**UNIT –IV**

**PARTIAL DIFFERENTIATION**

**I Taylor’s Theorem:**

If f:[a,b] →R is such that

a) fn-1 is continuous on [a,b]

b) fn-1 is derivable on(a,b) or f(n) exists on (a,b) and PєZ+ then there exist a point cє(a,b) such that
 f(b) = f(a)+

i). Schlomitch –Roche’s form of remainder

 Rn = 

(ii). lagrange’s form of remainder put p=n we get

Rn = 

(iii). Cauchy’s form remainder:

Put p= 1, we get

Rn = 



**Note :** f(x) = f(a) + (x – a) f1(a) + f(a) + …..…… is called T.S. for f(x) about x = a.

**V. Maclaurin’s Theorem:-**

 If f:[0,x]→R is such that (i). f(n-1) is continuous on [0,x] (ii) f(n-1) is derivable on (0,x) and PєZ+ then there exists a real number є (0,1) such that

f(x) = f(0)+xf1(0)+x2/2! F11(0)+--------+xn-1- fn-1(0)+ Rn

1. Roche’s form of remainder Rn

Rn = 

(ii). legrange’s form remainder

Put p=n, we get Rn = xn/n! fn(Ѳx)

(iii). Cauchy’s form of remainder put p=1, we get

Rn = 

**Note** : f(x)= f(0)+xf1(0)+x2/2!+f11(0)+……..+xn/n!fn(0)+…….. is called Maclaurin’s series expansion of f(x)

**Examples**

**1. Verify Taylor’s Theorem for f(x) = (1-x)5/2 with Lagrange’s form of remainder upto 2 terms in the interval [0,1].**

Sol:Consider f(x) = (1-x)5/2 in [0,1]

i) . f(x), f1(x) are continuous in [0,1]

ii). f11(x) is differentiable in [0,1]

Thus f(x) satisfies the conditions of Taylor’s theorem

We consider Taylor’s theorem with Lagrange form of remainder f(x) = f(0)+xf1(0)+x2/2!f11(θ x) with 0<θ<1 ------(1)

n=p=2, a=0 and x=1

f(x) = (1-x)5/2 f(0) =1

f1(x) = -5/2(1-x)3/2 => f1(0)= -5/2

f11(x) = 15/4(1-x) ½ => f11(θx)= 15/4(1-θx) ½

f11(θ)= 15/4(1-θ) ½ and f(1) = 0

from (1) we have f(x) = f(0)+xf1(0)+x2/2!f11(θ x)

substituing the above values, we get

0= 1+1(-5/2)+12/2!(15/4)(1-θ) ½

θ=9/25 = 0.36

θ lies between 0 and 1.

Thus the Taylors theorem is verified.

**2) & hence deduce that **

Soln:- let f(x)= log(1+ex). Then f(0)= log2

Diff. succecively w.r.to x, we get









f111(0) = 0







Sub f(0), f1(0), f11(0), etc , in Maclaurin’s series f(x) = f(0)+xf1(0)+x2/2!f11(0)+----

We obtain 

 

**Deduction : -**

Diff the result in (\*) w.r.to x, we get



**3). Write Taylor’s series for f(x) = (1-x)5/2 with Lagrange’s form of remainder upto 3 terms in the interval [0,1].**

Sol: Given f(x) = (1-x)5/2 clearly f11(x) is continuous on [0,1] and f111(x) is derivable on (0,1).

Thus f(x) satisfies the conditions of Taylor’s theorem

We consider Taylor’s theorem with Lagrange form of remainder f(x) = f(0)+xf1(0)+x2/2!f11(0)+x3/3! f111(θ x) where 0<θ<1

Given interval [0,x]= [0,1] => x=1

f(x) = (1-x)5/2 f(0) =1

f1(x) = -5/2(1-x)3/2 => f1(0)= -5/2

f11(x) = -5/2.-3/2(1-x) ½ = 15/4(1-x) ½

f11(0) = 15/4(1) =15/4

f111(x)= 15/8(1-x)-1/2

thus f(x) = f(0)+xf1(0)+x2/2!f11(0)+x3/3! f111(θ x) where 0<θ<1

 =1-x.5/2+x2/2(15/4)+x3/6(-15/8)+-----

 = 1-5x/2+15x2/8-5x3/16+--

**Exercise Problems:**

**Taylor’s and Maclaurin’s Theorem**

1. S.T
2. sinx = x - + - ……………..

1. cosx = 1- + - ……………..

1. log (1 + x ) = x - + - ……………..

1. e x = 1 + + + + ……….

1. Sinhx = x + + -+……………..

1. e(sinx) = 1 + + - + ……….

1. = + - + …………

1. π/4 + x) ] = 2x + x3 + x6 + ………..

1. Expand sin x in ascending powers of (x - π/2 )
2. Expand f(x) = 2x3 + 7x2 + x – 6 in powers of (x -2)
3. S.T ex = 1 + + + …….. + +

1. Expand f(x) = f(0) + x f1(0) + f11(x) find the value of as x tends to 1 where f(x) =(1- x)(5/2)

1. Calculate the approximate value of to four decimal place using Taylor’ theorem

**Exercise**

**2. FUNCTION OF SEVERAL VARIABLES**

**Jacobian (J) :** Let U = u (x , y) , V = v(x , y) are two functions of the independent variables x , y. The jacobian of ( u , v ) w.r.t (x , y ) is given by

J ( ) = =

Note : 

Similarly if U = u(x, y , z ) , V = v (x, y , z) , W = w(x, y , z)

Then the Jacobian of u , v , w w.r.to x , y , z is given by

J ( ) = =

**Solved Problems:**

1. **If x + y2 = u , y + z2 = v , z + x2 = w find** 

Sol : Given x + y2 = u , y + z2 = v , z + x2 = w

 We have = =

 = 1(1-0) – 2y(0 – 4xz) + 0

 = 1 – 2y(-4xz)

 = 1 + 8xyz

* = =

1. **S.T the functions u = x + y + z , v = x2 + y2 + z2 -2xy – 2yz -2xz and w = x3 + y3 + z3 -3xyz are functionally related. (’07 S-1)**

Sol: Given u = x + y + z

 v = x2 + y2 + z2 -2xy – 2yz -2xz

 w = x3 + y3 + z3 -3xyz

we have

 =

 =

 =6

c1 => c1 –c2

c2 => c2 –c3

 =6

=6[2(x - y) (y2 + xy – xz -z2 )-2(y - z)(x2 + xz – yz - y2)]

=6[2(x - y)( y – z)(x + y + z) – 2(y – z)(x – y)(x + y + z)]

=0

 Hence there is a relation between u,v,w.

1. **If x + y + z = u , y + z = uv , z = uvw then evaluate (’06 S-1)**

Sol: x + y + z = u

 y + z = uv

 z = uvw

 y = uv – uvw = uv(1 – w)

 x = u – uv = u (1 – v)

 =

 =

R2 => R2 + R3

 =

 = uv[ u –uv +uv]

 = u2v

1. **If u = x2 – y2 , v =2xy where x = r cos , y = r sin S.T = 4r3 (’07 S-2)**

Sol: Given u = x2 – y2  , v = 2xy

 =r2cos2 – r2sin2 = 2rcos r sin

 = r2 (cos2 – sin2 = r2 sin2

 = r2 cos2

 = =

 = (2r)(2r)

 = 4r2 [rcos22 + r sin22 ]

 =4r2(r)[ cos22 + sin22 ]

 =4r3

**5. If u = , v = , w = find (’08 S-4)**

Sol: Given u = , v = , w =

 We have

 =

 ux = yz(-1/x2) = , uy = , uz =

 = , xz(-1/y2) = ,

 = , = , = xy (-1/z2) =

 =

 = . .

 =

 = 1[-1(1-1) -1(-1-1) + (1+1) ]

 = 0 -1(-2)+(2)

 =2 + 2

 =4

**Assignment**

Calculate if x = , y = , z = and u = r sin cos , v = r sin sin ,w = r cos

**6. If x = er sec , y = er tan P.T . = 1 ( ’08 S-2 )**

Sol: Given x = er sec , y = er tan

 = , =

 = er sec = x , = ersec tan

 = er tan = y , = er sec2

 x2 – y2 = e2r (sec2 - tan2 )

* 2r = log (x2 – y2 )
* r = ½ log (x2 – y2 )

 = ½ (2x) =

 = ½ (-2y) =

 = = =

* = ,= sin-1( )

 = y () =

 = (1/x) =

 = = e2r sec2 - y er sec tan

 = e2r sec [sec2 - tan2 ] = e2r sec

 =

 =[ - ]

 = = =

 . = 1

**Functional Dependence**

Two functions u and v are functionally dependent if their Jacobian

J ( ) = = = 0

If the Jacobian of u, v is not equal to zero then those functions u, v are functionally independent.

\*\* **Maximum & Minimum for function of a single Variable:**

To find the Maxima & Minima of f(x) we use the following procedure.

1. Find (x) and equate it to zero
2. solve the above equation we get x0,x1 as roots.
3. Then find f11(x).

If f11(x)(x = x0) > 0, then f(x) is minimum at x0

If f11(x)(x = x0) , < 0, f(x) is maximum at x0 . Similarly we do this for other stationary points.

**PROBLEMS:**

1. **Find the max & min of the function f(x) = x5 -3x4 + 5 (’08 S-1)**

Sol : Given f(x) = x5 -3x4 + 5

 f1(x) = 5x4 – 12x3

for maxima or minima f1(x) =0

5x4 – 12x3 = 0

 X =0 , x= 12/5

 f11(x) = 20 x3 – 36 x2

 At x = 0 => f11(x) = 0. So f is neither maximum nor minimum at s = 0

At x = (12/5) f11(x) =20 (12/5)3 – 36(12/5)

 =144(48-36) /25 =1728/25 > 0

 So f(x) is minimum at x = 12/5

The minimum value is f(12/5) = (12/5)5 -3(12/5)4 + 5

\*\* **Maxima & Minima for functions of two Variables**:

**Working procedure:**

1. Find and Equate each to zero. Solve these equations for x & y we get the pair of values (a1,b1) (a2,b2) (a3 ,b3) ………………

1. Find = , n = 
2. IF n –m2 > 0 and < 0 at (a1,b1) then f(x ,y) is maximum at (a1,b1) and maximum value is f(a1,b1) .

1. IF n –m2 > 0 and  > 0 at (a1,b1) then f(x ,y) is minimum at (a1,b1) and minimum value is f(a1,b1) .

1. IF n –m2 < 0 and at (a1,b1) then f(x ,y) is neither maximum nor minimum at (a1,b1). In this case (a1,b1) is saddle point.
2. IF n –m2 = 0 and at (a1,b1) , no conclusion can be drawn about maximum or minimum and needs further investigation. Similarly we do this for other stationary points.

**PROBLEMS:**

1. **Locate the stationary points & examine their nature of the following functions.** (’07 S -2 )

 u =x4 + y4 -2x2 +4xy -2y2, (x > 0, y > 0)

Sol: Given u( x ,y) = x4 + y4 -2x2 +4xy -2y2

 For maxima & minima = 0, = 0

 = 4x3 -4x + 4y = 0  x3 – x + y = 0 -------------------> (1)

 = 4y3 +4x - 4y = 0  y3 + x – y = 0 -------------------> (2)

 Adding (1) & (2) ,

 x3 + y3 = 0

 ****= x = – y -------------------> (3)

1.  x2 – 2x  x = 

Hence (3)  y = 0, 

 l = = 12x2 – 4 , m = = ( ) = 4 & n = = 12y2 – 4

 ln – m2 = (12x2 – 4 )( 12y2 – 4 ) -16

 At (, ) , ln – m2 = (24 – 4)(24 -4) -16 = (20) (20) – 16 > 0

 The function has minimum value at (, )

 At (0,0) , ln – m2 = (0– 4)(0 -4) -16 = 0

 (0,0) is not a extrem value.

1. **Investigate the maxima & minima if any of the function**

 **f(x) = x3y2(1-x-y).**

 (‘08 S – 4)

Sol: Given f(x) = x3y2 (1-x-y) = x3y2- x4y2 – x3y3

 = 3x2y2 – 4x3y2 -3x2y3 = 2x3y – 2x4y -3x3y2

For maxima & minima = 0 and = 0

* 3x2y2 – 4x3y2 -3x2y3 = 0 => x2y2(3 – 4x -3y) = 0 ---------------> (1)
* 2x3y – 2x4y -3x3y2 = 0 => x3y(2 – 2x -3y) = 0 ----------------> (2)

 From (1) & (2) 4x + 3y – 3 = 0 ----------------X2

 2x + 3y - 2 = 0 -----------------X3

 ---------------------------------------

 2x = 1 => x = ½

4 ( ½) + 3y – 3 = 0 => 3y = 3 -2 , y = (1/3)

 l =  = 6xy2-12x2y2 -6xy3

(1/2,1/3) = 6(1/2)(1/3)2 -12 (1/2)2(1/3)2 -6(1/2)(1/3)3 = 1/3 – 1/3 -1/9 = -1/9

 m = = ( ) =6x2y -8 x3y – 9x2y2

 (1/2 ,1/3) = 6(1/2)2(1/3) -8 (1/2)3(1/3) -9(1/2)2(1/3)3 = =

 n = = 2x3 -2x4 -6x3y

 (1/2,1/3) = 2(1/2)3 -2(1/2)4 -6(1/2)3(1/3) = - - = -

 ln- m2 =(-1/9)(-1/8) –(-1/12)2 = - = = > 0

The function has a maximum value at (1/2 , 1/3)

1. **Find three positive numbers whose sum is 100 and whose product is maximum.**

(’08 S-1)

Sol: Let x ,y ,z be three +ve numbers.

 Given x + y + z = 100

* Z = 100 – x – y

 Let f (x,y) = xyz =xy(100 – x – y) =100xy –x2y-xy2

For maxima or minima = 0 and = 0

 =100y –2xy-y2 = 0 => y(100- 2x –y) = 0 ----------------> (1)

 = 100x –x2 -2xy = 0 => x(100 –x -2y) = 0 ------------------> (2)



 100 -2x –y = 0

 200 -2x -4y =0

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 -100 + 3y = 0 => 3y =100 => y =100/3

 100 – x –(200/3) = 0 => x = 100/3

l =  =- 2y

(100/3 , 100/3 ) = - 200/3

m =  = ( ) = 100 -2x -2y

 (100/3 , 100/3 ) = 100 –(200/3) –(200/3) = -(100/3)

 n =  = -2x

 (100/3 , 100/3 ) = - 200/3

 ln -m2 = (-200/3) (-200/3) - (-100/3)2 = (100)2 /3

The function has a maximum value at (100/3 , 100/3)

 i.e. at x = 100/3, y = 100/3  z = 

 The required no. are x = 100/3, y = 100/3, z = 100/3

1. **Find the maxima & minima of the function f(x) = 2(x2 –y2) –x4 +y4 (’08 S-3)**

Sol: Given f(x) = 2(x2 –y2) –x4 +y4 = 2x2 –2y2 –x4 +y4

 For maxima & minima = 0 and = 0

 = 4x - 4x3 = 0 => 4x(1-x2) = 0 => x = 0 , x = ± 1

 = -4y + 4y3 = 0 => -4y (1-y2) = 0 =>y = 0, y = ± 1

l = = 4-12x2

m = =  = 0

n = = -4 +12y2

we have ln – m2 = (4-12x2)( -4 +12y2 ) – 0

 = -16 +48x2 +48y2 -144x2y2

 = 48x2 +48y2 -144x2y2 -16

1. At ( 0 , ± 1 )

ln – m2 = 0 + 48 - 0 -16 =32 > 0

l = 4-0 = 4 > 0

f has minimum value at ( 0 , ± 1 )

f (x ,y ) = 2(x2 –y2) –x4 +y4

f ( 0 , ± 1 ) = 0 – 2 – 0 + 1 = -1

The minimum value is ‘-1 ‘.

1. At ( ± 1 ,0 )

 ln – m2 = 48 + 0 - 0 -16 =32 > 0

 l = 4-12 = - 8 < 0

f has maximum value at ( ± 1 ,0 )

 f (x ,y ) = 2(x2 –y2) –x4 +y4

f ( ± 1 , 0 ) =2 -0 -1 + 0 = 1

The maximum value is ‘1 ‘.

1. At (0,0) , (± 1 , ± 1)

 ln – m2 < 0

 l = 4 -12x2

 (0 , 0) & (± 1 , ± 1) are saddle points.

 F has no max & min values at (0 , 0) , (± 1 , ± 1).

**Assignment**

1. Find the maximum value of x,y,z when x + y + z = a .

[ Ans: ]

**\*Extremum** : A function which have a maximum or minimum or both is called

 ‘extremum’

**\*Extreme value** :- The maximum value or minimum value or both of a function is

 Extreme value.

**\*Stationary points:** - To get stationary points we solve the equations = 0 and

 = 0 i.e the pairs (a1, b1), (a2, b2) ………….. Are called

 Stationary.

\***Maxima & Minima for a function with constant condition** :**Lagrangian Method**

 Suppose f(x , y , z) = 0 ------------(1)

 ( x , y , z) = 0 ------------- (2)

F(x , y , z) = f(x , y , z) + ( x , y , z) where is called Lagrange’s constant.

1. = 0 => + = 0 --------------- (3)

= 0 => + = 0 --------------- (4)

 = 0 => + = 0 --------------- (5)

1. Solving the equations (2) (3) (4) & (5) we get the stationary point (x, y, z).
2. Substitute the value of x , y , z in equation (1) we get the extremum.

**Problem:**

1. **Find the minimum value of x2 +y2 +z2 given x + y + z =3a (’08 S-2)**

Sol: u = x2 +y2 +z2

 = x + y + z - 3a = 0

 Using Lagrange’s function

F(x , y , z) = u(x , y , z) + ( x , y , z)

For maxima or minima

 = + = 2x + = 0 ------------ (1)

 = + = 2y + = 0 ------------ (2)

 = + = 2z + = 0 ------------ (3)

 (1) , (2) & (3)

 = -2x = -2y = -2z

 = x + x + x - 3a = 0

 = a

 = y =z = a

Minimum value of u = a2 + a2 + a2 =3 a2

**OBJECTIVES**

1. The value of c of Rollen’s theorem for f(x)= in (0,) is

1. (b) (c) (d)

1. Using which mean value theorem , we can calculate approximately the value

of in the easier way

 (a) Cauchy’s (b) Lagrange’s (c) Taylor’s II order(d) Rolle’s

3. The value of Cauchy’s mean value theorem for f(x)= and g(x)= defined on [a,b] ,0<a<b is

 (a) (b) (c) (d)

4. If f(x) is continuous in[a,b], for every value of x in (a,b), f(a)=f(b), there exists at least one value c of x in (a,b) such that (c)=\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

 (a) 0 (b) a + b (c) c (d) b

5. Lagrange’s mean value theorem for f(x)= sec x in (0,2) is

 (a) applicable (b) not applicable due to non-differentiability

 (c) applicable and c= (d) not applicable due to discontinuity

6.f(a+h)=f(a) + h(a)+ (a) + -----------+ (a+) is called

 (a)Taylor’s theorem with Lagrange form of remainder

 (b) Cauchy’s theorem with Lagrange form of remainder

 (c) Reiman’s theorem with Lagrange form of remainder

 (d) Lagrange’s theorem with Lagrange form of remainder

7. if f(x)=f(0)+---------(0). then the series is called

 (a) Maclaurin’s Series (b) Taylor’s Series

(c) Cauchy’s Series (d) Lagrange Series

8. The value of Rolle’s theorem in(-1,1) for f(x)=-x is

 (a)0 (b) (c) (d) )

9. The value of x so that =(x) when a<x<b given f(x)= ,a=1,b=4 is

 (a) (b) (c) (d)

10. The value of c of Cauchy’s mean value theorem for the function f(x) = , g(x)= in the
 interval [1,2] is

 (a) (b) (c) (d)

11. If f(0)=0,(0)=1,(0)=1,(0)=1, then the maclaurin’s expansion of

 f(x) is given by

 (a) x + + +------ (b) x + - +------

(c) - x - -+------ (d) x - ++------

12. The value of c of Rolle’s theorem in [,2] for f(x)= + is

 (a) (b) (c) 1 (d)

13. Lagrange’s mean value theorem for f(x)=secx in (0,2) is

 (a) not applicable due to discontinuity (b) applicable & c= (c)not applicable due to non differentiable (d) applicable

14.In the Taylor’s theorem, the cauchy’s form of remainder is

 (a) (b) (a+

(c) (d)

15. The value of c in Rolle’s theorem for f(x)=sinax in(0,a) is

 (a) (b) (c) (d)

16. The value of c in Rolle’s theorem for f(x)= in (-1,1) is

 (a) 0 (b)0.5 (c) 0.25 (d) -0.5

17. The value of c in Rolle’s theorem for f(x)=-x in (0,1)

 (a)0 (b)0.5 (c)0.25 (d)-0.5

18.The value of c in Lagrange’s mean value theorem for f(x)= in (0,1) is

 (a) log(e-) (b) loge (c) log(e+1) (d)log(e-1)

19.The value of c in Cauchy MVT for f(x)= and g(x)= in (3,7) is

 (a)4 (b)5 (c)4.5 (d)6

20.The value of if f(x)= & f(x+h)=f(x)+h

 (a)-0.5 (b)0.25 (c)0 (d)0.5

21.The value of c in Cauchy’s mean value theorem for f(x)= and g(x)= in

 (1,4) is

 (a)1.5 (b)2 (c)2.5 (d)3

22.The value of c in Lagrange’s mean value theorem for f(x)=logx in [1,e] is

 (a) (b)e+1 (c)e-1 (d)e

23. Lagrange’s mean value theorem is not applicable to the function f(x) in

 [-1,1] because

 (a) f(-1) f(1) (b) f is not continuous in [-1,1]

 (c) f is not derivable in (-1,1) (d) f is not a bijective function

24.Lagrange’s MVT is not applicable to the function defined on [-1,1] by f(x)=xsin (x

 (a)f(-1)=f(1) (b) f is not continuous in [-1,1]

(c) f is not derivable in (-1,1) (d) f is not a one-to-one function

25.The value of c for Lagrange’s MVT for the function f(x)=cosx in [0,] is

 (a) (b)

(c) (d)

26.The value of c for Rolle’s theorem for f(x)=(x-a)(x-b) in [a,b] is

 (a) - (b) (c) a+ b (d)

27. The value of c of Lagrange’s mean value theorem for f(x)=(x-2)(x-3) in

 [0,1] is

 (a) 0.5 (b) 1 (c) 2.5 (d) 2

28. The value of c of Rolle’s theorem for f(x)=(x-1)(x-2) in [0,3] is

 (a) 1.5 (b) 2.5 (c) 3 (d) 2

29. The value of c of Cauchy’s mean value theorem for f(x)=sinx and g(x)=cosx

 in [0,]

 (a) (b) (c) (d)

30. Maclaurin’s expansion for log(1+x) is

 (a) x - + - +------------------- (b) x + + + +------------------

 (c) x + + + +------------------- (d) x - + - +----------------

31. Maclaurin’s expansion of cosx is

 (a) (b)

(c) (d)

32. The expansion of in powers of (x-1)

 (a) e (b)

(c) e (d)

33. The expansion for sinx in powers of (x -) is

 (a) 1 - + ------------------

(b) x + + + -----------------------

(c) 1 + + + ------------------

(d) x - + + -----------------------

34. if u=x+y and v=xy then = --------------------------

35. if x=cosv , y= v then = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

36. =\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

37. if u= then = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

38. x = -------------------------

39. Two functions u and v are said to be functionally dependent if = \_\_\_\_\_\_\_\_\_\_\_\_\_

40. If u= and v= then = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

41. If u=siny,v=cosy then =\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

42. . = \_\_\_\_\_\_\_\_\_\_\_\_\_\_

43. If x=rcos,y=rsin then =\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

44. If u=3x+5y and v=4x-3y then =\_\_\_\_\_\_\_\_\_\_\_\_

45. If u= and v=xy then = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

46. If u= - 2y ,v=x+y then = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

(a) (b) 2(x+1) (c) 3(x+1) (d) None

47. If u(1-v)=x, uv=y then . =

(a) 0 (b) 1 (c) xy (d) None

48. If u= , v= x + y then . =

(a) 0 (b) 1 (c) xy (d) None

49. Are u=x , v=2x functionally dependent? If so what is ?

(a) yes,1 (b) yes,0 (c) No,0 (d) None

50. If u=y ,v= then is

(a)5 (b)4 (c) (d)

**(Assignment Questions)**

**{Mean Value Theorems Functions of Several Variables}**

1. Verify Rolle’s Theorem For f(x)=log {} On [a,b] a>0

1. Verify Rolle’s Theorem For f(x)=2x3+x2-4x-2 On [- , ]

1. P.T < Tan-1b – Tan-1a <  where 0 < a <b &hence deduce

 < Tan-1 (2 )<

1. Verify mean value Theorem for f(x) = and g(x) = in [a,b]

1. Using mean value theorem, P.T < log(1+x) < x x>0

1. Verify Rolle’s theorem for f(x) = in (0,

1. State Maclaurins’s Theorem with Lagrange’s form of remainder for f(x) = cosx
2. If x+y2=u , y+z2=v ,z+ x2=w find .

1. If x+y+z=u, y+z= uv, z=uvw then evaluate .

1. S.T the functions u=x+y+z, v=x2+y2+z2-2xy-2zx and w=x3+y3+z3-3xyz are functionally related.
2. Find the max & min values of the function f(x)=x5$-$3x4+5.
3. Find three positive numbers whose sum is 100 and whose product is maximum.
4. Locate the stationary points & examine their nature of the following functions u=x4+y4-2x2+4xy-2y2 (x>0,y>0).
5. If u= ,v= , w= , find .

1. Verify Cauchy’s mean value theorem for f(x)=logx , g(x)=1/x on [1,e] .