secondary circuit, is in the electrical form, in an Induction motor part of its energy in the rotor circuit is in electrical part converted into mechanical form.

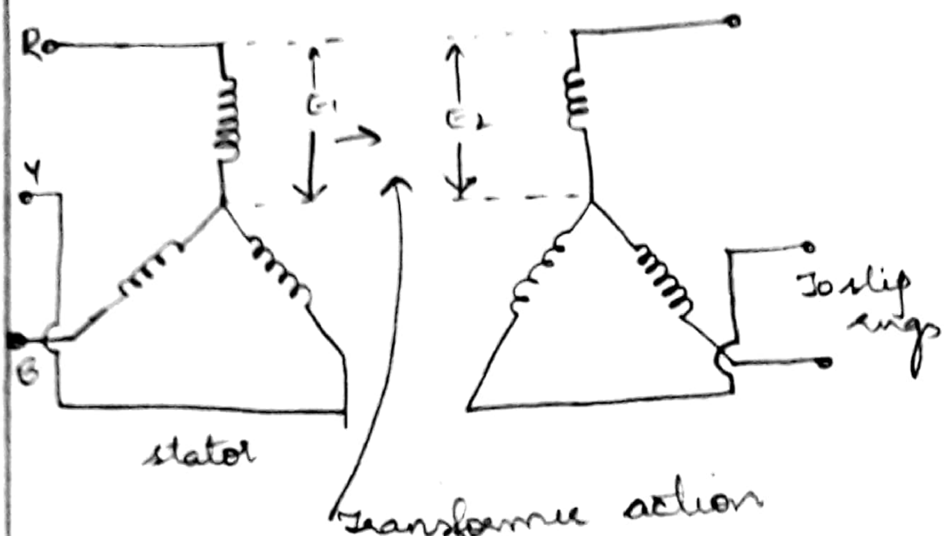



Fig:- Induction motor as transformer

Induction motor can be treated as transformer shown in fig. with the slip ring induction motor with star connected stator and rotor is shown.

If, $E_1$ = stator emf per phase in volts.

$E_2$ = Rotor induced emf per phase in volts at start when motor is at standstill.

→ According to normal transformer, there exists a fixed relation b/w $E_1$ and $E_2$ called transformation ratio.

∴ At start when $\boxed{N=0,\ S=1}$

we get, $\frac{E_2}{E_1} = K = \boxed{\frac{\text{Rotor turns / phase}}{\text{stator turns / phase}}}$

PROBLEMS :-

For a 4-pole, 50Hz induction motor ratio of stator to rotor turns is 2. On a certain load, its speed is observed to be 1455 r.p.m when connected to 415V supply. Calculate (i) Frequency of rotor e.m.f in running condition.

(ii) magnitude of induced emf in the rotor at standstill.
(iii) magnitude of induced emf in the rotor under running condition. Assume star connected stator.

Sol Given data :-

$K = \frac{\text{Rotor turns}}{\text{stator turns}} = \frac{1}{2} = 0.5$ and

$P = 4,\ f = 50Hz,\ N = 1455\ rpm,\ E_{1\,line} = 415V$

$$N_s = \frac{120f}{P} = \frac{120\times 50}{4} = 1500\ r.p.m$$

For given load, N = 1455 r.p.m.

$$s = \frac{N_s - N}{N_s} = \frac{1500-1455}{1500} = 0.03 \text{ i.e, } 3\%.$$

(i) $f_r = sf = 0.03\times 50 = 1.5Hz.$

(ii) At standstill, induction motor acts as a transformer so

$$\frac{E_{2ph}}{E_{1ph}} = \frac{\text{Rotor turns}}{\text{stator turns}} = K$$

But ratio of stator to rotor turns is 2 i.e.

$\frac{N_1}{N_2} = 2 \quad \therefore \frac{N_2}{N_1} = \frac{1}{2} = K$ and $E_{1\,line} = 415V.$

$\therefore$ The given values are always line values unless and until specified stated as per phase.

As star connection $E_{line} = \sqrt{3}\,E_{ph}$

$$E_{1ph} = \frac{E_1}{\sqrt{3}} = \frac{415}{\sqrt{3}}$$

$\therefore E_{1ph} = 239.6V$

$\therefore E_{2ph}/E_{1ph} = 1/2 \Rightarrow E_{2ph} = \frac{1}{2}\times 239.6 = 119.8V.$
(Rotor induced emf on standstill)

(iii) In running condn.

$E_{2r} = sE_2 = 0.03\times 119.8 = 3.59V.$

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## TORQUE EQUATION :-

The torque produced in the induction motor depends on

1. The part of rotating magnetic field which reacts with rotor and is responsible to produced induced emf in rotor.
2. The magnitude of rotor current in running conditions.
3. The power factor of the rotor circuit in running condition.

Mathematically the relationship can be expressed as,

$$T \propto \phi I_{2r} \cos\phi_{2r} \quad \text{——— (1)}$$

$\phi$ = Flux responsible to produce induced emf

$I_{2r}$ = Rotor running current

$\cos\phi_{2r}$ = Running pf of rotor

Flux $\phi$ produced by stator is propotional to $E_1$ i.e, stator voltg

$$\phi \propto E_1 \quad \text{——— (2)}$$

$E_1$ & $E_2$ are related to each other through ratio of stator turns to rotor turns i.e, K.

$$\therefore \frac{E_2}{E_1} = K. \quad \text{——— (3)}$$

Using eq. (3) in eq (2),

$$E_2 \propto \phi \quad \text{——— (4)}$$

In eq (1), $\phi$ can be replaced by $E_2$

$$I_{2r} = \frac{E_{2r}}{Z_{2r}} = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \quad \text{——— (5)}$$

$$\cos\phi_{2r} = \frac{R_2}{Z_{2r}} = \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}} \quad \text{——— (6)}$$

Using eq (4), (5), (6) in eq. (1)

$$T \propto E_2 \cdot \frac{sE_2}{\sqrt{R_2^2+(sX_2)^2}} \cdot \frac{R_2}{\sqrt{R_2^2+(sX_2)^2}}$$

$$T \propto \frac{sE_2^2 R_2}{R_2^2+(sX_2)^2} \quad N\text{-}m$$

$$T = \frac{KsE_2^2R_2}{R_2^2+(sX_2)^2} \qquad \text{———— (7)}$$

where K is constant of proportionality

K is proved to be $\frac{3}{2\pi n_s}$ for 3-$\phi$ induction motor

$$K = \frac{3}{2\pi n_s} \quad ; \quad n_s = \text{synchronous speed in r.p.s}$$

$$n_s = \frac{N_s}{60}$$

Using eq (8) in (7) we get

$$\boxed{T = \frac{3}{2\pi n_s} \cdot \frac{sE_2^2 R_2}{R_2^2+(sX_2)^2} \quad N\text{-}m}$$

## STARTING TORQUE :-

Starting torque is the torque produced by an induction motor at start. At start, N=0 and slip s=1.

So putting s=1 in above eq,

$$\boxed{T_{st} = \frac{3}{2\pi n_s} \cdot \frac{E_2^2 R_2}{R_2^2+X_2^2}} \qquad \text{———— (10)}$$

From eq (10), it is clear that by changing $R_2$, the starting torque $T_{st}$ can be controlled.

## CONDITION FOR MAXIMUM TORQUE :-

The torque depends on slip at which motor is running. The supply voltage to the motor is usually rated and constant and there exists a fixed ratio b/w $E_1$ and $E_2$. Hence $E_2$ is also constant. ||ly $R_2$, $X_2$, $n_s$ are constant for induction motor.

While finding the condition for maximum torque, the only parameter which controls the torque is slip s. mathematically for the maximum torque, we can write,

$$\frac{dT}{ds} = 0$$

where $T = \dfrac{K s E_2^2 R_2}{R_2^2 + (s X_2)^2}$

$$\frac{dT}{ds} = \frac{(K s E_2^2 R_2)\frac{d}{ds}(R_2^2 + s^2 X_2^2) - (R_2^2 + s^2 X_2^2)\frac{d}{ds}(K s E_2^2 R_2)}{(R_2^2 + s^2 X_2^2)^2}$$

$$K s E_2^2 R_2 [2 s X_2^2] - (R_2^2 + s^2 X_2^2)(K E_2^2 R_2) = 0$$

$$\therefore 2 s^2 K X_2^2 E_2^2 R_2 - R_2^2 K E_2^2 R_2 - K s^2 X_2^2 E_2^2 R_2 = 0$$

$$K s^2 X_2^2 E_2^2 R_2 - R_2^2 K X_2^2 R_2 = 0$$

$$s^2 X_2^2 - R_2^2 = 0$$

$$s^2 = \frac{R_2^2}{X_2^2} \Rightarrow \boxed{s = \frac{R_2}{X_2}}$$ neglecting -ve slip

$$\boxed{S_m = \frac{R_2}{X_2}}$$

This is slip at which torque is max and is denoted as $S_m$. It is the ratio of standstill per phase values of resistance and reactance of rotor, when torque produced by the induction motor is at its maximum.

## MAGNITUDE OF MAXIMUM TORQUE :-

This can be obtained by substituting $s_m = \frac{R_2}{X_2}$ in torque denoted by $T_m$.

$$T_m = \frac{k s_m E_2^2 R_2}{R_2^2 + (s_m X_2)^2}$$

$$T_m = \frac{k\left[\frac{R_2}{X_2}\right] E_2^2 R_2}{R_2^2 + \left[\left[\frac{R_2}{X_2}\right] X_2\right]^2}$$

$$\boxed{T_m = \frac{k E_2^2}{2 X_2}} \quad N\text{-}m$$

From the expression, we observe

1. It is inversely proportional to the rotor reactance.
2. It is directly proportional to the square of the rotor induced emf at standstill.
3. The maximum torque is not dependent on the rotor resistance $R_2$. But the slip at which it occurs i.e speed at which it occurs depends on the value of rotor resistance $R_2$.

## PROBLEMS :-

1. A 3-$\phi$, 400V, 50Hz, 4 pole induction motor has a star connected stator winding. The rotor resistance and reactance are $0.1\Omega$ and $1\Omega$ resp. The full load speed is 1440 rpm. Calculate the torque developed on full load by the motor. Assume stator to rotor ratio as 2:1

Sol. Given data :-

$P = 4$, $f = 50Hz$, $R_2 = 0.1\Omega$, $X_2 = 1\Omega$, $N = 1440$ r.p.m.

$$\frac{\text{Stator turns}}{\text{Rotor turns}} = \frac{2}{1}$$

$$\therefore \quad K = \frac{E_2}{E_1} = \frac{\text{Rotor turns}}{\text{stator turns}} = \frac{1}{2} = 0.5$$

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ r.p.m}$$

$E_1$ line $= 400V$. (stator line voltage)

$$\therefore \quad E_{1ph} = \frac{E_1 \text{ line}}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94V$$

But $\frac{E_{2ph}}{E_{1ph}} = 0.5 = K$

$\therefore \quad E_{2ph} = 0.5 \times 230.94 = 115.47V$

Full load slip, $s = \frac{N_s - N}{N_s} = \frac{1500 - 1440}{1500} = 0.04$

$n_s$ = synchronous speed in r.p.s

$$= \frac{N_s}{60} = \frac{1500}{60} = 25 \text{ rps}$$

$$T = \frac{3}{2\pi n_s} \cdot \frac{sE_2^2 R_2}{R_2^2 + (sX_2)^2} = \frac{3}{2\pi \times 25} \times \frac{0.04 \times (115.47)^2 \times 0.1}{(0.1)^2 + (0.04 \times 1)^2}$$

$$\boxed{T = 87.81 \text{ N-m}}$$

Q. A 400V, 4 pole, 3-$\phi$, 50Hz star connected induction motor has a rotor resistance and reactance per phase equal to $0.01\,\Omega$ and $0.1\,\Omega$ resp. Determine (i) Starting torque
(ii) slip at which maximum torque will occur
(iii) speed at which maximum torque occur
(iv) maximum torque (v) full load torque if full load slip is 4%. Assume ratio of stator to rotor turns as 4.

Sol. Given data:-

$P = 4$, $f = 50Hz$, stator turns / rotor turns $= 4$,

$R_2 = 0.01\Omega$, $X_2 = 0.1\Omega$.

$E_{1line}$ = stator line voltage $= 400V$.

$$E_{1ph} = \frac{E_1 line}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94 \quad \text{star connected.}$$

$$K = \frac{E_{2ph}}{E_{1ph}} = \frac{\text{Rotor turns}}{\text{stator turns}} = \frac{1}{4}$$

$$E_2 = \frac{1}{4} \times E_{1ph} \Rightarrow \frac{230.94}{4} = 57.735V$$

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ r.p.m.}$$

(i) At start, $s = 1$

$$T_{st} = \frac{KE_2^2 R_2}{R_2^2 + X_2^2} \quad \text{where } k = 3/2\pi n_s$$

$$n_s = \frac{N_s}{60} = \frac{1500}{60} = 25 \text{ r.p.m}$$

$$K = \frac{3}{2\pi \times 25} = 0.01909$$

$$\boxed{T_{st} = 63.031 \text{ N-m}}$$

(ii) slip at which maximum torque occurs is

$$s_m = \frac{R_2}{X_2} = \frac{0.01}{0.1} = 0.1$$

$$\boxed{\% s_m = 10\%}$$

(iii) speed at which max torque occurs,

$$N = N_s(1 - s_m) = \boxed{1350 \text{ rpm} = N}$$

(iv)
$$\boxed{T_m = \frac{KE_2^2}{2X_2} = 318.16 \text{ N-m}}$$

(v) $s_f = 0.04$, as $\% s_f = 4\%$.

$$T_{f.l} = \frac{ks_f E_2^2 R_2}{R_2^2 + (s_f X_2)^2} = \boxed{219.52 \text{ N-m} = T_{fl}}$$

## TORQUE – SLIP CHARACTERISTICS :-

As induction motor is loaded from no load to full load, its speed decreases hence slip increases. Due to the increased load, motor has to produce more torque to satisfy load demand. T depends on slip. The behaviour of motor can be judged by plotting a curve in which torque produced against slip.

The curve obtained by plotting torque against slip from $s = 1$ (at start) to $s = 0$ (at synchronous speed) is called torque slip characteristics.

$$T \propto \frac{sR_2}{R_2^2 + (sX_2)^2} \qquad [\ E_2 \text{ is constant as supply voltage const})$$

To judge the torque-slip char, slip range is divided into two parts and analyse independently.

### (i) LOW SLIP REGION :-

In this region, s is small. so $(sX_2)^2$ is small compared to $R_2^2$ that can be neglected.

$$T \propto \frac{sR_2}{R_2^2} \propto s \implies \boxed{T \propto s}$$

So as load ↑, N ↓, s ↑. This increases torque which satisfies load demand. Hence graph is straight line in nature.

At $N = N_s$, $s = 0$ hence $T = 0$, as no torque is generated at $N = N_s$, motor stops if it tries to achieve $N_s$.

∴ Torque increases linearly in this region.

### (ii) HIGH SLIP REGION :-

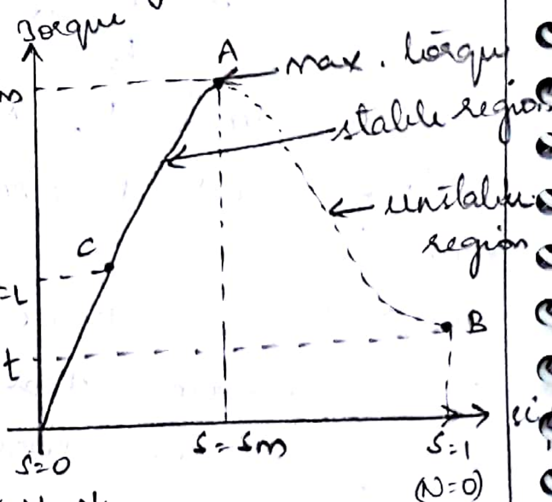
In this region slip is high i.e slip approaches to 1. Here $R_2^2$ is small as compared to $(sX_2)^2$. Hence neglecting $R_2^2$, we get

$$T \propto \frac{s R_2}{(s X_2)^2} \propto \frac{1}{s} \Rightarrow \boxed{T \propto \frac{1}{s}}$$

As T is inversely proportional to s, nature is like rectangular hyperbola.

→ Now when load ↑, load demand ↑ but N↓, s↑.
As $T \propto \frac{1}{s}$, ∴ T↓ as s↑. But torque must increase to satisfy load demand.

- As T↓ as loading effect, N↓ further, s↑. Again T↓ as $T \propto \frac{1}{s}$, hence same load acts as an extra load due to reduction in torque produced. Hence speed further stops. Eventually motor comes to standstill condition.
- The motor cannot continue to rotate at any point in this high slip region called unstable region of operation.
- So torque slip characteristics has two parts.
  1. Straight line called stable region of operation.
  2. Rectangular hyperbola called unstable region of operation.
- In low slip region, s↑, T↑ linearly. Every motor has its own limit to produce a torque. The max torque, the motor can produce as load increases is $T_m$ which occurs at $s = s_m$. So linear behaviour continues till $s = s_m$.
- If load is increased beyond the limit, motor slip acts dominantly pushing motor into high slip region. Due to unstable cond$^n$, motor comes to standstill cond$^n$ at such a load.




OA = stable region
AB = unstable
B = starting torque
A = max. torque
C = full load torque

Hence Tm which motor can produce is also called breakdown torque or pull out torque. So range s=0 to s=sm is called low slip region, known as stable region of operation. Motor always operate at a point in this region.
Range s=sm to s=1 is called high slip region which is rectangular hyperbola, called unstable region of operation. motor cannot continue to rotate at any point in this region.
As S=1, N=0 i.e at start, motor produces a torque called starting torque denoted as Tst.

## LOSSES IN INDUCTION MOTOR:-

The various power losses in an Induction motors are
(i) Constant losses
(ii) Variable losses

## ~~Core~~ CONSTANT LOSSES :-

These are classified as core losses and mechanical losses. Core losses occur in stator core and rotor core. These are also called iron losses. These losses include eddy current losses and hysterisis losses.
The eddy current losses are minimized by using laminated construction while hysterisis losses are minimized by selecting high grade silicon steel as the material for stator and rotor. The iron losses depends on the frequency. The stator frequency is always supply freq. hence stator iron losses are dominant. As against this in rotor circuit, the freq is very very small which is slip times the supply frequency. Hence rotor iron losses are very small and generally neglected, in the running condition.
The mech. losses include frictional losses at the bearing and windage losses. The friction changes with speed but practically the drop in speed is very small hence these losses are assumed to be the part of constant losses.

## VARIABLE LOSSES :-

This include the copper losses in stator and rotor winding due to current flowing in the winding. As current changes as load changes, these losses are said to be variable losses. Generally stator iron losses are combined with copper losses at a particular load to specify total stator losses at particular load condition.

Rotor copper loss = $3 I_{2R}^2 R_2$

$I_{2R}$ - Rotor current per phase at particular load.

$R_2$ - Rotor resistance per phase.

## POWER FLOW DIAGRAM FOR AN INDUCTION MOTOR:-

Induction motor converts an electrical power supplied to it into mechanical power. The various stages in this conversion is called power flow in an induction motor.

The 3-$\phi$ supply given to the stator is the net electrical i/p (stator i/p)

$$\boxed{P_{in} = \sqrt{3} V_L I_L \cos\phi}$$

$V_L, I_L$ - line values of supply.

- The part of this power is utilized to supply the losses in the stator which are stator core as well as copper losses. The remaining power is delivered to the rotor magnetically through the air gap with the help of rotating magnetic field. This is called rotor input denoted as $P_2$.

$$\boxed{P_2 = P_{in} - \text{stator losses (core + copper)}}$$

- The rotor is not able to convert its entire i/p to the mech. as it has to supply rotor losses. Rotor losses are rotor copper losses denoted as $P_c$ as iron losses are very small.

$$\boxed{P_c = 3 \times I_{2R}^2 \times R_2}$$

After supplying these losses, the remaining part of $P_2$ is converted into mech which is called gross mech. power denoted by $P_m$.

$$P_m = P_2 - P_c$$

This power is delivered to the load connected to the shaft. But during this mech. transmission, part of $P_m$ is utilised to provide mechanical losses like friction and windage. Finally the power is available to the load at the shaft. This is called net output of the motor denoted as Pout. called as shaft power.

$$P_{out} = P_m - \text{mech. losses}.$$

The rating of the motor is specified in terms of Pout when load condition is full load condition. This is expressed in horse powers and called H.P rating of the motor.

All the above stages are diagramatically shown called as power flow diagram of an induction motor.

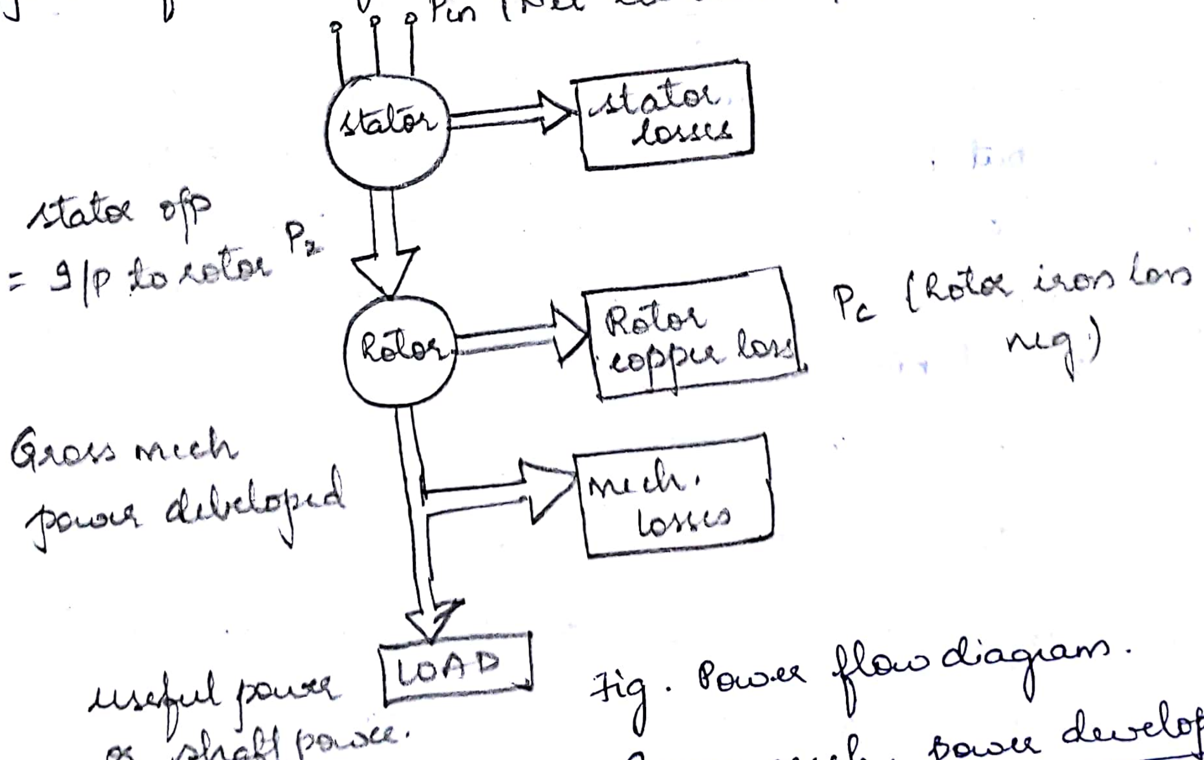



Fig. Power flow diagram.

$$\text{Rotor } \eta = \frac{\text{Rotor o/p}}{\text{Rotor i/p}} = \frac{\text{Gross mech. power developed}}{\text{Rotor I/p}} = \frac{P_m}{P_2}$$

$$\text{Net motor } \eta = \frac{\text{Net o/p at shaft}}{\text{Net electrical i/p to motor}} \Rightarrow \boxed{\%\eta = \frac{P_{out}}{P_{in}} \times 100}$$

RELATIONSHIP BETWEEN $P_2$, $P_c$ AND $P_m$ :-

The rotor input $P_2$, rotor copper loss $P_c$ and gross mechanical power developed $P_m$ are related through the slip s.

Let $T$ = Gross torque developed by motor in N-m

$P = T \times \omega$

$\omega$ = Angular speed $= \frac{2\pi N}{60}$, N = speed in rpm.

Now i/p to the rotor $P_2$ is from stator side through rotating magnetic field which is rotating at synchronous speed $N_s$. So, torque developed by rotor can be expressed interms of power input $P_2$ and angular speed at which power is inputed i.e, $\omega_s$ as

$$P_2 = T \times \omega_s \quad \text{where } \omega_s = \frac{2\pi N_s}{60} \text{ rad/sec}$$

$$P_2 = T \times \frac{2\pi N_s}{60} \quad \text{where } N_s \text{ is in r.p.m} \quad \text{——(1)}$$

The rotor tries to deliver this torque to the load. So rotor output is gross mechanical power developed $P_m$ and torque T. But rotor gives output at speed N and not $N_s$. So from output side $P_m$ and T can be related through angular speed $\omega$ and not $\omega_s$.

$$\therefore P_m = T \times \omega, \quad \text{where } \omega = \frac{2\pi N}{60}$$

$$P_m = T \times \frac{2\pi N}{60} \quad \text{——(2)}$$

The difference b/w $P_2$ and $P_m$ is rotor copper loss $P_c$.

$$P_c = P_2 - P_m = T \times \frac{2\pi N_s}{60} - T \times \frac{2\pi N}{60}$$

$$\therefore P_c = T \times \frac{2\pi}{60}(N_s - N) = \text{Rotor copper loss} \quad \text{——(3)}$$

Dividing (3)/(1)

$$\frac{P_c}{P_2} = \frac{T \times \frac{2\pi}{60}(N_s - N)}{T \times \frac{2\pi}{60} \times N_s} = \frac{N_s - N}{N_s}$$

$$\frac{P_c}{P_2} = s \quad \text{as} \quad \frac{N_s - N}{N_s} = \text{slip } s$$

$\therefore$ $\boxed{\text{Rotor copper loss } P_c = s \times \text{Rotor input } P_2}$

Now, $P_2 - P_c = P_m$

$$P_2 - sP_2 = P_m$$

$$\boxed{(1-s)P_2 = P_m}$$

Relationship can be expressed as

$$\boxed{P_2 : P_c : P_m = 1 : s : 1-s}$$

$$\frac{P_c}{P_m} = \frac{s}{1-s}, \quad \frac{P_2}{P_c} = \frac{1}{s} \text{ and so on.}$$

** The torque produced by rotor is gross mechanical torque and due to mechanical losses entire torque cannot be available to drive load. The load torque is net o/p torque called shaft torque (or) useful torque and denoted as $T_{sh}$.

$$T_{sh} = \frac{P_{out}}{\omega} = \frac{P_{out}}{\frac{2\pi N}{60}} \quad \text{and } T_{sh} < T_g \text{ due to mech. losses.}$$

EXAMPLE :-

The useful torque of a 3-$\phi$, 50Hz, 8 pole induction motor is 190 N-m. The rotor frequency is 1.5Hz. Calculate the rotor copper losses if mechanical losses are 700 W.

sol. $T_{sh} = 190 Nm$, $f_r = 1.5Hz$

mechanical loss = 700W

$$f_r = sf$$

$$S = \frac{1.5}{50} = 0.03$$

$$P_{out} = T_{sh} \times \omega$$

$$N = N_s(1-s)$$

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{8} = 750 \text{ r.p.m}$$

Rotor $N = 750(1-0.03)$

$$N = 727.5 \text{ rpm}$$

$$P_{out} = T_{sh} \times \frac{2\pi N}{60} \Rightarrow \frac{190 \times 2\pi \times 727.5}{60}$$

$$P_{out} = 14474.88W$$

From power flow diagram,

$$P_m - \text{mech losses} = P_{out}$$

$$P_m - 700 = 14474.88$$

$$P_m = 15174.88W$$

From $P_2 : P_c : P_m$ is $1 : s : 1-s$

$$\frac{P_c}{P_m} = \frac{s}{1-s}$$

$$P_c = P_m \times \frac{s}{1-s} \Rightarrow 15174.88 \times \frac{0.03}{(1-0.03)}$$

$$\boxed{P_c = 469.326W}$$

2. The full load power input to 4 pole, 50 Hz three phase induction motor is 50 kW, running at 1440 rpm. Calculate its full load efficiency if stator losses are 1000W and frictional losses are 650W.

Sol. Given data

$P = 4$, $f = 50\,Hz$, $P_{in} = 50\,kW$, $N = 1440\,rpm$

Stator losses = 1000W

Frictional losses = 650W

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500\ r.p.m$$

$$s = \frac{N_s - N}{N_s} = \frac{1500-1440}{1500} = 0.04$$

$P_2$ = Rotor input = $P_{in}$ − stator losses

$= 50 \times 10^3 - 1000$

$= 49 \times 10^3\ W$.

Now $P_2 : P_c : P_m$ is $1 : s : 1-s$

$$\frac{P_2}{P_c} = \frac{1}{s}$$

$$P_c = s \times P_2 = 0.04 \times 49 \times 10^3 = 1960\,W$$

$$P_m = P_2 - P_c = 49 \times 10^3 - 1960 = 47040\,W$$

$P_{out} = P_m$ − Frictional losses $= 47040 - 650$

$= 46390\ W$

$$\%\eta = \frac{P_{out}}{P_{in}} \times 100$$

$$= \frac{46390}{50 \times 10^3} \times 100 = 92.78\%$$

As slip is full load, efficiency is full load efficiency.

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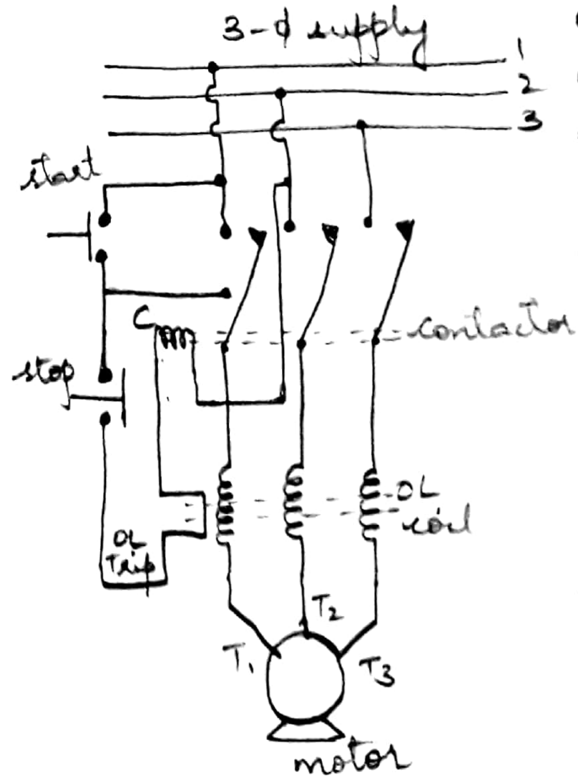
## STARTING METHODS FOR CAGE MOTORS:-

The followings three methods are commonly used for starting squirrel cage induction motors.

### (a) DIRECT ON LINE (DOL) STARTING:-

It is also called across the line starting and is used for small motors below about 5 kw. It is customary to employ a straight single-throw switch or a contactor as shown in fig.

- The starting current is high i.e, about 4 to 7 times the full-load current, the actual value depending on the size and design of the motor. Such a high starting current causes a relatively large voltage drop in the cables and ~~causes a relatively~~ thereby affects the supplies to other consumers on the same system.
- If the supply is isolation as in factories having their own generators, there is no such limit and motors of several hundred kw rating are started by direct-on line (DOL) method.
- Large motors fed from a common supply are started with reduced voltage by using any of the following two methods.



## (b) AUTO TRANSFORMER STARTER :-

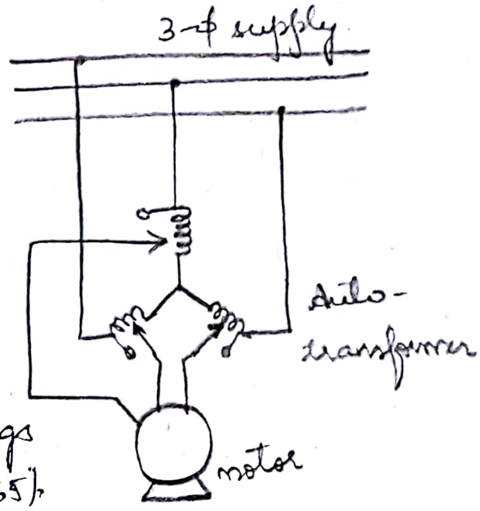
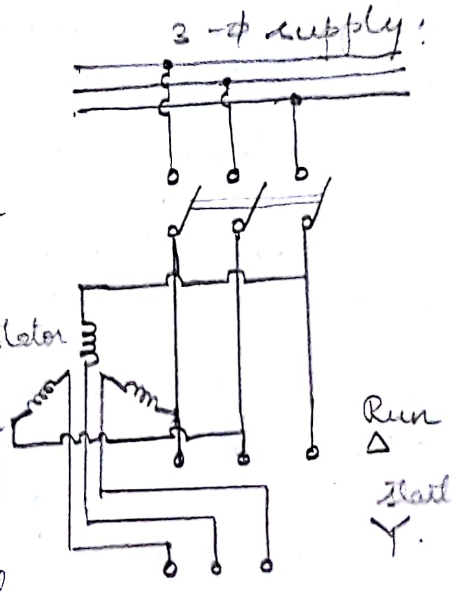



Fig 2. shows a 3-ϕ star connected auto-transformer used for applying reduced voltage across motor stator during starting period. In this way, starting current drawn by the motor can be reduced to any desired value.

- The auto-transformer usually has tappings which reduce the line voltages to 50%, 65% and 80% of the normal value.
- Since torque developed by the motor varies as the square of the applied voltage, the starting torque is considerably reduced.
- For ex if 50% tap is used, the starting torque developed by the motor would be $(0.5)^2$ or $1/4$ (or) 25% of the full load torque. For 65% and 80% taps, the starting torques would be $(0.65)^2 = 42.2\%$ and $(0.8)^2 = 64\%$ of the full load torque resp.
- Generally, a double throw switch is used for connecting the auto-transformer in the circuit for starting purposes.
- After the motor has run up to speed, the switch is moved directly to the supply.

## (c) STAR-DELTA STARTER :-

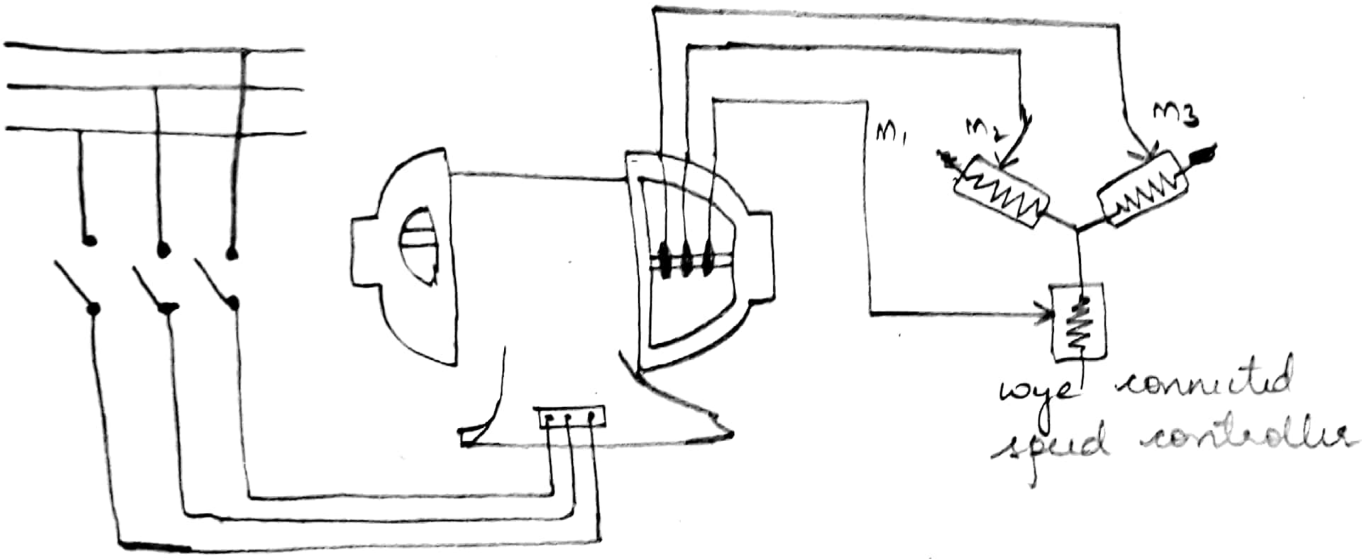



- In this method, the stator winding of the motor is connected in star for starting and in delta for normal running. The voltage of each phase at starting is reduced to $1/\sqrt{3}$ of the line voltage.
- The current in each phase is also reduced by the same factor so that

line current at starting becomes $1/\sqrt{3} \times 1/\sqrt{3} = 1/3$ of the current which the motor would have taken if connected directly across the supply.

- Since $T \propto V^2$, the starting torque is reduced to $(1/\sqrt{3})^2 = 1.3$ = 13.3% of the normal value.
- The change-over from star-starting to delta-running is made by a double-throw switch with interlocks to prevent motor starting with the switch in the RUN position.

STARTING OF SLIP RING MOTORS :-




- Fig. shows the schematic diagram of the circuit used for starting wound-rotor or slip-ring induction motor. Variable resistance can be added into the rotor circuit for reducing the starting current and improving the starting torque.
- The rotor windings are connected to the Y-connected external resistors through slip rings and brushes.
- The starter resistance is usually of the face-plate type, the starter arms forming the star point.
- The motor is started with all the starter resistance circuit in thus giving maximum starting torque.
- As motor speed increases, the starter resistance is gradually decreases. when motor comes upto its full-load speed, the starting resistance is reduced to zero and the slip rings are short circuited