D.C Motors

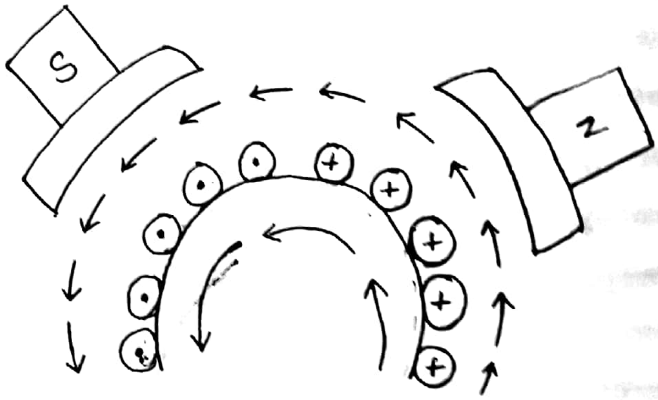
D.C Motor principle :-

A machine that Converts d.c power into mechanical power is known as a d.c. motor. Its operation is based on the principle that when a current carrying conductor is placed in a magnetic field, the Conductor Experiences a mechanical force. The direction of this force is given by Fleming's Left hand rule and magnitude is given by ;

$$F = BIl \text{ newtons.}$$

Basically, there is no Constructional difference between a d.c motor and a d.c. generator. The same d.c machine can be run as a generator and motor.

Working of D.C Motor :-



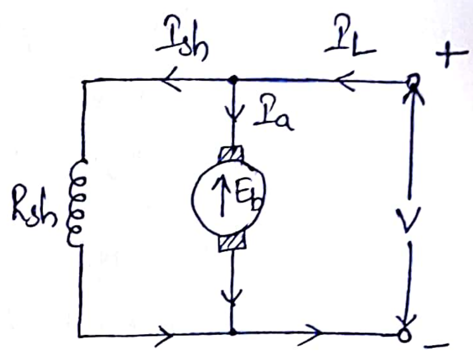
Consider a part of multipolar d.c. motor as shown. When the terminals of the motor are connected to an External Source of d.c supply ;

(i) The field magnets are Excited developing alternate N and S poles.

(ii) the armature conductors carry currents. All conductors under N-pole carry currents in one direction while all the conductors under S-pole carry currents in the opposite direction.

Refering to above fig. and applying Fleming's Left hand rule,
it is clear that force on Each conductor is tending to rotate
the armature in anticlockwise direction. All these forces add together
to produce driving torque which sets the armature rotating.
When the conductor moves from one side of a brush to the
other, the current in that conductor is reversed and at the
same time it comes under the influence of next pole which is
of opposite polarity. Consequently, the direction of force on the
... the same.

Voltage Equation of D.C Motor :-



Let in a d.c motor

V = applied voltage

E_b = back emf

R_a = armature resistance

I_a = armature current.

Since back emf E_b acts in opposition to the applied voltage V, the net voltage across the armature circuit is $V - E_b$. The armature current I_a is given by ;

$$I_a = \frac{V - E_b}{R_a}$$

$$\boxed{V = E_b + I_a R_a} \quad \text{——— ①}$$

Power Equation :-

If eq.① is multiplied by I_a throughout, we get,

$$\boxed{VI_a = E_b I_a + I_a^2 R_a}$$

VI_a = Electric power supplied to armature (armature i/p)

$E_b I_a$ = power developed by armature (armature o/p)

$I_a^2 R_a$ = Electric power wasted in armature (arm. cu loss).

Thus out of the armature i/p, a small portion is wasted as $I_a^2 R_a$ and the remaining portion $E_b I_a$ is converted into mechanical power within the armature.

Condition for maximum power :-

The mechanical power developed by the motor is $P_m = E_b I_a$.

Now, $P_m = VI_a - I_a^2 R_a$

Since V & R_a are fixed, power developed by the motor depends upon armature current. For max. power, dP_m/dI_a should be zero.

$$\therefore \frac{dP_m}{dI_a} = V - 2I_aR_a = 0$$

$$I_aR_a = V/2 .$$

Now, $V = E_b + I_aR_a$

$= E_b + V/2$ $\quad | \because I_aR_a = V/2 .$

$$\boxed{E_b = V/2}$$

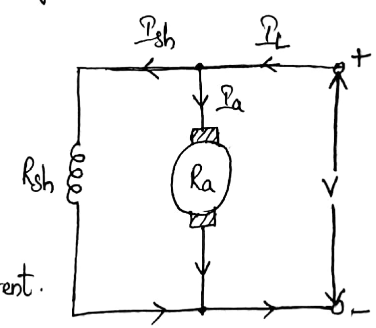
Hence mechanical power developed by the motor is maximum when back e.m.f is equal to half the applied voltage.

Types of D.C. Motors :-

- DC Motors
  - Seperately Excited
  - Self - Excited
    - Series
    - Shunt
    - Compound
      - Long Shunt
      - Short Shunt

① Shunt - wound motor :-

Shunt - wound motor in which the field winding is connected in parallel with the armature. The current through the shunt field winding is not the same as the armature current.

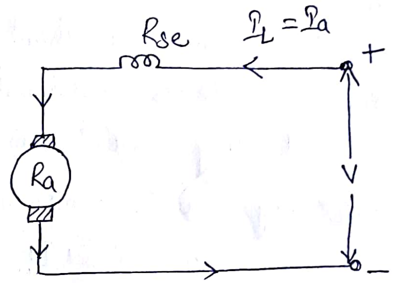


Shunt field windings are designed to produce the necessary mmf by means of a relatively large no. of turns of wire having high resistance. Therefore, shunt field current is relatively small compared with the armature current.

$$I_L = I_a + I_{sh}$$
$$I_{sh} = V/R_{sh}.$$
and $V = E_b + I_a R_a + V_{brush}$ $\boxed{\phi \propto I_{sh}}$

(ii) Series - wound motor :-

In which the field winding is connected in series with the armature. Series field winding carries the armature current. Since, current passing through a series field winding is the same as the armature current.

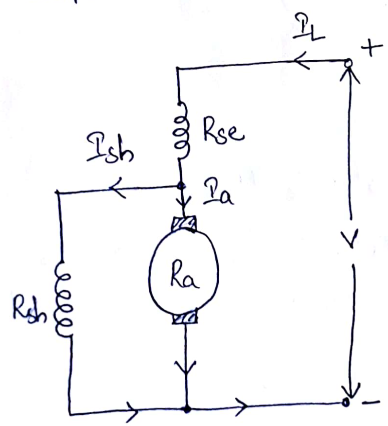


Therefore, a series field winding has a relatively small no. of turns of thick wire and therefore, will possess a low resistance.

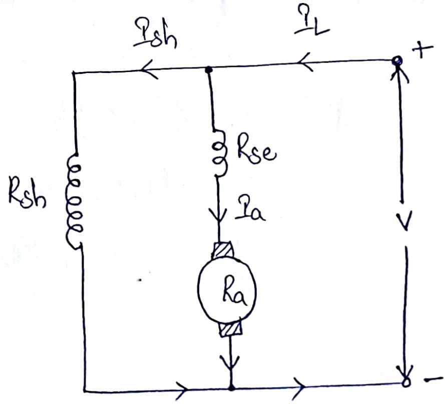
$$I_L = I_{se} = I_a$$
and $V = E_b + I_a R_a + I_{se} R_{se} + V_{brush}.$

$$V = E_b + I_a (R_a + R_{se}) + V_{brush}.$$ $\boxed{\phi \propto I_{se} \propto I_a}$

(iii) Compound - wound motor :-



Short - shunt Connection



Long - shunt Connection.

Compound - wound motor which has two field windings ; one connected in parallel with the armature and the other in series with it. There are two types of compound motor connections. When the shunt field winding is directly connected across the armature terminals, it is called short - shunt connection. When the shunt winding is so connected that it shunts in series combination of armature and series field, it is called long - shunt connection.

For long shunt compound motor,

$$I_L = I_{se} + I_{sh}$$

But $I_{se} = I_a$

$$\therefore \quad I_L = I_a + I_{sh}$$

and $I_{sh} = \dfrac{V}{R_{sh}}$

and $V = E_b + I_a R_a + I_{se} R_{se} + V_{brush}$

$$= E_b + I_a (R_a + R_{se}) + V_{brush}$$

For short shunt compound motor,

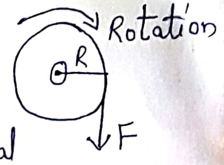
$$I_L = I_{se}$$

and $I_L = I_a + I_{sh}$

$$V = E_b + I_{se} R_{se} + I_a R_a + V_{brush}$$

$$= E_b + I_L R_{se} + I_a R_a + V_{brush}.$$

Torque Equation of a DC Motor :-



The turning or twisting force about an axis is called torque. Consider a wheel of radius R metres acted upon by a circumferential force F newtons as shown.

The wheel is rotating at a speed of N r.p.m. Then angular speed of the wheel is,

$$\omega = \frac{2\pi N}{60} \text{ rad/sec.}$$

So workdone in one revolution is,

$$W = F \times \text{Distance travelled in one revolution}$$

$$= F \times 2\pi R \text{ joules.}$$

And $$P = \text{power developed} = \frac{\text{Workdone}}{\text{Time}}$$

$$= \frac{F \times 2\pi R}{\text{Time for 1 rev}} = \frac{F \times 2\pi R}{\left(\frac{60}{N}\right)} = (F \times R) \times \left(\frac{2\pi N}{60}\right)$$

$$\therefore \; P = T \times \omega \text{ watts}$$

where T = Torque in N.m.

ω = Angular speed in rad/sec.

Let T_a be the gross torque developed by the armature of the motor. It is also called armature torque. The gross mechanical power developed in the armature is $E_b I_a$, ~~as~~ then

$$\text{Power in armature} = \text{Armature torque} \times \omega$$

$$\therefore \; E_b I_a = T_a \times \frac{2\pi N}{60}.$$

But E_b in a motor is given by,

$$E_b = \frac{\phi P N Z}{60 A}$$

$$\therefore \frac{\phi P N Z}{60A} \times I_a = T_a \times \frac{2\pi N}{60}$$

$$\therefore T_a = \frac{1}{2\pi} \phi I_a \times \frac{PZ}{A}$$

$$\boxed{T_a = 0.159\, \phi I_a \cdot \frac{PZ}{A} \text{ Nm}}$$

$$E_b = \frac{P\phi \ldots}{60A}$$

$$\frac{P\phi Z}{A} = \frac{60 E_b}{N}$$

Sub in power eq.

$$T_a = 0.159 \times \left(\frac{60 \times E_b}{N}\right) \times I_a$$

$$\boxed{T_a = 9.55 \times \frac{E_b I_a}{N}} \text{ Nm}$$

$$T_{sh} = 9.55 \times \frac{o/p}{N} \text{ Nm}$$

Speed Regulation :-

The speed regulation for a d.c motor is defined as the ratio of change in speed corresponding to no load and full load condition to speed corresponding to full load.

Mathematically it is expressed as,

$$\boxed{\%\ \text{speed regulation} = \frac{N_{\text{no load}} - N_{\text{full load}}}{N_{\text{full load}}} \times 100}$$

D.C Motor characteristics :-

(i) Torque – Armature current characteristics (T Vs I_a):

The graph showing the relationship between the torque and the armature current is called a torque – armature current characteristic. There are also called electrical characteristics.

(ii) Speed – Armature current characteristics (N Vs I_a) :

The graph showing the relationship between the speed and armature current characteristics.

(iii) Speed – Torque characteristics (N Vs T) :

The graph showing the relationship between the speed and the torque of the motor is called speed – torque characteristics of the motor. There are also called mechanical characteristics.

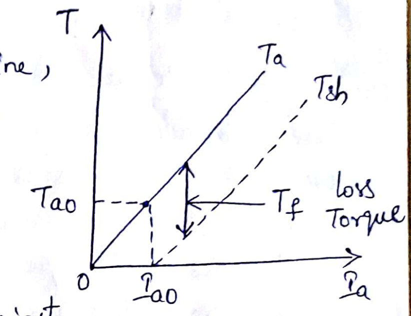
Characteristics of D.C shunt Motor :-

(i) Torque – Armature current characteristics :-

For a d.c motor $T \propto \phi I_a$.

For a constant values of R_{sh} and supply voltage V, I_{sh} is also constant and hence flux is also constant.

$\therefore \boxed{T_a \propto I_a}$

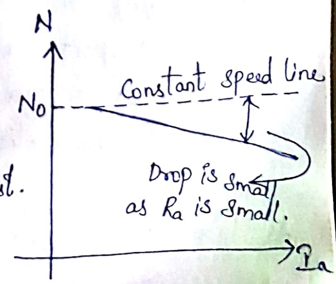


The equation represents a straight line, passing through the origin, as shown.
As load increases, armature current increases, increasing the torque developed linearly.
Now if shaft torque is plotted against armature current, it is known that shaft torque is less than the armature torque and the difference between the two is loss torque T_f as shown.
On no load $T_{sh} = 0$ but armature torque is present which is just enough to overcome stray losses shown as T_{ao}. The current required is I_{ao} on no load to produce T_{ao} and hence T_{sh} graph has an intercept of I_{ao} on the current axis.
To generate high starting torque, this type of motor requires a large value of armature current at start. This may damage the motor hence d.c shunt motors can develop moderate starting torque and hence suitable for such applications where starting torque requirement is moderate.

(ii) Speed – Armature current characteristics :-

From the speed equation we get,

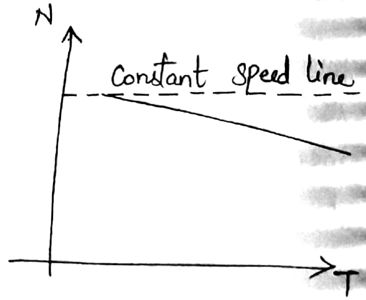
$$N \propto \frac{V - I_a R_a}{\phi} \propto V - I_a R_a \quad \text{as } \phi \text{ is const.}$$



As load increases, the armature current increases and hence drop I_aR_a also increases.
Hence for constant supply voltage, $V - I_aR_a$ decreases and hence speed reduces. But as R_a is very small, for change in I_a from no load to full load, drop I_aR_a is very small and hence drop in speed is also not significant from no load to full load. So the characteristics is slightly droping.

(iii) <u>Speed – Torque characteristics</u> :-

These characteristics can be derived from above two characteristics. This graph is similar to speed – armature current characteristics as torque is proportional to the armature current. This curve shows that the speed almost remains constant through torque changes from no load to full load conditions.



→ <u>Characteristics of D.C Series Motor</u> :-

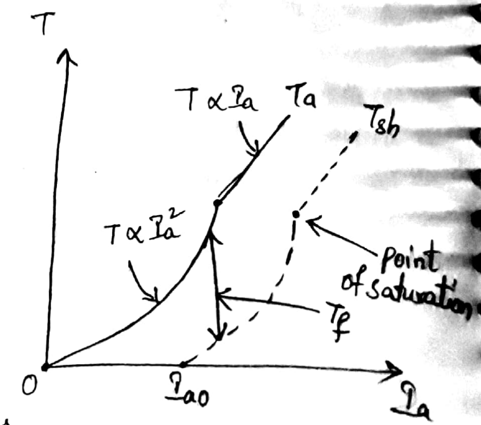
(i) <u>Torque – Armature current characteristics</u> :-

In case of series motor the series field winding is carrying the entire armature current. So flux produced is proportional to the armature current.

$$\therefore \phi \propto I_a$$

Hence $$\boxed{T_a \propto \phi I_a \propto I_a^2}$$

Thus torque in case of series motor is proportional to the square of the armature current. This relation is parabolic in nature as shown.



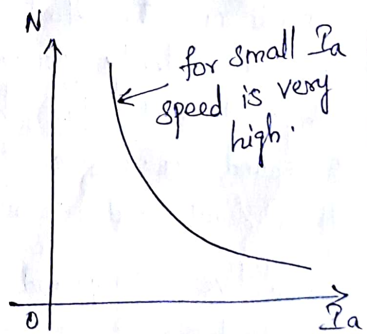
As load increases, armature current increases and torque produced increases

Proportional to the square of the armature current upto a certain limit.

As the entire I_a passes through the series field, there is a property of an electromagnet called saturation, may occur. Saturation means though the current through the winding increases, the flux produced remains constant. Hence after saturation the characteristics take the shape of straight line as flux becomes constant, as shown. The difference between T_a and T_{sh} is loss Torque T_f as shown.

At start as $T \propto I_a^2$, these types of motors can produce high torque for small amount of armature current hence the series motors are suitable for the applications which demand high starting torque.

(ii) Speed – Armature current characteristics :-



From the speed equation we get,

$$N \propto \frac{E_b}{\phi} \propto \frac{V - I_aR_a - I_aR_{se}}{I_a}$$

as $\phi \propto I_a$ in case of series motor.

Now the values of R_a and R_{se} are so small that the effect of change in I_a on speed overrides the effect of change in $V - I_aR_a - I_aR_{se}$ on the speed.

Hence in the speed equation, $E_b \cong V$ and can be assumed constant. So speed equation reduces to,

$$\boxed{N \propto \frac{1}{I_a}}$$

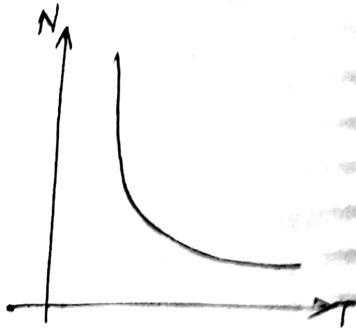
So speed – torque current characteristics is rectangular hyperbola type as shown.

(iii) **Speed - Torque characteristics :-**

In case of series motors,

$T \propto I_a^2$ and $N \propto \frac{1}{I_a}$

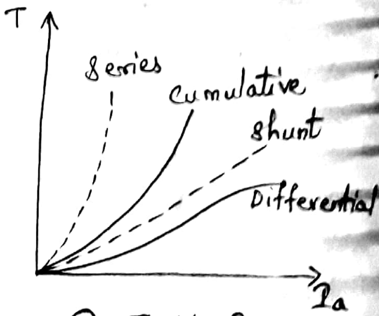
Hence we can write, $\boxed{N \propto \frac{1}{\sqrt{T}}}$



Thus a torque increases when load increases, the speed decreases. On no load, torque is very less and hence speed increases to dangerously high value. Thus the nature of the speed - torque characteristics is similar to the nature of the speed - armature current characteristics.

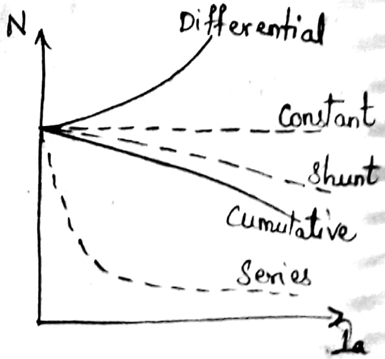
→ **Characteristics of D.C Compound Motor :-**

Compound motor characteristics basically depends on the fact whether the motor is cumulatively compound or differential compound. All the characteristics of the compound motor are the combination of the shunt and series characteristics.

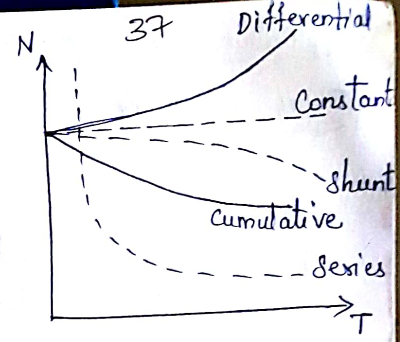


(a) T Vs I_a

Cumulative compound motor is capable of developing large amount of torque at low speeds just like series motor. However it is not having a disadvantage of series motor even at light or no load. The shunt field winding produces the definite flux and series flux helps the shunt field flux to increase the total flux level.



(b) N Vs I_a



(c) N Vs T

So cumulative compound motor can run at a reasonable speed and will not run with dangerously high speed like series motor, on light or no load condition.

In differential compound motor, as two fluxes oppose each other, the resultant flux decreases as load increases, thus the machine runs at a higher speed with increase in the load. This property is dangerous as on full load, the motor may try to run with dangerously high speed. So differential compound motor is generally not used in practice.

The various characteristics of both the types of compound motors cumulative and the differential as shown in fig. (a), (b) and (c).

The exact shape of these characteristics depends on the relative contribution of series and shunt field windings. If the shunt field winding is more dominant then the characteristics take the shape of the shunt motor characteristics. While if the series field winding is more dominant then the characteristics take the shape of the series characteristics.

Applications of D.C. Motors :-

Types of Motor	characteristics	Applications
Shunt	Speed is fairly constant and medium starting torque.	1. Blowers & fans 2. Lathe machines 3. Machine tools 4. Milling machines. 5. Drilling machines
Series	High starting torque. No load condition is dangerous. Variable speed.	1. Cranes 2. Hoists, Elevators 3. Trolleys 4. Conveyors 5. Electric locomotives
Cumulative Compound	High starting torque. No load condition is allowed.	1. Rolling mills 2. Punches 3. Shears 4. Heavy planers 5. Elevators
Differential Compound	Speed increases as load increases.	Not suitable for any practical application.

Problem :- A 4 pole, 250 V, d.c. series motor has a wave connected armature with 200 conductors. The flux per pole is 25 mWb when motor is drawing 60A from the supply. Armature resistance is 0.15 Ω while series field winding resistance is 0.2 Ω. Calculate the speed under this condition.

Sol :- $P = 4$, $Z = 200$, $A = 2$, $\phi = 25 \times 10^{-3}$ Wb

$I_a = I_L = 60A$, $R_a = 0.15\ \Omega$, $R_{se} = 0.2\ \Omega$

$\therefore\ V = E_b + I_a R_a + I_a R_{se}$

$$250 = E_b + 60\,(0.15 + 0.2)$$

$$\therefore E_b = 229\ V.$$

Now $E_b = \dfrac{\phi P N Z}{60 A}$.

$$\therefore 229 = \frac{25 \times 10^{-3} \times 4N \times 200}{60 \times 2}$$

$$\therefore N = 1374\ r.p.m.$$

Problem:- A 250V, d.c. shunt motor takes a line current of 20A. Resistance of shunt field winding is 200Ω and resistance of the armature is 0.3Ω. Find the armature current and the back e.m.f.

Sol:- $V = 250\ V$, $I_L = 20A$, $R_a = 0.3\Omega$, $R_{sh} = 200\Omega$

$$I_L = I_a + I_{sh}$$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{250}{200} = 1.25A$$

$$\therefore I_a = I_L - I_{sh} = 20 - 1.25 = 18.75\ A$$

$$V = E_b + I_a R_a$$

$$\therefore E_b = V - I_a R_a = 250 - 18.75 \times 0.3 = 244.375V.$$

Problem:- A d.c. series motor is running with a speed of 800 r.p.m. while taking a current of 20A from the supply. If the load is changed such that the current drawn by the motor is increased to 50A, calculate the speed of the motor on new load. The armature and series field winding resistances are 0.2Ω and 0.3Ω respectively. Assume the flux produced is proportional to the current. Assume supply voltage as 250V.

Sol :- For load 1, $N_1 = 800$ rpm, $I_1 = I_{a1} = 20A$

For load 2, $I_2 = I_{a2} = 50A$

$$R_a = 0.2\,\Omega, \quad R_{se} = 0.3\,\Omega$$

From voltage equation $V = E_{b1} + I_{a1} R_a + I_{se1} R_{se}$

But $I_1 = I_{a1} = I_{se1} = 20A$.

$$\therefore 250 = E_{b1} + 20(0.2 + 0.3)$$

$$\therefore E_{b1} = 240V.$$

and $V = E_{b2} + I_{a2} R_a + I_{se2} R_{se}$

$$\therefore 250 = E_{b2} + 50(0.2 + 0.3)$$

$$\therefore E_{b2} = 225\,V.$$

From the speed equation,

$$N \propto \frac{E_b}{\phi}$$

Now $\phi \propto I_{se} \propto I_a$.

$$\therefore \frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}} \times \frac{\phi_2}{\phi_1}.$$

$$\frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}} \times \frac{I_{a2}}{I_{a1}}.$$

$$\therefore N_2 = N_1 \times \frac{E_{b2}}{E_{b1}} \times \frac{I_{a1}}{I_{a2}} = 800 \times \frac{225}{240} \times \frac{20}{50} = 300 \text{ r.p.m}$$

Efficiency :-

$$\eta = \frac{o/p}{i/p}$$

$$\eta_g = \frac{o/p}{o/p + \text{Total losses}} \text{ —— for generators}$$

$$\eta_m = \frac{i/p - \text{Total losses}}{i/p} \text{ —— for motors}$$

For generators,

$$\text{power } o/p = VI\cos\phi \text{ —— for a.c generators}$$

$$= VI \text{ —— for d.c generators}$$

where $\cos\phi$ = power factor.

for motors,

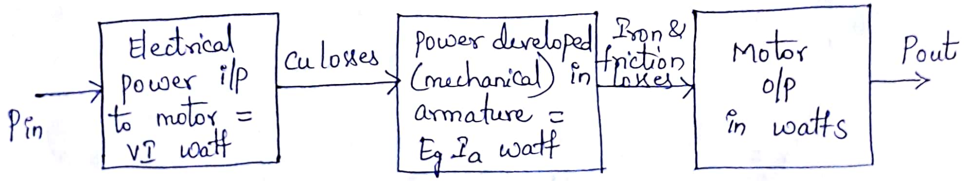
$$\text{power } i/p = VI\cos\phi \text{ —— for a.c motors}$$

$$= VI \text{ —— for d.c. motors.}$$

Losses in a D.C machine :-

Losses in a D.C machine whether it is a motor or a generator are same.

Power stages for a motor



Problem :- A 6 pole, 500 volts, wave connected shunt motor has 1200 armature conductors and useful flux/pole of 20 mWb. Armature and field resistance are 0.5 ohms and 250 ohms. What will be the speed and torque developed by the motor when it draws 20 Amps from supply? Neglect armature reaction. If magnetic & mechanical losses are 900 watts find (i) Useful torque (ii) Efficiency at this load.

Sol :- $P = 6$, $V = 500V$, Wave wound so $A = 2$

$Z = 1200$, $\phi = 20\,mWb$, $R_a = 0.5\Omega$, $R_{sh} = 250\Omega$, $I_L = 20A$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{500}{250} = 2A.$$

$$\therefore I_a = I_L - I_{sh} = 20 - 2 = 18A.$$

Now $V = E_b + I_a R_a$

$$500 = E_b + 18 \times 0.5$$

$$\therefore E_b = 491\,V$$

And $E_b = \frac{\phi P N Z}{60 A}$

$$491 = \frac{20 \times 10^{-3} \times 6 \times N \times 1200}{60 \times 2}$$

$$\therefore N = 409.167 \text{ rpm.}$$

The elec. Equivalent of mech. power developed is,

$$P_m = E_b I_a = 491 \times 18 = 8838\,W.$$

$$T_g = \frac{P_m}{\omega} = \frac{P_m}{\left(\frac{2\pi N}{60}\right)} = \frac{8838}{\left(\frac{2\pi \times 409.167}{60}\right)} = 206.26\,N\text{-}m$$

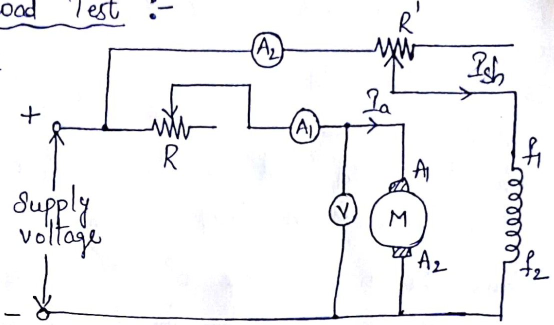
$$P_{out} = P_m - \text{Mech. losses} = 8838 - 900 = 7938\,W.$$

$$\therefore \text{Useful torque} = T_{sh} = \frac{P_{out}}{\omega} = \frac{7938}{\left(\frac{2\pi \times 409.167}{60}\right)} = 185.26\,Nm$$

$$P_{in} = V I_L = 500 \times 20 = 10000 \text{ W}$$

$$\% \eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{7938}{10000} \times 100 = 79.38\%$$

Swinburne's Test or No load Test :-



This is indirect method of testing d.c. motors in which flux remains practically constant i.e. specially in case of shunt & compound motors. Without actually loading the motor the losses and hence efficiency at different loads can be found out.

The motor is run on no load at its rated voltage. At the starting some resistance is connected in series with the armature which is cut when motor attains sufficient speed. Now the speed of the motor is adjusted to the rated speed with the help of shunt field rheostat as shown.

The no load armature current I_a is measured by ammeter A_1 whereas the shunt current is measured by ammeter A_2.

If V is the supply voltage then motor i/p at no load will be,

$$\boxed{\text{Power i/p at no load} = V(I_a + I_{sh}) \text{ watts}}$$

There will be cu loss in the field winding which will be given as,

$$\boxed{\text{Field cu loss} = V \times I_{sh}}$$

Let R_a be the resistance of armature,

$$\boxed{\text{Armature cu loss} = I_a^2 \cdot R_a}$$

Thus the stray losses which includes iron, friction & windage losses can be obtained as, Stray losses = i/p at no load − Field cu losses − No load armature cu losses

$\therefore \boxed{\text{Stray losses} = V(I_a + I_{sh}) - (V \times I_{sh}) - (I_a^2 R_a) = W_a}$

In the field and armature windings there will be cu loss due to flow of current which will increase the temperature of the field and armature winding when the motor is loaded. This increase in temperature will affect their resistances. Thus the new value of field resistance R_{sh}' and that of armature R_a' can be found by considering that rise in temperature as about 40°c.

If α_1 = Resistance temperature coefficient of copper at room temperature

$$\boxed{R_a' = R_a(1 + \alpha_1 \times 40)}$$

At room temperature the shunt field winding resistance will be,

$$R_{sh} = \frac{V}{I_{sh}}$$

$$\therefore \boxed{R_{sh}' = R_{sh}(1 + \alpha_1 \times 40)}$$

Now shunt winding current, $I_{sh}' = \frac{V}{R_{sh}'}$.

$$\boxed{\text{New field Cu loss} = I_{sh}'^2 \times R_{sh}}$$

Now if we want to find the efficiency of the motor at say $\frac{1}{4}$th full load. It can be calculated as follows,

Let $I_{F.L}$ = Full load current of motor

W_F = Field cu loss

W = stray losses.

Load current at $\frac{1}{4}$th full load $= \frac{I_{F.L}}{4}$.

$\therefore$ Motor i/p at $\frac{1}{4}$th full load $= V \times \frac{I_{F.L}}{4}$ watts.

Armature current at $\frac{1}{4}$th full load, $I_a' = \frac{I_{F.L}}{4} - I_{sh}'$.

Armature cu loss at $\frac{1}{4}$th full load $= I_a'^2 . R_a = \left(\frac{I_{F.L}}{4} - I_{sh}'\right)^2 . R_a$.

$\therefore$ Motor o/p at $\frac{1}{4}$th full load $=$ Motor i/p at $\frac{1}{4}$th load − Losses.

$$= \left[V \times \frac{I_{F.L}}{4}\right] - \left[\frac{I_{F.L}}{4} - I_{sh}'\right]^2 R_a - W_F - W$$

$$= \left[V \times \frac{I_{F.L}}{4}\right] - I_a'^2 R_a - W_F - W.$$

Efficiency at $\frac{1}{4}$th full load, $\eta = \frac{o/p}{i/p} = \frac{i/p - \text{Losses}}{i/p}$.

$$\therefore \boxed{\eta = \frac{\left[V \times \frac{I_{F.L}}{4}\right] - I_a'^2 R_a - W_F - W}{V \times \frac{I_{F.L}}{4}}}$$

This is the efficiency of motor when the load on motor is $\frac{1}{4}$th of full load which can be found without loading the motor. The efficiencies at other loads can be calculated similarly.

Problem:- A 440V d.c shunt motor takes a no load current of 2.5 A. The resistance of the shunt field and the armature are 550Ω and 1.2Ω respectively. The full load line current is 32A. Find the full load o/p and the efficiency of the motor.

Sol:- No load current $I = 2.5$ A,

No load i/p $= V.I = 440 \times 2.5 = 1100$ W

$$I_{sh} = \frac{V}{R_{sh}} = \frac{440}{550} = 0.8 \text{ A}.$$

In d.c shunt motor, $I = I_{sh} + I_a$

$\therefore I_a = I - I_{sh} = 2.5 - 0.8 = 1.7A.$

No load armature cu loss $= I_a^2 R_a = (1.7)^2 \times 1.2 = 3.468$ watts.

Constant losses = No load i/p − No load armature cu loss

$= 1100 - 3.468$

$= 1096.532\ W.$

Now, full load line current i.e. $I = 32A$

$I = I_{sh} + I_a.$

$I_a = I - I_{sh} = 32 - 0.8 = 31.2A.$

Full load armature cu loss $= I_a^2 \cdot R_a = (31.2)^2 \times 1.2 = 1168.128\ W.$

Total losses = Full load armature cu loss + Constant losses

$= 1168.128 + 1096.532 = 2264.66\ W.$

Full load motor i/p $= V \cdot I = 440 \times 32 = 14080\ W.$

Full load motor o/p = i/p − losses $= 14080 - 2264.66$

$= 11815.34\ W.$

% Efficiency at full load $= \dfrac{\text{Full load o/p}}{\text{Full load i/p}} \times 100.$

$= \dfrac{11815.34}{14080} \times 100$

$= 83.91$

$\therefore$ Efficiency of motor at full load $= 83.91\%.$

Brake Test :-

Another method of testing the d.c motor is brake test method. This is a direct method of testing the motor. In this method the motor is put on the direct load by means of a belt and pulley arrangement. By adjusting the tension of belt, the load is adjusted to give the various values of currents. The load is finally adjusted to get full load current. The power developed gets wasted against the friction between belt and shaft. Due to the braking action of belt the test is called brake test.

The fig.ⓐ shows the experimental setup for performing brake test on a d.c. shunt motor while the fig.ⓑ shows the belt and pulley arrangement mounted on the shaft of the motor.

The tension in the belt can be adjusted using the handle. The tension in kg can be obtained from the spring balance readings.

Let R = Radius of pulley in metre

N = Speed in r.p.m.

W_1 = Spring balance reading on tight side in kg.

W_2 = Spring balance reading on slack side in kg.

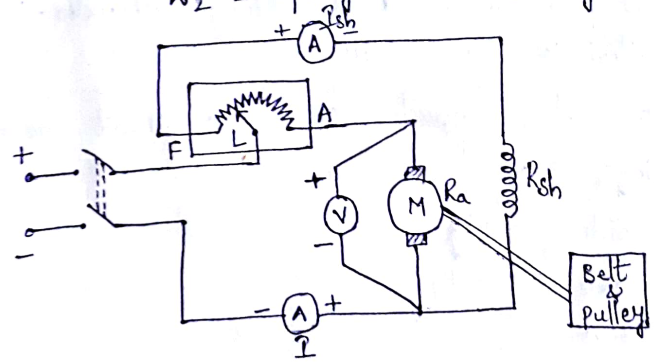


fig ⓐ Experimental setup

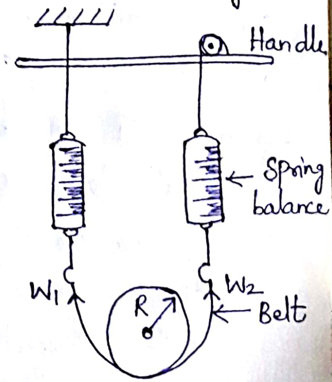


fig ⓑ Belt and pulley arrangement

So net pull on the belt due to friction at the pulley is the difference between the two spring balance readings.

$$\boxed{\text{Net pull} = W_1 - W_2 \text{ kg} = 9.81 (W_1 - W_2) \text{ N}}$$

As radius R and speed N are known, the shaft torque developed can be obtained as,

$$\boxed{T_{sh} = \text{Net pull} \times R = 9.81 (W_1 - W_2) R \text{ N-m}}$$

Hence the o/p power can be obtained as,

$$\boxed{P_{out} = T_{sh} \times \omega = 9.81 (W_1 - W_2) R \times \frac{2\pi N}{60} \text{ Watts}}$$

Now let, V = Voltage applied in volts

I = Total line current drawn in amperes.

then $\boxed{P_{in} = VI \text{ watts}}$

Thus if the readings are taken on full load condition then the efficiency can be obtained as,

$$\boxed{\% \eta = \frac{P_{out}}{P_{in}} \times 100}$$

Adjusting the load step by step till full load, no. of readings can be obtained. The speed can be measured by tachometer. Thus all the motor characteristics can be plotted.

Problem:- In a brake test conducted on a d.c shunt motor the full load readings are observed as,

Tension on tight side = 9.1 kg

Tension on slack side = 0.8 kg

Total current = 10A

Supply voltage = 110V

Speed = 1320 r.p.m , The radius of pulley is 7.5 cm

Calculate its full load efficiency.

Sol :- $W_1 = 9.1 kg$, $W_2 = 0.8 kg$, $I = 10A$, $V = 110V$, $R = 7.5 cm$

$$T_{sh} = (W_1 - W_2)\, 9.81 \times R = (9.1 - 0.8) \times 9.81 \times 0.075$$

$$= 6.1067 \text{ Nm}$$

$$P_{out} = T_{sh} \times \omega = T_{sh} \times \frac{2\pi N}{60}$$

$$= \frac{6.1067 \times 2\pi \times 1320}{60} = 844.133 W$$

$$P_{in} = VI = 110 \times 10 = 1100 W.$$

$$\% \eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{844.133}{1100} \times 100$$

$$= 76.74\%$$

problem :- A 120V d.c shunt motor has an armature resistance of 0.2Ω and a field resistance of 60Ω. It runs at 1800 r.p.m taking a full load current of 40A. Find the speed on half load condition.

Sol :- $V = 120V$, $R_a = 0.2\Omega$, $R_{sh} = 60\Omega$

For full load, $I_{L1} = 40A$, $N_1 = 1800$ r.p.m.

For shunt motor, $I_{sh} = \frac{V}{R_{sh}} = \frac{120}{60} = 2A$.

For full load, $I_{a1} = I_{L1} - I_{sh} = 40 - 2 = 38A$

$$\therefore E_{b1} = V - I_{a1} R_a = 120 - 38 \times 0.2 = 112.4 V$$

For half load, $T_2 = \frac{1}{2} T_1$

and $T \propto \phi I_a \propto I_a$ as ϕ is constant.

$$\therefore \frac{T_1}{T_2} = \frac{I_{a1}}{I_{a2}}$$

$$\frac{T_1}{0.5 T_1} = \frac{I_{a1}}{I_{a2}}$$

$\therefore I_{a2} = 19A.$

$\therefore E_{b2} = V - I_{a2} R_a = 120 - 19 \times 0.2 = 116.2V$

Use speed equation, $N \propto E_b$ as ϕ is constant.

$$\therefore \frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}}$$

$$\frac{1800}{N_2} = \frac{112.4}{116.2}$$

$$\therefore N_2 = 1860.85 \text{ r.p.m.}$$

Problem:- A 4-pole d.c. generator with a shunt field resistance of 100Ω and armature resistance of 1Ω has 378 wave connected conductors in its armature. The flux per pole is 0.02 wb. If a load resistance of 10Ω is connected across the armature terminals and the generator is driven at 1000 r.p.m, calculate the power absorbed by the load.

Sol:- $P=4$, $R_{sh}=100\Omega$, $R_a = 1\Omega$, $Z=378$, $\phi = 0.02$ wb, $R_L = 10\Omega$, $N=1000$ rpm, Wave i.e. $A=2$.

$$I_{sh} = \frac{V_t}{R_{sh}} = \frac{V_t}{100} A, \quad I_L = \frac{V_t}{R_L} = \frac{V_t}{10} A$$

$$\therefore I_a = I_L + I_{sh} = \frac{V_t}{100} + \frac{V_t}{10} = 0.11 V_t.$$

$$E_g = \frac{\phi P N Z}{60A} = \frac{0.02 \times 4 \times 1000 \times 378}{60 \times 2} = 252V$$

Now, $E_g = V_t + I_a R_a = V_t + (0.11 V_t) \times 1$

$$252 = 1.11 V_t$$

$$\therefore V_t = 227.027 \text{ volts.}$$

$$\therefore I_L = \frac{V_t}{R_L} = 22.7027A.$$

$\therefore P_L$ = power absorbed by the load $= V_t I_L$

$= 227.027 \times 22.7027 = 5154.1265$ W.

Problem :- The armature of a 6 pole, d.c shunt motor takes 300 A at the speed of 400 revolutions per minute. The flux per pole is 75 mWb. The no. of turns is 500. The torque lost in windage, friction and iron losses can be assumed a 2.5%. Calculate : (i) Torque developed by the armature (ii) shaft torque (iii) Shaft power in KW.

Sol :- $P = 6$, $I_a = 300A$, $N = 400\ r.p.m$, $\phi = 75\ mWb$

Turns = 500, $T_{lost} = 2.5\%$ of T_a.

Two Conductors Constitute one turn. hence,

$$Z = 2 \times \text{Turns} = 2 \times 500 = 1000.$$

Assume lap connected armature i.e. $A = P$.

(i) $$T_a = \frac{1}{2\pi} \phi I_a \times \frac{PZ}{A} = \frac{1}{2\pi} \times 75 \times 10^{-3} \times 300 \times \frac{6 \times 1000}{6}$$

$$= 3580.986\ Nm.$$

(ii) $$T_{lost} = 2.5\% \text{ of } T_a = \frac{2.5}{100} \times 3580.986 = 89.5246\ Nm.$$

$$\therefore T_{sh} = T_a - T_{lost} = 3491.461\ Nm.$$

(iii) $$P_{out} = T_{sh} \times \omega = T_{sh} \times \frac{2\pi N}{60}$$

$$= \frac{3491.461 \times 2\pi \times 400}{60} = 146.25\ KW.$$